TENSOR MORPHOLOGICAL PROFILE FOR HYPERSPECTRAL IMAGE CLASSIFICATION

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ABSTRACT

This paper proposes a novel multi-dimensional morphology descriptor, tensor morphology profile (TMP), for hyperspectral image classification. TMP is a general framework to extract the multi-dimensional structures in high-dimensional data. The n^{th} -order morphology profile is proposed to work with the n^{th} -order tensor, which can capture the inner high order structures. This is different with the traditional mathematical morphology operations which are usually limited to two-dimensional data. By treating hyperspectral images a tensor, it is possible to extend the morphology to high dimensional data so that the powerful morphological tools can be used to analyze the hyperspectral images with spectral-spatial information fused. Experimental results on two commonly used hyperspectral images show that the tensor morphological profile consistently performs better than the extended morphological profile for hyperspectral image classification.

Index Terms— Mathematical morphology, tensor modeling, hyperspectral imaging classification

1. INTRODUCTION

Hyperspectral image classification addresses the problem of land-cover classes identification and thematic map generation, which has extensive applications in precision agriculture, mine exploration, environment monitoring, etc [1]. It usually consists of several key steps, sampling, data preprocessing, feature extraction, model construction. Among these steps, feature extraction is of significant importance and aims to find the most compact and informative set of features, improving the accuracy and efficiency of classification tasks. Although traditional classifier with the pixel-wise spectral response has good generalization to unseen data, the created thematic map often suffers from the salt and pepper noises due to noises existing in hyperspectral data [2, 3, 4]. Besides, in images of urban areas, simply relying on spectra might not be able to distinguish those classes with different spatial structures but made of similar materials. Therefore, it is necessary to introduce the spatial information in hyperspectral image classification, so as to build better modelling of local structures in the image, and facilitate more accurate land-cover and object classification.

Fig. 1: Pavia University dataset in tensor representation: (a) raw hyperspectral tensor rendered with false color; (b-d) binary hyperspectral tensor with a threshold of 0.25, 0.5 and 0.75. These images indicate that different classes are with different high-dimensional structures which can be extracted by mathematical morphology.

During the past several years, spectral-spatial feature extraction has attracted increasing attention in hyperspectral image classification [5, 6]. While traditional spectral features are extracted at the single pixel level in hyperspectral images, spectral-spatial feature extraction methods use spatial neighborhood to calculate features. Several typical image processing methods have been extended to multiple dimensions to extract spectral-spatial features in hyperspectral images, such as 3D discrete wavelet [7], 3D Gabor wavelet [8], 3D scattering wavelet [9], and local binary patterns [10]. Other novel spectral-spatial features based on other theories include spectral saliency [11], spherical harmonics [12], and affine invariant descriptors [13]. Alternatively, some researchers tend to extract spectral and spatial features separately. One of the highly cited methods is the *extend morphology profile*. Mathematical morphology is suitable to extract or suppress objects and structures in images and thus can be used for typical image processing tasks such as image filtering and image segmentation, image measurement, etc. Benediktsson et al. [14] pioneered the application of mathematical morphology in hyperspectral image classification. Fauvel et al. [15] fused

the spatial and spectral information by stacking the extended morphology profile and original spectral response vector, and then it was fed into the SVM classifier. Since then, EMP has become a benchmark in terms of spectral-spatial features in hyperspectral image classification and a number of variations have been proposed [16, 6].

In spite of these efforts, mathematical morphology is still restricted to two-dimensional images and cannot be applied to multivariate images directly, which is due to lack of ordering relationship between two vectors [17]. In other words, defining maximum or minimum between two vectors is ambiguous. A number of researchers attempt to solve this problem by developing different vector ordering scheme [18]. However, none of them has been commonly accepted. It is urgent to extend morphology to multivariate images in a simple way so that this powerful tool can be used to deal with the increasing number of multiple multivariate images such as HSI, MRI, videos, etc.

