H_∞ Consensus of Event-based Multi-agent Systems with Switching Topology

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Abstract

This paper is concerned with the consensus problem for multi-agent systems with switching topology. A novel distributed control strategy is proposed to reduce the frequency of controller update and save network [resources](#). Based on communication uncertainty of practical networks, partly information exchange and switching topology which subjects to a heterogeneous Markov chain are considered in controller design. An H_∞ consensus criterion is derived by [using linear matrix inequality](#) and Lyapunov [methods](#). According to this consensus criterion, a sufficient condition on designing H_∞ state-feedback controller is presented. Finally, a simulation example is given to illustrate the effectiveness of proposed event-based control strategy.

Keywords: Multi-agent [system](#); Consensus; Event-triggered control; Markov switching.

1. Introduction

In recent years, consensus problem of multi-agent systems has been widely studied [due to its broad applications in](#) multivehicle formation control, wireless sensor networks[5, 9, 21] [and so on](#). There are several literatures [4, 10, 11,

13, 14, 15, 34] have focused on consensus problem. Nowadays, two dominant methods, continuous control and periodic sampling control, have been used to solve consensus problem. However, continuous control strategy needs continuous controller update, and in periodic sampling control strategy, the actuation is more frequency than necessary. Therefore, many researchers have suggested that event-triggered control [1, 6, 16, 18, 19, 25, 27, 28] could be a promising technique to reduce controller's update and computation cost because control signals are kept constant until triggering condition is violated. Owing to the above advantages, event-triggered control strategy has been firstly proposed in [2] that could be applied to solve consensus problems of first-order multi-agent systems. Following this thought, an event-based control scheme presented in [22] has released the threshold of event triggering condition to be state-independent, thus continuous monitoring between neighbor agents could be avoided. Ref. [26] solves the consensus problem of both first-order and second-order heterogeneous multi-agent systems with a distributed event-triggered consensus protocol. In [35], two event-based control strategies under continuous and discrete communication among neighbor agents are considered, by which the consensus of general linear multi-agent systems can be achieved. Different from [35], the consensus problem of multi-agent systems is studied in [27] by using observer-based output feedback event-triggered control strategy, where states of agents are unavailable to us and it has been observed by output signals. In [8], a novel distributed event-triggered sampled-data transmission strategy is proposed to achieve consensus of multi-agent systems. By using this method, the consensus of multi-agent systems can be transformed into the stability of a time-delay systems.

In aforementioned work, the communication topology of multi-agent systems is assumed to be fixed. However, in practice, it is difficult to avoid network uncertainty, such as the changes of system components. Thus, the research on consensus of multi-agent systems with switching topology is of great practical significance, and it has been extensively studied in literatures[3, 17, 20, 24, 29, 30, 31, 32, 33]. In [20], consensus problem is studied for multi-agent systems

where agents communicate via a stochastic information network, and the communication topology is governed by a periodic switching graph. Both fixed and switching network topologies are considered in [17], and average consensus of multi-agent systems can be achieved. The consensus problem of first-order multi-agent systems is investigated in [33] with fixed and switching topologies, where the switching topologies are described by a finite-time Markov chain. In [32], the mean square consentability problem is presented for second-order multi-agent systems with Markov switching topology. The formation problem for lipschitz nonlinear multi-agent systems with random jumping process is considered in [24], where the random jumping process is modeled as a Markov chain. The sampled-data leader-following consensus problem is studied in [3] for lipschitz nonlinear systems, where random switching network topologies are modeled by Markov process and communication delay is solved by proposed variable periodic sampling control protocol. In most of the articles referenced above, the dynamics of the agents are limited to be first-order/second-order systems, control strategies are restricted to be continuous control strategies and the Markov switching chain is assumed to be homogeneous. This motivates us to investigate the H_∞ consensus problem for general linear multi-agent systems with heterogeneous Markov switching topology via event-driven controller.

The main contributions of this paper are summarized as follows:

1. A novel event-based control strategy has been proposed. By using the proposed control strategy, a global H_∞ consensus of discrete-time multi-agent systems can be achieved. Moreover, computation cost and actuation update of agents can be reduced.
2. Considering the uncertainty of practical networks, a switching topology which subjects to heterogeneous Markov chain has been presented to describe the communication among agents.
3. Partly information exchange has been considered among neighbor agents, which is more reasonable compared with full information exchange in practical networks.

Notations. $\mathbb{R}^{n \times n}$ means the set of $n \times n$ real matrices. $\mathbb{N}$ denotes the set

of positive integers. $B^\dagger$ denotes generalized inverse of B . $A \otimes B$ denotes the Kronecker product of matrices A and B . I_N represents the identity matrix of dimension N . The superscript $\top$ stands for matrix transposition. $\mathbb{E}\{\cdot\}$ stands for the mathematical expectation. Symbol $\|\cdot\|$ represents Euclidean norm.

2. Problem Statement

The communication topology can be described by a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, where the set of nodes is $\mathcal{V} = \{1, 2, \dots, N\}$, the set of edges is $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. The set of neighbors of agent i is denoted by $\mathcal{N}_i = \{j : (j, i) \in \mathcal{E}\}$. The adjacent matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$ with nonnegative adjacency elements a_{ij} . If $j \in \mathcal{N}_i$, $a_{ij} = 1$, otherwise, $a_{ij} = 0$. The degree matrix is described by $\mathcal{D} = \text{diag}\{\mathcal{D}_1, \mathcal{D}_2, \dots, \mathcal{D}_N\}$ with $\mathcal{D}_i = \sum_{j \in \mathcal{N}_i} a_{ij}$. The matrix $\mathcal{L} = \mathcal{D} - \mathcal{A}$ is Laplacian matrix[7].

The multi-agent systems including N agents are described by the following discrete-time linear system

$$x_i(k+1) = Ax_i(k) + Bu_i(k) + C\nu_i(k), \quad (1)$$

where $x_i(k) \in \mathbb{R}^n$ represents the state value of agent i , $u_i(k) \in \mathbb{R}^m$ denotes the control input, $\nu_i(k)$ is the disturbance of the system, $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$ and $C \in \mathbb{R}^{n \times 1}$ are constant matrices.

The transmission signal received at node i from node j can be described by

$$y_{ij}(k) = E_{ij}\hat{x}_j(k) + G_{ij}\vartheta_{ij}(k), \quad (2)$$

where $\vartheta_{ij}(k)$ is channel noise, and satisfies $\mathbb{E}\{\vartheta_{ij}(k)\} = 0$, $E_{ij} \in \mathbb{R}^{n \times n}$ and $G_{ij} \in \mathbb{R}^{n \times n}$ are known constant matrices with appropriate dimensions. The sequence of event-triggered instants of agent i is denoted by $k_0^i, k_1^i, \dots$. Each k_l^i is defined when the triggering condition is satisfied, for $i \in \mathcal{V}$, $l = 0, 1, 2, \dots$. Define $\hat{x}_j = x_j(k_{l'(k)}^j)$, $k \in [k_l^i, k_{l+1}^i)$, where $l'(k) = \arg \max_{\epsilon \in \mathbb{N}} \{\epsilon | k \geq k_\epsilon^j\}$.

Remark 1. The information transmission between neighbor agents can be given by Eq.(2), which considers partial information exchange among agents

and the channel noise which is produced by signal transmission. If matrix E_{ij} is unfilled rank, there is only partial information of agent j 's state has been transmitted to agent j , otherwise, full information exchange between neighbors will occur. Specially, if there is full information exchange and no channel noise, i.e., E_{ij} is non-singular matrix and $\vartheta_{ij}(k) = 0$, the information transfer from agent j to agent i can be simplified as $y_{ij}(k) = \hat{x}_j(k)$ [2, 22].

