APPLYING A CRESH AGGREGATE LABOUR INDEX TO GENERATE AGE-WAGE PROFILES.

Ross Guest
Griffith University
Australia

Bjarne S. Jensen
University of Southern Denmark
Denmark

ABSTRACT

This paper shows how a CRESH labour index can generate more realistic optimal wage profiles than traditional (restrictive) functional forms. The CRESH index function allows for age-specific elasticities of substitution that are implied by a proper choice of CRESH parameters. The ability to generate plausible optimal age-wage profiles can be useful in, for example, calibrating demographic macroeconomic models. The CRESH analysis also provides one explanation for the well-established divergence of actual relative wages by age from the relative age-specific intensity parameters of a simple additive labour index. Moreover, the CRESH labour index may explain the increasing relative wages for middle aged workers as a result of employing larger numbers of older workers (population aging).

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Corresponding author

Professor Ross Guest

Department of Accounting, Finance and Economics

Griffith University, Qld, 4222

Email: r.guest@griffith.edu.au

1. Introduction

Age-specific relative wages are not entirely explained by relative age-specific intensity parameters of a simple additive labour index. Explanations include seniority-based wage schemes (Lazear, 1981) and the payment of ‘efficiency wages’ (Yellen, 1984). Relative cohort size is also a factor, due not only to competition for jobs (Easterlin, 1978) but also to imperfect substitution of labour by age (Lam, 1989). The latter idea – imperfect substitution of labour by age - is the subject of the empirical application in this paper. Intuitively, imperfect substitution among workers of different ages is likely as job skills and attributes differ with age, including physical capacity, judgement, maturity, ability to assimilate new knowledge and interpersonal skills. Indeed empirical studies have generally found finite age-specific elasticities of substitution (Mitchell, 1988; Card and Lemieux, 2001; Roger and Wasmer, 2009), albeit with considerable variation in magnitude.

The empirical application here relaxes the traditional assumption in the studies above that labour substitution elasticities are constant across all ages. This is done by adopting a CRESH labour index where the elasticity parameters can be calibrated to generate optimal age-wage profiles that are closer to the actual age wage profiles than is the case with constant elasticity parameters. This can be useful, for example, in calibrating demographic macroeconomic models to explain changing relative wages of age groups. In the few applications of demographic models that have adopted imperfect labour substitution by age (Prskawetz et al., 2008; Guest, 2007), there has, however, been little or no empirical evidence for the choice of the age-specific labour elasticities, hence requiring extensive sensitivity exercises. The calibration procedure here provides an evidence-based method for the empirical determination of the parameters for labour demands and their substitution elasticities.

2. The CRESH labour index and age-wage profiles

Hanoch (1971) introduced the globally regular CRESH¹ implicit production (aggregator) function, which is more flexible than CES by allowing variable Allen-Uzawa elasticities of substitution among pairwise production inputs. There has however been relatively little use of this functional form due partly to empirical difficulties in estimating/calibrating/validating its parameters.

An aggregate CRESH labour index is given implicitly by:

$$F(L, L_1, L_2, \dots, L_M) = \gamma \sum_{i=1}^M \alpha_i \left[\frac{L_i}{L} \right]^{\rho_i} - 1 = 0 \quad (1)$$

where L_i is labour input (cohort) of age i , L is the quantitative size (number) of the aggregate labour index, α_i are intensity (weight) parameters and ρ_i are substitution parameters; they are subject to parameter restrictions, cf. Hanoch (1971, p.700), and γ is a “general efficiency parameter”, $\sum_{i=1}^M \alpha_i = 1$

is here imposed as an “identifying” restriction :

$$\gamma > 0; \quad \forall i: \alpha_i > 0, \quad \sum_{i=1}^M \alpha_i = 1; \quad \forall i: 0 < \forall \rho_i < 1 \text{ or } \forall \rho_i < 0 \quad (2)$$

A CES family of special cases of (1-2) is given by:

$$\forall i: \rho_i = \rho: L = \gamma \left[\sum_{i=1}^M \alpha_i L_i^\rho \right]^{\frac{1}{\rho}}; \quad \forall i: \rho_i = 0: L = \gamma \prod_{i=1}^M L_i^{\alpha_i} \quad (3)$$

Behind the implicit function $F(L, L_1, L_2, \dots, L_M)$ stated (1), there exist an “explicit” function,

