Chapter 13

Transformations of Teaching and Learning through Digital Technologies

Vince Geiger	<i>Australian Catholic University</i>
Nigel Calder	<i>University of Waikato</i>
Hazel Tan	<i>Monash University</i>
Esther Loong	<i>Deakin University</i>
Jodie Miller	<i>Australian Catholic University</i>
Kevin Larkin	<i>Griffith University</i>

Abstract

This chapter is a critical synthesis of research related to the transformations that take place when digital technologies are incorporated into teaching and learning practices. In developing this synthesis, research from all levels of education was reviewed with a focus on the opportunities digital technologies offer for cognitive, pedagogical, affective and professional change. The chapter is structured in alignment with Pierce and Stacey's (2010) map of pedagogical opportunities in which three dimensions for educational transformation were identified: tasks, classroom, and subject. A discussion of future directions for research into technology enhanced mathematics education concludes the review.

Keywords

Mathematics, Technology, Digital Technologies, Digital Tools, Computers, Digital pedagogies, Virtual objects, ICT

1 Introduction

This chapter is a critical review of Australasian research, over the period 2012-2015, related to the transformations that take place when digital technologies are incorporated into teaching and learning practice. The review considers research conducted at all levels of education and focuses on opportunities digital technologies offer for cognitive, pedagogical, affective and professional change to the and teaching and learning of mathematics.

In the previous RiMEA chapter on digital technologies (Geiger et al., 2012), the following issues were identified as foci for future research:

- How digital technologies influence learning trajectories
- The influence of digital technologies on developing mathematical dialogue and argumentation
- The role of digital technologies in promoting and enhancing social interaction within learning communities
- How digital technologies can be integrated into student-centred inquiry learning approaches
- The equity issues which emerge with the wide-spread deployment of digital technologies and how these can be addressed

Many of these issues receive attention within the current review, along with new and emerging research themes. It might be expected that these themes were influenced by the process of curriculum reform associated with the advent of the Australian Curriculum, although some commentators, for example Goos (2012), raise concerns that this may have been a lost opportunity for the integration of digital technologies into mathematics classrooms.

Fundamental to the progress of the field is the development of theoretical frameworks as a foundation for research. A number of studies reviewed in this chapter have contributed to new knowledge by extending frameworks previously utilised to frame research into the use of digital tools within mathematics education including transactional distance theory (Larkin and Jamieson-Proctor 2013, 2015), TPACK (Handal et al. 2013; Larkin et al. 2012) and affordances (Brown and Stillman, 2014). Others have used theoretical frames drawn from outside of mathematics education in new ways, such as Pierce and Stacey's (2013) use of Roger's (1995) framework for the diffusion of innovation. In some cases, more than one framework has been used to generate new theoretical insight, for example, Goos (2013) employed both the *Master-Servant-Partner-Extension of self* (MSPE) framework (Goos et al. 2000) and Valsiner's (1997) zone theory of child development to provide complementary perspectives on the evolution of teachers' technology influenced pedagogical identities. A number of chapters related to digital technologies in the RiMEA series have been organised around the types of technologies available at that time, such as, computers, multimedia, and the internet (Goos and Cretchley 2004), while others have taken a more thematic approach, for example, learning contexts and curricular design; learners and learning; teachers and teaching; and gender and affect (Geiger et al. 2012). This chapter is structured via the dimensions identified in the *map of pedagogical opportunities* (Figure 13.1), developed by Pierce and Stacey (2010) (henceforth referred to as *the pedagogical map*), which positions technology as an agent that provides opportunities for three types of educational transformation: curriculum, assessment, and pedagogy. These transformations are made possible by the functionalities of digital technologies. While this map was initially created with a focus on the pedagogical opportunities afforded by mathematics analysis software, we see the framework as more broadly relevant to any digital technology that offers teachers and/or learners the technological functional capacities that allow for the dimensions of:

- *Tasks* they will set for their students
- Their *classroom* interaction
- The *subject* being taught

Within each of these dimensions, transformations are seen to take place within specific classroom activity. In the case of *Tasks*, for example, digital tools provide opportunity to change the: learning of pen-and-paper skills; investigation of real world data; exploration of regularity and variation; simulation of real situations; and the linking of representations. Transformations that take place within these types of activities, and others identified in research related to emerging technologies, will be outlined and discussed.

Figure 13.1 Pedagogical map for mathematics analysis software (Pierce and Stacey, 2010, p. 6)

In the sections that follow, we review the role of digital technologies in promoting or mediating these types of transformations from the perspectives of *learners and learning* and *teachers and teaching*.

2 Tasks

In this section, we discuss research on the opportunities provided by digital technologies to transform tasks used for learning and teaching mathematics. Tasks related to activities presented in the *pedagogical map*, such as *learning pen-and-paper skills*, *investigating real world data*, *exploring regularity and variation*, *simulating of real situations*, and the *linking of representations*, are considered along with innovative students activities made possible by the availability of digital technologies.

2.1 Learners and learning

Research related to tasks from the perspective of learners and learning coalesce around five different aspects: *linking representations; using real data for exploring regularity and variation; simulating real or fantasy situations through game; and designing virtual objects.*

2.1.1 Linking representations

The effectiveness of Apps used on different types of technologies (e.g., computers, tablets, some calculators) for transforming the learning of students within early childhood and primary education has been one focus of research into the use of digital technologies to link mathematical representations. Goodwin and Gould (2014), for example, found that the App *Race in the Outback* was effective in developing young children's numeral identification and ability to match numerals to quantity skills. This was effected through the linking of multiple representations of pictures of Australian animals and symbolic representations of number in the App. The potential of digital tools to mediate the linking of representations was also investigated by Yeh (2013), who found that the use of an online environment for integrating learning about 3D objects with direction, location and movement helped Year 4 students develop the capacity to mentally and visually construct virtual 3D objects.

In her research within secondary contexts, Brown (2015a) explored the potential of graphing calculators to link representations. Using data drawn from two Year 11 mathematical methods classes, and one Year 9 mathematics class, she identified 16 affordances that a Technology-Rich Teaching and Learning Environment (TRTLE) could offer student learning. These affordances were supported through technology by the linking of representations included in activities such as translating a situation (e.g., a task presented by teacher) into a strategy; perceiving relevant affordances of technology; choosing affordance bearers (e.g., the GRAPH key on the Graphics Calculator); applying mathematical and technical knowledge; interpreting mathematical output; and responding to the output. In contrast to the potential of digital tools to afford learning opportunities, Brown (2015b) also found that “the transformational power of technology was not realised by many students” (p. 440) despite having the technical and mathematical knowledge, due to the lack of perception of key affordances of the graphics calculator.

2.1.2 Using real data for exploring regularity and variation

In a study of 27 students (12 years old), Allmond and Makar (2014) described the use of Tinkerplots (Konold and Miller 2005) software that represented life-related data using box plots in an inquiry based teaching unit. Two investigations were described; an inquiry on estimating the time taken for a paper helicopter to drop, and a second inquiry comparing jumps of origami animals. Students collected data and created hand-drawn dot plots as part of both activities to visually identify point and interval estimates to account for variability. Data were then imported into Tinkerplots for further analyses and exploration where students attempted to standardise interval estimates and box plots. Findings included that the software: alleviated computational boundaries allowing for a focus on analysis; supported students in operating accurately and efficiently; and provided a flexible platform for the exploration of data.