The event-triggered transmission scheme is designed as in [18]

$$e_i^\top(k)\Phi(r(k))e_i(k) \leq \varepsilon(r(k))\omega_i^{r(k)\top}(k)\Phi(r(k))\omega_i^{r(k)}(k), \quad (3)$$

where $\omega_i^{r(k)}(k) = \sum_{j \in \mathcal{N}_i^{r(k)}} a_{ij}(x_i(k) - x_j(k))$, $e_i(k) = \hat{x}_i(k) - x_i(k)$ is the state measurement error, $\hat{x}_i(k) = x_i(k_l^i)$, for $k \in [k_l^i, k_{l+1}^i)$. $\varepsilon(r(k)) > 0$ denotes the threshold, $\Phi(r(k)) \in \mathbb{R}^{n \times n} > 0$ represents the weighting matrix.

Remark 2. If $e_i^\top(k)\Phi(r(k))e_i(k) \leq \varepsilon(r(k))\omega_i^{r(k)\top}(k)\Phi(r(k))\omega_i^{r(k)}(k)$ is fulfilled, no event is triggered, then the next event trigger is generated when $e_i^\top(k)\Phi(r(k))e_i(k) > \varepsilon(r(k))\omega_i^{r(k)\top}(k)\Phi(r(k)) \times \omega_i^{r(k)}(k)$ is satisfied. The threshold ε and the weighting matrix Φ are adjustable parameters to achieve desired performance and triggering instants can be determined by the combination of these two parameters. When ε approaches to zero, which means events will be triggered continuously, the event-triggered transmission strategy proposed in this paper can be simplified as continuous control strategy [15, 13].

The consensus control law based on state feedback can be constructed as

$$u_i(k) = K^{r(k)} \sum_{j \in \mathcal{N}_i^{r(k)}} [y_{ij} - E_{ij}\hat{x}_i(k)], \quad (4)$$

where $K^{r(k)} \in \mathbb{R}^{m \times n}$ is the feedback gain matrix.

The switching topology of multi-agent systems is subject to a discrete-time Markov chain $\{r(k), k \geq 0\}$, which takes values in a finite set $S = \{1, 2, \dots, n_0\}$ with the following transition probabilities

$$P\{r(k+1) = t | r(k) = s\} = \pi_{st}(k), \forall s, t \in S, \quad (5)$$

where $\pi_{st}(k) \geq 0$, $\forall s, t \in S$ is the transition rate from s to t and $\sum_{t=1}^{n_0} \pi_{st}(k) = 1$.

Assume that $\Pi(k)$ evolves in a polytope with vertice $\Pi^{(m)}, m = 1, 2, \dots, m_0$. Specifically, define $\Pi(k)$ is of the form

$$\Pi(k) = \sum_{m=1}^{m_0} \alpha_m(k) \Pi^{(m)}, \quad (6)$$

where $\Pi^{(m)} = [\pi_{st}^m], m = 1, 2, \dots, m_0$ are given matrices, $\sum_{m=1}^{m_0} \alpha_m(k) = 1$ and $\alpha_m(k) \geq 0$.

Remark 3. In this paper, the [topology switching of multi-agent systems is subject](#) to a Markov chain (5), which has a time-varying probability transition matrix (6). If the probability transition matrix is time-variant, the Markov chain is said to be heterogeneous, otherwise, the Markov chain is homogeneous [33, 3].

For multi-agent systems, define the performance variable z as

$$z^{r(k)}(k) = \sum_{i=1}^N \sum_{j \in \mathcal{N}_i^{r(k)}} \|\hat{x}_i(k) - \hat{x}_j(k)\|^2, \quad (7)$$

then, one can further obtain

$$\begin{aligned} z^{r(k)}(k) &= 2\hat{x}^\top(k) \bar{\mathcal{L}}(r(k)) \hat{x}(k) \\ &= \hat{x}^\top(k) [\bar{\mathcal{L}}^\top(r(k)) + \bar{\mathcal{L}}(r(k))] \hat{x}(k) \\ &= [x(k) + e(k)]^\top [\bar{\mathcal{L}}^\top(r(k)) + \bar{\mathcal{L}}(r(k))] [x(k) + e(k)] \\ &\leq 2x^\top(k) [\bar{\mathcal{L}}^\top(r(k)) + \bar{\mathcal{L}}(r(k))] x(k) \\ &\quad + 2e^\top(k) [\bar{\mathcal{L}}^\top(r(k)) + \bar{\mathcal{L}}(r(k))] e(k) \\ &= 4x^\top(k) \bar{\mathcal{L}}(r(k)) x(k) + 4e^\top(k) \bar{\mathcal{L}}(r(k)) e(k), \end{aligned} \quad (8)$$

where $\mathcal{L}(r(k))$ is the Laplacian matrix of communication topology, and $\bar{\mathcal{L}}(r(k)) = \mathcal{L}(r(k)) \otimes I_n$, $x(k) = [x_1^\top(k), \dots, x_N^\top(k)]^\top$, $\hat{x}(k) = [\hat{x}_1^\top(k), \dots, \hat{x}_N^\top(k)]^\top$, $e(k) = [e_1^\top(k), \dots, e_N^\top(k)]^\top$.

Definition 1. Given a positive scalar γ , the protocol (4) is said to achieve global consensus with a guaranteed H_∞ performance γ for the agents in (1), if the following two requirements are held:

1. The multi-agent systems (1) with systems disturbance $\nu_i = 0$ and channel

noise $\vartheta_{ij} = 0$ can reach global consensus in the sense of

$$\lim_{k \rightarrow \infty} \|x_i(k) - x_j(k)\| = 0, \forall i, j = 1, 2, \dots, N. \quad (9)$$

2. Under the zero-initial condition, the performance variable z satisfies

$$\mathbb{E}\left\{\sum_{k=0}^{\infty} z^{r(k)}(k)\right\} \leq \frac{1}{\gamma} \left(\sum_{k=0}^{\infty} \|\nu(k)\|^2 + \sum_{k=0}^{\infty} \sum_{i=1}^N \sum_{j \in \mathcal{N}_i^{r(k)}} \mathbb{E}\{\|\vartheta_{ij}(k)\|^2\} \right), \quad (10)$$

where $\nu(k) = [\nu_1^\top(k), \dots, \nu_N^\top(k)]^\top$.

3. Main Results

In this section, a sufficient event-based control strategy for H_∞ consensus of multi-agent systems (1) with Markov switching topology is provided. From (1) and (4), the closed-loop system can be constructed as

$$\begin{aligned} x_i(k+1) &= Ax_i(k) + BK^{r(k)} \sum_{j \in \mathcal{N}_i^{r(k)}} [E_{ij}\hat{x}_j(k) + G_{ij}\vartheta_{ij}(k) \\ &\quad - E_{ij}\hat{x}_i(k)] + C\nu_i(k) \\ &= Ax_i(k) - BK^{r(k)} \sum_{j \in \mathcal{N}_i^{r(k)}} E_{ij}(\hat{x}_i(k) - \hat{x}_j(k)) \\ &\quad + BK^{r(k)} \sum_{j \in \mathcal{N}_i^{r(k)}} G_{ij}\vartheta_{ij}(k) + C\nu_i(k) \\ &= Ax_i(k) - BK^{r(k)} \sum_{j=1}^N E_{ij}\mathcal{L}_{ij}(r(k))\hat{x}_j(k) \\ &\quad + BK^{r(k)} \sum_{j \in \mathcal{N}_i^{r(k)}} G_{ij}\vartheta_{ij}(k) + C\nu_i(k) \\ &= Ax_i(k) - BK^{r(k)} \sum_{j=1}^N E_{ij}\mathcal{L}_{ij}(r(k))x_j(k) \\ &\quad - BK^{r(k)} \sum_{j=1}^N E_{ij}\mathcal{L}_{ij}(r(k))e_j(k) \\ &\quad + BK^{r(k)} \sum_{j \in \mathcal{N}_i^{r(k)}} G_{ij}\vartheta_{ij}(k) + C\nu_i(k), \end{aligned} \quad (11)$$

where $\mathcal{L}(r(k)) = [\mathcal{L}_{ij}(r(k))] \in \mathbb{R}^{N \times N}$ is the Laplacian matrix of communication topology.