$L = f(L_1, L_2, \dots, L_M)$ that is homogenous of degree one in the labour inputs L_i . By implicit

differentiation of (1):

$$\frac{\partial F}{\partial L_i} = \frac{\gamma \alpha_i \rho_i (L_i/L)^{\rho_i-1}}{L}; \quad \frac{\partial F}{\partial L} = \frac{\gamma \sum_{i=1}^M \alpha_i \rho_i (L_i/L)^{\rho_i}}{L} \quad (4)$$

¹ Constant Ratio of Elasticities Homothetic (Hanoch, 1971).

and using (4), the marginal productivities of labour inputs (cohorts) L_i , $\partial L / \partial L_i$ become:

$$\frac{\partial L}{\partial L_i} = \frac{\partial f}{\partial L_i} = - \frac{\partial F / \partial L_i}{\partial F / \partial L} = \frac{\alpha_i \rho_i (L_i / L)^{\rho_i - 1}}{\sum_{i=1}^M \alpha_i \rho_i (L_i / L)^{\rho_i}}, \quad i = 1, \dots, M \quad (5)$$

Then the labour cost shares are given by:

$$\varepsilon_{L_i} = \frac{\partial L}{\partial L_i} \frac{L_i}{L} = \frac{\alpha_i \rho_i \lambda_i^{\rho_i}}{\sum_{i=1}^M \alpha_i \rho_i \lambda_i^{\rho_i}}, \quad \lambda_i = L_i / L \quad ; \quad \sum_{i=1}^M \varepsilon_{L_i} = 1 \quad (6)$$

where the elasticities, ε_{L_i} , (cost shares) sum to one by the degree of homogeneity of one, (1).

Hence the CRESH relative wages (relative marginal productivities, $\partial L / \partial L_i$, marginal rates of substitution) become by (5) - with $\lambda_i = L_i / L$ and $w_i / w_j = \partial f / \partial L_i / \partial f / \partial L_j$:²

$$\frac{w_i}{w_j} = \frac{\alpha_i \rho_i (L_i / L)^{\rho_i - 1}}{\alpha_j \rho_j (L_j / L)^{\rho_j - 1}} = \frac{\alpha_i \rho_i \lambda_i^{\rho_i - 1}}{\alpha_j \rho_j \lambda_j^{\rho_j - 1}}, \quad i \neq j \quad (7)$$

The special cases of (1), (7), for the CES family (3) are,

$$\text{Cobb Douglas: } \frac{w_i}{w_j} = \frac{\alpha_i}{\alpha_j} \left[\frac{\lambda_i}{\lambda_j} \right]^{-1}, \quad \text{CES: } \frac{w_i}{w_j} = \frac{\alpha_i}{\alpha_j} \left[\frac{\lambda_i}{\lambda_j} \right]^{\rho - 1}, \quad \text{Additive: } \frac{w_i}{w_j} = \frac{\alpha_i}{\alpha_j}, \quad i \neq j \quad (8)$$

Unless (1) is just an additive index ($\rho_i = 1 \forall i$), the relative wages in (7) depend not only on the

relative intensity parameters, α_i / α_j , but also on the labour index ratios, $\lambda_i = L_i / L$, and the substitution

parameters, ρ_i . The changes in relative wages w_i / w_j , (7), are smaller the higher is the value of ρ_i .

Intuitively, the more flexible a labour input (a higher value of ρ_i) is, the smaller is the change in its relative wage required to clear the labour market for the given change in supply of the labour input, L_i .

To clearly see these implications of exogenous labour supply, L_i , we restate (7) formally as,

² Note that in cases where $\rho_i \neq 1, \forall i : L \neq \sum \alpha_i L_i$, and hence $\sum \alpha_i \lambda_i \neq 1$.

$$\frac{w_i}{w_j} = \frac{\alpha_i \rho_i}{\alpha_j \rho_j} \frac{(L_i)^{\rho_i-1}}{(L_j)^{\rho_j-1}} L^{\rho_j-\rho_i}, \quad i \neq j \quad (9)$$

By using the elasticity rules for composite functions, we get from (9), (6), (2), the elasticity of the wage ratio, w_i/w_j , with respect to the labour input, L_i :

$$E\left[\frac{w_i}{w_j}, L_i\right] = -1 + \rho_i + (\rho_j - \rho_i)\varepsilon_{L_i} < 0; \quad E\left[\frac{w_j}{w_i}, L_i\right] = 1 - \rho_i + (\rho_i - \rho_j)\varepsilon_{L_i} > 0 \quad (10)$$

