Guiding light via geometric phases

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Known methods for transverse confinement and guidance of light can be grouped into a few basic mechanisms, the most common being metallic reflection, total internal reflection and photonic-bandgap (or Bragg) reflection^{1–5}. All of them essentially rely on changes of the refractive index, that is on scalar properties of light. Recently, processes based on “geometric Berry phases”, such as manipulation of polarization states or deflection of spinning-light rays^{6–9}, have attracted considerable interest in the contexts of singular optics and structured light^{10,11}. Here, we disclose a new approach to light waveguiding, using geometric Berry phases and exploiting polarization states and their handling. This can be realized in structured three-dimensional anisotropic media, in which the optic axis lies orthogonal to the propagation direction and is modulated along it and across the transverse plane, so that the refractive index remains constant but a phase distortion can be imposed on a beam. In addition to a complete theoretical analysis with numerical simulations, we present a proof-of-principle experimental demonstration of this effect in a discrete element implementation of a geometric phase waveguide. The mechanism we introduce shows that spin-orbit optical interactions can play an important role in integrated optics and paves the way to an entire new class of photonic systems that exploit the vectorial nature of light.

Waveguides are central to modern photonics and optical communications. Besides the standard optical fibers –based on total internal reflection (TIR) and graded-index (GRIN) refractive potential– and the hollow-metal-pipes for microwaves¹, several more complex structures have been investigated, ranging from photonic-bandgap systems^{2–5} to “slot” waveguides¹², plasmonic waveguides¹³, coupled-resonators^{14,15}, grating-mediated¹⁶ and Kapitza-effect waveguides¹⁷. Despite such variety, all light-guiding mechanisms investigated hitherto rely on variations, sudden or gradual, of the refractive index or –generally– the dielectric permittivity. Even when anisotropic materials are employed to realize waveguides, as for example in liquid crystals¹⁸, light confinement is based on the transverse modulation of the refractive index experienced by extraordinary waves through the nonuniform orientation of the optic axis with respect to the wave-vector. A fundamental question is whether the guided propagation of light can be achieved at all in structures without perturbations of the refractive index. As we shall prove, this is indeed possible provided that the transverse trapping is purely based on vectorial effects, that is, it relies on spin-orbit interactions between wave propagation and polarization states of light¹⁹: otherwise stated, an entirely new mechanism for light confinement.

Spin-orbit photonic interactions are strictly related to geometric Berry phases. In the context of optics, the latter are phase retardations linked exclusively to the geometry of the transformations imposed to light

by the medium and independent of the optical path length¹⁹. This concept has been already implemented in optical elements with various architectures, including patterned dielectric gratings, liquid crystals and metasurfaces^{20–24}. These devices exploit the medium anisotropy to modulate the polarization state of light in a space-varying manner across the plane transverse to propagation. This, in turn, gives rise to a spatially inhomogeneous Pancharatnam-Berry (PB) phase^{8,9,25}, resulting in a reshaped optical wavefront. Hence, a PB optical element (PBOE) behaves as a phase mask, despite exhibiting a constant refractive index and a transversely-uniform optical path length, that is a flat geometry.

In this work we disclose the possibility of guiding electromagnetic waves (and in particular light) by exploiting PB phases in a bulk, that is an extended continuous medium supporting beam propagation over several Rayleigh distances. As we will show, at variance with PBOEs this requires birefringent materials whose optic axis is modulated both in the transverse plane and in the longitudinal coordinate along the propagation direction, a medium structured in three-dimensions (3D).

We investigate light propagation along the axis z of an inhomogeneous uniaxial dielectric. The optic axis $\hat{u}$ is assumed to be space-varying, but lying everywhere in the xy plane transverse to propagation. Its point-wise orientation is described by the angle $\theta(x, y, z)$ between $\hat{u}$ and the y axis in the laboratory frame (see Fig. 1a). We also assume that the principal values $\epsilon_{\parallel}$ and $\epsilon_{\perp}$ of the (relative) permittivity tensor are uniform, corresponding

FIG. 1. Geometric-phase waveguide concept. **a**, Reference system with orientation of the optic axis $\mathbf{u}$ and ordinary/extraordinary (o/e) field directions in the laboratory frame xyz ; the angle θ between $\mathbf{u}$ and the axis y varies from point to point. **b**, Geometric phase acquired by a plane wave, CP at the input, propagating along z in a transversely homogeneous medium with $\theta = \pi/4$ as a function of the birefringence retardation $\delta(z)$, relative to the case with $\theta = 0$. The geometric phase sign is fixed by the CP input handedness (blue and red lines). If θ is uniform along z (dashed lines), the geometric phase reaches a maximum (in the example $\pi/2$) when $\delta = \pi$ and then decreases to zero for $\delta = 2\pi$. If the angle θ is suddenly inverted at $\delta = \pi$ (solid lines), the phase grows monotonically. If θ is sinusoidally modulated along z (solid lines with circles), the phase increases monotonically at a slightly lower rate than in the previous case. When launching a light beam along z in a periodic uniaxial medium with $|\theta|$ larger on axis than in the outer regions, the cumulative geometric phase leads to a guiding effect for an input LCP or RCP, depending on the initial θ modulation sign. **c**, 3D illustration of a continuously modulated geometric-phase waveguide: the orientation of the optic axis is longitudinally sinusoidal and transversely Gaussian. We sketch nine sections within a modulation period, with the black rods representing the optic axis distribution and the colors corresponding to θ ; the guided light beam is represented as a red arrow. **d**, A circular plane wave input beam propagates along the structure with varying polarizations which alternate between LCP and RCP, with the same periodicity of the medium (e_x and e_y are normalized electric fields in the time domain). Star and triangle show the geometric phase evolution of two points in the transverse plane (x_1, y_1, z) and (x_2, y_2, z) located so that $\theta_2 - \theta_1 = \pi/4$. After one period the overall phase difference is π , consistently with the graph in **b** (solid lines).

to constant refractive indices $n_o = \sqrt{\epsilon_{\perp}}$ and $n_e = \sqrt{\epsilon_{\parallel}}$ for ordinary and extraordinary eigenwaves, respectively.

Let us first recall that, in a homogeneous uniaxial with plane waves propagating with wave-vector along z , the ordinary and extraordinary eigenfields have amplitudes $\psi_o(z) = e^{ik_0 n_o z} \psi_o(0)$ and $\psi_e(z) = e^{ik_0 n_e z} \psi_e(0)$, respec-

tively, with $k_0 = 2\pi/\lambda$ the vacuum wavenumber and λ the wavelength. In other words, the two waves propagate independently of each other and acquire a relative phase retardation $\delta(z) = k_0 \Delta n z$ versus propagation, where $\Delta n = n_e - n_o$ is the birefringence. If we now turn from the usual ordinary/extraordinary linear polar-

ization basis to the left/right (L/R) circular polarization (CP) basis, the same evolution for LCP/RCP wave amplitudes is described by (see Methods for a derivation):