Denote

$$\Psi^{r(k)} = \begin{bmatrix} \mathcal{L}_{11}(r(k))E_{11} & \mathcal{L}_{12}(r(k))E_{12} & \cdots & \mathcal{L}_{1N}(r(k))E_{1N} \\ \mathcal{L}_{21}(r(k))E_{21} & \mathcal{L}_{22}(r(k))E_{22} & \cdots & \mathcal{L}_{2N}(r(k))E_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{L}_{N1}(r(k))E_{N1} & \mathcal{L}_{N2}(r(k))E_{N2} & \cdots & \mathcal{L}_{NN}(r(k))E_{NN} \end{bmatrix}, \quad (12)$$

and $\theta^{r(k)} = [\theta_1^{r(k)\top}, \dots, \theta_N^{r(k)\top}]^\top$, where $\theta_i^{r(k)} = BK^{r(k)} \sum_{j \in \mathcal{N}_i^{r(k)}} G_{ij} \vartheta_{ij}(k)$.

Therefore, Eq. (11) can be rewritten in compact form as

$$\begin{aligned} x(k+1) &= \left[I_N \otimes A - \Psi^{r(k)} (I_N \otimes (BK^{r(k)})) \right] x(k) \\ &\quad - \Psi^{r(k)} \left(I_N \otimes (BK^{r(k)}) \right) e(k) \\ &\quad + \theta^{r(k)} + (I_N \otimes C) \nu(k), \end{aligned} \quad (13)$$

which can be converted to a simplified form

$$x(k+1) = \mathcal{H}^{r(k)} \zeta(k) + \theta^{r(k)}, \quad (14)$$

where $\mathcal{H}^{r(k)} = [I_N \otimes A - \Psi^{r(k)} (I_N \otimes (BK^{r(k)})) , -\Psi^{r(k)} (I_N \otimes (BK^{r(k)})) , I_N \otimes C]$, $\zeta(k) = [x^\top(k), e^\top(k), \nu^\top(k)]^\top$.

3.1. Consensus Analysis

In the following, we first establish an H_∞ consensus criterion for multi-agent systems (1) with heterogeneous switching topology.

Theorem 1. Assume that the communication topologies are described by directed graphs. Given some constants $\gamma > 0$, $\varepsilon(s) > 0$ and matrices $K^s > 0$ ($s = 1, 2, \dots, n_0$). Consider multi-agent systems (1) with controller (4), under the event-triggered transmission scheme (3), the overall systems can achieve global consensus with an H_∞ performance γ , if there exist matrices $P(s) > 0$ and $\Phi(s) > 0$, such that

$$\begin{bmatrix} -P^m(s) & P^m(s) \mathcal{L}_{ii}^n(s) BK^s G_{ij} \\ G_{ij}^\top K^s B^\top \mathcal{L}_{ii}^l(s) P^m(s) & -I_n \end{bmatrix} < 0, \quad (15)$$

and

$$\begin{bmatrix} M^s & \mathcal{H}^{s\top} \bar{\mathcal{L}}^{l\top}(s) \bar{P}^m(s) \\ \bar{P}^m(s) \bar{\mathcal{L}}^n(s) \mathcal{H}^s & -\bar{P}^m(s) \end{bmatrix} < 0, \quad (16)$$

$$s = 1, 2, \dots, n_0; \quad l, m, n = 1, 2, \dots, m_0,$$

where $P^m(s) = \sum_{t=1}^{n_0} \pi_{st}^m P(t)$, $\bar{P}^m(s) = I_N \otimes P^m(s)$, $\mathcal{L}^r(s) = \sum_{t=1}^{n_0} \pi_{st}^r \mathcal{L}(t)$, $\bar{\mathcal{L}}^r(s) = \mathcal{L}^r(s) \otimes I_n$, for $r = l, n$. $M^s = \text{diag}\{\bar{\mathcal{L}}^\top(s) (\varepsilon(s) \bar{\Phi}(s) - \bar{P}(s)) \bar{\mathcal{L}}(s) + 4\gamma \bar{\mathcal{L}}(s), -\bar{\Phi}(s) + 4\gamma \bar{\mathcal{L}}(s), -I_N\}$, $\bar{\Phi}(s) = I_N \otimes \Phi(s)$, $\bar{\mathcal{L}}(s) = \mathcal{L}(s) \otimes I_n$, $\bar{P}(s) = I_N \otimes P(s)$.

Proof. Consider Lyapunov function for closed-loop systems (11) as

$$V(k) = \omega^{r(k)\top}(k) \bar{P}(r(k)) \omega^{r(k)}(k), \quad (17)$$

where $\omega^{r(k)}(k) = \bar{\mathcal{L}}(r(k))x(k)$. Calculating the difference of Lyapunov function along the system and taking the mathematical expectation, we have

$$\begin{aligned} \Delta V &= \mathbb{E} \left\{ V(k+1) | \omega^{r(k)}(k), r(k) = s \right\} - V(k) \\ &= \mathbb{E} \left\{ \omega^{s+1\top}(k+1) \bar{P}(r(k+1)) \omega^{s+1}(k+1) \right\} - \omega^{s\top}(k) \bar{P}(s) \omega^s(k) \\ &= \zeta^\top(k) \mathcal{H}^{s\top} \bar{\mathcal{L}}^\top((r(k+1)) \bar{P}(r(k+1)) \bar{\mathcal{L}}(r(k+1)) \mathcal{H}^s \zeta(k) \\ &\quad + \mathbb{E} \left\{ \theta^{s\top} \bar{\mathcal{L}}^\top(r(k+1)) \bar{P}(r(k+1)) \bar{\mathcal{L}}(r(k+1)) \theta^s \right\} \\ &\quad - x^\top(k) \bar{\mathcal{L}}^\top(s) \bar{P}(s) \bar{\mathcal{L}}(s) x(k), \end{aligned} \quad (18)$$

where

$$\begin{aligned} \bar{P}(r(k+1)) &= \sum_{t=1}^{n_0} \pi_{st}(k) \bar{P}(t) = \sum_{t=1}^{n_0} \sum_{m=1}^{m_0} \alpha_m(k) \pi_{st}^m \bar{P}(t) \\ &= \sum_{m=1}^{m_0} \alpha_m(k) \bar{P}^m(s). \end{aligned} \quad (19)$$