Thus by (10), increasing L_i will always decrease the CRESH relative wages of L_i , but the higher ρ_i is, the smaller is the relative decline in w_i/w_j ; a higher ρ_j has a similar effect on diminishing the decline in w_i/w_j as ρ_i . Moreover, a larger L_i will always increase the CRESH relative wages of the other labour groups L_j , (compared to L_i); the higher ρ_i is, the larger is the relative increase in w_j/w_i , but the effect of higher ρ_j gives a smaller increase in w_j/w_i as a result of larger L_i , cf. (10).

The Allen-Uzawa substitution elasticities, (σ_{ij}) , between factor pairs (ij) are for CRESH, (1-2), cf. Hanoch (1971, p.699) – in our notation – given by (cf. Table 3) :

$$\sigma_{ij} = \frac{1}{(1-\rho_i)(1-\rho_j)\bar{\rho}} = \sigma_{ji}; \quad \bar{\rho} = \sum_{i=1}^M \frac{\varepsilon_{L_i}}{1-\rho_i}, \quad i \neq j \quad (11)$$

$$\sigma_{ii} = \frac{1}{(1-\rho_i)} \left[\frac{1}{(1-\rho_i)\bar{\rho}} - \frac{1}{\varepsilon_{L_i}} \right] \quad (12)$$

where (σ_{ii}) are the “total substitution elasticity” terms; the variable $(\bar{\rho})$ is a weighted average of the parameters $1/(1-\rho_i)$, with the respective cost shares (L_i) as variable weights. Clearly, especially larger values of ρ_i and ρ_j give a larger σ_{ij} . The restrictions (2) imply that $\sigma_{ij} > 0$, (11), i.e. all L_i , $i=1, \dots, M$ are substitutes. The common element, $1/\bar{\rho}$, in (11) implies that the ratio of the elasticities, σ_{ij} , have an invariant “constant” pattern, hence the term “CRESH”:

$$\forall k \neq i, j: \frac{\sigma_{ik}}{\sigma_{jk}} = \frac{1 - \rho_j}{1 - \rho_i}, \rho_i > \rho_j: \sigma_{ik} > \sigma_{jk}; \quad \frac{\sigma_{ij}}{\sigma_{kl}} = \frac{(1 - \rho_k)(1 - \rho_l)}{(1 - \rho_i)(1 - \rho_j)} \quad (13)$$

The corresponding own-price/cross-price factor demand (compensated) elasticities are (cf. Table 4),

$E(L_i, w_i) = (\partial L_i / \partial w_i)(w_i / L_i)$; $E(L_j, w_j) = (\partial L_j / \partial w_j)(w_j / L_j)$ are then by (11-12),

$$E(L_i, w_i) = \sigma_{ii} \varepsilon_{L_i}; \quad E(L_i, w) = \sigma_{ij} \varepsilon_{L_j}; \quad E(L_j, w_i) = \sigma_{ji} \varepsilon_{L_i} \quad (14)$$

The aim of the simulations below is to examine the effect of alternative sets of substitution parameters, ρ_i , on the optimal relative wage profile (7) for a given year, and to choose the set of ρ_i that generates a wage profile (7) that fits closely the observed relative wage profiles for that year.

2.1 Data and calibration

The labour cost shares ε_{L_i} are based on data for w_i and L_i . They are given as data for a base year, here chosen as 2011, and Australia is chosen as a reference (illustrative) country. All data values for 2011 are given in Table 1. The values for w_i (column 2, Table 1) are calculated from the Australian Bureau of Statistics, Catalogue 6310. for the year 2011. The values of L_i (in thousands, column 1) – which enter the calculated λ_i (column 5) – are given exogenously, and are based on population projections for Australia, using United Nations World Population Prospects, 2010 Revision, and labour force participation rates for Australia from ILO (2013).³

The observed wage w_i of the age-groups for both 2001 and 2011 are plotted in Fig. 1a for Australia; the calculated labour index ratios, $\lambda_i = L_i / L$, obtained from L_i and L , (1), are given in Fig. 1b, cf. Tables 1-2. As Fig. 1a shows, the relative wage for middle aged workers (age 35 to 55) has increased slightly in the decade from 2001 to 2011. Over the same period the labour index ratios, λ_i , for middle aged workers has fallen (cf. Fig 1b). The analysis here suggests a link between the