In a study using generic digital tools, Geiger (2013) described a task where Year 8 students, in order to monitor their levels of physical activity as part of their Physical Education (PE) program, each wore a pedometer for one week. The number of steps students recorded in a day was entered into a spreadsheet that was on public display in the classroom. Students used the chart facilities of the software to explore the variability that lay within the data for different variables, for example, days of the week and gender. Additionally, students made use of arithmetic calculators to convert the number of steps travelled by individuals over a week into kilometres. Geiger reported on the high level of engagement among students and how the combination of task and technology allowed the teacher to move to a more investigative teaching approach.

While also reporting on high levels of motivation and engagement among students working with Web-based real-life scenarios and real data into the mathematics lessons, Loong and Herbert (2012) found that these experiences did not translate into an appreciation for the use of mathematics in everyday life. They argued that there must be greater emphasis on students working with “messy” data in order that they develop the contextual and strategic know-how needed to apply mathematics in the real world.

2.1.3 Simulating real or fantasy situation through games

Some research reported on the effect of using software that simulates real and fantasy situations as a way to transform learning. Lowrie et al. (2013), for example, investigated the use of a game (The Legend of Zelda: Phantom Hourglass) that contained challenging spatial features that needed to be interpreted and decoded with respect to the game storyline. The research suggested there were gender differences in the type of games students enjoyed and in the approaches adopted to play the game. Male players generally utilised a trial and error approach and routinely accessed the spatial features of the games (including the dual screens) to monitor thinking and to record the pathways explored. Female players tended to read more text, use more deliberate movements, and utilised less experimental methods to determine where they wanted to go. The authors suggested that games such as this could be used as a stimulus for operating in out-of-game environments where similar skills were needed.

Games also featured in a study by Attard and Curry (2012) of Year 3 students' iPad supported mathematics learning. They claimed App-based games provided the interactivity, feedback, and information needed for higher levels of participation and so enhanced the learning of mathematics. Consistent with these findings, Calder and Campbell (2015) found similar outcomes with reluctant 16-18 year old learners when using app games within numeracy and literacy programs. Further evidence of the positive effect games-based mathematics instruction can have on student engagement is found in a study by O'Rourke et al. (2013), where researchers made use of *Handheld Game Console Nintendo DS* technology to explore the perceptions of Year 4/5 students, from nine schools in Western Australia, in relation to the development of mental computation skills. While there was anecdotal evidence of improved mastery of mathematical skills, the study provided stronger evidence for increased levels of engagement in mathematics. Student participants reported that they were challenged, had developed strategies for success, and were able to make independent choices – factors that the authors claimed impacted on students' self-improvement approaches, self-determination, and self-esteem. While these findings are promising, Calder (2015) cautions that when evaluating the positive impact of games on student learning, the importance of appropriate pedagogy when using apps should not be overlooked.

2.1.4 Designing virtual objects

Past studies have also investigated the potential of tasks that require students to create virtual objects using digital media for learning mathematics. In a recently published study, Yeh and Chandra (2015) analysed how students designed a 3D virtual world using LOGO programs in a 10 week long teaching experiment. Groups of three Year 5 students worked in a team and chose a design project based on the creation of a virtual world. In the design and construction of the virtual world, students: explored relationships between geometrical objects (e.g., creating a simple 3D tree using a cone and a cylinder); made use of maps and plans; and utilised mathematical skills associated with number, measurement and scale, patterns and generalisation, and chance. The researchers reported that through employing trial and error strategies, students were able to see patterns and make generalisations. Further, they claimed that students' mathematical abstraction and logical thinking and reasoning were enhanced through the process of designing and creating their own virtual 3D world.

In a different approach to building students' mathematical capabilities by involving them in the process of design, Calder (2012) reported the use of *Scratch* programming software by 10 year olds to develop mathematics activities for their buddy class of 5-year olds. Scratch utilised a building block command structure with LOGO elements for students to create environments with graphic, audio and video elements. The study revealed that the immediate, visual and onscreen feedback, in addition to support from their teachers, served to modify and transform students' thinking about mathematics as they designed mathematics activities.

2.1.5 Summary for learners and learning – tasks

The studies discussed in the preceding section show the variety of approaches and technologies that can be brought to bear within tasks in creating pedagogical opportunities that enhance

mathematics learning. Aspects of the *pedagogical map* related to *tasks* such as *investigating real world data*, *exploring regularity and variation*, *simulating of real situations*, and the *linking of representations* are all evident in current research; however, there did not appear to be any research relevant to the *learning pen-and-paper skills aspect*. The research reviewed here also investigated aspects of tasks not covered by the task aspect of the *pedagogical map*, such as technologies that allow students to operate within virtual worlds; either created for them or designed by them. These new themes are worthy of further investigation.

2.2 Teachers and teaching

In this section we review literature related to technology enhanced mathematics tasks from the perspective of teachers and teaching. The themes that emerge from the research considered here represent a departure from the types of activity outlined in the *pedagogical model* under tasks, as they are concerned with the *challenges faced by teachers in designing tasks in ICT-rich environments* and how digital technologies are vital for *transforming tasks in pre-service teacher education* – while at the same time being mindful of the *task* aspects of the *pedagogical map*.

2.2.1 Challenges faced by teachers in setting tasks in ICT-rich environments

In order to design tasks that provide pedagogical opportunities, teachers need to be able to see the affordances of technologies for mathematics learning (Brown, 2015a) and have the motivation and the means (e.g., knowledge and skills; availability of hardware, software and support; professional development) to design such tasks.

Early career teachers (ECTs) represent a particular case. Attard and Orlando (2014) conducted four case studies of ECTs' use of technology within mathematics classes. They found that there was an unrealistic expectation, from established teachers, that ECTs were highly skilled with integrating technology into their teaching. Further, while ECTs seemed more familiar with the use of IWB, they found the use of iPads more challenging. Despite reporting that ECTs were aware of the potential advantages offered by the iPads available within their schools, the researchers reported that Apps were mainly used for drill and practice purposes. Attard and Orlando saw this behaviour as a reliance on familiar resources and a reluctance to develop their own tasks, or adapt other resources, to use with iPads – a situation they termed a *pedagogical dilemma*.

Established teachers can also face such a dilemma. Zuber and Anderson (2013) used a mixed-methods approach to investigate the experiences and beliefs of 28 mathematics teacher at five secondary schools during the second year of the New South Wales digital education revolution laptop program. Teachers were grouped into categories the “Non Adopters”, “Cautious Adopters” and “Early Adopters” according to their self-reported classroom use of laptops. Overall, it was found that most teachers did not allow frequent student use of laptops in the classroom and so they rarely developed technology active tasks. This appeared to be due to a belief that “students authentically learn mathematics only using pen and paper” (p. 279). Additionally, “Cautious Adopters” and “Non Adopters” argued that laptops exacerbate classroom management problems, especially for lower-achieving students.

In a study where new technologies were more readily embraced, Pierce and Stacey (2013) observed four “early majority” teachers (those who accept the necessity for change but wish to move forward carefully and thoughtfully) who did not have a special interest in technology. They found that these teachers gradually integrated new uses for mathematics analysis software (e.g., spreadsheets, function graphers, symbolic algebra manipulation and dynamic geometry) into their existing repertoire of teaching approaches, typically by adapting existing tasks. Teachers viewed these new technologies as incremental improvements to earlier calculators and so it was important to introduce them into the classroom in order to keep students up to date with technology. Pierce and Stacey comment that this positive intention comes with the challenge of accommodating rapid changes in software and hardware design. The constant need to learn and teach new technical skills at the expense of looking for more opportunities to approach mathematics concepts in new ways is a dilemma that must be addressed when introducing new digitally enhanced approaches to mathematics teaching and learning.

