Extended Carmeli's Synchronization Measure for Noise Reduction Assessment Methodology

P. Celka

¹ Griffith University, School of Engineering, Gold Coast, Australia

Abstract—Designing a Quality measure for noise reduction methods for real life data is a significant challenge. The classical mean square error fails at dealing with fine details of signal patterns. We have used and developed a new way of measuring the quality of the noise reduction scheme using a synchronization measure in the state space. We have applied the method to highly noisy ECG signal measured on rowers. Performance of the new measure is assessed on synthetic data and results on rower's ECG signal before and after noise reduction is presented.

Keywords— Synchronisation, state space, noise reduction, performance measure

I. INTRODUCTION

Synchronization of biological signals is now known to be a key factor for information processing and extraction. For these reasons, scientists and engineers have been working at understanding the underlying mechanisms of synchronization in biological systems. Biomedical engineers have similarly been working towards developing signal processing methods that could capture the synchronization level in multidimensional signals. N:m phase locking, mean phase locking value, coherence and mutual information had provided and still provide excellent tools for performing these tasks, which are now parts of clinical, physiological and psychological research daily life practice.

A recent synchronization measure has been developed by Carmeli *et al.* [1] in the context of electroencephalographic signal processing. The method is based on a spatial state-space analysis of the data using principal component analysis, together with a new entropy-like synchronization index. It is the aim of this paper to extend the use of this index for spatio-temporal data and show its usefulness in the context of noise reduction assessment methodology [2]. Validation results on synthetic data are provided and compared with the original Carmeli's measure. We finally show the results on low signal-to-noise ratio electrocardiogram noise reduction for rowers' subjects.

II. METHOD

The ECG and EMG signals have been recorded using a 16 bit resolution PowerLab © system and tape 3M dot-like sensors. A band-pass filter with cut-off frequencies 0.1Hz and 1000Hz has been used for the anti-aliasing filtering. Sampling frequency was set to 4000Hz for both signals. Signals were then resampled at $F_s=1000\text{Hz}$ for further digital processing. The EMG sensors are disposed on the opposite chest site from the ECG ones in order to have a maximum correlation with the ECG movement and muscle activity. Five rowers were selected to perform the tests (age 25 ± 3 years; i.e. mean $\pm$ standard deviation). The following protocol was designed: 1min resting, 2min rowing at 15stroke/min, 2min resting, 2min rowing at 25strokes/min, and finally 2min resting. Data were then analysed and processed using the Matlab© software. We finally use the EMG signal to reduce the muscle and motion artifact in the ECG signal using a normalized least mean square adaptive algorithm [2].

III. COMPUTING A STATE SPACE SYNCHRONISATION INDEX

A. Motivation

Most noise reduction validation methods are based on the computation of a mean square error (MSE) or some function of it. The MSE is computed between a given or constructed signal that serves as a reference signal, and the noise reduced signal. The difficulty with this assessment method comes from reliance on the stationarity assumption of the PQRST complexes. This assumption is violated in all circumstances due to the very nature of the time variability of the heart function. Also, the MSE is by no means a good measure when subtle amplitude fluctuations or phase information have to be retrieved. In order to overcome this difficulty, we have used a state space approach. This approach is based on a nonlinear technique called embedding and al-

allows us to reconstruct the state space trajectory of the underlying dynamic system from which we have measured a signal, i.e. the ECG. The trajectory in the state space is time/frequency independent and thus enables us to compare different PQRST complexes with different time-frequency patterns and be more robust against stationarity. We have used an embedding dimension of $m=2$ and an embedding delay J/Fs estimated using the first zero-crossing of the auto-covariance function. The average embedding delay was about $\tau=1/2*40=12.5ms$. Assuming that the typical bandwidth of an ECG is $40Hz$. The delay embedding actually performs a natural sub-sampling of the signals and thus a time re-scaling by a factor of J when the sampling frequency is kept constant. In order to compare different embedded PQRST complexes, we have zero-padded the signals such that they have the same number of samples.

Each PQRST complex $QRS_i(k)$, for $i=1, \dots, K$, was automatically detected using a method similar to the one proposed by Afonso. We have used a $600ms$ window centred around each peak-R waves ($L=0.6/Fs$ samples) to define the signals $QRS_i(k)$. The $m=2$ dimensional state space vector corresponding to $QRS_i(k)$ is $s_i(k)=(x_i(k), y_i(k))^T$ with $x_i(k)=QRS_i(k)$, and $y_i(k)=QRS_i(k-J)$ the first and second coordinate respectively. The time index k now runs from 1 to $L-(m-1)J$ due to the embedding.