By using the same method, one can obtain $\bar{\mathcal{L}}(r(k+1)) = \sum_{r=1}^{m_0} \alpha_r(k) \bar{\mathcal{L}}^r(s)$, $r = l, n$. Then, it follows from (18) that

$$\begin{aligned} \Delta V &= \zeta^\top(k) \mathcal{H}^{s\top} \sum_{l=1}^{m_0} \alpha_l(k) \bar{\mathcal{L}}^{l\top}(s) \sum_{m=1}^{m_0} \alpha_m(k) \bar{P}^m(s) \sum_{n=1}^{m_0} \alpha_n(k) \bar{\mathcal{L}}^n(s) \mathcal{H}^s \zeta(k) \\ &\quad + \mathbb{E} \left\{ \theta^{s\top} \sum_{l=1}^{m_0} \alpha_l(k) \bar{\mathcal{L}}^{l\top}(s) \sum_{m=1}^{m_0} \alpha_m(k) \bar{P}^m(s) \sum_{n=1}^{m_0} \alpha_n(k) \bar{\mathcal{L}}^n(s) \theta^s \right\} \\ &\quad - x^\top(k) \bar{\mathcal{L}}^\top(s) \bar{P}(s) \bar{\mathcal{L}}(s) x(k). \end{aligned} \quad (20)$$

According to Schur complement and LMI (15), the second term of (20) can be upper bounded by

$$\begin{aligned}
& \mathbb{E} \left\{ \theta^{s\top} \sum_{l=1}^{m_0} \alpha_l(k) \bar{\mathcal{L}}^{l\top}(s) \sum_{m=1}^{m_0} \alpha_m(k) \bar{P}^m(s) \sum_{n=1}^{m_0} \alpha_n(k) \bar{\mathcal{L}}^n(s) \theta^s \right\} \\
&= \sum_{l=1}^{m_0} \alpha_l(k) \sum_{m=1}^{m_0} \alpha_m(k) \sum_{n=1}^{m_0} \alpha_n(k) \mathbb{E} \{ \theta^{s\top} \bar{\mathcal{L}}^{l\top}(s) \bar{P}^m(s) \bar{\mathcal{L}}^n(s) \theta^s \} \\
&= \sum_{l=1}^{m_0} \alpha_l(k) \sum_{m=1}^{m_0} \alpha_m(k) \sum_{n=1}^{m_0} \alpha_n(k) \sum_{i=1}^N \mathbb{E} \{ \theta_i^{s\top} \mathcal{L}_{ii}^l(s) P^m(s) \mathcal{L}_{ii}^n(s) \theta_i^s \} \\
&\leq \sum_{l=1}^{m_0} \alpha_l(k) \sum_{m=1}^{m_0} \alpha_m(k) \sum_{n=1}^{m_0} \alpha_n(k) \sum_{i=1}^N \sum_{j \in \mathcal{N}_i^s} \mathbb{E} \{ \vartheta_{ij}^\top(k) G_{ij}^\top K^{s\top} B^\top \mathcal{L}_{ii}^l(s) \\
&\quad \times P^m(s) \mathcal{L}_{ii}^n(s) B K^s G_{ij} \vartheta_{ij}(k) \} \\
&\leq \sum_{l=1}^{m_0} \alpha_l(k) \sum_{m=1}^{m_0} \alpha_m(k) \sum_{n=1}^{m_0} \alpha_n(k) \sum_{i=1}^N \sum_{j \in \mathcal{N}_i^s} \mathbb{E} \{ \|\vartheta_{ij}(k)\|^2 \},
\end{aligned} \tag{21}$$

for $\sum_{m=1}^{m_0} \alpha_m(k) = 1$, one has

$$\begin{aligned}
& \mathbb{E} \left\{ \theta^{s\top} \sum_{l=1}^{m_0} \alpha_l(k) \bar{\mathcal{L}}^{l\top}(s) \sum_{m=1}^{m_0} \alpha_m(k) \bar{P}^m(s) \sum_{n=1}^{m_0} \alpha_n(k) \bar{\mathcal{L}}^n(s) \theta^s \right\} \\
&\leq \sum_{i=1}^N \sum_{j \in \mathcal{N}_i^s} \mathbb{E} \{ \|\vartheta_{ij}(k)\|^2 \}.
\end{aligned} \tag{22}$$

In light of (22), it follows from (20) that

$$\begin{aligned}
\Delta V &\leq \zeta^\top(k) \mathcal{H}^{s\top} \sum_{l=1}^{m_0} \alpha_l(k) \bar{\mathcal{L}}^{l\top}(s) \sum_{m=1}^{m_0} \alpha_m(k) \bar{P}^m(s) \sum_{n=1}^{m_0} \alpha_n(k) \bar{\mathcal{L}}^n(s) \mathcal{H}^s \zeta(k) \\
&\quad - x^\top(k) \bar{\mathcal{L}}^\top(s) \bar{P}(s) \bar{\mathcal{L}}(s) x(k) + \sum_{i=1}^N \sum_{j \in \mathcal{N}_i^s} \mathbb{E} \{ \|\vartheta_{ij}(k)\|^2 \}.
\end{aligned} \tag{23}$$

From the event-triggered transmission scheme (3), it is clear that for $k \in \{k_l^i, k_l^{i+1}\}$, we obtain

$$e^\top(k) \bar{\Phi}(s) e(k) \leq \varepsilon(s) \omega^{s\top} \bar{\Phi}(s) \omega^s = \varepsilon(s) x^\top(k) \bar{\mathcal{L}}^\top(s) \bar{\Phi}(s) \bar{\mathcal{L}}(s) x(k). \tag{24}$$

Hence, from (23) and (24), it follows that

$$\begin{aligned}
\Delta V &\leq \sum_{l=1}^{m_0} \alpha_l(k) \sum_{m=1}^{m_0} \alpha_m(k) \sum_{n=1}^{m_0} \alpha_n(k) \zeta^\top(k) \mathcal{H}^{s\top} \bar{\mathcal{L}}^{l\top}(s) \bar{P}^m(s) \bar{\mathcal{L}}^n(s) \mathcal{H}^s \zeta(k) \\
&\quad - x^\top(k) \bar{\mathcal{L}}^\top(s) \bar{P}(s) \bar{\mathcal{L}}(s) x(k) + \sum_{i=1}^N \sum_{j \in \mathcal{N}_i^s} \mathbb{E} \{ \|\vartheta_{ij}(k)\|^2 \} \\
&\quad + \varepsilon(s) x^\top(k) \bar{\mathcal{L}}^\top(s) \bar{\Phi}(s) \bar{\mathcal{L}}(s) x(k) - e^\top(k) \bar{\Phi}(s) e(k) \\
&= \sum_{l=1}^{m_0} \alpha_l(k) \sum_{m=1}^{m_0} \alpha_m(k) \sum_{n=1}^{m_0} \alpha_n(k) \zeta^\top(k) \left\{ \mathcal{H}^{s\top} \bar{\mathcal{L}}^{l\top}(s) \bar{P}^m(s) \bar{\mathcal{L}}^n(s) \mathcal{H}^s \right. \\
&\quad \left. + M^s \right\} \zeta(k) + \sum_{i=1}^N \sum_{j \in \mathcal{N}_i^s} \mathbb{E} \{ \|\vartheta_{ij}(k)\|^2 \} - \gamma z^s + \nu^\top(k) \nu(k).
\end{aligned} \tag{25}$$

Let

$$\lambda_0 = \max \left\{ \lambda_{\max} \left\{ \mathcal{H}^{s\top} \bar{\mathcal{L}}^{l\top}(s) \bar{P}^m(s) \bar{\mathcal{L}}^n(s) \mathcal{H}^s + M^s \right\} \right\}, \tag{26}$$

then by applying Schur complement to (16), one can readily obtain $\lambda_0 < 0$.