$$\begin{aligned}\psi_L(z) &= e^{i\bar{n}k_0 z} \left[\cos\left(\frac{\delta}{2}\right) \psi_L(0) - i \sin\left(\frac{\delta}{2}\right) e^{i2\theta} \psi_R(0) \right] \\ \psi_R(z) &= e^{i\bar{n}k_0 z} \left[\cos\left(\frac{\delta}{2}\right) \psi_R(0) - i \sin\left(\frac{\delta}{2}\right) e^{-i2\theta} \psi_L(0) \right],\end{aligned}\quad (1)$$

where $\bar{n} = (n_o + n_e)/2$ is the average refractive index. Equations (1) point out that the two forward-propagating circular waves evolve with a common phase $\bar{n}k_0 z$ and, in addition, periodically exchange handedness (that is LCP becomes RCP and vice versa) acquiring an additional phase factor $\pm 2\theta$ (+/- for initial RCP/LCP, respectively). This extra phase clearly has a geometric nature and is an example of PB phase^{8,9,25}. The two CP waves completely interchange after a propagation distance $z_{\text{coh}}/2$ such that $\delta(z_{\text{coh}}/2) = \pi$, then the process reverts and the optical field retrieves its initial state in $z = z_{\text{coh}}$ where $\delta(z_{\text{coh}}) = 2\pi$ (see Fig. 1b). Hence, the geometric phase in such uniform medium oscillates but does not accumulate over distance. While in PBOEs the propagation can be halted when $\delta(z) = \pi$ by properly arranging the medium length (or its birefringence), so as to obtain a non vanishing PB phase at the output, in a system with extended propagation length (e.g. a waveguide) the described geometric phase appears to play no significant role.

A simple equalization approach to build-up a geometric phase along z consists of introducing a periodic modulation in order to counteract its recurring cancellation, analogous to dispersion-compensation in fibers and quasi-phase-matching in nonlinear optics^{26,27}. This requires to periodically invert the sign of θ along z with the same spatial period as the “natural” interchange described above, so that the PB phase will keep adding up (with the same positive or negative sign depending on the input polarization) and accumulate monotonically over distance (Fig. 1b). That is, we must have $\theta(z) = \theta(z + \Lambda)$ where $\Lambda = z_{\text{coh}} = 2\pi/(\Delta n k_0) = \lambda/\Delta n$ (the average value of $\theta(z)$, even if nonzero, plays no role). The resulting PB phase can then be exploited to control light over an extended propagation length. In particular, to achieve light confinement the phase retardation needs to be larger on the beam axis than in the outer regions, as in TIR or GRIN optical fibers. Such a phase modulation across the beam gives rise to a focusing effect able to counteract the natural diffraction and leading to transverse confinement and guidance. By combining longitudinal (z) and transverse (xy plane) modulations of θ , an overall 3D structure described by $\theta(x, y, z) = \sigma(z)\Gamma(x, y)$ is obtained, with $\sigma(z) = \sigma(z + \Lambda)$ a periodic function to yield a monotonic growth of the geometric phase and $\Gamma(x, y)$ a transverse profile which defines the waveguide cross-section. A sample sketch of such a structure is in Fig. 1c. We stress that in this inhomogeneous anisotropic

medium the optic axis $\hat{u}$ remains always orthogonal to the propagation direction, so no refractive index changes contribute to guiding.

We developed a full analytic theory of the afore described PB guiding mechanism in the frame of the slowly-varying envelope approximation (see Methods). The main results, in a simpler geometry with one transverse coordinate x for the sake of simplicity, can be summarized in a dynamical equation for the wave amplitude A corresponding to a given CP input:

$$i \frac{\partial A}{\partial z} = -\frac{1}{2\bar{n}k_0} \frac{\partial^2 A}{\partial x^2} + V(x)A \quad (2)$$

where $V(x) = \pm(\pi/\Lambda)\Gamma(x)$ with +/- for input LCP/RCP, respectively. Quadratic terms in Γ have been omitted for simplicity (see Methods for the complete expression). Equation (2) is fully equivalent to a 1D Schrödinger equation for a particle oscillating in a potential $V(x)$ (with z playing the role of time) or to the standard (paraxial) Helmholtz equation for light propagating in a GRIN medium with refractive index $n(x)$ such that $V(x) = -k_0[n^2(x) - \bar{n}^2]/(2\bar{n})$. Depending on the sign of $\Gamma(x)$, either LCP or RCP perceive a trapping potential and get confined, while the orthogonal CP undergoes defocusing and diffract even faster than normal, confirming once again the spin-orbit nature of this interaction. For the focused CP component the structure behaves as a standard GRIN waveguide with index profile $n(x)$ (the CP state refers to the input, as the circular polarization continuously evolves between left and right during propagation, see Fig. 1d). Examples of the calculated effective potential and corresponding guided modes are given in Fig. 2a-c. In order to check the validity of our theory we carried out finite-difference time-domain (FDTD) numerical simulations of light propagation in the waveguide structure, solving the full Maxwell equations in the space-variant birefringent medium. Figure 2d-f provides examples of the obtained results, in excellent agreement with the theoretical predictions.

We demonstrated this novel approach to guiding light with a proof-of-principle experiment. Rather than using the continuous structure described above, we realized a simpler “discrete-element” PB-waveguide consisting of equally spaced PBOEs alternating with a homogeneous isotropic dielectric (air); each PBOE is essentially equivalent to a thin slice of the PB-waveguide and acts as a focusing element, that is, a geometric-phase lens (GPL)^{9,28-30}. In other words, we mimicked the operation of the PB-waveguide with a sequence of equally spaced converging lenses, exclusively using PB phases (Fig. 3a).

Our GPLs were thin films of a birefringent uniaxial, nematic liquid crystals, having a transverse distribution of the optic axis given by $\theta = \alpha(x^2 + y^2)$, with α a constant, see Fig. 3b. Neglecting diffraction within the finite GPL thickness, the action of each lens on a CP input beam is described by Eqs. (1). For $\delta = \pi$, the polarization handedness is inverted at the output and

FIG. 2. **Theory and simulations.** **a-b**, Effective photonic potential $V(x)2\bar{n}/k_0$ versus x and maximum θ angle Γ_0 (we assumed a Gaussian distribution for the transverse orientation by setting $\Gamma = \Gamma_0 \exp(-x^2/w_D^2)$) perceived by the defocused (**a**, LCP input) and the confined (**b**, RCP input) waves, respectively. The terms proportional to Γ^2 are accounted for (see Methods). **c**, Corresponding fundamental guided mode; represented is the field amplitude versus x and Γ_0 . **d-f**, FDTD simulations for $\Gamma_0 = 15^\circ$ when the input beam is LCP (**d**) and RCP (**e**), respectively; the color scale gives the local light intensity; the red lines give the beam radius evolution for a homogeneous medium, that is for ordinary diffraction. **f**, Evolution of the confined beam polarization state within a modulation period. Here $\lambda = 1 \mu\text{m}$, $n_o = 1.5$, $n_e = 1.7$, $\sigma(z)$ is sinusoidal and the transverse distribution has $w_D = 5 \mu\text{m}$.

the outgoing wave acquires a geometric phase $\pm 2\theta(x, y)$, which is equivalent to the phase of a thin lens having focal length $f = \pm \frac{\pi}{2\alpha\lambda}$. Hence, the GPL acts as a focusing element for one CP handedness but defocusing for the opposite one. Since the circular polarization handedness is inverted at each GPL, in order to balance out diffraction throughout the structure, we flipped the sign of α at each step, resulting in a longitudinal θ oscillation as for the continuous PB waveguide case. The fundamental mode of our discrete waveguide, the shape-preserving Gaussian beam that propagates with period equal to the distance d between opposite lenses, has a beam waist $w_0 = \sqrt{\lambda[(4f-d)^2/(2n_i\pi)]}$ centered between subsequent GPLs, where n_i is the refractive index of the isotropic medium between elements.