B. A modified S index

In order to assess the performance of the noise reduction scheme, we have to 'compare' the original and noise reduced complexes with a so-called PQRST *template* $QRS_T(k)$ in the state space: a state space validation measure. Similarly to $QRS_i(k)$, we define $s_T(k)=(x_T(k), y_T(k))^T$. This template was manually extracted from the signals during the first rest period (see Section II for the protocol). The state space validation measure is based on a distance between the two vectors $s_i(k)$ and $s_T(k)$: a synchronization measure S .

Our state space validation measure is based on a multi-dimensional synchronisation index $0 < S < 1$ developed by Carmeli *et al.* [1]. This method is based on the analysis of the eigenvalues of the auto-covariance matrix of the multi-dimensional signal. This method, while originally proposed in a multidimensional state space context, suffers from two drawbacks for our application. The first is the fact that a constant phase shift between the state space coordinates reduces S , while we expect it to be phase shift invariant: i.e. it is not invariant under time shift. In our context, synchronization is thought to be more like a phase locking, rather than an instantaneous time-locking of the trajectories in the state space. The second drawback is that the embedding of a signal artificially creates correlated components and thus biases the synchronisation index S . This fact has already been pointed out by Carmeli *et al.* [1]. Indeed, the so-called

Takens embedding is constructed using delayed copies of the signals, which are thus automatically correlated with each other. In order to avoid this problem, we compute the synchronization index S using similar state space coordinates: along coordinates x and y separately (see eq. (3)). A detailed analysis of the S index is beyond the scope of this work, but we illustrate these facts by performing two experiments with the following two signals:

$$u(t) = \text{Re} \left(e^{j\omega_1 t - 10(t-\tau_1)^2} + e^{j\omega_2 t - (t-\tau_2)^2} \right) \quad (1)$$

$$v(t) = \text{Re} \left(e^{j\omega_1 t + \phi - 10(t-\tau_1)^2} + e^{j\omega_2 t + \phi - (t-\tau_2)^2} \right) + \eta(t) \quad (2)$$

Where $\omega_1 = 2\pi 10 \text{ rad/s}$, $\omega_2 = 2\pi 100 \text{ rad/s}$, $\tau_1 = 50 \text{ ms}$, $\tau_2 = 70 \text{ ms}$, $t = k/F_s$ ($k=1, \dots, L$), and $F_s = 3000 \text{ Hz}$. The signal $u(t)$ plays the role of $QRS_T(k)$ and $v(t)$ of $QRS_i(k)$. Both signals were generated over a duration of 200 ms , and $\eta(t)$ is a Gaussian white noise signal. The SNR was set to 20 dB . Note that the constant phase factor is applied only to the harmonic part of the signal $v(t)$.

We have first tested the effect of the phase on the S index using an embedding of $m=2$ and a lag of $J=1$, for a phase ϕ that varies from 0 to $\pi/2$. The result is displayed in Figure 1 and shows that Carmeli's S index (dash-dot line) is sensitive to a constant phase shift. The fact that $S < 1$ for $\phi=0$ shows the influence of the noise.

Fig.1 Carmeli's S index (dash-dot line) and the modified S index (solid line) for an increasing phase difference ϕ

Fig. 2 Carmeli's S index (dash-dot line) and the modified S index (solid line) for an increasing embedding dimensions m and lag J . We have chosen $\phi = \pi/8$ to see the embedding parameters' influence exclusively. Note that all the lines for the modified S index are almost superimposed