Consider $\sum_{m=1}^{m_0} \alpha_m(k) = 1$, from (25), we have

$$\begin{aligned}
&\mathbb{E} \{ V(k+1) \} - V(k) + \gamma z^s - \nu^\top(k) \nu(k) \\
&\leq \sum_{l=1}^{m_0} \alpha_l(k) \sum_{m=1}^{m_0} \alpha_m(k) \sum_{n=1}^{m_0} \alpha_n(k) \lambda_0 \|\zeta(k)\|^2 + \sum_{i=1}^N \sum_{j \in \mathcal{N}_i^s} \mathbb{E} \{ \|\vartheta_{ij}(k)\|^2 \} \\
&\leq \lambda_0 \|\zeta(k)\|^2 + \sum_{i=1}^N \sum_{j \in \mathcal{N}_i^s} \mathbb{E} \{ \|\vartheta_{ij}(k)\|^2 \}
\end{aligned} \tag{27}$$

Assuming that system disturbance ν_i and channel noise ϑ_{ij} are zero, Eq.(27) can be simplified as

$$\mathbb{E} \{ V(k+1) \} - V(k) \leq -\gamma z^s(k). \tag{28}$$

Therefore, it follows from (28) that $\Delta V \leq 0$, implying that $\|x_i(k) - x_j(k)\| \rightarrow 0$, as $k \rightarrow \infty$. That is, the first condition in Definition 1 holds.

On the other hand, it follows from (27) that

$$\mathbb{E} \{ V(k+1) \} - V(k) + \gamma z^s(k) - \nu^\top(k) \nu(k) \leq \sum_{i=1}^N \sum_{j \in \mathcal{N}_i^s} \mathbb{E} \{ \|\vartheta_{ij}(k)\|^2 \}. \tag{29}$$

Summing up both sides of (29) over time period $[0, +\infty)$, one can obtain

$$\begin{aligned} & \mathbb{E}\{V(\infty)\} - \mathbb{E}\{V(0)\} + \gamma \mathbb{E}\left\{\sum_{k=0}^{\infty} z^s(k)\right\} - \sum_{k=0}^{\infty} \|\nu(k)\|^2 \\ & \leq \sum_{k=0}^{\infty} \sum_{i=1}^N \sum_{j \in \mathcal{N}_i^s} \mathbb{E}\{\|\vartheta_{ij}(k)\|^2\}. \end{aligned} \quad (30)$$

Since $x(0) = 0$, it is clear that $V(0) = 0$. Consider $\mathbb{E}\{V(\infty)\} > 0$, we have

$$\mathbb{E}\left\{\sum_{k=0}^{\infty} z^s\right\} \leq \frac{1}{\gamma} \left(\sum_{k=0}^{\infty} \|\nu(k)\|^2 + \sum_{k=0}^{\infty} \sum_{i=1}^N \sum_{j \in \mathcal{N}_i^s} \mathbb{E}\{\|\vartheta_{ij}(k)\|^2\} \right). \quad (31)$$

Therefore the global H_{∞} consensus problem of multi-agent systems (1) is solved.

Notice that in Theorem 1, the switching process of communication topology is subject to a Markov chain. If communication topology is fixed, it is a special case of the system considered in Theorem 1, and we have the following corollary.

Corollary 1. Assume that the communication graph is directed. Given some constants $\gamma > 0$, $\varepsilon > 0$ and a matrix $K > 0$. Consider multi-agent systems (1) with the controller

$$u_i(k) = K \sum_{j \in \mathcal{N}_i} [y_{ij} - E_{ij} \hat{x}_i(k)], \quad (32)$$

under the event-triggered transmission scheme

$$e_i^{\top}(k) \Phi e_i(k) \leq \varepsilon \omega_i^{\top}(k) \Phi \omega_i(k), \quad (33)$$

the overall system can achieve global consensus with [an](#) H_{∞} performance γ if there exist matrices $P > 0$ and $\Phi > 0$, such that

$$\begin{bmatrix} -P & * \\ G_{ij}^{\top} K^{\top} B^{\top} \mathcal{L}_{ii} P & -I_n \end{bmatrix} < 0, \quad (34)$$

and

$$\begin{bmatrix} M & * \\ \bar{P} \bar{\mathcal{L}} \mathcal{H} & -\bar{P} \end{bmatrix} < 0 \quad (35)$$

where $\omega_i(k) = \sum_{j \in \mathcal{N}_i} a_{ij}(x_i(k) - x_j(k))$, $M = \text{diag}\{\bar{\mathcal{L}}^{\top}(\varepsilon \bar{\Phi} - \bar{P}) \bar{\mathcal{L}} + 4\gamma \bar{\mathcal{L}}, -\bar{\Phi} + 4\gamma \bar{\mathcal{L}}, -I_N\}$, $\bar{P} = I_N \otimes P$, $\bar{\mathcal{L}} = I_N \otimes \mathcal{L}$, $\varepsilon > 0$ is the threshold, $\Phi > 0$ is the weighting matrix and $\bar{\Phi} = I_N \otimes \Phi$.

Proof. From (1) and (32), the closed-loop systems can be given by

$$\begin{aligned} x_i(k+1) = & Ax_i(k) - BK \sum_{j=1}^N E_{ij} \mathcal{L}_{ij} x_j(k) - BK \sum_{j=1}^N E_{ij} \mathcal{L}_{ij} e_j(k) \\ & + BK \sum_{j=1}^N G_{ij} \vartheta_{ij} + C \nu_i(k), \end{aligned} \quad (36)$$

where $\mathcal{L} = [\mathcal{L}_{ij}] \in \mathbb{R}^{N \times N}$.

Denote

$$\Psi = \begin{bmatrix} \mathcal{L}_{11} E_{11} & \mathcal{L}_{12} E_{12} & \cdots & \mathcal{L}_{1N} E_{1N} \\ \mathcal{L}_{21} E_{21} & \mathcal{L}_{22} E_{22} & \cdots & \mathcal{L}_{2N} E_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{L}_{N1} E_{N1} & \mathcal{L}_{N2} E_{N2} & \cdots & \mathcal{L}_{NN} E_{NN} \end{bmatrix}, \quad (37)$$

Eq.(36) can be written in a compact form as

$$x(k+1) = [I_N \otimes A - \Psi(I_N \otimes (BK))]x(k) - \Psi(I_N \otimes (BK))e(k) + (I_N \otimes C)\nu(k) + \theta, \quad (38)$$

where $\theta = [\theta_1^\top, \dots, \theta_N^\top]^\top$ and $\theta_i = BK \sum_{j \in \mathcal{N}_i} G_{ij} \vartheta_{ij}(k)$. Define $\mathcal{H} = [I_N \otimes A - \Psi(I_N \otimes (BK)), -\Psi(I_N \otimes (BK)), I_N \otimes C]$, Eq.(38) can be abbreviated to

$$x(k+1) = \mathcal{H}(k)\zeta(k) + \theta, \quad (39)$$

where $\zeta(k)$ is defined below (14). The performance variable is defined by

$$z(k) = \sum_{i=1}^N \sum_{j \in \mathcal{N}_i} \|\hat{x}_i(k) - \hat{x}_j(k)\|^2. \quad (40)$$

Choose a Lyapunov function as

$$V(k) = \omega^\top(k+1) \bar{P} \omega(k+1), \quad (41)$$

where $\omega(k) = \mathcal{L} \otimes I_n$. Then, the difference of Lyapunov function is given by

$$\begin{aligned} \Delta V = & \mathbb{E}\{V(k+1)|\omega(k)\} - V(k) \\ = & \zeta^\top k \mathcal{H}^\top \bar{\mathcal{L}}^\top \bar{P} \bar{\mathcal{L}} \mathcal{H} \zeta(k) + \mathbb{E}\{\theta^\top \bar{\mathcal{L}}^\top \bar{P} \bar{\mathcal{L}} \theta\} - x^\top(k) \bar{\mathcal{L}}^\top \bar{P} \bar{\mathcal{L}} x(k). \end{aligned} \quad (42)$$