For our experiments, we set up a sequence of five GPLs, rotating every other one by π around the y axis so as to produce alternating signs of α ; moreover, exploiting the electro-optic response of nematic liquid crystals, the lenses were electrically tuned to $\delta = \pi$ (see Methods for details). We characterized light propagation in the structure for both LCP and RCP inputs and compared it with free-space propagation. As can be seen in Figs. 3c-e, the experimental results are in very good agreement with the polarization-dependent waveguiding predicted for the continuous case: only one input CP handedness was con-

fined in the PB-waveguide with a shape-preserving mode, whereas the opposite polarization was radiated off almost immediately. The acquired data matched well the theory, as shown in Fig. 3f.

In conclusion, we have shown, both theoretically and experimentally, that geometric Pancharatnam-Berry phases can be used for transverse confinement of electromagnetic waves, thus introducing an entirely new light-guiding principle that exploits spin-orbit optical interactions and the vectorial nature of electromagnetic radiation. Besides its fundamental interest, the proposed approach is technologically relevant for future integrated optics systems, including those involving metasurfaces^{23,24}. The development of novel generations of PB guided-wave photonics and manipulation of light is envisioned in dielectrics and metamaterials for the whole ultraviolet to terahertz spectrum.

FIG. 3. **Experiment.** **a**, Experimental setup: five equally-spaced electrically-tuned GPLs form a discrete-element geometric-phase waveguide. A 532 nm continuous-wave Gaussian beam was adjusted in transverse size with a telescope so as to match the fundamental mode of the waveguide, circularly polarized with a quarter-wave plate (QWP) and then launched into the waveguide. Beam profiles at various intermediate positions z along the propagation and at the output were acquired by a movable CCD camera and used to reconstruct the mode parameters of the beam after each GPL. **b**, Distribution of the optic axis and corresponding microscope image of a GPL between crossed polarizers; dark fringes correspond to regions where the optic axis is aligned parallel to one of the polarizers. **c-e**, Data-reconstructed evolution of the propagating beam for each of the following cases: **c**, guided mode obtained for RCP input; **d**, divergent beam obtained for LCP input; **e**, free-space diffracting beam for the same input parameters. The color scale gives the local light intensity. Horizontal lines and L1-L5 labels indicate the GPL positions within the discrete sequence (dashed lines mark removed GPLs). **f**, Beam radius versus z in the guided case. Dots are the measurement data, the blue solid lines are the Gaussian-beam fits for those dots between two subsequent GPLs, the blue shaded areas being the fit confidence regions at one standard deviation; the black dashed line is the theoretical prediction based on the ABCD method, also accounting for the imperfections of the Gaussian beam (as defined by the M^2 parameter) and of the GPLs; the dashed green line corresponds to the predicted beam evolution for an ideal Gaussian input, with $M^2 = 1$. Vertical dashed lines give the positions of the GPLs in the waveguide. **g**, Acquired intensity beam profiles (I versus x, y) at the input plane of each GPL in the structure for the guided case; the scale-bar corresponds to 400 μm .

I. METHODS

A. Dynamics of circularly polarized waves through a uniform uniaxial medium

Plane-wave light propagation along z in a uniform uniaxial medium can be described by a two-component Jones vector $\psi(z)$ (in the bra-ket notation $|\psi\rangle$). We then define a 2×2 transmission Jones matrix $\mathbf{T}(z, \theta)$ linking the vector ψ at the input plane $z = 0$ with that at any given distance z , as a function of the angle θ formed by the optic axis with the y axis. The two components of ψ represent the amplitudes of two orthogonal polarizations which define the chosen basis of the representation. Here in particular we employ the (LCP, RCP) basis. In this Letter, we adopt the notation $\hat{L} = (\hat{x} - i\hat{y})/\sqrt{2}$ (LCP) and $\hat{R} = (\hat{x} + i\hat{y})/\sqrt{2}$ (RCP), corresponding to the source-point-of-view convention on CP states. The rotation operator $\mathbf{R}(\theta)$ around the $\hat{z}$ axis is diagonal in the CP basis and can be written as

$$\mathbf{R}_{LR}(\theta) = \begin{pmatrix} e^{-i\theta} & 0 \\ 0 & e^{i\theta} \end{pmatrix}, \quad (3)$$

with the subscript “LR” denoting the basis. The transmission matrix $\mathbf{T}_{LR}(z, \theta)$ has the form

$$\mathbf{T}_{LR}(z, \theta) = \mathbf{R}_{LR}^{-1}(\theta) \cdot \mathbf{P}^{-1} \cdot \begin{pmatrix} e^{ik_0 n_o z} & 0 \\ 0 & e^{ik_0 n_e z} \end{pmatrix} \cdot \mathbf{P} \cdot \mathbf{R}_{LR}(\theta), \quad (4)$$

where $\mathbf{P} = (1, 1; -i, i)/\sqrt{2}$ is the matrix changing basis from $\{\hat{x}, \hat{y}\}$ to $\{\hat{L}, \hat{R}\}$. A straightforward derivation leads from Eq. (4) to Eq. (1).

Let us consider the solution for a purely circular polarization at the input, for e.g. $\psi_R(0) = 1$ and $\psi_L(0) = 0$. From Eq. (1) we get $\psi_R(z) = e^{i\tilde{n}k_0 z} \cos(\frac{\delta}{2})$ and $\psi_L(z) = -ie^{i\tilde{n}k_0 z} \sin(\frac{\delta}{2})e^{2i\theta}$. From the latter expression we can calculate the phase difference between two states corresponding to two different orientations for the optic axis, say θ_1 and θ_2 , respectively. Following Pancharatnam’s original concept^{25,31}, the phase delay $\Delta\phi(\theta_1, \theta_2)$ is

$$\begin{aligned} \Delta\phi(\theta_1, \theta_2) &= \arg [\langle \psi(\theta_1) | \psi(\theta_2) \rangle] \\ &= \arg \left[\cos^2 \left(\frac{\delta}{2} \right) + \sin^2 \left(\frac{\delta}{2} \right) e^{2i(\theta_2 - \theta_1)} \right] \end{aligned} \quad (5)$$

Expression (5) results in the transverse phase delay plotted in Fig. 1b (dashed lines) in the homogeneous limit. When the optic axis distribution is flipped at $\Lambda/2$, the phase delay can be easily obtained from Eq. (4) (solid lines in Fig. 1b). When the optic axis distribution is sinusoidally modulated along z , the accumulated $\Delta\phi(\theta_1, \theta_2)$ can be numerically calculated partitioning the medium in several layers, each of them short enough to make the variations of θ negligible within each layer, as shown in Fig. 1b (solid lines with circles).