³ An assumption was required for labour aged over 65 years because the ILO data provide a single participation rate for the over 65 population. It was assumed that this rate applied to the population aged 65 to 69 and that the participation rate for the population aged over 69 was zero.

change in the labour index ratios and the change in the relative wages of middle aged workers. No such link is possible in standard models which use an additive labour index since in that case relative wages are independent of the labour input ratios and labour index ratios, cf. (8). For the CRESH labour index applied here, the effect of labour index ratios on relative wages is due to imperfect labour substitution by age where the substitution parameters are age-specific rather than constant across all ages.

The intensity (weight) parameters, α_i , in the CRESH labour index (1) are calibrated as follows. From (7), we get :

$$\frac{\alpha_i}{\alpha_j} = \frac{\varepsilon_{L_i} \lambda_j^{\rho_j} \rho_j}{\varepsilon_{L_j} \lambda_i^{\rho_i} \rho_i} \quad (15)$$

Given any choice of ρ_i together with the other variables of (15), the ratios α_i/α_j (15) are summed for $i=1,...,n$, which allows α_1 to be calculated, and hence all α_i , for $i=1,...,n$, are obtained. By this calibration procedure, the associated values of the CRESH intensity parameters (weights), α_i can be calculated to any particular pattern for the set of parameters, ρ_i .

The final CRESH parameters to be carefully chosen are the substitution parameters ρ_i . The method for selecting them is described as follows. An initial set of ρ_i determines α_i , as described above. In order to validate the calibrated ρ_i and α_i , parameters outside the base year (2011), we insert the calibrated values of (ρ_i, α_i) from (2011) into the relative wages (7), but now use the calculated values for the labour index ratios λ_i for a year other than 2011, which we chose to be 2001 - such that the intervening years allow some change in the values of λ_i . This gives an optimal relative wage profile (7) for the year 2001, which is then compared with actual (observed) wage profile for 2001. The values for ρ_i can be adjusted and the procedure repeated in order to produce an *optimal (simulated) relative wage profile* (7) that closely fits the *observed (actual) wage profile* (2001), cf. Fig 1a. The various patterns of ρ_i that were tested in this way are described in Section 3 below.

3. Results

The simulated (optimal) CRESH relative age-wage profile for 2001 is given in Table 2, along with the required data and parameters. The profile given in Table 2 is the result of simulating various alternative sets of values for ρ_i as described above. The aim is to find a pattern of ρ_i that generates a relative wage profile that fits the observed relative wage profile in the chosen year of 2001. Initially the CES forms (3) with alternative constant values of $\rho_i = \rho$, were applied. See Figs. 2 to 4, where all relative wage profiles, w_i/w_1 , are seen. The Cobb Douglas case, $\rho = 0.0$ with calibrated $\alpha_i = \varepsilon_i$ (2011), is seen in Fig. 2. Figs. 3 and 4 show the plots for constant elasticities of substitution by age of 2 ($\rho = 0.5$), and 5 ($\rho = 0.8$), respectively (their associated calibrated α_i for 2011 are not shown here). The best CES fit is found with the constant substitution elasticity of 5 (Fig. 4).

The next step was to experiment with CRESH values of ρ_i that vary by age. Various patterns were applied, including an approximate U-shape pattern (Fig. 6a), a rising pattern (Fig. 6b) and a falling pattern (Fig. 6c). A slightly better fit (at younger and older ages) than in the CES=5 case resulted from the approximate U-shape pattern of ρ_i (Fig. 6a), and these age-specific ρ_i and substitution elasticities σ_{ij} are given in Table 3. The values of ρ_i , the associated calibrated α_i , labour input ratios, λ_i , and cost shares, ε_i , for 2001 are shown in Table 2. The values of the weights, α_i , also given in Table 1, rise to a peak in the 40-44 year age group and then gradually decline. The corresponding optimal relative wage rates, w_i/w_1 , for 2001 are given in Table 2 and plotted in Fig. 5.