2.2.2 Transforming tasks in pre-service teacher education

There is now great potential for technology embedded learning and teaching activities to transform teaching within pre-service education courses. The range of technologies utilised is broad and includes video-based digital tools, online Apps, electronic whiteboards, and online quizzes. Digital technologies are used to enhance the learning experience of pre-service teachers and to prepare them for utilising technological tools when they become fully active within the profession.

Martin (2012) reported on a study of the effectiveness of on-line quizzes as a formative and summative assessment instrument on mathematics subject content within a first-year primary teacher education program. The quizzes were included as a “hurdle” task within the course, that is, a pass grade or higher was necessary for a student to complete the subject. The quizzes were administered to student teachers at the end of the previous semester by way of preparation for the upcoming unit. During the semester, pre-service teachers were allowed to take the quizzes at any time and were encouraged to collaborate with peers, or others, when completing items online. When completing the online quizzes, immediate feedback and directions for further help on the content were provided. Pre-service teachers reported that the online quizzes were generally a useful tool for self-assessment and for improving their mathematical understanding.

Providing models of “good practice” that are readily accessible to pre-service teachers has been a long-term challenge in teacher preparation programs. In attempting to address this problem, Beswick and Muir (2013) used video excerpts of an experienced teacher engaging with four Grade 8 students in a course consisting of 176 pre-service teachers. The videos were used as stimulus for discussions about the connection between theory and practice within mathematics education. Although many participants struggled to see beyond obvious aspects of teaching, approximately three-quarters of the 176 pre-service teachers either agreed or strongly agreed that the videos: provided good examples of mathematics/ numeracy practice; increased their understanding of effective mathematics/numeracy practice; provided a link between theory and practice; provided them with useful strategies to use in the classroom; and helped them to reflect on their own teaching beliefs and practices. The researchers also highlighted the importance of

the quality of prompts following the video excerpts in directing the pre-service teachers' focus and discussion.

Researchers have also reported efforts to enhance the capabilities of primary pre-service teachers related to integrating digital technologies into their developing teaching practice. Bate et al. (2013) reported on the outcomes of a thirteen-week university mathematics education unit that was redesigned to model the use of technologies in tasks within mathematics classrooms as part of the \$8 million Teaching Teachers for the Future initiative. The learning sequence involved attention to: concrete (physically building or creating objects); representational (using ICT to extrapolate concepts through accessing large data sets or using virtual manipulatives), and abstract (using ICT to manipulate large data sets or generalising from pattern generation). The researchers reported that throughout the course, pre-service teachers' conceptions of digital technologies changed from viewing technologies as motivational and/or drill and practice tools to envisioning possibilities of incorporating technologies into tasks meant for collaborative problem solving and student-centred discovery. The authors also claimed that pre-service teachers exhibited increased discernment toward digital technology integration by developing the ability to critique available software.

In a separate study, McDonald (2012) conducted research with third year students enrolled in a preparatory mathematics education course that embedded digital technologies such as Fun With Construction (2011), Smartboard notebook, various interactive websites, and productivity tools such as PowerPoint and Excel. Pre-service primary teachers were required to work in pairs to design an original interactive Smartboard application to support student learning for a specific mathematics concept. The results of pre- and post- program surveys, in addition to post-presentation reflection questions, indicated that confidence with using IWBs and digital technologies to teach mathematics had increased and attitudes towards the use of technology as a means of promoting student engagement and enjoyment were more positive. At the same time, pre-service teachers anticipated that there would be challenges when attempting to use digital tools in schools including access to software and hardware, networking issues, the principal's opinion of using technology in the classroom, and time needed for students to prepare for the use of digital technologies.

2.2.3 Summary of teachers and teaching – tasks

In the research studies examined above, innovative approaches were reported in relation to tasks utilised to enhance pre-service teacher mathematics learning or as preparation for integrating digital technologies into their future classroom practice. It would seem that because of the nature of this research the task aspects of the pedagogical map were not as relevant as for learners and learning. This provides opportunity, however, to identify those aspects that are unique to teachers and teaching. While most studies reported positive outcomes, they also remind us of the potential challenges pre-service, early career, and experienced teachers face when they attempt to introduce technology active approaches to teaching and learning into schools. This challenge is strongly related to the beliefs and attitudes of current and future teachers about the nature of mathematics learning and the role of technology in this enterprise. Another challenge, identified in the research of Larkin (2015), is how teachers can discriminate between high and low quality digital resources, given the plethora of free Apps and commercial software now available. While

there is encouragement from research that it is possible to develop teachers' capacities to make such judgements, this remains an important area for ongoing research.

3 Classroom

This section addresses research related to the classroom and consequently the dimensions from the *pedagogical map* of *changes in classroom social dynamics* and the *classroom didactic contract*. Thus the focus is on how learning mathematics plays out in individual and collective relationships between students, teachers and digital tools. In exploring this focus, we embrace a broad view of what constitutes a classroom by examining research related to both traditional proximate learning environments (i.e., face to face contexts) and non-proximate forms such as online courses, forums and communities. Past RiMEA chapters devoted to the role of digital technologies in mathematics education have considered teachers, learners, and digital tools as separate entities in the acts of teaching and learning (Geiger et al., 2012). In this chapter, we take a more integrated approach by looking at technology active classrooms from the perspective of *learners and learning* and *teachers and teaching*.

3.1 Learners and learning

Two themes emerged when considering research related to technology active classrooms from the perspective of Learners and Learning. These themes related to the *classroom* aspects of the *pedagogical map*: *changing interactions and digital tools* and *peer interactions in the online space*. Research findings are drawn from primary, secondary, and tertiary environments.

3.1.2 Changing interactions and digital tools

Tan (2012) explored the relationship between students' beliefs about mathematics and their interactions with graphing calculators through the lens of the *Master-Servant-Partner-Extension of self* (MSPE) framework (Geiger 2009; Goos et al. 2003). The study involved 964 Singaporean and 176 Victorian secondary students who were surveyed using an online questionnaire. Findings included that students' ways of interacting with advanced calculators were associated with their beliefs about mathematics and their experience of mathematics teaching. The use of calculators as *Master* (being subservient to technology) was associated with what Tan terms a high Separate Knowing-Surface Approach, and the use of calculators as *Collaborator* (high level of sophistication) was associated with a high Connected Knowing-Deep Approach.

Technology can be used to enhance and transform student learning when engaging in mathematical modelling. Redmond et al. (2012) conducted a study with both primary (Year 4 and 5) and middle school (Year 8 and 9) students using TI-Nspire handheld technology and digital probes when engaging in a mathematical modelling challenge. Students' survey data indicated that collaboration was necessary in order to collect data using the available technology. It was noted by the authors, however, that such comments were more prevalent among the older cohort of students.

By contrast, a study conducted by Whyburn and Way (2012) on the use of interactive white boards (IWB) with primary school students found that interactions between the teacher and student were limited, with the teacher asking closed questions with students seated at their desks for the duration of instruction with the IWB. Using *Bubble Dialogue* templates, group interviews and lesson observations to gather data on student views, a theme-based description of student perceptions was developed. The majority of students' comments related to engagement factors rather than deep learning and understanding, and the lesson observations revealed teacher-centred approaches, with limited strategies to promote student interaction and higher-order thinking. The findings highlight the need for supporting the professional learning of teachers even though they may have been using an IWB for several years.

3.1.3 Peer interactions in the online space

In a study that considered how middle school students communicated with each other, and with the teacher when online in mathematics class, Mojica-Casey et al. (2014) found that students reported both positive and negative experiences. Students commented that while the use of ICT during mathematics lessons provided greater autonomy when communicating with peers, there was potential for all attention to be directed toward the computer screen, lowering the degree of interaction with others.