B. Spin-dependent photonic potential

In the paraxial limit (i.e., neglecting the longitudinal field components) and for small birefringence ($\Delta n \ll 1$), the Maxwell equations for the electric field $\mathbf{E} = E_x \hat{x} + E_y \hat{y}$ in two dimensions (i.e., with no field evolution across y) can be cast as

$$\begin{aligned} \frac{\partial^2}{\partial z^2} \begin{pmatrix} E_x \\ E_y \end{pmatrix} &= -\frac{\partial^2}{\partial x^2} \begin{pmatrix} E_x \\ E_y \end{pmatrix} + \\ &-k_0^2 \begin{pmatrix} \epsilon_{xx}(x, z) & \epsilon_{xy}(x, z) \\ \epsilon_{yx}(x, z) & \epsilon_{yy}(x, z) \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix} \end{aligned} \quad (6)$$

We would like to adopt now the slowly-varying-envelope approximation (SVEA) so as to obtain a simpler first-order partial differential equation in the evolution coordinate z . SVEA is usually based on the presence of two very different spatial scales, that is a short scale of order λ and a long scale given by the Rayleigh length $z_R = \pi \tilde{n} w^2 / \lambda$, where w is the transverse spatial scale of the problem (the beam radius and/or the transverse spatial modulations of the medium). In our case, however, we also have an intermediate scale $\Lambda = \lambda / \Delta n$, with $\lambda \ll \Lambda \ll z_R$ for reasonable values of Δn and w . It is therefore not convenient to apply the SVEA directly to Eq. (6), as the field components E_x and E_y undergo a relatively rapid evolution on scale Λ because of birefringence. It is more convenient to switch first to the space-varying ordinary/extraordinary wave basis, with a rotation by the angle $\theta(x, z)$ around the z axis. This basis change diagonalizes the effect of birefringence and hence allows one to adopt the SVEA in an optimal way, but at the same time it generates the space-varying geometric phases which affect the resulting slow-envelope dynamics. After applying the SVEA and neglecting higher-order terms in λ/Λ and Λ/z_R , we obtain our final dynamical equations for the slowly-varying amplitudes (see Supplementary Material for a full derivation). We find in particular that the two components of the field can be decoupled from each other if one adopts the CP basis for the slow amplitudes (corresponding to the input states of the polarizations). The resulting dynamical equation for each of these amplitudes A is

$$\begin{aligned} i \frac{\partial A}{\partial z} &= -\frac{1}{2\tilde{n}k_0} \frac{\partial^2 A}{\partial x^2} + \frac{1}{4\tilde{n}k_0} \left[\left(\frac{d\Gamma}{dx} \right)^2 + \frac{4\pi^2}{\Lambda^2} \Gamma^2(x) \right] A \\ &\pm \frac{\pi}{\Lambda} \Gamma(x) A, \end{aligned} \quad (7)$$

where $\pm = +$ for the amplitude A corresponding to input LCP and $\pm = -$ for the amplitude corresponding to the input RCP. The guided modes of the system can then be obtained by setting $A(x, z) = e^{i\beta z} A(x, 0)$, with the effective propagation constant β acting as an eigenvalue (hence $\Lambda \ll 1/\beta$ must hold for consistency). Compared to the simpler model presented in the main text, the photonic potential contains a contribution related with the Kapitza effect¹⁷ due to both transverse ($\frac{d\Gamma}{dx}$) and longitudinal ($\frac{d\sigma}{dz}$) modulations of the optic axis orientation θ (see Supplementary Material for further details).

Thus, neglecting the terms proportional to the square of Γ , the two circular polarizations perceive photonic potentials of the same transverse profile but opposite signs, as stated by Eq. (2). Equation (7) shows that a higher-order contribution due to a non-adiabatic modulation of the geometric phase (term in square brackets) acts in the same way on both components; when $w_D = 5 \mu\text{m}$ such term can be neglected for Γ_0 up to 360° , as apparent in Fig. 2a-b. The Kapitza term increases for narrower distributions of θ (Supplementary Fig. 1 shows how the trapping potential gets strongly distorted for $\Gamma_0 = 360^\circ$ when $w_D = 0.5 \mu\text{m}$).

C. FDTD simulations

For the FDTD numerical simulations we employed the open-source code MEEP³² to solve the full Maxwell equations in two dimensions, with no approximations. In all simulations we assumed a Gaussian shaped orientation of the optic axis across x in the form $\Gamma(x) = \Gamma_0 \exp(-x^2/w_D^2)$. The excitation was a continuous-wave source launched in $x = z = 0$ with a width of $3 \mu\text{m}$ across x , turned on at $t = 0$ and infinitely narrow across z . The modulated uniaxial medium was placed in $z_0 = 2 \mu\text{m}$ with modulation $\sigma(z) = \sin[\frac{2\pi\Delta n}{\Lambda}(z - z_0)]$. The refractive indices n_o and n_e were taken equal to 1.5 and 1.7, respectively. The simulations confirm that an input RCP is confined in the anisotropic structure, whereas an LCP input is expelled towards the edges (Supplementary Fig. 3). The polarization of the simulated confined wave undergoes small variations in the transverse plane (Supplementary Fig. 2) and is not perfectly periodic along z . Such small discrepancies between simulations and our analytic theory are clearly due to higher-order terms in λ/Λ and Λ/z_R which have been neglected in the analytic theory. For $w_D = 5 \mu\text{m}$ light confinement improves with the maximum rotation Γ_0 (Supplementary Fig. 3); however, for $\Gamma_0 = 90^\circ$ even in the defocusing case a small amount of power is trapped on axis owing to the higher order terms (Kapitza effect) previously neglected in Eq. (7). Further simulations show how light trapping/repulsion both increase as the width w_D gets smaller, in agreement with theory (Supplementary Fig. 5). We also studied numerically the effects of shifting $\sigma(z)$ by changing z_0 , obtaining a perfect agreement with theory. In fact, when $z_0 = z_{\text{coh}}/2$ the two polarization states exchange their roles, that is, LCP at the input gets trapped whereas a launched RCP undergoes defocusing (Supplementary Fig. 6); when $z_0 = z_{\text{coh}}/4$ the beam evolution does not depend on the input polarization, with power equally shared by confined and radiated modes. These results demonstrate that the PB-phase waveguide infringes translational symmetry along the propagation axis, at variance with standard (TIR or GRIN) waveguides. Finally, we numerically ascertained the role of small mismatches between the modulation period Λ and the length z_{coh} , an important issue in actual implementa-

tions of the proposed waveguides. Keeping all the parameters fixed except for Λ , the global behavior of the two polarization states, i.e., LCP defocused and RCP guided, is robust against Λ variations up to 50%.