By using Schur complement to (34), the second term of (42) satisfies

$$\mathbb{E}\{\theta^\top \bar{\mathcal{L}}^\top \bar{P} \bar{\mathcal{L}} \theta\} \leq \sum_{i=1}^N \sum_{j \in \mathcal{N}_i} \mathbb{E}\{\|\vartheta_{ij}(k)\|^2\}. \quad (43)$$

From (42) and (43), and according to event-triggered transmission scheme (33), it follows that

$$\begin{aligned} \Delta V \leq & \zeta^\top(k) \mathcal{H}^\top \bar{\mathcal{L}}^\top \bar{P} \bar{\mathcal{L}} \mathcal{H} \zeta(k) - x^\top(k) \bar{\mathcal{L}}^\top \bar{P} \bar{\mathcal{L}} x(k) + \sum_{i=1}^N \sum_{j \in \mathcal{N}_i} \mathbb{E} \{ \|\vartheta_{ij}(k)\|^2 \} \\ & + \varepsilon x^\top(k) \bar{\mathcal{L}}^\top \bar{\Phi} \bar{\mathcal{L}} x(k) - e^\top(k) \bar{\Phi} e(k). \end{aligned} \quad (44)$$

In the view of Schur complement equivalence, linear matrix inequality (35) can be rewritten as

$$\mathcal{H}^\top \bar{\mathcal{L}}^\top \bar{P} \bar{\mathcal{L}} \mathcal{H} + \text{diag} \{ \bar{\mathcal{L}}^\top (\varepsilon \bar{\Phi} - \bar{P}) \bar{\mathcal{L}} + 4\gamma \bar{\mathcal{L}}, -\bar{\Phi} + 4\gamma \bar{\mathcal{L}}, -I_N \} < 0. \quad (45)$$

Denote $\tilde{\lambda}_0 = \lambda_{\max} \{ \mathcal{H}^\top \bar{\mathcal{L}}^\top \bar{P} \bar{\mathcal{L}} \mathcal{H} + \text{diag} \{ \bar{\mathcal{L}}^\top (\varepsilon \bar{\Phi} - \bar{P}) \bar{\mathcal{L}} + 4\gamma \bar{\mathcal{L}}, -\bar{\Phi} + 4\gamma \bar{\mathcal{L}}, -I_N \} \}$, it is clear that $\tilde{\lambda}_0 < 0$. Then, it follows from (44) that

$$\begin{aligned} & \mathbb{E} \{ V(k+1) \} - V(k) + \gamma z(k) - \nu^\top(k) \nu(k) \\ & \leq \tilde{\lambda}_0 \|\zeta(k)\|^2 + \sum_{i=1}^N \sum_{j \in \mathcal{N}_i} \mathbb{E} \{ \|\vartheta_{ij}(k)\|^2 \} \\ & \leq \sum_{i=1}^N \sum_{j \in \mathcal{N}_i} \mathbb{E} \{ \|\vartheta_{ij}(k)\|^2 \}. \end{aligned} \quad (46)$$

Regardless of disturbance and channel noise, i.e., $\nu_i = 0$, $\vartheta_{ij} = 0$, it is obvious that

$$\mathbb{E} \{ V(k+1) \} - V(k) \leq -\gamma z(k). \quad (47)$$

Therefore, one can obtain that $\Delta V < 0$. It shows that the first condition in Definition 1 is satisfied.

Then, summing (46) over time period $[0, +\infty)$, one has

$$\begin{aligned} & \mathbb{E} \{ V(\infty) \} - \mathbb{E} \{ V(0) \} + \gamma \mathbb{E} \left\{ \sum_{k=0}^{\infty} z(k) \right\} - \sum_{k=0}^{\infty} \|\nu(k)\|^2 \\ & \leq \sum_{k=0}^{\infty} \sum_{i=1}^N \sum_{j \in \mathcal{N}_i} \mathbb{E} \{ \|\vartheta_{ij}(k)\|^2 \}. \end{aligned} \quad (48)$$

Under zero initial conditions, one has $V(0) = 0$. Consider $V(\infty) > 0$, Eq.(48) guarantees

$$\mathbb{E} \left\{ \sum_{k=0}^{\infty} z(k) \right\} \leq \frac{1}{\gamma} \left(\sum_{k=0}^{\infty} \|\nu(k)\|^2 + \sum_{k=0}^{\infty} \sum_{i=1}^N \sum_{j \in \mathcal{N}_i} \mathbb{E} \{ \|\vartheta_{ij}(k)\|^2 \} \right). \quad (49)$$

This completes the proof.

Based on Theorem 1, a sufficient condition on existence of the state feedback controller is given in the next section.

3.2. Controller Design

In this section, an explicit calculation has been given for designing the controller gain K^s ($s = 1, 2, \dots, n_0$) in (4). Then, the main result of controller design has been given in Theorem 2.

Theorem 2. Assume that the communication topologies are described by directed graph. Given some constants $\gamma > 0$, $\varepsilon(s) > 0$ ($s = 1, 2, \dots, n_0$). Consider multi-agent systems (1) and controller (4) with the state feedback controller gain $K^s = B^\dagger \tilde{P}^{-1}(s)Q^s$, under the event-triggered transmission scheme (3), the overall systems can achieve global consensus with [an](#) H_∞ performance γ , if there exist matrices $P(s) > 0$, $Q^s > 0$ and $\Phi(s) > 0$, such that

$$\begin{bmatrix} -\tilde{P}(s) & * \\ G_{ij}^\top Q^{s\top} \tilde{\mathcal{L}}_{ii}(s) & -I_n \end{bmatrix} < 0, \quad (50)$$

and

$$\begin{bmatrix} M^s & * \\ (\tilde{\mathcal{L}}(s) \otimes I_n) \tilde{\mathcal{H}}_1^{s\top} & -I_N \otimes \tilde{P}(s) \end{bmatrix} < 0, \quad (51)$$

$$s = 1, 2, \dots, n_0,$$

where M^s is defined below (16), $\tilde{\mathcal{H}}_1^s = [I_N \otimes (\tilde{P}(s)A) - \Psi^s(I_N \otimes Q^s), -\Psi^s(I_N \otimes Q^s), I_N \otimes (\tilde{P}(s)C)]$, $\tilde{P}(s) = \sum_{t=1}^{n_0} \bar{\pi}_{st} P(t)$, $\tilde{\mathcal{L}}(s) = \sum_{t=1}^{n_0} \bar{\pi}_{st} \mathcal{L}(t)$, and $\bar{\pi}_{st} = \max\{\pi_{st}^m\}$, $m = 1, 2, \dots, m_0$.

Proof. Denote $\tilde{P}(s)BK^s = Q^s$, the closed-loop multi-agent systems (11) can be rewritten as

$$x(k+1) = \tilde{\mathcal{H}}^s \zeta(k) + \tilde{\theta}^s, \quad (52)$$

where $\tilde{\mathcal{H}}^s = [I_N \otimes A - \tilde{\Psi}^s(I_N \otimes (\tilde{P}^{-1}(s)Q^s)), -\tilde{\Psi}^s(I_N \otimes (\tilde{P}^{-1}(s)Q^s)), I_N \otimes C]$, $\tilde{\theta}^s = \tilde{P}^{-1}(s)Q^s \times \sum_{j \in \mathcal{N}_i^s} G_{ij} \vartheta_{ij}(k)$, $s = 1, 2, \dots, n_0$.