The approximate U-shape pattern of ρ_i implies that middle age workers, who have relatively low values for ρ_i , have a mix of attributes that make them harder to substitute /allocate than somewhat younger or older workers. This has implications for their relative wages. The lower values of ρ_i for middle age workers implies that a given change in their relative labour supplies results in a relatively large change in their wages in order to maintain full employment. Population ageing in most advanced countries has seen the population share of middle aged workers begin to fall in recent

decades (compare λ_i for ages 30 to 50 in Tables 2 and 1 for example) which will continue for the next decade or two according to standard projections. The analysis here indicates that this will continue to increase the relative wage for middle aged workers. This follows from (7) and (10).

Population ageing over coming decades implies a continuation of the recent increase in L_i and λ_i for $i=9-11$ (ages 55 to 64), as well as decreases in L_i and λ_i for $i=1-2$ (ages 15 to 24), as can already be seen from our tables, here in reverse order, from Table 2 and Table 1. The consequences of these labour supply changes follow directly from the elasticities (comparative statics), (10), and are evident in comparing the relative wages and CRESH calculations in Table 1 and 2 (which have the same α_i and ρ_i parameters). Compared to the calculated CRESH relative wages 2001, the increases (all w_i are reduced relative to w_1) in the observed relative wages for young ages (15-24) in 2011, and also the decreases in the observed relative wages w_i/w_1 for older cohorts (55-64) in 2011 stand out clearly – consistent with (10) and the related discussion above.

4. Conclusion

This paper presents a new approach for calibrating the parameters of a CRESH labour index which can generate more realistic optimal wage profiles than the more restrictive traditional functional forms. The CRESH function allows for age-specific elasticities of substitution which are implied by the choice of parameters. This is useful, for example, in demographic macroeconomic models; and it provides one explanation for the well-established divergence of actual relative wages by age from the ratio of intensity parameters by age that arises from a simple additive labour index. For example, the increasing relative wage for middle aged workers could be due to their relatively low substitution parameters in combination with a reduced labour force share as the population ages.

The methods and analysis here conducted for Australia could be extended to observed wage profiles and demographic data for many other countries. International comparisons of such analyses may contribute to a better understanding of the economic implications of population ageing and suggest policies towards more flexibility in optimizing the allocation of labour resources across ages.

The CRESH labour index is an instrument for generating optimal (Pareto efficient) age-wage profiles that can be compared with the available data.

Table 1. 2011: data for wage rates, labour cost shares and parameters

age (i)	L_i	w_i (\$)	w_i/w_1	ε_i	λ_i	ρ_i	α_i
15-19	657	563	1.00000	0.03441	0.69936	0.60	0.04823
20-24	1142	709	1.25933	0.07530	1.21521	0.80	0.05465
25-29	1248	1005	1.78508	0.11665	1.32798	0.80	0.07885
30-34	1124	1196	2.12433	0.12504	1.19617	0.70	0.10692
35-39	1116	1222	2.17052	0.12681	1.18734	0.70	0.10900
40-44	1091	1230	2.18472	0.12474	1.16032	0.60	0.12902
45-49	1089	1243	2.20782	0.12591	1.15894	0.50	0.15871
50-54	999	1215	2.15808	0.11281	1.06234	0.60	0.12303
55-59	799	1208	2.14565	0.08972	0.84979	0.95	0.07480
60-64	502	1065	1.89165	0.04968	0.53372	0.95	0.06443
>64	226	901	1.60036	0.01892	0.24021	0.95	0.05237
sum				1.00000			1.00000

Table 2. 2001: Optimal values, CRESH: (5), (6), (7), (9)

observed							
age (i)	L_i	w_i/w_1 (7)	w_i/w_1	ε_i (6)	λ_i	ρ_i	α_i
15-19	638	1.00000	1.00000	0.04023	0.87964	0.60	0.04823
20-24	944	1.36140	1.55102	0.08107	1.30200	0.80	0.05465
25-29	987	1.94684	1.96939	0.12123	1.36153	0.80	0.07885
30-34	941	2.27220	2.26276	0.13485	1.29772	0.70	0.10692
35-39	932	2.32249	2.39286	0.13665	1.28648	0.70	0.10900
40-44	976	2.25595	2.32398	0.13897	1.34693	0.60	0.12902
45-49	889	2.35287	2.32398	0.13192	1.22594	0.50	0.15871
50-54	761	2.37631	2.33929	0.11414	1.05022	0.60	0.12303
55-59	445	2.39032	2.29847	0.06712	0.61402	0.95	0.07480
60-64	187	2.15037	2.08929	0.02532	0.25744	0.95	0.06443
>64	74	1.83114	1.94388	0.00852	0.10169	0.95	0.05237
sum				1.00000			1.00000

Table 3. Substitution elasticities - $\sigma_{i,i}$, $\sigma_{i,j}$: (6), (11). Based on 2001 data.