We have also tested the influence of embedding parameters on the Carmeli's S index, using a set of embedding dimension $1 \leq m \leq 20$ and a set of lag $1 \leq J \leq 5$. The constant phase was set to $\pi/8$. The result is shown in Figure 2 and shows that Carmeli's S index (dash-dot line) varies as a function of both parameters m and J in a quite complex way, revealing the care that should be taken when using this measure in state space. We thus propose to modify the S index by: 1) introducing constant phase compensation between coordinates of the respective state space vectors and 2) computing the S index along each pair of state space coordinates separately. This is done by computing the instantaneous phases $\Phi_{x_{i(T)}}(k)$ and $\Phi_{y_{i(T)}}(k)$ of each signal $x_{i(T)}(k)$ and $y_{i(T)}(k)$ respectively, using the Hilbert transform and time averaging values $\bar{\Phi}_{x_{i(T)}}(k) = \langle \Phi_{x_{i(T)}}(k) \rangle$ and $\bar{\Phi}_{y_{i(T)}}(k) = \langle \Phi_{y_{i(T)}}(k) \rangle$ (The symbol $\langle . \rangle$ is meant for the time-average of the signal). Each signal is then converted to its analytical form $x_{i(T)}^a(k)$ and $y_{i(T)}^a(k)$ respectively. A set of two vectors is constructed as follows:

$$\mathbf{X}_i^a(k) = \begin{pmatrix} x_T^a(k), x_i^a(k) \end{pmatrix}^T; \quad \mathbf{Y}_i^a(k) = \begin{pmatrix} y_T^a(k), y_i^a(k) \end{pmatrix}^T \quad (3)$$

We then construct the 4×4 transformation matrix $\mathbf{U}_i$:

$$\mathbf{U}_i = \begin{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{-j(\bar{\Phi}_{x_T} - \bar{\Phi}_{x_i})} \end{pmatrix} & \mathbf{0} \\ \mathbf{0} & \begin{pmatrix} 1 & 0 \\ 0 & e^{-j(\bar{\Phi}_{y_T} - \bar{\Phi}_{y_i})} \end{pmatrix} \end{pmatrix} \quad (4)$$

with $\mathbf{0}$ the 2×2 zero matrix, and transform the vectors

$\mathbf{X}_i^a(k)$ and $\mathbf{Y}_i^a(k)$:

$$\begin{pmatrix} \tilde{\mathbf{X}}_i^a(k) \\ \tilde{\mathbf{Y}}_i^a(k) \end{pmatrix} = \mathbf{U}_i \begin{pmatrix} \mathbf{X}_i^a(k) \\ \mathbf{Y}_i^a(k) \end{pmatrix} \quad (5)$$

The following covariance matrices are then constructed,

using the real part of the vectors $\tilde{\mathbf{X}}_i(k) = \text{Re}(\tilde{\mathbf{X}}_i^a(k))$

and $\tilde{\mathbf{Y}}_i(k) = \text{Re}(\tilde{\mathbf{Y}}_i^a(k))$

$$\mathbf{C}_{xx_i} = \left(\sum_{k=1}^{L-(m-1)J} \tilde{\mathbf{X}}_i(k) \tilde{\mathbf{X}}_i^T(k) \right) / (L - (m-1)J) \quad (6)$$

$\mathbf{C}_{yy_i}$ is computed the same way. The eigenvalue matrices are: $\Lambda x_i = \text{diag}(\lambda x_{i,l})$ and $\Lambda y_i = \text{diag}(\lambda y_{i,l})$ for $l=1,2$. We then normalize the eigenvalue matrices with respect to the total power along each coordinates: $\tilde{\Lambda} x_i = \Lambda x_i / \text{Tr}(\Lambda x_i)$

and ($\text{Tr}()$ is the trace operator), and finally compute the synchronisation index along each coordinates x_i and y_i :

$$S(x_i) = 1 + \text{Tr}(\tilde{\Lambda} x_i \ln(\tilde{\Lambda} x_i)) / \ln(2) \quad (7)$$

$S(y_i)$ is computed similarly. An average synchronisation index is computed for each state space coordinate of all PQRST complexes, respectively $S_A(x) = \sum_i S(x_i) / K$ and $S_A(y) = \sum_i S(y_i) / K$, and the final S index is computed as an average over all state space coordinates: i.e. x and y :

$$S = (S_A(x) + S_A(y)) / 2 \quad (8)$$

Due to the normalisation of the eigenvalues, the S index is independent of the amplitude of the signals. The modified S index computes a state space average of synchronization index between individual state space coordinates. Instead, the original Carmeli's S index looks at the synchronization between all the state space coordinates. Figure 2 and Figure 3 (solid lines) show that the proposed modified S index is phase- and embedding-parameters independent, which is the expected result. The modified S index is sensitive to the noise level: it decreases with a decrease of the SNR, but it does so in a quite uniform manner across all values of m and J . To make the method more user and noise independent we have estimated J with the first zero-crossing of the autocovariance function which allows for *re-sampling* the signals so that they have the minimum redundancy. When the embedding delay J is so chosen, the modified S index is time-scale invariant. The embedding dimension m has been estimated using a modified version of the false nearest neighbors [3]. We use these two methods for the rest of the paper when estimating S . From this perspective, S is a complementary performance measure of the classical mean square error (MSE), which is time-scale, time-shift and amplitude invariant.