In this paper, we show that it is possible to design a multi-dimensional morphology which works better than two-dimensional morphology. Instead of treating HSI as two-dimensional images with vectorized values at each pixel, we borrow the concept of tensor modeling and design the morphology on the tensor structures so as to avoid the ordering problem. In Fig. 1, the hyperspectral image is represented as a tensor and different color indicates the value of a voxel. It can be noted that different classes are with distinguishable structures in both spatial and spectral dimension. This can be further investigated when the data cube is converted into binary data via a thresholding process. Each class consists of discriminative high dimensional binary structures mixed with isolated points, which can be analysed by set operations in mathematical morphology.

2. METHODOLOGY

Mathematical morphology was first proposed in the 1960s and afterwards it keeps involving to solve new practical problems in image analysis [17]. This method aims to analyse the shape and form of objects based on set theory, integral geometry, and lattice algebra. By treating the pixels in the image as subsets in two dimension space, set operations such as set union and intersection can be used to study the properties of objects in images. Here, we will mainly focus on the object space and ordering problem which are crucial to the development of tensor morphology.

2.1. Notation and Theoretical Foundation

MM is firstly developed on two dimensional images, and the object space of MM evolves from images to Euclidean sets and then comes to a more general concept of the complete lattice. Supposing two sets of data: the spatial unit of images and the set consisting of intensity values represented by D_f

and I , an image is a function mapping the set of intensity value to the spatial units. Therefore, different types of images mainly rely on the definition of set I . For a grayscale image, it can be defined as:

$$f : D_f \subset Z^2 \rightarrow \{0, 1, \dots, t_{max}\} \quad (1)$$

where t_{max} is the maximum value of image intensity. When t_{max} equals 1, f becomes a binary image.

After having the object space ready, it is possible to define the basic operations in MM - erosion and dilation. They employ the structuring elements (SE) to enhance or alleviate structures based on the specific requirements from users. In terms of set theoretical operations, the erosion and dilation on binary images are defined as:

$$\begin{aligned} \varepsilon_B(X) &= \bigcap_{b \in B} X_{-b} \\ \delta_B(X) &= \bigcup_{x \in X} B_x = \bigcup_{b \in B} X_b \end{aligned} \quad (2)$$

where X and B are image set and SE, and their elements are represented as x and b . Note that translation of set is used here, and B_x means that set B is translated so that element x in X is the origin of B . The same to X_b . When applying MM on grayscale images, the equation becomes:

$$\begin{aligned} [\varepsilon_B(f)](x) &= \min_{b \in B} f(x + b) \\ [\delta_B(f)](x) &= \max_{b \in B} f(x + b) \end{aligned} \quad (3)$$

Based on erosion and dilation, a series of tools can be developed such as opening and closing. These two processes can remove specific structures and noises without destroying the original primary structures. The results of processing are called morphological profiles.

$$\begin{aligned} \gamma_B(I) &= \delta_B[\varepsilon_B(I)] \\ \phi_B(I) &= \varepsilon_B[\delta_B(I)] \end{aligned} \quad (4)$$

The ordering problem is involved with multichannel or multivariate images. In this case, the set of intensity is a vector rather than a scalar. An image with m channels or bands can be denoted as:

$$f : D_f \subset Z^2 \rightarrow \{I^m | I = \{0, 1, \dots, t_{max}\}\} \quad (5)$$

where $I^m = (I_1, I_2, \dots, I_m)$. In other word, the function f maps each location p in a image to a vector.

$$f(p) = (f_1(p), f_2(p), \dots, f_m(p)), f_i(p) \quad (6)$$

Then defining equation 3 on the new object space is difficult. The hyperspectral image usually consists of multiple bands in which a pixel is expressed in a vector which represents the spectral responses at different wavelength. Applying MM on hyperspectral images will have the same problem.