D. Fabrication of the geometric phase lenses

Geometric phase lenses and similar PBOEs can be realized with a variety of techniques and materials^{20–22,33,34}. Liquid crystals²¹ and liquid crystal polymers³³ are the most suitable for visible and near infrared illumination. Our GPLs were fabricated using polarization holography in combination with photo-alignment of nematic liquid crystal³⁵. Planar cells were realized with two glass substrates held parallel at a separation of $6 \mu\text{m}$, previously coated with a convenient photoaligning surfactant³⁵. Such substrates were exposed to collimated ultraviolet light with an inhomogeneous distribution of the linear polarization state. The polarization pattern was realized by coaxial superposition of two beams (473 nm diode-pumped solid-state laser) with orthogonal circular polarizations and various phase-front curvatures. The interference of such beams, rather than an intensity modulation, produces a pattern of linear polarizations, with the angle of the polarization plane proportional to the point-wise phase difference between the beams. After exposure of the coated glass slides, nematic liquid crystals (mixture E7 from Merck) were introduced in the cell by capillarity and aligned with a correspondingly inhomogeneous orientation of the optic axis (molecular director). The required half-wave phase retardation of the GPLs was finely adjusted by applying a 10 kHz square-wave electric voltage ($\approx 2.5 \text{ V}$ peak-peak) as in other liquid-crystal-based PBOEs³⁶. Each GPL can also be optically “switched off” by applying the voltage giving full-wave retardation ($\approx 4.0 \text{ V}$). Five GPLs were fabricated, with focal lengths 15.17, 15.98, 14.83, 15.17 and $15.69 \pm 0.01 \text{ cm}$, respectively, for light of wavelength 532 nm. The dispersion of the focal length values is due to the imperfect repeatability of the exposure conditions and insufficient stability of the fabrication setup.

E. Beam characterization within the structure

In order to reconstruct the beam parameters inside the GPL structure, the propagating beam at 532 nm was sampled by a CCD camera placed at various propagation distances between the lenses. Moreover, to improve the measurement accuracy of the local beam parameters, additional beam profiles were collected at given distances from each lens. This was accomplished by either switching off the GPLs following the one under measurement or by physically removing the remaining lenses from the sequence. The beam radii $w(z)$ were obtained through Gaussian fits of the acquired profile images and used to reconstruct the modal parameters within and at

the output of the waveguide. The obtained radius evolution was then compared with the theoretical predictions from ABCD Gaussian propagation and numerical simulations. A more realistic description of the beam was obtained with a non-unitary beam-quality factor M^2 of the confined beams after each lens ($M_1^2 = 1.05 \pm 0.01$, $M_2^2 = 1.18 \pm 0.02$, $M_3^2 = 1.19 \pm 0.01$, $M_4^2 = 1.15 \pm 0.04$, and $M_5^2 = 1.32 \pm 0.04$, uncertainties at 95% confidence level), suitably modifying the propagation equations³⁷ for the simulations. The gradual increase of the M^2 parameter after each step of the discrete structure can be ascribed to degradation of the beam profile (as visible in Fig. 3g) due to the noisy patterns of the GPLs.

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This work was jointly conceived by A.A., C.P.J., G.A. and L.M.; S.S. designed and carried out the experiment, with the help and supervision of B.P., E.S. and L.M.; A.A. and C.P.J. developed the theory and performed the numerical simulations, with the help and supervision of G.A.; all authors discussed the results and contributed to the manuscript.

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SUPPLEMENTARY INFORMATION

S1. LIGHT PROPAGATION IN A PERIODIC SYSTEM ENCOMPASSING A ROTATION OF THE OPTIC AXIS IN THE TRANSVERSE PLANE

We consider light propagation in inhomogeneous anisotropic dielectrics, in particular uniaxials; nonetheless, our results can be readily generalized to biaxial crystals. We take a medium whose dielectric properties vary across the transverse coordinate x owing exclusively to a rotation of the optic axis in the plane xy orthogonal to the propagation coordinate z . We consider finite wavepackets with wavevector parallel to $\hat{z}$; hence, corresponding to electric fields oscillating orthogonally and parallel to the optic axis, respectively, the two independent eigenvalues $\epsilon_\perp$ and $\epsilon_\parallel$ of the relative permittivity tensor are constant in space. We define the birefringence $\Delta n = \sqrt{\epsilon_\parallel} - \sqrt{\epsilon_\perp} = n_e - n_o$ and assume that the distribution of the optic axis is purely planar and transverse to $\hat{z}$, such that at each z the rotation can be described by a standard 2D operator acting in xy :

$$\mathbf{R}(\theta) = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}, \quad (\text{S1})$$

where the angle θ is defined with respect to the y axis of a Cartesian reference system in the laboratory frame, with $\theta = 0$ corresponding to a dielectric permittivity $\epsilon_\parallel$ for electric fields along y . We study forward propagating light waves in the presence of a periodic modulation of θ along z . To this extent we set

$$\theta(x, z) = \sigma(z)\Gamma(x) = \left(\sum_{p=-\infty}^{\infty} \sigma_p e^{\frac{i2\pi pz}{\Lambda}} \right) \Gamma(x), \quad (\text{S2})$$

with $\sigma(z)$ a function periodic with Λ and $\Gamma(x)$ the transverse distribution of optic axis orientation, the latter being uniform across y . Hereafter, we examine a purely sinusoidal modulation $\sigma_p = \sigma_1 \delta_{1,p} + \sigma_{-1} \delta_{-1,p}$, with $\delta_{p,p'}$ the Kronecker's delta and $\sigma_{-1} = \sigma_1^*$ in order for σ to be real valued. Moreover, we deal with the case $\Lambda = \lambda/\Delta n$ in which resonant effects are expected to occur.

In the paraxial limit (neglecting longitudinal field components) and for small birefringence ($\Delta n \ll 1$), Maxwell equations in two dimensions (i.e., no field evolution across y) can be cast as

$$\frac{\partial^2}{\partial z^2} \begin{pmatrix} E_x \\ E_y \end{pmatrix} = -\frac{\partial^2}{\partial x^2} \begin{pmatrix} E_x \\ E_y \end{pmatrix} - k_0^2 \begin{pmatrix} \epsilon_{xx}(x, z) & \epsilon_{xy}(x, z) \\ \epsilon_{yx}(x, z) & \epsilon_{yy}(x, z) \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}, \quad (\text{S3})$$

where $\mathbf{E} = E_x \hat{x} + E_y \hat{y}$ is the electric field.

In order to investigate the propagation of an electromagnetic (optical) wave in such a system, we make use of the transformation $\boldsymbol{\varphi} = \mathbf{R} \cdot \mathbf{E}$ and write $\boldsymbol{\varphi} = (E_o; E_e)$, with E_o and E_e the pointwise ordinary and extraordinary polarization components of the electric field, respectively. In the rotated reference system the two-dimensional dielectric tensor is diagonal, specifically $\boldsymbol{\epsilon}_D = (\epsilon_\perp, 0; 0, \epsilon_\parallel)$; after setting $\mathbf{T} = (0, -1; 1, 0)$, Eq. (S3) becomes

$$\frac{\partial^2 \boldsymbol{\varphi}}{\partial z^2} + 2 \frac{\partial \theta}{\partial z} \mathbf{T} \cdot \frac{\partial \boldsymbol{\varphi}}{\partial z} = - \left\{ \frac{\partial^2 \boldsymbol{\varphi}}{\partial x^2} - \left[\left(\frac{\partial \theta}{\partial x} \right)^2 + \left(\frac{\partial \theta}{\partial z} \right)^2 \right] \boldsymbol{\varphi} \right\} - k_0^2 \boldsymbol{\epsilon}_D \cdot \boldsymbol{\varphi} - \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial z^2} \right) \mathbf{T} \cdot \boldsymbol{\varphi} - 2 \frac{\partial \theta}{\partial x} \mathbf{T} \cdot \frac{\partial \boldsymbol{\varphi}}{\partial x}. \quad (\text{S4})$$

Equation (S4) shows that a scalar potential proportional to $(\partial \theta / \partial x)^2 + (\partial \theta / \partial z)^2$ acts on both field components. Additionally, other terms (containing the operator $\mathbf{T}$) couple ordinary and extraordinary polarizations: due to the rotation of the optic axis, the ordinary and extraordinary components are no longer independent, but can exchange energy during propagation. Noteworthy, it is apparent from Eq. (S4) that no refractive index gradients affect propagation as the latter is entirely governed by the power exchange between ordinary and extraordinary components, in turn leading to a complex evolution of the overall polarization state.