Promoting and managing productive interactions between peers has also been noted as a challenge for tertiary instruction in mathematics education. Larkin and Jamieson-Proctor (2013, 2015) considered how transactional distance theory (TDT) can be used in the design of an online mathematics education methods course for first year pre-service teachers that aimed to promote interaction between learners. The researchers found this framework to be helpful in understanding the balance between dialogue and structure for learners. Further, they claim there was a positive increase in student attitudes towards mathematics and a reduction in anxiety related to this subject. In a study that supports these findings, Galligan and Hobohm (2013) investigated the use of screencasts as an assessment tool for undergraduate education students. They found that the screencasts provide a catalyst for student/student interaction, thus reducing transactional distance.

Taking a socio-cultural approach, Goos and Geiger (2012) examined the peer-to-peer interactions between students enrolled in the third year of a pre-service mathematics education course. The research considered how software can promote student learning of mathematics and collaboration (e.g., Internet based networks for working with mathematics objects) and assist collaboration but not necessarily encourage mathematics learning (e.g., databases or virtual communities in online education). The researchers argued that technology-mediated collaboration can lead to different forms of knowledge and relationships between students. Thus, digital technology can transform the way students interact with each other and technology while constructing personal knowledge within a community of mathematics learners.

3.1.4 Summary of learners and learning – classroom

Each of the studies reported above relates to *changes in classroom social dynamics* and the *classroom didactic contract* in some form. Research into the peer interactions in online space,

however, represents a huge shift in what can be considered a classroom and, as a consequence, very different types of social dynamics need to be negotiated between peers as well as new and unique didactic contracts established between teachers and learners.

3.2 Teachers and teaching

The themes that emerged from literature focused on teachers and teaching were: affordances that support interactions; transforming learning communities with digital pedagogy; and using digital technology to enhance interactions in research contexts.

3.2.1 Affordances of digital technologies for supporting teacher-student interactions

In a study of affordances in technology-rich teaching and learning environments (TRTLE), Brown (2013) provided a theoretical frame for understanding the role of teachers. Results of the study indicated that secondary mathematics teachers engaged in seven roles when interacting with students: evaluator; interpreter; hermeneutic facilitator; enabler; promoter; scaffolder; and follower. These roles were strongly linked to the management of students' enactment of affordances. The roles identified from this study provide opportunity for teachers to promote an engaging and interactive environment for the teaching and learning mathematics.

In a case study of one senior secondary teacher's initial attempts to adopt a Flipped Classroom approach, Muir and Chick (2014) reported that teaching practice shifted from a teacher-directed approach to one where classroom time was more focused on one-on-one tutoring – dramatically changing the way the teacher had previously interacted with his students. The authors suggest that this initial finding provides incentive for further research to be conducted into the new teacher-student interactions that emerge from this innovative approach.

3.2.2 Transforming learning communities with digital pedagogy

There were a number of studies concerned with reducing the barriers between lecturers and students, thus providing students enrolled in online course with a 'similar' experience to those students who attend face-to-face courses.

Higher education is transforming the delivery of learning in mathematics with the development of new technologies, with blended learning environments becoming increasingly common as a mode of delivery. Building on Engelbrecht and Harding's (2005) desirable attributes for teaching online, Prieto and Holmes (2014) conducted a study with postgraduate students (practicing teachers $n=60$) in which they investigated the factors that contributed to a quality online environment. The researchers reported that they moved from an asynchronous delivery to a blended approach, where the interactions with students were transformed to virtual face-to-face experiences. This was achieved through delivering content to enable: (a) students' self-regulation and engagement with digital resources (books, short videos, interactive blogs, discussion forums); and (b) student directed learning (assessment tasks with some providing feedback). Students indicated that engaging in discussion forums was the preferred mode of interaction within this community. Overall the participants who agreed to partake in the study accessed

more of the online materials and had more interactions in the discussion forums, than those students enrolled in the course but not participants in the study.

Galligan et al. (2012) report on a study that considered the impact of digital technology in reducing the “distance” between first year university students studying within an online environment. Students reported that interactions with other online participants provided opportunity to demonstrate personal mathematical understandings, and for others to provide feedback and additional explanations. Additionally, the researchers claimed that these interactions created a ‘humanness’ within the environment. It would appear, however, that the level of engagement for distance students was lower than those studying face to face, possibly due to the asynchronous adoption and delays in communication and support for students.

In a different take on tertiary learning within online environments, Muir (2012) conducted mathematics lessons with pre-service teachers using ‘Second Life’ to provide pre-service teachers with the opportunity to focus on the teaching of mathematics in a virtual environment without the concern of impact upon the learning of real students. It allowed the instructor to both model the teacher’s role and also to act as a facilitator to debrief, reflect and discuss the pedagogy involved in the learning experience. The author claimed that Second Life provided an alternative to more traditional approaches in that this virtual environment can be ‘controlled’ by the instructor, providing the opportunity to focus on particular topics, misconceptions, learning difficulties, and pedagogical approaches. Feedback from participants indicated that despite some limitations, it had potential for enhancing their practical experience as pre-service teachers.

3.2.3 Using digital technologies to support interactions in research

Larkin and Jorgensen’s (2014) research on primary school students’ attitudes and emotions towards mathematics extends the role of digital tools in transforming teacher-student interactions to include researchers. Within this research, students (Year 3 and Year 6) used iPads to record their personal aspects of learning mathematics. Results indicate that while 64% of students (n=105) recorded a video diary, most students only recorded one entry over the ten weeks. Despite this low rate, the researches claim that this method was more successful than traditional methods, such as surveys or interviews, for collecting this type of data. This study adds to methodological approaches to research involving young students and provides an alternative to the way in which interactions can occur between teachers, researchers and students.

3.2.4 Summary of teachers and teaching – classroom

In this section, studies have each dealt with *changes in classroom social dynamics* and the redefinition of the *classroom didactic contract*. Online teaching environments appear to bring with them the challenges of isolation and disconnection of teachers and learners to their communities – hence the importance of attempting to create a more “human” experience (Galligan et al. 2012) within such environments. The notion that digital tools can catalyse a renegotiation of a different type of classroom contract between research, teachers and students (Larkin and Jorgensen 2014) is also a new and novel direction for further research.

4 Subject

This section is concerned with research that aligns digital pedagogical media with particular mathematical content areas. In previous reviews (e.g., Geiger et al. 2012; Goos and Cretchley, 2004), there has been considerable reference to digital media that were developed to enhance learning in particular content areas. For instance, research that involved Computer Algebra Systems (CAS) in algebra or *Fathom* in exploratory data analysis. Pierce and Stacey's *pedagogical map* indicates that the potential for change in the *subject* being taught lies in three directions: *exploit contrast of ideal and machine mathematics*; *rebalance emphasis on skill, concepts, and applications*; and *build metacognition and overview*. Australasian research that related directly to specific mathematical content areas is considered with these directions in mind. As in previous sections, research literature is viewed through the twin lenses of *learners and learning* and *teachers and teaching*. Within these two areas, the review is organised under topic headings: numeracy and modelling; number and algebra; statistics and probability; and geometry and measurement.

4.1 Learners and learning

4.1.1 Numeracy and modelling

Several studies focussed on the development of the ability to apply mathematics to the real world through attention to numeracy or mathematical modelling approaches. Redmond et al. (2012) sought to compare the reflective writings of two cohorts of students (Year 4/5 and Year 8/9) who participated in a series of mathematical modelling challenges. Whilst the reflections of the younger cohort were results oriented, the older cohort's reflections spoke more to the affective domain, group processes, the use of technology, and the acquisition of mathematical knowledge. This study supports the idea that with scaffolding, middle years students can engage in reflective practice to develop mathematical modelling skills.