2.2. Extended Morphological Profile

The ordering problem in multivariate images can be solved through the marginal process, which ignores the inner correlation between channels and processes each channel of the image separately. Since hyperspectral images consist of many bands, marginal process will suffer from heavy computational burden and redundant information. Extended morphological profile employs principle component analysis (PCA) to transform the original hyperspectral images into fewer number of bands while maintaining the majority of spectral variance. Firstly a series of morphology operations are applied on a single component to extract multi-scale spatial information with different sizes of structuring elements:

$$\Omega^{(n)}(I) = [\gamma^{(n)}(I), \dots, \gamma^{(1)}(I), I, \phi^{(1)}(I), \dots, \phi^{(n)}(I)] \quad (7)$$

where $\gamma^{(n)}(I)$ and $\phi^{(n)}(I)$ are the opening and closing operations with a disk-shape structural element of size n , respectively. Then the morphological profiles are obtained on each of the p primary components:

$$\hat{\Omega}_p^{(n)}(I) = [\Omega_1^{(n)}(I), \Omega_2^{(n)}(I), \dots, \Omega_p^{(n)}(I)] \quad (8)$$

In the last step, the morphological profiles are stacked with the spectral response to form the spectral-spatial feature.

From the process, two limitations of EMP can be observed. Firstly, the unsupervised dimension reduction methods cannot guarantee the completeness of spatial information associated with different classes. Applying PCA on hyperspectral images maintains the largest spectral variance at each pixel but it may not contain all spatial clues belonging to different classes. Secondly, since EMP and spectral response are with different characteristics, directly stacking them into a representation leads to the imbalance of spatial and spectral information belonging to the same samples.

2.3. Tensor Morphological Profile

A tensor is a multidimensional array and its order or mode is the number of indices which needs to specify an element in the array [19]. For instance, a N^{th} order tensor can be represented as $\mathcal{T} \in R^{D_1 \times D_2 \times \dots \times D_N}$. Therefore, a hyperspectral image can be arranged as a third order tensor with modes corresponding to image columns, rows and wavelengths [20]. But in perspective of object space of MM, a tensor is a function rather than a subset of Euclidian space. It maps a subset of Euclidean space to a range of intensity values. A grayscale tensor with mode N can be formalised as:

$$\mathcal{T} : D_{\mathcal{T}} \subset Z^{D_1 \times D_2 \times \dots \times D_N} \rightarrow \{0, 1, \dots, i_{max}\} \quad (9)$$

where $D_{\mathcal{T}}$ is the defined domain for tensor $\mathcal{T}$ and i_{max} is the maximum value of tensor intensity. The difference between equation 5 and the definition here is that the former

Fig. 2: Extended morphological profiles versus tensor morphological profiles on a region of Pavia University dataset. The second to fourth rows correspond to TMP at different bands. Different columns illustrate opening and closing with SE of different sizes.

definition treats the data as two-dimensional sets plus $n - 1$ -dimensional intensity set. In contrast, the later one treats data as n -dimensional data along with one-dimensional set accounting for the intensity.

For the sake of understanding the practical meaning of MM on a tensor, it can be further partitioned to multiple binary tensors so that the set operations can be applied. The thresholding process on a intensity set is defined as:

$$CS_h I(x) = \begin{cases} 0 & \text{if } I(x) < h \\ 1 & \text{if } I(x) \geq h \end{cases} \quad (10)$$

$CS_h I(x)$ is named as cross section which represents the output with a threshold of h . In order to contain the whole information of the original data, it is necessary to apply multiple times thresholding steps and set $h = 0, 1, 2, \dots, i_{max}$. Interestingly, there is a relationship among these cross sections:

$$CS_{i_{max}} I(x) \subseteq CS_{i_{max}-1} I(x) \subseteq \dots \subseteq CS_0 I(x) \quad (11)$$

Thus a grayscale tensor can be decomposed into the sum of its cross-sections:

$$\mathcal{T}(x) = \sum_{h=1}^{i_{max}} [CS_h(\mathcal{T})](x) = \max\{h \mid [CS_h(\mathcal{T})](x) = 1\} \quad (12)$$

The thresholding step not only plays a intermediate role to connect the grayscale tensor and its binary representation, but also provides the practical meaning of applying MM on tensor. The binary morphology is able to extract the high dimensional structures inside the binary tensor and remove those isolated structures which are mainly due to the random noises. Then these high dimensional set based features can be combined to represent the feature of grayscale tensors. This is equivalent to the following equations:

$$\begin{aligned} [\varepsilon_B(\mathcal{T})](x) &= \min_{b \in B} \mathcal{T}(x + b) \\ [\delta_B(\mathcal{T})](x) &= \max_{b \in B} \mathcal{T}(x + b) \end{aligned} \quad (13)$$