For $P^m(s) \leq \tilde{P}(s)$, $\mathcal{L}^r(s) \leq \tilde{\mathcal{L}}(s)$, $r = l, n$, following (18), the difference of Lyapunov function can be bounded by

$$\begin{aligned} \Delta V &= \mathbb{E}\{V(k+1)|\omega^{r(k)}(k), r(k) = s\} - V(k) \\ &\leq \zeta^\top(k) \tilde{\mathcal{H}}^{s^\top} (\tilde{\mathcal{L}}^\top(s) \otimes I_n) (I_N \otimes \tilde{P}(s)) (\tilde{\mathcal{L}}(s) \otimes I_n) \tilde{\mathcal{H}}^s \zeta(k) \\ &\quad + \mathbb{E}\left\{\tilde{\theta}^{s^\top} (\tilde{\mathcal{L}}^\top(s) \otimes I_n) (I_N \otimes \tilde{P}(s)) (\tilde{\mathcal{L}}(s) \otimes I_n) \tilde{\theta}^s\right\} \\ &\quad - x^\top(k) \tilde{\mathcal{L}}^\top(s) \tilde{P}(s) \tilde{\mathcal{L}}(s) x(k), \end{aligned} \quad (53)$$

according to LMI (50), the second term of (53) is bounded by

$$\begin{aligned} &\mathbb{E}\left\{\tilde{\theta}^{s^\top} (\tilde{\mathcal{L}}^\top(s) \otimes I_n) (I_N \otimes \tilde{P}(s)) (\tilde{\mathcal{L}}(s) \otimes I_n) \tilde{\theta}^s\right\} \\ &\leq \sum_{i=1}^N \sum_{j \in \mathcal{N}_i^s} \mathbb{E}\left\{\vartheta_{ij}(k) G_{ij}^\top Q^{s^\top} \tilde{\mathcal{L}}_{ii}(s) \tilde{P}^{-1}(s) \tilde{\mathcal{L}}_{ii}(s) Q^s G_{ij} \vartheta_{ij}(k)\right\} \\ &\leq \sum_{i=1}^N \sum_{j \in \mathcal{N}_i^s} \mathbb{E}\{\|\vartheta_{ij}(k)\|^2\}, \end{aligned} \quad (54)$$

thus, one has

$$\begin{aligned} \Delta V &\leq \zeta^\top(k) \tilde{\mathcal{H}}^{s^\top} (\tilde{\mathcal{L}}^\top(s) \otimes I_n) (I_N \otimes \tilde{P}(s)) (\tilde{\mathcal{L}}(s) \otimes I_n) \tilde{\mathcal{H}}^s \zeta(k) \\ &\quad + \sum_{i=1}^N \sum_{j \in \mathcal{N}_i^s} \mathbb{E}\{\|\vartheta_{ij}(k)\|^2\} - x^\top(k) \tilde{\mathcal{L}}^\top(s) \tilde{P}(s) \tilde{\mathcal{L}}(s) x(k). \end{aligned} \quad (55)$$

By using Schur complement, LMI (51) can be written as

$$\tilde{\mathcal{H}}_1^{s^\top} (\tilde{\mathcal{L}}^\top(s) \otimes I_n) (I_N \otimes \tilde{P}^{-1}(s)) (\tilde{\mathcal{L}}(s) \otimes I_n) \tilde{\mathcal{H}}_1^s + M^s < 0. \quad (56)$$

Considering event-triggered transmission scheme (3), from (55) and (56), one has

$$\begin{aligned} &\mathbb{E}\{V(k+1)\} - V(k) + \gamma z^s - \nu^\top(k) \nu(k) \\ &\leq \zeta^\top(k) \left\{ \tilde{\mathcal{H}}_1^{s^\top} (\tilde{\mathcal{L}}^\top(s) \otimes I_n) (I_N \otimes \tilde{P}^{-1}(s)) (\tilde{\mathcal{L}}(s) \otimes I_n) \tilde{\mathcal{H}}_1^s + M^s \right\} \zeta(k) \\ &\quad + \sum_{i=1}^N \sum_{j \in \mathcal{N}_i^s} \mathbb{E}\{\|\vartheta_{ij}(k)\|^2\} \\ &\leq \sum_{i=1}^N \sum_{j \in \mathcal{N}_i^s} \mathbb{E}\{\|\vartheta_{ij}(k)\|^2\}. \end{aligned} \quad (57)$$

Then, similar to the proof in Theorem 1, one can conclude that the multi-agent systems can achieve global consensus with a guaranteed H_∞ performance γ . This completes the proof.

Remark 4. In Theorem 2, γ is a known prescribed constant. However, in many cases, we need to find an optimal γ . This problem can be converted into the following optimization problem:

$$\begin{aligned} \min_{\gamma > 0, P(s) > 0, Q^s} \quad & -\gamma \\ \text{subject to} \quad & \text{Eq.(50) and (51).} \end{aligned} \quad (58)$$

In the following corollary, a state feedback controller is designed for achieving global consensus of multi-agent systems with fixed communication topology. Thus, it can be seen that the systems in Corollary 2 is just a special case of the systems in Theorem 2.

Corollary 2. Assume that the communication graph is directed. Given some constants $\gamma > 0$ and $\varepsilon > 0$. Consider multi-agent systems (1) and state feedback controller (32) with controller gain $K = B^\dagger P^{-1}Q$, under the event-triggered transmission scheme (33), the overall systems can achieve global H_∞ consensus with a performance γ , if there exist matrices $P > 0$, $Q > 0$ and $\Phi > 0$, such that

$$\begin{bmatrix} -P & * \\ G_{ij}^\top Q^\top \mathcal{L}_{ii} & -I_n \end{bmatrix} < 0, \quad (59)$$

and

$$\begin{bmatrix} \Xi & * & * & * \\ 0 & -\bar{\Phi} + 4\gamma\bar{\mathcal{L}} & * & * \\ 0 & 0 & -I_N & * \\ \mathcal{L} \otimes (PA) - \bar{\mathcal{L}}\Psi(I_N \otimes Q) & -\bar{\mathcal{L}}\Psi(I_N \otimes Q) & \mathcal{L} \otimes PC & -I_N \otimes P \end{bmatrix} < 0, \quad (60)$$

where $\Xi = \bar{\mathcal{L}}^\top (\varepsilon\bar{\Phi} - \bar{P})\bar{\mathcal{L}} + 4\gamma\bar{\mathcal{L}}$, $\bar{\mathcal{L}}$, $\bar{P}$ and Ψ are defined in Corollary 1.