	L_1	L_2	L_3	L_4	L_5	L_6	L_7	L_8	L_9	L_{10}	L_{11}
L_1	-60.877	2.534	2.534	1.689	1.689	1.267	1.014	1.267	10.136	10.136	10.136
L_2	2.534	-56.610	5.068	3.379	3.379	2.534	2.027	2.534	20.273	20.273	20.273
L_3	2.534	5.068	-36.177	3.379	3.379	2.534	2.027	2.534	20.273	20.273	20.273
L_4	1.689	3.379	3.379	-22.466	2.253	1.689	1.352	1.689	13.515	13.515	13.515
L_5	1.689	3.379	3.379	2.253	-22.141	1.689	1.352	1.689	13.515	13.515	13.515
L_6	1.267	2.534	2.534	1.689	1.689	-16.723	1.014	1.267	10.136	10.136	10.136
L_7	1.014	2.027	2.027	1.352	1.352	1.014	-14.350	1.014	8.109	8.109	8.109
L_8	1.267	2.534	2.534	1.689	1.689	1.267	1.014	-20.637	10.136	10.136	10.136
L_9	10.136	20.273	20.273	13.515	13.515	10.136	8.109	10.136	-216.87	81.090	81.090
L_{10}	10.136	20.273	20.273	13.515	13.515	10.136	8.109	10.136	81.090	-708.864	81.090
L_{11}	10.136	20.273	20.273	13.515	13.515	10.136	8.109	10.136	81.090	81.090	-2267.405
$\sum_{j=1}^M \varepsilon_{L_j} \sigma_{ji}$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table 4. Conditional factor demand elasticities, $E(L_i, w_j)$: (13). Based on 2001 data.

	w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	w_9	w_{10}	w_{11}	$\sum_{j=1}^M E(L_i, w_j)$
L_1	-2.449	0.205	0.307	0.228	0.231	0.176	0.134	0.145	0.680	0.257	0.086	0.000
L_2	0.102	-4.589	0.614	0.456	0.462	0.352	0.267	0.289	1.361	0.513	0.173	0.000
L_3	0.102	0.411	-4.386	0.456	0.462	0.352	0.267	0.289	1.361	0.513	0.173	0.000
L_4	0.068	0.274	0.410	-3.030	0.308	0.235	0.178	0.193	0.907	0.342	0.115	0.000
L_5	0.068	0.274	0.410	0.304	-3.026	0.235	0.178	0.193	0.907	0.342	0.115	0.000
L_6	0.051	0.205	0.307	0.228	0.231	-2.324	0.134	0.145	0.680	0.257	0.086	0.000
L_7	0.041	0.164	0.246	0.182	0.185	0.141	-1.893	0.116	0.544	0.205	0.069	0.000
L_8	0.051	0.205	0.307	0.228	0.231	0.176	0.134	-2.355	0.680	0.257	0.086	0.000
L_9	0.408	1.643	2.458	1.823	1.847	1.409	1.070	1.157	-14.557	2.053	0.691	0.000
L_{10}	0.408	1.643	2.458	1.823	1.847	1.409	1.070	1.157	5.443	-17.947	0.691	0.000
L_{11}	0.408	1.643	2.458	1.823	1.847	1.409	1.070	1.157	5.443	2.053	-19.309	0.000
$\sum_{j=1}^M \varepsilon_{L_j} E(L_j, w_i)$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Figure 1a. Wages (\$) **Figure 1b. Labour index (CRESH) ratios, $\lambda[i]$**

Figure 2. Age-wage profile, Cobb-Douglas, year 2001

Figure 3. Age-wage profile, CES=2, year 2001

Figure 4. Age-wage profile, CES=5, year 2001

Figure 5. Age-wage profile, CRESH, year 2001

Figure 6a. Values of $p[i]$ Figure 6b. Values of $p[i]$ Figure 6c. Values of $p[i]$

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