Taking a different perspective, Geiger et al. (2015) investigated the role of digital technologies in promoting effective numeracy teaching and learning. Drawing on data from a case study of a single teacher and her class of Year 8 students, possibilities for technology integration into classroom numeracy practice were illustrated and discussed. This research provided evidence of the positive influence of digital tools on students' development of skills, mathematical knowledge, dispositions, and orientation towards using mathematics critically.

4.1.2 Number and algebra

While there has been a focus on numeracy among Australasian educational jurisdictions, there has been continuing research into basic skills, such as number sense, that are an important foundation for the study of more abstract topics such as algebra.

In a study that investigated underlying skills in number sense such as counting and spatial skills among four year old children, Edens and Potter (2013) examined participants' patterns for engaging in spontaneous mathematical activities (primarily block construction and computer games) involving numerosity in free-play activity. Findings indicated that children who spontaneously focused on numerosity were advanced in their counting skills. Teacher rating of student motivation and interest was also correlated with improved counting and spatial skills, while child self-reports of persistence in mathematics correlated with improved spatial skills.

Research was conducted by Spencer (2013) into how iPads can support learning in number within the early years and made use of the App "Know Number Free" to assist young children to recognise and count numbers. The author claims the advantages of using iPads in promoting student learning in number include the personalisation of learning, providing opportunity for kinaesthetic and play-based learning, and motivation.

A study that linked understanding in number and algebra was conducted by Cooper (2012) and investigated the variety of ways pre-service primary teachers used an applet to demonstrate zero pairs as a mechanism for understanding algebra. In addition, Cooper also examined how the applet could be used as a tool for creating representational models, transforming the engagement, and bridging the gap between concrete and symbolic representations.

4.1.3 Statistics and probability

When considering the judgements that students make about differences in data sets, Stack and Watson (2013) discussed some of the difficulties students have in conceptualising probabilistic and statistical concepts. In this research, they observed that students developed visual and intuitive understandings of the issues of sampling, randomness and populations. Further, the researchers argue that to create meaningful learning experiences for students requires considerable teacher knowledge and skill, especially in regards to drawing out key themes from a range of diverse activities. In a complementary report that investigated opportunities for the early development of informal inference with *Tinkerplots*, Fitzallen and Watson (2014) presented two examples that challenged the placement of inference in the school curriculum. The first study investigated Grade 5 and 6 students' understanding of covariation; the second was the study of Grade 10 students' understanding of resampling. Common to both studies were the opportunities for the development of informal inference. In both cases, many students based their conclusions on visual aspects such as the shape of a simulated distribution rather than relying upon statistical measures for these conclusions.

In another study that used *Tinkerplots* as a digital tool to support research, Prodromou (2012) examined a pedagogical theory of introducing the classicist and the frequentist approach to probability by investigating important elements in Year 9 students' learning process while working with a combinatorial problem. Results indicated that after students had seen the systematic construction of the event space via combinatorial analysis, they viewed the sample space as an essential property that regulated the results of the distribution of each sum's theoretical frequency.

4.1.4 Geometry and measurement

Research within geometry and measurement was also conducted by a number of researchers. Yeh (2013) documented how an online learning environment, for investigating geometry and spatial abilities, transformed the ways that young children can mentally and visually construct virtual 3D objects using movements in both egocentric and fixed frames of reference (FOR). The research also included a focus on how the digital environment afforded the opportunity to integrate learning about 3D shapes with direction, location and movement. Findings suggested that Year 4 (aged 9) children can develop the capacity to construct a cube using egocentric and fixed FOR separately or in combination.

While findings that have implications for student learning are valuable, Lowrie et al. (2014) conducted research with the aim of providing insight into the challenges students face when attempting to respond to graphic-rich mathematics tasks – often used to make judgements about students' mathematical knowledge. This research compared Singaporean Grade 6 students' performance and strategy preference on two graphic-rich mathematics tasks, which were presented via pencil-and-paper and iPad modes. These researchers found statistically significant differences between students' performances on the two tasks, one in favour of the paper mode and the other in favour of the iPad. However, the students who possessed higher spatial ability were more likely to solve the tasks correctly in either mode. The implications of the study are timely given the fact that high-stakes tests are likely to be presented in a digital form in coming years.

4.1.5 Summary of learners and learning – subject

While this section does not appear to cover research that addresses the *exploit contrast of ideal and machine mathematics* aspect of the pedagogical map, there are certainly instances where digital technologies have allowed for the *rebalance emphasis on skill, concepts, and applications* with students. Research on modelling and numeracy (e.g., Geiger et al. 2015) offers two examples where applications of mathematics are facilitated by the use of technology and the work within statistics and probability (e.g., Stack and Watson 2013) demonstrates how digital tools can be used to promote deeper understanding of concepts such as resampling. Perhaps because of the focus on *learners and learning*, there have been no instances of research related to *build metacognition and overview* and this remains an important avenue for future research as this is a key teacher responsibility.

4.2 Teachers and teaching

4.2.1 Number and algebra

Goodwin and Gould (2014), investigating the use of technology to support number development in preschool students, identified features of instruction related to the use of Apps that supported or inhibited early number knowledge development. Features such as giving instantaneous feedback were effective for particular groups of students, while they also found that teachers needed to expand and consolidate students' on-screen learning.

The use of Computer Algebra Systems (CAS) continues to be a focus in ongoing research. Heid et al. (2013), examined the influence of CAS on the role of algebra in the school curriculum, and

indicated that changes occur not only in the tasks, but also effect instruction and ways of working within mathematics classrooms. They contend that school classrooms that incorporated CAS encouraged new explorations of mathematical invariants, active linking of dynamic representations, engagement with real data, and simulations of real and mathematical relationships. Further, CAS created opportunities to both extend some algebraic procedures while also introducing and assisting the exploration of new structures, offering a range of opportunities for the enrichment of multiple views of algebra.

In another study that focused on the use of CAS by students transitioning to tertiary mathematics, Varavsky (2012) reported findings of research which included two versions of Year 12 intermediate-level mathematics in the state of Victoria: one where students learn and are examined with CAS and another where students can only use scientific calculators. Their study compared the performance of 1 240 students as they transitioned to traditional university-level mathematics according to whether they learned intermediate mathematics with or without the assistance of a CAS. Varavsky (2012) concluded that students without CAS demonstrated improved performance; however, the most important factor affecting student first-year university performance was whether or not they had completed advanced-level mathematics studies in secondary school rather than whether or not they had used CAS.

Other work has explored the benefits of instructional approaches that link algebraic, graphic, and numeric representations through the use of software such as *GeoGebra*. For example, Aveni et al. (2014) investigated how a class of Year 9 students reacted to *GeoGebra* when using this software for the first time. The authors claim that as a result, there was a shift in students' use of more formal language and, while students were not immediately comfortable with the dynamic environment, demonstrated an enhanced understanding of linear relationships.

4.2.2 Statistics and probability

Digital tools were central to investigations into teachers' approaches to statistical inference and to assessment at secondary and tertiary levels.

Tinkerplots was utilised in an action research project involving two Year 10 teachers in the investigation of the implications of resampling (Watson and Chance 2012). The authors claim that the use of this software was effective in assisting students to build intuitions about the effect of sample size and within-group variability and notions of inference based on resampling. Watson (2013) also reported that students were able to gain intuition about what is required to accept or reject a claim made for a difference of two populations, without formal testing procedures. Further, the concrete-visual approach of *Tinkerplots* facilitated such intuition (Watson, 2014).