Proof. Consider $PBK = Q$, Eq. (42) can be rewritten as

$$\Delta V \leq \zeta^\top(k) \tilde{\mathcal{H}}^\top \bar{\mathcal{L}}^\top \bar{P} \tilde{\mathcal{L}} \tilde{\mathcal{H}} \zeta(k) + \mathbb{E}\{\tilde{\theta}^\top \bar{\mathcal{L}} \bar{P} \tilde{\mathcal{L}} \tilde{\theta}\} - x^\top(k) \bar{\mathcal{L}}^\top \bar{P} \bar{\mathcal{L}} x(k), \quad (61)$$

where $\tilde{\mathcal{H}} = [I_N \otimes A - \Psi(I_N \otimes (P^{-1}Q)), -\Psi(I_N \otimes (P^{-1}Q)), I_N \otimes C]$, $\tilde{\theta} = P^{-1}Q \sum_{j \in \mathcal{N}_i} G_{ij} \vartheta_{ij}(k)$. Then, applying Schur complement to (59), the second term of (61) can be presented as

$$\begin{aligned} \mathbb{E}\{\tilde{\theta}^\top \bar{\mathcal{L}}^\top \bar{P} \bar{\mathcal{L}} \tilde{\theta}\} &\leq \sum_{i=1}^N \sum_{j \in \mathcal{N}_i} \mathbb{E}\{\vartheta_{ij}^\top(k) G_{ij}^\top Q^\top \mathcal{L}_{ii} P^{-1} \mathcal{L}_{ii} Q G_{ij} \vartheta_{ij}(k)\} \\ &\leq \sum_{i=1}^N \sum_{j \in \mathcal{N}_i} \mathbb{E}\{\|\vartheta_{ij}(k)\|^2\}, \end{aligned} \quad (62)$$

thus, it follows from (61) that

$$V \leq \zeta^\top(k) \tilde{\mathcal{H}}^\top \bar{\mathcal{L}}^\top \bar{P} \bar{\mathcal{L}} \tilde{\mathcal{H}} \zeta(k) + \sum_{i=1}^N \sum_{j \in \mathcal{N}_i} \mathbb{E}\{\|\vartheta_{ij}(k)\|^2\} - x^\top(k) \bar{\mathcal{L}}^\top \bar{P} \bar{\mathcal{L}} x(k). \quad (63)$$

Using Schur complement, linear matrix inequality (60) can be transformed into

$$\tilde{\mathcal{H}}_1^\top \bar{\mathcal{L}}^\top \bar{P}^{-1} \bar{\mathcal{L}} \tilde{\mathcal{H}}_1 + \text{diag}\{\bar{\mathcal{L}}^\top (\varepsilon \bar{\Phi} - \bar{P}) \bar{\mathcal{L}} + 4\gamma \bar{\mathcal{L}}, -\bar{\Phi} + 4\gamma \bar{\mathcal{L}}, -I_N\} < 0, \quad (64)$$

where $\tilde{\mathcal{H}}_1 = [I_N \otimes (PA) - \Psi(I_N \otimes Q), -\Psi(I_N \otimes Q), I_N \otimes (PC)]$, $\bar{P}^{-1} = I_N \otimes P^{-1}$. Therefore, considering event-triggered transmission scheme (33) and (64), it follows from (61) that

$$\begin{aligned} &\mathbb{E}\{V(k+1)\} - V(k) + \gamma z(k) - \nu^\top(k) \nu(k) \\ &\leq \zeta^\top(k) \left\{ \tilde{\mathcal{H}}_1^\top \bar{\mathcal{L}}^\top \bar{P}^{-1} \bar{\mathcal{L}} \tilde{\mathcal{H}}_1 + \text{diag}\{\bar{\mathcal{L}}^\top (\varepsilon \bar{\Phi} - \bar{P}) \bar{\mathcal{L}} + 4\gamma \bar{\mathcal{L}}, \right. \\ &\quad \left. -\bar{\Phi} + 4\gamma \bar{\mathcal{L}}, -I_N\} \right\} \zeta(k) + \sum_{i=1}^N \sum_{j \in \mathcal{N}_i} \mathbb{E}\{\|\vartheta_{ij}(k)\|^2\} \\ &\leq \sum_{i=1}^N \sum_{j \in \mathcal{N}_i} \mathbb{E}\{\|\vartheta_{ij}(k)\|^2\}. \end{aligned} \quad (65)$$

Then, similar to the proof in Corollary 1, one can conclude that the multi-agent systems can achieve asymptotically consensus with a guaranteed H_∞ performance γ . This completes the proof of the corollary.

4. Simulation

In this section, an example is presented to validate the effectiveness of the proposed control strategy. Considering the multi-agent systems with five agents,

each agent can be described as follows

$$x_i(k+1) = \begin{bmatrix} 1.2 & 0.5 & 0.2 \\ -0.1 & 0.9 & 0 \\ 0.2 & 0.3 & 0.8 \end{bmatrix} x_i(k) + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} u_i(k) + \begin{bmatrix} 0.3 \\ 0.2 \\ 0.2 \end{bmatrix} \nu_i(k),$$

where $\nu_i(k) = e^{-2k}$. The parameters of transmission equation (2) are given by

$$E_{ij} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad G_{ij} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \vartheta_{ij}(k) = N(0, 1/k),$$

where channel noise $\vartheta_{ij}(k)$ satisfies normal distribution, $\mathbb{E}\{\vartheta_{ij}(k)\} = 0$, and $\mathbb{D}\{\vartheta_{ij}(k)\} = 1/k$. The heterogeneous switching topology switches among three different graphs, which is shown in Fig.1, and the Laplacian matrices of graphs are given in following, respectively. The evolution of Markov chain has been shown in Fig.2, which represents the switching of communication topology.

$$\mathcal{L}^1 = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ -1 & 2 & 0 & -1 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}, \quad \mathcal{L}^2 = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 2 & -1 & -1 \\ -1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\mathcal{L}^3 = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 2 & 0 & -1 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}.$$

The parameters of probability transition (6) are given by

$$\Pi^1 = \begin{bmatrix} 0.4 & 0.3 & 0.3 \\ 0.3 & 0.4 & 0.3 \\ 0.5 & 0.2 & 0.3 \end{bmatrix}, \quad \Pi^2 = \begin{bmatrix} 0.5 & 0.2 & 0.3 \\ 0.3 & 0.3 & 0.4 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}, \quad \Pi^3 = \begin{bmatrix} 0.3 & 0.3 & 0.4 \\ 0.5 & 0.3 & 0.2 \\ 0.3 & 0.3 & 0.4 \end{bmatrix}.$$

Fig 1. Communication topologies of multi-agent systems

Fig 2. Evolution of Markov Chain

By solving (50) and (51), the controller gain are

$$K^1 = [0.6154 \quad -0.0094 \quad 0.2782],$$

$$K^2 = [0.4805 \quad -0.0067 \quad 0.2367],$$

$$K^3 = [0.4693 \quad -0.0068 \quad 0.2138].$$

By using Matlab LMI toolbox, the optimization problem in Remark 4 can be solved and H_∞ performance level γ can be calculated as 0.1029.

Simulation results have been presented in Fig.3 and Fig.4. From Fig. 3, one can observe that multi-agent systems (1) can achieve H_∞ consensus asymptotically by using controller (4). Fig. 4 shows the event-triggering instants of multi-agent systems. The vertical line represents the inter-event intervals of agents. One can find that event happens only when triggering condition is vi-

Fig 3. The states of MASs using event-triggered control

Fig 4. Event of agents.

olated. This shows that event-based control strategy can reduce the frequency of controller's update and CPU usage effectively.

5. Conclusion

In this paper, H_∞ consensus of multi-agent systems based on event-triggered control has been considered. The partly information exchange and time-varying Markov jumping topology are proposed to describe environmental disturbances and sudden changes of system components. A novel control strategy is proposed, and it has been shown that the overall system can achieve H_∞ performance asymptotically.

6. Acknowledgments

This work is supported by the National Natural Science Foundation of China (61272064, 61273026, 61174064), Shanghai Pujiang Program (14PJ1409000), the Fundamental Research funds for Central Universities and the Open Research

fund for Key Laboratory of Embedded System and Service Computing, Ministry of Education, Tongji University.

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