IV. INFLUENCE OF THE ECG PERTURBATIONS ON THE S INDEX

The first set of assessment includes the influence of the movement and muscle disturbance on the S index on the noise reduced signals. Indeed, the movement and muscle artefacts distort the PQRST complex and should be reflected on the S index. The clinicians are interested by the analysis of these waves and in particular the time intervals RR, PQ and QT for the location of the P, Q, R and T waves). In order to assess this, we have performed a series of tests using the adaptive algorithm [2] on three different level of output noise as presented in this section. We have performed an automatic PQR and T wave detection on each of the following signals: 1) the noise-free synthetic ECG

generated from a dynamical model [4], 2) the ECG cleaned with the method in [2], 3) the ECG cleaned using a local nonlinear projection (LNLP) method [5,6]. We have fixed the linear FIR filter order to $N=5$ and $\mu=0.0049$ for the adaptive NLMS algorithm.

The LNLP method and the adaptive methods were then used to reduce the disturbance, and further compared in terms of the following clinical parameters mean square errors: $\sigma_{a,nl}^P = \text{sqr}t(<(P - P_{a,nl})^2>)$ where $P=QT, PQ$ and RR ; and P_a is meant for the P time intervals after noise reduction using the adaptive scheme. P_{nl} is meant for the P time intervals after noise reduction using the LNLP. Table 1 reveals that the adaptive algorithm is actually outperforming the LNLP in terms of recovering the PQRT waves and subsequently the time series QT, PQ and RR. Table 2 shows that the S index is lower for the LNLP as compared with the adaptive algorithm. This clearly indicates that the S index reflects the quality of the PQRT wave recovering during the process of noise reduction.

Table 1 Clinical parameters fluctuations as a function of the SNR [dB]. Time series interval standard deviations are expressed in ms

	SNR=-2.5	SNR=1.5	SNR=5
σ_{nl}^{PQ}	34.9	50.0	42.0
σ_a^{PQ}	10.9	7.5	6.7
σ_{nl}^{QT}	36.6	33.5	31.3
σ_a^{QT}	12.1	10.0	6.6
σ_{nl}^{RR}	$5.1 \cdot 10^{-3}$	$6.3 \cdot 10^{-3}$	$5.6 \cdot 10^{-3}$
σ_a^{RR}	$5.1 \cdot 10^{-3}$	$6.3 \cdot 10^{-3}$	$5.4 \cdot 10^{-3}$

Table 2 S index as a function of the SNR [dB]. S_a and S_{nl} refer to the S index computed for the adaptive and the LNLP respectively, while S_o has been computed on the clean ECG

	SNR=-2.5	SNR=1.5	SNR=5
S_o	0.92	0.90	0.94
S_{nl}	0.66	0.62	0.68
S_a	0.91	0.90	0.95

V. EXPERIMENTAL RESULTS

We have performed the noise reduction technique presented in [2] and evaluated the performance based on the S index. We have also computed the Gain in SNR using classical MSE estimation using an average QRS signal as a ref-

erence template. We have computed the average S index in three conditions before (o) and after the noise reduction (c): 1) at rest S^R , 2) during the first part of the protocol S^1 , and 3) during the second part of the protocol S^2 . We obtained: $S_o^R=0.957$, $S_c^R=0.97$, $S_o^1=0.84$, $S_c^1=0.866$, $S_o^2=0.69$, $S_c^2=0.81$.

VI. CONCLUSIONS

We have developed a new way of measuring the performance of noise reduction method when applied to real life signals. The method is based on a nonlinear embedding of the signals and a synchronization index S in the state space. Results show the interesting feature of the modified S index: time-shift, frequency and amplitude invariance. We will continue this work using well known tools from group theoretical approach with similitude group-transform invariants.

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Address of the corresponding author:

Author: Patrick Celka
Institute: School of Engineering
Street: Parkland Drive
City: Southport, Queensland
Country: Australia
Email: p.celka@griffith.edu.au