In attempting to meet the significant challenge of assessing students understanding of statistical concepts at tertiary level, Neumann et al. (2012) report on the use of interactive computer-based simulations in informal and formal assessment. The authors reported that students believed the simulations improved their understanding of statistics and its practical application, gave them a way to practice statistics, motivated them to complete course tasks, and was interesting and

engaging. Students attributed their perceived improvement to the immediate feedback provided by the simulation software.

4.2.3 Summary of teachers and teaching – subject

Again, there was no research reported on the aspect *exploit contrast of ideal and machine mathematics*, perhaps an indication that this element is somewhat opportunistic in nature, rather than something teachers and/or researchers plan for. Most research in this section was related to *rebalance emphasis on skill, concepts, and applications* with the work of Heid et al. (2013) indicating clear advantages for teachers to create lessons in which students explore mathematics at a conceptual level, and engage with real data through the use of CAS. This potential should be considered with some caution given Varavsky's finding that the most important factor affecting student first-year university performance was whether or not students had completed advanced-level mathematics studies in secondary school rather than their use of CAS. Research in statistics education appears to address the aspect of *build metacognition and overview* as, for example, the work of Watson and Chance (2012) required students to begin with life related data sets in developing a conceptual understanding of resampling – approaching this topic from a different point than may be the case in many classrooms.

5 Conclusions and Future Directions for Research

This review has shown that there is a broad research agenda around technology and mathematics education. This agenda includes investigations into the use of a wide variety of digital tools for the purpose of improving the teaching and learning of mathematics. Pierce and Stacey's (2010) map of pedagogical opportunities has been used to guide the review of literature in this field by considering *tasks, classroom* and *subject* and the finer grained aspects of these dimensions. While not a specific focus of this review, it seems apparent that the various aspects of the *pedagogical map* are more applicable to either *learners and learning* or *teachers and teaching*, suggesting that the development of a pedagogical map for each of these perspectives might be a worthwhile research exercise. There were also areas where the map might be extended, especially in relation to online learning and the opportunities provided by virtual learning environments, somewhat unsurprisingly, since the *pedagogical map* was developed to identify and describe opportunities made available by proximate technologies situated within conventional learning spaces. The extension of the pedagogical map to accommodate a broader range of technologies is an important future research enterprise. These suggestions are part of a larger issue of developing theoretical frameworks that can be used to guide research in the area of digital technologies and mathematics education.

The use of mobile digital technologies is gaining increasing prominence in both the broader community and also in mathematics classrooms. In addition, the plethora of Apps available for these technologies, including games that potentially embed aspects of mathematics that can be exploited by teachers to promote student learning, require further investigation.

A somewhat overlooked aspect of the role of digital tools in mathematics education is the use of technology in teacher pre-service and in-service learning. This review provides insights into the challenges facing teachers when introducing or expanding technology integrated mathematics

classrooms. While some studies indicate there is great potential for technology to mediate teacher professional learning, it has been observed that teachers can adopt technologies in a manner that reinforces their current practice rather than transforms it (Pierce and Bardini 2015).

Teaching and learning with online environments has clearly taken a foothold and will likely expand – in secondary and tertiary contexts at least. How new and emerging technologies mediate productive student-student and student-teacher interaction within these environments is an ongoing theme within research. These environments demand new types of interaction and collaboration between students and students and students and teachers, especially as digital tools become “smarter”.

How to improve teaching and learning within specific subject disciplines is an ongoing topic of research, especially as product development in mathematics enabled technologies continues. While research into the use of existing technologies such as CAS, *Tinkerplots*, and *GeoGebra* are of current interest; new or as yet unknown technologies, offering different affordances and constraints, will continue to emerge into the future.

In considering how and in which direction the field might progress, attention must also be paid to the equity issues that emerge when teaching and learning with digital tools (Bardini et al. 2013), particularly in rural and remote communities (e.g., Goodwin and Gould, 2014; Lowrie and Jorgensen, 2012). Issues of access remain a point of division between the advantaged and the disadvantaged in all aspects of education and thus work must be done to minimise inequity exacerbated by differing levels of access to digital technologies.

This review has provided insight into future directions for research into technology enhanced mathematics education including the need to:

- ☐ develop principles that help teachers discern the quality of Apps and other software
- ☐ provide insight into how to participate in and manage interactions and relationships in online environments
- ☐ explore the potential of virtual worlds and technologies that allow students to design or influence technology formed environments
- ☐ investigate new modes of instruction, for example, blended learning and flipped classroom approaches
- ☐ document the role of digital technologies in pre-service and in-service mathematics teacher education
- ☐ explore the notion of *futures* in relation to new digital tools
- ☐ investigate new ways of structuring teaching approaches that leverage off the affordances of digital tools.

References

Allmond, S., & Makar, K. (2014). From hat plots to box plots in Tinkerplots: Supporting students to write conclusions which account for variability in data. In K. Makar, B. da Sousa,

- & R. Gould (Eds.), *Proceedings of the Ninth International Conference on Teaching Statistics* (pp. 1-6). International Association for Statistical Education. Retrieved from http://icots.info/9/proceedings/pdfs/ICOTS9_2E1_ALLMOND.pdf
- Attard, C., & Curry, C. (2012). Exploring the Use of iPads to Engage Young Students with Mathematics. In J. Dindyal, L. P. Cheng, & S. F. Ng (Eds.), *Mathematics education: Expanding horizons. Proceedings of the 35th annual conference of the Mathematics Education Research Group of Australasia* (pp. 75-82). Singapore: MERGA.
- Attard, C., & Orlando, J. (2014). Early career teachers, mathematics and technology: Device conflict and emerging mathematical knowledge. In J. Anderson, M. Cavanagh, & A. Prescott (Eds.), *Curriculum in focus: Research guided practice. Proceedings of the 37th annual conference of the Mathematics Education Research Group of Australasia* (pp. 71-78). Sydney: MERGA.
- Aventi, B., Serow, P., & Tobias, S. (2014). Linking geogebra to explorations of linear relationships. In J. Anderson, M. Cavanagh, & A. Prescott (Eds.), *Curriculum in focus: Research guided practice. Proceedings of the 37th annual conference of the Mathematics Education Research Group of Australasia* (pp. 79-86). Sydney: MERGA.
- Bardini, C., Oldenburg, R., Stacey, K., & Pierce, R. (2013). Technology prompts new understandings: The case of equality. In V. Steinle, L. Ball, & C. Bardini (Eds.), *Mathematics education: Yesterday, today and tomorrow. Proceedings of the 36th annual conference of the Mathematics Education Research Group of Australasia* (pp. 82-89). Melbourne: MERGA.
- Bate, F. G., Day, L., & Macnish, J. (2013). Conceptualising changes to pre-service teachers' knowledge of how to best facilitate learning in mathematics: A TPACK inspired initiative. *Australian Journal of Teacher Education*, 38(5), 14-30.
- Beswick, K., & Muir, T. (2013). Making connections: Lessons on the use of video in pre-service teacher education from the trial of particular protocol. *Mathematics Teacher Education and Development*. 15(2), 27-51.
- Brown, J. P. (2015a). Complexities of digital technology use and the teaching and learning of function. *Computers & Education*, 87, 112-122.
- Brown, J. P. (2015b). Visualisation tactics for solving real world tasks. In G. Stillman, W. Blum, & M.S. Biembengut (Eds.), *Mathematical modelling in education research and practice: Cultural, social and cognitive influences* (pp. 431-442). Cham, Switzerland: Springer.
- Brown, J. P. (2013). Teaching roles in technology-rich teaching and learning environments. In V. Steinle, L. Ball & C. Bardini (Eds.), *Mathematics education: Yesterday, today and tomorrow. Proceedings of the 36th annual conference of the Mathematics Education Research Group of Australasia* (pp. 106-113). Melbourne: MERGA.
- Brown, J., & Stillman, G. (2014). Affordances: Ten years on. In J. Anderson, M. Cavanagh, & A. Prescott (Eds.), *Curriculum in focus: Research guided practice. Proceedings of the 37th annual conference of the Mathematics Education Research Group of Australasia* (pp. 111-118). Sydney: MERGA.
- Calder, N. (2012). The Layering of mathematical interpretations through digital media. *Educational Studies in Mathematics*, 80(1), 269-285.
- Calder, N. (2015). Apps: Appropriate, applicable and appealing? In T. Lowrie & R. Jorgensen (Eds.), *Digital games and mathematics learning: Potential, promises and pitfalls* (pp. 233-250). The Netherlands: Springer.
- Calder, N. S., & Campbell, A. (2015). "You play on them. They're active." Enhancing the mathematics learning of reluctant teenage students. In M. Marshman, V. Geiger, & A.