In the slowly varying envelope approximation (SVEA), we first set $E_o = \psi_o(x, z) e^{ik_0 n_o z}$ and $E_e = \psi_e(x, z) e^{ik_0 n_e z}$ in Eq. (S4). Exploiting the system periodicity in z we then introduce the following further transformation:

$$\psi_j(x, z) = A_j(x, z) B_j(x, z) \quad (j = e, o), \quad (\text{S5})$$

where $A_j(x, z)$ are the slow-varying envelopes for the two wave components and $B_j(x, z)$ are periodic functions of z accounting for the effect of the medium modulations with period Λ . It is convenient to write the latter ones in an

exponential form as follows:

$$B_j(x, z) = e^{i \sum_{p=-\infty}^{\infty} \int \beta_p^{(j)}(x, z) dz} = \exp \left[\sum_{p=-\infty}^{\infty} \frac{\Lambda}{2\pi p} \beta_p^{(j)}(x) e^{\frac{i2\pi pz}{\Lambda}} \right] \quad (\text{S6})$$

where we set $\beta_p^{(j)}(x, z) = \beta_p^{(j)}(x) e^{\frac{i2\pi pz}{\Lambda}}$ and the term $p = 0$ must be excluded from the sum. Inserting these ansatzes in Eq. (S4) and expanding in powers of Λ all terms one obtains a hierarchy of coupled equations for the amplitudes $A_j(x, z)$ and the coefficients $\beta_p^{(j)}(x)$. Neglecting from these equations all terms with powers of the period Λ equal to or larger than 1 and higher-order terms in λ/Λ (which is equal to Δn at resonance), we obtain the following coupled equations for the slow amplitudes:

$$i \frac{\partial A_o}{\partial z} = -\frac{1}{2\bar{n}k_0} \frac{\partial^2 A_o}{\partial x^2} + \frac{1}{2\bar{n}k_0} \left[\overline{\sigma^2(z)} \left(\frac{d\Gamma}{dx} \right)^2 + \Gamma^2 \overline{\left(\frac{d\sigma}{dz} \right)^2} \right] A_o + \frac{2\pi}{\Lambda} \sigma_{-1} \Gamma(x) A_e, \quad (\text{S7})$$

$$i \frac{\partial A_e}{\partial z} = -\frac{1}{2\bar{n}k_0} \frac{\partial^2 A_e}{\partial x^2} + \frac{1}{2\bar{n}k_0} \left[\overline{\sigma^2(z)} \left(\frac{d\Gamma}{dx} \right)^2 + \Gamma^2 \overline{\left(\frac{d\sigma}{dz} \right)^2} \right] A_e + \frac{2\pi}{\Lambda} \sigma_1 \Gamma(x) A_o, \quad (\text{S8})$$

where $\overline{m(z)} = \frac{1}{\Lambda} \int_0^\Lambda m(z) dz$ and $\bar{n} = (n_e + n_o)/2$.

After inspection of Eqs. (S7-S8), three salient terms stand out:

- a Kapitza-like potential, proportional to the squared transverse derivative of the distribution Γ of optic axis orientation; its magnitude is modulated by the mean square of the periodic modulation $\sigma(z)$;
- an effective transverse potential with profile proportional to Γ^2 ; its magnitude is modulated by the mean square of the longitudinal derivative of the periodic modulation $\sigma(z)$;
- a phase-matched coupling between ordinary and extraordinary waves via the fundamental harmonics $\sigma_{\pm 1}$ of the periodic modulation.

The equations (S7-S8) can be recast in a more compact form as

$$i \frac{\partial \mathbf{A}}{\partial z} = \mathbf{L}_{\text{ISO}} \cdot \mathbf{A} + \mathbf{L}_{\text{ANI}} \cdot \mathbf{A}, \quad (\text{S9})$$

where $\mathbf{A} = (A_o; A_e)$ and we introduced the isotropic (matrix) operator

$$\mathbf{L}_{\text{ISO}} = \frac{1}{2\bar{n}k_0} \left\{ -\frac{\partial^2}{\partial x^2} + \left[\overline{\sigma^2} \left(\frac{d\Gamma}{dx} \right)^2 + \Gamma^2 \overline{\left(\frac{d\sigma}{dz} \right)^2} \right] \right\} \mathbf{I}, \quad (\text{S10})$$

and the anisotropic operator

$$\mathbf{L}_{\text{ANI}} = \frac{2\pi}{\Lambda} \Gamma(x) \begin{pmatrix} 0 & \sigma_{-1} \\ \sigma_1 & 0 \end{pmatrix}. \quad (\text{S11})$$

The presence of $\mathbf{L}_{\text{ANI}}$ in Eq. (S9) accounts for the power exchange between ordinary and extraordinary components.

Let us now take $\sigma(z) = \sin(\frac{2\pi z}{\Lambda})$, that is $\sigma_1 = -i/2$ and $\sigma_{-1} = i/2$; then $\mathbf{L}_{\text{ANI}} = -i\frac{\pi}{\Lambda} \Gamma(x) \mathbf{T}$ and its eigenvalues are $\pm i$ with the two circular polarizations $(1; \pm i)/\sqrt{2}$ for eigenvectors (plus and minus correspond to RCP and LCP in our convention, respectively). Therefore, when the optic axis is modulated along z with period equal to the beat length, the localized wave solutions are circularly polarized. By using the transformation

$$\mathbf{A}_{oe} = \mathbf{P} \cdot \mathbf{A}_{LR} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -i & i \end{pmatrix} \begin{pmatrix} A_L \\ A_R \end{pmatrix}, \quad (\text{S12})$$

the system of equations separates into two independent scalar equations

$$i \frac{\partial A_L}{\partial z} = -\frac{1}{2\bar{n}k_0} \frac{\partial^2 A_L}{\partial x^2} + \frac{1}{4\bar{n}k_0} \left[\left(\frac{d\Gamma}{dx} \right)^2 + \frac{4\pi^2}{\Lambda^2} \Gamma^2(x) \right] A_L + \frac{\pi}{\Lambda} \Gamma(x) A_L, \quad (\text{S13})$$

$$i \frac{\partial A_R}{\partial z} = -\frac{1}{2\bar{n}k_0} \frac{\partial^2 A_R}{\partial x^2} + \frac{1}{4\bar{n}k_0} \left[\left(\frac{d\Gamma}{dx} \right)^2 + \frac{4\pi^2}{\Lambda^2} \Gamma^2(x) \right] A_R - \frac{\pi}{\Lambda} \Gamma(x) A_R, \quad (\text{S14})$$