Here the SE B is a binary tensor with the same mode to $\mathcal{T}$ and it corresponds to the flat SE in EMP. It is straightforward to derive opening and closing:

$$\begin{aligned}\gamma_B(\mathcal{T}) &= \delta_B[\varepsilon_B(\mathcal{T})] \\ \phi_B(\mathcal{T}) &= \varepsilon_B[\delta_B(\mathcal{T})]\end{aligned}\quad (14)$$

At last, the tensor morphological profiles are obtained by applying opening and closing on a tensor with multiple SE of size n :

$$\Omega^{(n)}(\mathcal{T}) = \left[\gamma^{(n)}(\mathcal{T}), \dots, \gamma^{(1)}(\mathcal{T}), \mathcal{T}, \phi^{(1)}(\mathcal{T}), \dots, \phi^{(n)}(\mathcal{T}) \right] \quad (15)$$

The output $\Omega^{(n)}(\mathcal{T})$ fuses the spatial-spectral information in high dimensions. Note that choosing a suitable SE is crucial to TMP but it is beyond the scope of this paper. To show the difference between EMP and TMP, an example on Pavia University is shown in Fig. 2.

3. EXPERIMENTS AND RESULTS

We have designed two experiments comparing EMP and TMP with two widely used hyperspectral datasets, Pavia University and Indian Pines datasets. These two datasets are typical urban and rural hyperspectral images with abundant spectral information, as well as different spatial resolution. Pavia University dataset is captured with the ROSIS sensor in Pavia, northern Italy. It has 103 spectral bands covering the spectral range from 0.43 to 0.86 μm . The resolution is very high with 1.3 m per pixel. There are 9 classes in the ground truth. Available training and testing samples are given with the dataset which can be found in [6]. The second dataset Indian Pine is with a much lower spatial resolution of 20 m per pixel. It is acquired with AVIRIS sensor over the agricultural Indian Pine test site in Northwestern Indiana. The number of the spectral band is 200 (20 bands was removed due to water absorption). 16 classes are considered in the experiment. As no available training and testing samples available in advance, 5% per class are randomly selected as the training data and the rest of samples are used for testing.

We have compared three feature extraction methods which were raw spectral response (SPE), extended morphological profile (EMP) and tensor morphological profile (TMP). While a disk-shaped SE was used in EMP, a cylinder-shaped SE was adopted in TMP. In the experiment, the radius of the cylinder was set to 1, 3, 5, 7 and the height was 3, 5, 9, 17. The shape of SE is designed based on the characteristics of spatial and spectral dimensions. A linear SVM was adopted in the classification step. The classified thematic maps are shown in Fig. 3 for both datasets. From the figure, it can be noted that the spectral-spatial methods achieve much smoother results than raw spectral method. Furthermore, due to TMP fusing the spectral and spatial information in high dimension and extracting the inner structures, it successfully

Fig. 3: Classification results on Pavia University and Pavia University dataset with different methods. From left to right: spectral feature, extended morphological profile and tensor morphological profile.

classifies a few samples when EMP failed. In Table 1, we also show the statistical results in terms of overall accuracy, average accuracy and Kappa coefficient. The results indicate that TMP consistently performs better than EMP.

Table 1: Overall accuracy (OA), average accuracy (AA) and Kappa coefficient (κ) on Pavia University and Indian Pines datasets using spectral feature (SPE), extended morphological profile (EMP) and tensor morphological profile (TMP).

	SPE	EMP	TMP	SPE	EMP	TMP
OA	73.3	86.4	90.2	72.0	83.2	88.6
AA	84.1	84.5	86.7	65.2	81.9	87.4
κ	0.67	0.82	0.87	0.68	0.81	0.87

4. CONCLUSIONS

In this paper, a tensor morphological profile is proposed to extend mathematical morphology to high dimensional data. It partially solves the ordering problem in traditional mathematical morphology and can be used to extend spectral-spatial features with morphological operations. Experiments validate that TMP consistently performs better than original EMP. By developing different SE, it would be promising to improve the performance of TMP in hyperspectral image classification and even create possibilities in other computer vision tasks such as Magnetic Resonance Imaging or videos processing.

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