- Bennison (Eds.), *Mathematics education in the margins. Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia* (pp. 133–140). Sunshine Coast: MERGA.
- Cooper, T. E. (2012). Using virtual manipulatives with pre-service mathematics teachers to create representational models. *The International Journal for Technology in Mathematics Education*, 19(3), 105-115.
- Edens, K. M., & Potter, E. F. (2013). An exploratory look at the relationships among math skills, motivational factors and activity choice. *Early Childhood Education Journal*, 41(3), 235-243.
- Engelbrecht, J., & Harding, A. (2005). Teaching undergraduate mathematics on the internet. *Educational Studies in Mathematics*, 58(2), 253-276.
- Fitzallen, N., & Watson, J. (2014). Extending the curriculum with *TinkerPlots*: Opportunities for early development of informal inference. In K. Makar, B. de Sousa, & R. Gould (Eds.), *Sustainability in statistics education. Proceedings of the Ninth International Conference on Teaching Statistics* (pp. 1-6). Voorburg: International Statistical Institute.
- Fun with Construction. (2011). [Software]. Singapore: Heulab Pte. Ltd.
- Galligan, L., & Hobohm, C. (2013). Students using digital technologies to produce screencasts that support learning in mathematics. In V. Steinle, L. Ball, & C. Bordini (Eds.), *Mathematics education: Yesterday, today and tomorrow. Proceedings of the 36th annual conference of the Mathematics Education Research Group of Australasia* (pp. 322-329). Melbourne: MERGA.
- Galligan, L., Hobohm, C., & Loch, B. (2012). Tablet technology to facilitate improved interaction and communication with students studying mathematics at a distance. *Journal of Computers in Mathematics and Science Teaching*, 31(4), 363-385.
- Geiger, V. (2009). The master, servant, partner, extension-of-self framework in individual, small group and whole class contexts. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides. Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia* (pp. 201-208). Wellington: MERGA.
- Geiger, V. (2013). Mathematical applications, modelling and technology as windows into industry based mathematical practice. In A. Damlamian, J. F. Rodrigues, & R. Straesser (Eds.), *Educational interfaces between mathematics and industry: The 20th ICMI study* (pp. 271-278). New York: Springer.
- Geiger, V., Forgasz, H., Tan, H., Calder, N., & Hill, J. (2012). Technology in Mathematics education. In B. Perry, T. Lowrie, T. Logan, A. MacDonald, & J. Greenless (Eds.), *Research in mathematics education in Australasia 2008-2011* (pp. 111-142). The Netherlands: Sense Publishers.
- Geiger, V., Goos, M., & Dole, S. (2015). The role of digital technologies in numeracy teaching and learning. *International Journal of Science and Mathematics Education*, 13(5), 1115-1137.
- Goodwin, K., & Gould, P. (2014). Race in the outback: Investigating technology designed to support number development in a preschool serving an under-resourced community. In J. Anderson, M. Cavanagh, & A. Prescott (Eds.), *Curriculum in focus: Research guided practice. Proceedings of the 37th annual conference of the Mathematics Education Research Group of Australasia* (pp. 247–254). Sydney: MERGA.
- Goos, M. (2013). Technology integration in secondary school mathematics: The development of teachers' professional identities. In A. Clark-Wilson, O. Robutti, & N. Sinclair (Eds.), *The Mathematics Teacher in the Digital Era* (pp. 139-161). The Netherlands: Springer DOI 10.1007/978-94-007-4638-1_7

- Goos, M. (2012). Digital technologies in the Australian Curriculum: Mathematics - A lost opportunity? In B. Atweh, M. Goos, R. Jorgensen, & D. Siemon (Eds.), *Engaging the Australian Curriculum Mathematics - Perspectives from the field* (pp. 135-152). Retrieved from <http://www.merga.net.au/sites/default/files/editor/books/1/Chapter%207%20Goos.pdf>
- Goos, M., & Cretchley, P. (2004). Computers, multimedia, and the internet in mathematics education. In B. Perry, G. Anthony, & C. Diezmann (Eds.), *Research in Mathematics Education in Australasia 2000-2003* (pp. 151-174). Sydney: MERGA.
- Goos, M., Galbraith, P., Renshaw, P., & Geiger, V. (2003). Perspectives on technology mediated learning in secondary school mathematics classrooms. *Journal of Mathematical Behavior*, 22, 73-89.
- Goos, M., & Geiger, V. (2012). Connecting social perspectives on mathematics teacher education in online environments. *ZDM*, 44(6), 705-715.
- Handal, B., Campbell, C., Cavanagh, M., Petocz, P., & Kelly, N. (2013). Technological pedagogical content knowledge of secondary mathematics teachers. *Contemporary Issues in Technology and Teacher Education*, 13(1), 22-40.
- Heid, K. M., Thomas, M. O. T., & Zbiek, R. (2013). How might computer algebra systems change the role of algebra in the school curriculum? In K. Clements., A. Bishop., C. Keitel, J. Kilpatrick., & F. Leung. (Eds.), *Third International Handbook of Mathematics Education* (pp. 597-641). New York: Springer.
- Konold, C., & Miller, C. (2005). Tinkerplots (Software v1.0). Emeryville, CA: KCP Technologies.
- Larkin, K. (2015). "An app! An app! My kingdom for an app": An 18-month quest to determine whether apps support mathematical knowledge building. In T. Lowrie & R. Jorgensen (Eds.), *Digital games and mathematics learning: Potential, promises and pitfalls*. (Vol. 4, pp. 251-276): Netherlands: Springer.
- Larkin, K., & Jamieson-Proctor, R. (2013). Transactional distance theory (TDT): An approach to enhancing knowledge and reducing anxiety of pre-service teachers studying a mathematics education course online. In V. Steinle, L. Ball, & C. Bardini (Eds.), *Mathematics education: Yesterday, today and tomorrow. Proceedings of the 36th annual conference of the Mathematics Education Research Group of Australasia* (pp. 434-441). Melbourne: MERGA.
- Larkin, K., & Jamieson-Proctor, R. (2015). Using transactional distance theory to redesign an online mathematics education course for pre-service primary teachers. *Mathematics Teacher Education and Development*, 17(1), 44-61.
- Larkin, K., Jamieson-Proctor, R., & Finger, G. (2012). TPACK and pre-service teacher mathematics education: Defining a signature pedagogy for mathematics education using ICT and based on the metaphor "mathematics is a language". *Computers in the Schools*, 29(1), 207-226. doi:10.1080/07380569.2012.651424
- Larkin, L., & Jorgensen, R. (2014). Using video diaries to record student attitudes and emotions towards mathematics in year three and year six students. In J. Anderson, M. Cavanagh, & A. Prescott (Eds.), *Curriculum in focus: Research guided practice. Proceedings of the 37th annual conference of the Mathematics Education Research Group of Australasia* (pp. 373-380). Sydney: MERGA.
- Loong, E. Y. K., & Herbert, S. (2012). Student perspectives of web-based mathematics. *International Journal of Educational Research*, 53, 117-126.