where we used $\overline{\sigma^2(z)} = 0.5$ and $\overline{(d\sigma/dz)^2} = 2\pi^2/\Lambda^2$. The guided eigenmodes are then defined by setting $A_i(x, z) = e^{i\beta_0 z} A_i(x, 0)$, where β_0 is the propagation constant. The problem is then transformed into a standard eigenvalue problem, with the following equations:

$$-2k_0\bar{n}\beta_0 A_L = -\frac{\partial^2 A_L}{\partial x^2} + \left[\frac{1}{2} \left(\frac{d\Gamma}{dx} \right)^2 + \frac{2\pi^2}{\Lambda^2} \Gamma^2(x) \right] A_L + \frac{2k_0\bar{n}\pi}{\Lambda} \Gamma(x) A_L, \quad (\text{S15})$$

$$-2k_0\bar{n}\beta_0 A_R = -\frac{\partial^2 A_R}{\partial x^2} + \left[\frac{1}{2} \left(\frac{d\Gamma}{dx} \right)^2 + \frac{2\pi^2}{\Lambda^2} \Gamma^2(x) \right] A_R - \frac{2k_0\bar{n}\pi}{\Lambda} \Gamma(x) A_R, \quad (\text{S16})$$

Equations (S15) and (S16) are valid for left (LCP) and right (RCP) circularly polarized wavepackets at the input interface, respectively. It is noteworthy that RCP and LCP interchange roles if the modulation $\sigma(z)$ is shifted by half a period, in agreement with the intuitive picture provided in the main text.

A. Polarization-dependent effective index well

In standard (1+1)D graded-index waveguides, a generic transverse-electric mode u satisfies $-2k_0\bar{n}\kappa u = -\frac{\partial^2 u}{\partial x^2} - k_0^2 \Delta n^2 u$, with κ the propagation constant. Thus, from Eqs. (S15-S16) it is apparent that the two circular polarizations perceive the effective photonic potential $V_{\text{eff}} = -k_0 \Delta n_{\text{eff}}^2 / (2\bar{n})$ with the index distributions $\Delta n_{\text{eff}}^2(x)$ given by

$$\Delta n_{\text{eff},LCP}^2 = -\frac{2\bar{n}}{k_0} V_{\text{eff},LCP} = -\frac{1}{k_0^2} \left[\frac{1}{2} \left(\frac{d\Gamma}{dx} \right)^2 + \frac{2\pi^2}{\Lambda^2} \Gamma^2(x) \right] + \bar{n} \frac{\lambda}{\Lambda} \Gamma(x), \quad (\text{S17})$$

$$\Delta n_{\text{eff},RCP}^2 = -\frac{2\bar{n}}{k_0} V_{\text{eff},RCP} = -\frac{1}{k_0^2} \left[\frac{1}{2} \left(\frac{d\Gamma}{dx} \right)^2 + \frac{2\pi^2}{\Lambda^2} \Gamma^2(x) \right] - \bar{n} \frac{\lambda}{\Lambda} \Gamma(x), \quad (\text{S18})$$

The polarization independent term between square brackets in Eqs. (S17-S18) is a Kapitza-like equivalent photonic potential stemming from transverse and longitudinal modulation of the rotating optic axis. Since this photonic potential is independent from $k_0 = 2\pi/\lambda$, by itself it would support guided modes with wavelength-independent profile³⁸. The last terms on the RHS of Eqs. (S17-S18), with opposite signs as wave handedness reverses, are responsible for the strong dependence of light evolution on input polarization: the periodic rotation of the optic axis along z allows for an *accumulation* of the Berry phase during propagation, leading to the appearance of a potential proportional to Γ . Moreover, since phase-matching requires $\Lambda = \lambda/\Delta n$, such potential is directly proportional to the medium birefringence.

B. Bell-shaped orientation distribution of the optic axis

Hereafter we make explicit reference to optical frequencies. Nevertheless, since Maxwell equations are invariant when dividing all the length scales by a given factor and multiplying the frequency by the same factor, our results clearly hold valid regardless the electromagnetic band.

Supplementary Fig. S1 shows the effective index well $-\Delta n_{\text{eff}}^2$ (sign-inverted, so light is attracted by the dip) as computed from Eqs. (S17-S18) when the transverse distribution of the orientation angle $\Gamma(x)$ is bell-shaped and centered in $x = 0$. We assumed $\Gamma(x) = \Gamma_0 \exp(-x^2/w_D^2)$, with Γ_0 and w_D the maximum orientation angle and the width of the distribution, respectively. Using this simple ansatz we can address the role of each term in the effective index well, Eqs. (S17-S18). The term proportional to $(d\Gamma/dx)^2$ is a Kapitza-like term: when acting alone it yields quasi-modes, as detailed in Ref.¹⁷. This contribution to the index landscape, proportional to w_D^{-2} , increases as the angle distribution becomes narrower. Supplementary Fig. S1 illustrates two examples for $w_D = 0.5 \mu\text{m}$ and $w_D = 15 \mu\text{m}$, respectively: the Kapitza term becomes quite relevant for $w_D = 0.5 \mu\text{m}$ and large Γ_0 , with the appearance of two local maxima, symmetrically placed with respect to the axis $x = 0$. The term proportional to $\Gamma^2(x)$ gives rise to anti-guidance, i.e. light repulsion from the symmetry axis. The term is proportional to the square of the birefringence Δn , thus dominates for large anisotropies.

Finally, the last term breaks the degeneracy between the two opposite circular polarizations. This term contributes with opposite signs to the overall potential acting on RCP and LCP waves, respectively: in the absence of other contributions, when the RCP (LCP) wave is confined, the LCP (RCP) is repelled from the region close to the symmetry axis $x = 0$, i.e., it diffracts faster than in a homogeneous medium.

FIG. S1. **a-d** Transverse profile of the sign-inverted effective index distribution, proportional to the photonic potential $V_{\text{eff,LCP/RCP}}$ versus maximum reorientation angle Γ_0 for two different widths **(a-b)** $w_D = 0.5 \mu\text{m}$ and **(c-d)** $w_D = 15 \mu\text{m}$ of the transverse angular distribution. Input RCP waves are trapped in **a, c** and input LCP waves undergo defocusing in **b, d**, respectively, in agreement with the chosen initial section. **e-f** Profiles of the fundamental guided (RCP) mode when **(e)** $w_D = 0.5 \mu\text{m}$ and **(f)** $w_D = 15 \mu\text{m}$. Here $\lambda = 1 \mu\text{m}$, $n_o = 1.5$ and $n_e = 1.7$.

C. Finite-Difference-Time-Domain numerical simulations of the guiding case

In the numerical simulations we employed the open-source FDTD code named MEEP³² to solve the full Maxwell equations in two dimensions, assuming a Gaussian orientation of the optic axis across x , as above. The excitation was a continuous-wave source of wavelength $1 \mu\text{m}$ turned on at $t = 0$, infinitely narrow across z and launched in $x = z = 0$. The source had a Gaussian profile of width $3 \mu\text{m}$ across x and was point-like along z . The modulated uniaxial medium was placed in $z > z_0 = 2 \mu\text{m}$ with modulation $\sigma(z) = \sin\left[\frac{2\pi\Delta n}{\lambda}(z - z_0)\right]$. The refractive indices n_o and n_e were taken equal to 1.5 and 1.7, respectively.