- Lowrie, T., & Jorgensen, R. L. (2012). The tyranny of remoteness: Changing and adapting pedagogical practices in distance education. *International Journal of Pedagogies and Learning*, 7(1), 1-8. doi:10.5172/ijpl.2012.7.1.1
- Lowrie, T., Jorgensen, R., & Logan, T. (2013). Navigating and decoding dynamic maps. *Australian Journal of Educational Technology*, 29(5), 626-639.
- Lowrie, T., Ramful, A., Logan, T., & Ho, S. Y. (2014). Do students solve graphic tasks with spatial demands differently in digital form? In J. Anderson, M. Cavanagh, & A. Prescott (Eds.), *Curriculum in focus: Research guided practice. Proceedings of the 37th annual conference of the Mathematics Education Research Group of Australasia* (pp. 429-436). Sydney: MERGA.
- Martin, D. (2012). Rich assessment in a first-year, teacher education (primary) mathematics education subject. *International Journal of Pedagogies and Learning*, 7(1), 62-72.
- McDonald, S. (2012). The challenge to situate digital learning technologies in preservice teacher mathematics education. *Contemporary Issues in Technology and Teacher Education*, 12(4), 355-368.
- Mojica-Casey, M., Dekkers, J., & Thrupp, R. (2014). Research guided practice: Student online experiences during mathematics class in the middle school. In J. Anderson, M. Cavanagh, & A. Prescott (Eds.), *Curriculum in focus: Research guided practice. Proceedings of the 37th annual conference of the Mathematics Education Research Group of Australasia* (pp. 469-476). Sydney: MERGA.
- Muir, T. (2012). Virtual mathematics education: Using Second Life to model and reflect upon the teaching of mathematics. In J. Dindyal, L. P. Cheng, & S. F. Ng (Eds.), *Mathematics education: Expanding horizons. Proceedings of the 35th annual conference of the Mathematics Education Research Group of Australasia* (pp. 521-528). Singapore: MERGA.
- Muir, T., & Chick, H. (2014). Flipping the classroom: A case study of mathematics methods class. In J. Anderson, M. Cavanagh, & A. Prescott (Eds.), *Curriculum in focus: Research guided practice. Proceedings of the 37th annual conference of the Mathematics Education Research Group of Australasia* (pp. 485-492). Sydney: MERGA.
- Neumann, D. L., Hood, M. H., & Neumann, M. M. (2012). An evaluation of computer-based interactive simulations in the assessment of statistical concepts. *The International Journal for Technology in Mathematics Education*, 19(1), 17-23.
- O'Rourke, J., Main, S., & Ellis, M. (2013). 'It doesn't seem like work, it seems like good fun': Perceptions of primary students on the use of handheld game consoles in mathematics classes. *Technology, Pedagogy and Education*, 22(1), 103-120. doi:10.1080/1475939X.2012.733537
- Pierce, R., & Bardini, C. (2015). Computer algebra systems: Permitted but are they used? *Australian Senior Mathematics Journal*, 29(1), 32-42.
- Pierce, R., & Stacey, K. (2010). Mapping pedagogical opportunities provided by mathematics analysis software. *International Journal of Computers for Mathematics Learning*, 15(1), 1-20.
- Pierce, R., & Stacey, K. (2013). Teaching with new technology: Four 'early majority' teachers. *Journal of Mathematics Teacher Education*, 16(5), 323-347. doi:10.1007/s10857-012-9227-y
- Prieto, E., & Holmes, K. (2014). Online Students' perceptions of interactive tools to support postgraduate learning of mathematics. In J. Anderson, M. Cavanagh, & A. Prescott (Eds.), *Curriculum in focus: Research guided practice. Proceedings of the 37th annual conference of the Mathematics Education Research Group of Australasia* (pp. 525-533). Sydney: MERGA.
- Prodromou, T. (2012). The classicist and the frequentist approach to probability within a Tinkerplots2 combinatorial problem. In J. Dindyal, L. P. Cheng, & S. F. Ng (Eds.),

- Mathematics education: Expanding horizons. Proceedings of the 35th annual conference of the Mathematics Education Research Group of Australasia* (pp. 610-617). Singapore: MERGA.
- Redmond, T., Brown, R., Sheehy, J., & Kanasa, H. (2012). Exploring student reflective practice during a mathematical modelling challenge. In J. Dindyal, L. P. Cheng, & S. F. Ng (Eds.), *Mathematics education: Expanding horizons. Proceedings of the 35th annual conference of the Mathematics Education Research Group of Australasia* (pp. 641-647). Singapore: MERGA.
- Rogers, E. (1995). *Diffusion of innovations*. New York: The Free Press.
- Spencer, P. (2013). iPads: Improving numeracy learning in the early years. In V. Steinle, L. Ball, & C. Bordini (Eds.), *Mathematics education: Yesterday, today and tomorrow. Proceedings of the 36th annual conference of the Mathematics Education Research Group of Australasia* (pp. 610-617). Melbourne: MERGA.
- Stack, S., & Watson, J. (2013). Randomness, sample size, imagination and metacognition: Making judgments about differences in data sets. *Australian Mathematics Teacher*, 69(4), 23-30.
- Tan, H. (2012). Students' ways of knowing and learning mathematics and their ways of interacting with advanced calculators. In J. Dindyal, L. P. Cheng, & S. F. Ng (Eds.), *Mathematics education: Expanding horizons. Proceedings of the 35th annual conference of the Mathematics Education Research Group of Australasia* (pp. 704- 711). Singapore: MERGA.
- Valsiner, J. (1997). *Culture and the development of children's action: A theory of human development* (2nd ed.). New York: Wiley.
- Varsavsky, C. (2012). Use of CAS in secondary school: A factor influencing the transition to university-level mathematics? *International Journal of Mathematical Education in Science and Technology*, 43(1), 33-42.
- Watson, J., & Chance, B. (2012). Building intuitions about statistical inference based on resampling. *Australian Senior Mathematics Journal*, 26(1), 6-18.
- Watson, J. M. (2013). Resampling with TinkerPlots. *Teaching Statistics*, 35(1), 32-36.
- Watson, J. M. (2014). *TinkerPlots* as an interactive tool for learning about resampling. In T. Wassong, D. Frischmeier, P. R. Fischer, R. Hochmuth, & P. Bender (Eds.), *Using tools for learning mathematics and statistics* (pp. 421-436). Heidelberg: Springer Spektrum.
- Whyburn, L., & Way, J. (2012). Student perceptions of the influence of IWBs on their learning in mathematics. *Australian Educational Computing*, 27(1), 23-27.
- Yeh, A. (2013). Constructing a frame of a cube: Connecting 3D shapes with direction, location and movement. In V. Steinle, L. Ball, & C. Bordini (Eds.), *Mathematics education: Yesterday, today and tomorrow. Proceedings of the 36th annual conference of the Mathematics Education Research Group of Australasia* (pp. 690-697). Melbourne : MERGA.
- Yeh, A., & Chandra, V. (2015). Mathematics, programming, and STEM. In M. Marshman, V. Geiger, & A. Bennison (Eds.), *Mathematics education in the margins. Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia* (pp. 659-666). Sunshine Coast: MERGA.
- Zuber, E. N., & Anderson, J. (2013). The initial response of secondary mathematics teachers to a one-to-one laptop program. *Mathematics Education Research Journal*, 25(4), 279-298.