Supplementary Fig. S2 illustrates the propagation of an RCP wave input when $w_D = 5 \mu\text{m}$ and $\Gamma_0 = 15^\circ$, corresponding to a guiding potential (see Fig. 2 in the main text). As predicted, the natural diffractive spreading is compensated for by the effective waveguide resulting from Berry phase accumulation. At distances far enough from the input, in the stationary regime, the wavepacket acquires a periodic spatial distribution of its polarization and a nearly invariant profile. The polarization is generally elliptical; its trend can be examined by taking a single period $\lambda/\Delta n$ far enough from the input so that radiation (from imperfect coupling) is negligible. The polarization at the input is RCP, then it starts decreasing ellipticity; at about a quarter period it becomes linear, then elliptical again but with opposite handedness; at half period is nearly LCP, in excellent agreement with the theory. In the following half-period the polarization evolves in a similar manner, going from LCP to RCP. The polarization rotation versus propagation strictly resembles the behavior of a plane wave because it originates from the different phase velocities of the two linear eigenfield carriers (with $e^{ik_0 n_o z}$ and $e^{ik_0 n_e z}$, respectively).

Supplementary Fig. S3 compares the intensity evolution of propagating LCP and RCP wavepackets. In agreement with theory, one input polarization is subject to trapping, the other to a repulsive potential expelling light from the region around $x = 0$. The strength of either potentials increases with the maximum rotation Γ_0 : in the trapping case the waveguide eventually becomes multi-modal, as indicated by the appearance of breathing versus propagation; in the repulsive case the beam divergence increases with Γ_0 . For $\Gamma_0 \geq 90^\circ$ the polarization of the confined beam continues to oscillate along z , but in the presence of higher harmonics. In fact, in Eq. (S6) the functions $\beta_p(x, z)$ with $p \geq 1$ must be accounted for and correspond to shorter periodicity in both beam profile and polarization. Simultaneously, a higher-order contribution proportional to $\Lambda^2(\frac{d\Gamma}{dx}\frac{d^2\Gamma}{dx^2})^2$ appears in the overall photonic potential of the isotropic operator $\mathbf{L}_{\text{ISO}}$ defined by Eq. (S10)³⁹. Accordingly, even in the defocusing case a small portion of the wavepacket is trapped on-axis around $x = 0$ for $\Gamma_0 = 90^\circ$, as visible in Fig. S3. Owing to the additional modulating terms $\beta_p^{(j)}(x)$, further increases in Γ_0 (not shown) infringe the validity of Eqs. (S15-S16), leading to the generation of three-peaks for both input polarizations as well as appreciable changes of polarization state across x in the guided case.

Supplementary Fig. S4 shows how power is distributed between the two components E_x and E_y when light is guided: $|E_x|^2$ and $|E_y|^2$ distributions in the plane xz essentially coincide. The overall intensity is computed as

FIG. S2. Numerical snapshots of (a) e_y and (b) e_x in the plane xz once the stationary regime is reached in time; (c) corresponding average intensity map in xz ; the red solid lines correspond to beam diffraction when $\Gamma_0 = 0$, i.e., the homogeneous case. (d, g) Angular orientation of the polarization ellipse and (e, h) field ellipticity (arctangent of the ratio of minor to major axes) versus z in (d-e) $x = 0$ and (g-h) $x = 3 \mu\text{m}$, respectively. Polarization ellipses in (f) $x = 0$ and (i) $x = 3 \mu\text{m}$ plotted versus z in an interval $0 \leq z\Delta n/\lambda \leq 1$ far away from the excitation point, corresponding to $97 \mu\text{m} \leq z \leq 102 \mu\text{m}$. Here $w_D = 5 \mu\text{m}$, $\Gamma_0 = 15^\circ$, RCP input.

FIG. S3. Average beam intensity in the plane xz when the input is (a) LCP or (b) RCP. The maximum rotation Γ_0 is 0, 15, 45 and 90° , from left to right, respectively. Here $z_0 = 0$ and $w_D = 5 \mu\text{m}$.

$\bar{n}/(2Z_0) (|E_x|^2 + |E_y|^2)$ with Z_0 the vacuum wave impedance, thus ignoring impedance variations for the two polarizations.

FIG. S4. Evolution maps of (a) $|E_y|^2$, (b) $|E_x|^2$ and (c) overall average intensity in the propagation plane xz . The maximum rotation Γ_0 is equal to 0, 15 and 45° from left to right, respectively. The input wave is RCP. Here $w_D = 5 \mu\text{m}$.

D. Dependence on the transverse size of the effective waveguide

FIG. S5. Average intensity evolution in the propagation plane xz for (a) left- and (b) right-handed circularly polarized wavepackets. Here $\Gamma_0 = 45^\circ$ whereas the angular distribution width w_D is 3, 5, 10 and $15 \mu\text{m}$ from left to right, respectively.

Another important feature of the system is its dependence on the transverse extent of the orientation angle distribution. Both circular polarizations are shown in Supplementary Fig. S5. In agreement with theory, the smaller w_D the stronger is the repulsion of the defocused component from the perturbed region. Light spatial localization increases as w_D reduces, with intensity oscillations becoming slower in space as the θ distribution gets wider and wider. In line with Eqs. (S15-S16), light confinement undergoes the same trend as in standard waveguides based on total internal reflection.

E. Dependence on the input point

FIG. S6. Wavepacket evolution in xz for input LCP (a) and RCP (b) when the longitudinal modulation shift Φ is 0, $\pi/2$ and π , from left to right, respectively. Here $\Gamma_0 = 45^\circ$ and $w_D = 5 \mu\text{m}$.

Finally, we studied light propagation as the longitudinal modulation $\sigma(z)$ was shifted, that is, as the phase Φ in $\sigma = \sin \left[\frac{2\pi\Delta n}{\lambda}(z - z_0) + \Phi \right]$ was modified. Supplementary Fig. S6 shows the FDTD results: as predicted (Eqs. (S17-S18)), when the phase is inverted (i.e., $\Phi = \pi$) the two circular polarizations exchange roles, with RCP waves going from trapping to anti-guiding and the opposite for LCP; when $\Phi = \pi/2$, the intensity evolution remains the same regardless the input ellipticity: this agrees with Supplementary Fig. S2 showing quasi-linear polarization at a quarter period.

³⁸ The propagation constant remains wavelength dependent through the vacuum wavevector on the LHS of Eqs. (S15-S16)

³⁹ This contribution is $O(\Lambda^2)$ and was neglected when deriving Eqs. (S7-S8): in the complete expression the term $\sum_{p>1} \left(\int \frac{\partial \beta_p^{(j)}}{\partial x} dz \int \frac{\partial \beta_{-p}^{(j)}}{\partial x} dz \right)$ ($j = e, o$) appears within square brackets and brings in the role of all the fast scales on the slow scale.