XPath rewriting using multiple views

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Abstract. We study the problem of tree pattern query rewriting using multiple views for the class of tree patterns in $P^{//, \square}$. Previous work has considered the rewriting problem using a single view. We consider two different ways of combining multiple views, define rewritings of a tree pattern using these combinations, and study the relationship between them. We show that when rewritings using single views do not exist, we may use such combinations of multiple views to rewrite a query, and even if rewritings using single views do exist, the rewritings using combinations of multiple views may provide more answers than those provided by the union of the rewritings using the individual views. We also study properties of intersections of tree patterns, and present algorithms for finding rewritings using intersections of views.

1 Introduction

Query rewriting using views has many applications including data integration, query optimization, query caching, and support of physical data independence [5]. Given a query, it studies finding another query using only the views to produce correct answers to the original query. In the literature, two types of query rewritings are studied, namely, equivalent rewritings and contained rewritings. Given a view, V , and a query, Q , an equivalent rewriting produces all answers to the original query Q using view V , whereas a contained rewriting may produce a subset of the answers to Q using V . Both types of rewritings have been extensively studied in the relational database context [5].

Recently, XML query rewriting has attracted attention because of the rising importance of XML data [3, 14, 7, 6, 2, 9]. XPath lies in the center of all XML languages, where the major classes of XPath expressions that have been studied are tree patterns [1, 8]. Among previous work on rewriting XPath queries using views, Xu and Ozsoyoglu [14] studied the complexity of finding equivalent rewritings for four types of tree patterns studied in [8] and presented an approach for finding and minimizing such rewritings. Mandhani and Suciu [7] presented, in addition to cache organization, an efficient but incomplete method for finding equivalent rewritings of tree patterns involving $/$, $//$, $\square$ and $*$ when the patterns are assumed to be minimized and may have value-based predicates. In [6], Lakshmanan et al. studied maximal contained rewritings of tree patterns where

Fig. 1. Example XML tree t

both the views and queries are in $P^{[//, \cdot]}$, both in the absence and in the presence of non-recursive, non-disjunctive DTDs. As the name implies, the maximal contained rewriting is a rewriting that contains all other contained rewritings.

All of the above works focus on rewriting the query using a single view. In other words, no combination of views is considered for the purpose of rewriting. In this paper we study tree pattern rewriting using multiple views, and our work is motivated by the following observations. Suppose we have a set $\mathcal{V}$ of views and the query Q . Then

1. It is possible for Q to have no contained rewritings using any individual view in $\mathcal{V}$, but it can be (partially) answered using the combination of some of the views. As a simple example, the query $Q = \text{article}[\text{table}][\text{figure}]/\text{author}$ cannot be (partially) answered using either $V_1 = \text{article}[\text{table}]/\text{author}$ or $V_2 = \text{article}[\text{figure}]/\text{author}$, but it can be answered using $V_1 \cap V_2$, which is equivalent to Q .
2. Even if Q does have contained rewritings using V_1 , and/or Q does have contained rewritings using V_2 , there may be contained rewritings of Q using some combination of V_1 and V_2 (e.g., intersection) which provide strictly more answers to Q than the union of all contained rewritings of Q using the individual views. This is demonstrated by Example 1.
3. Even if a query Q does not have an equivalent rewriting using V according to the conventional definition of rewriting [6, 14, 7], it is still possible to find all of the correct answers to Q using V for all XML trees. This is demonstrated in Example 2.

The example below demonstrates observation 2.

Example 1. Consider the views

$V_1 = \text{paper}[\text{subsection}][\text{reference}]$,

$V_2 = \text{paper}[\text{example}][\text{reference}]$

and the query

$Q = \text{paper}[\text{subsection}][\text{example}][\text{reference}][\text{paper}]$.

It can be verified that

$V_1 \cap V_2 = \text{paper}[\text{subsection}][\text{example}][\text{reference}]$,

Fig. 2. Q has no equivalent rewriting using V according to conventional definition, but Q can be fully answered using V : $Q_1 \circ V \cap Q_2 \circ V = Q$

and evaluating the pattern `reference//paper` over the answers of $V_1 \cap V_2$ will produce the same answers as Q , that is, `reference//paper` is an equivalent rewriting of Q using $V_1 \cap V_2$. However, the maximal contained rewriting of Q using V_1 is

`reference//example/reference//paper`,

and the maximal contained rewriting of Q using V_2 is

`reference//subsection//example/reference//paper`.

As we show next, the rewriting using $V_1 \cap V_2$ produces strictly more answers than the union of the answers produced by the maximal contained rewritings using V_1 and V_2 individually. For the XML tree t shown in Fig.1, evaluating V_1 over t produces (the subtrees rooted at) n_1 and n_2 , and evaluating V_2 over t produces (the subtrees rooted at) n_2 and n_3 . Therefore, the maximal contained rewriting using V_1 or V_2 will produce no answers for Q , but the rewriting using $V_1 \cap V_2$ will produce the `paper` node under n_2 .

The next example demonstrates observation 3.

Example 2. Consider the query Q and view V shown in Fig.2 (a) and (b) respectively. It can be verified that Q has no equivalent rewritings using V according to the conventional definition. But given any XML tree t , we can find $Q(t)$ using the view as follows. We evaluate the query $Q_1 = x/x[e]/y$ over the subtrees in $V(t)$, and denote the results as $Q_1(V(t))$; we then evaluate $Q_2 = x/y$ over the subtrees in $V(t)$, and obtaining a set denoted as $Q_2(V(t))$. Finally, we take the intersection of $Q_1(V(t))$ and $Q_2(V(t))$. It can be verified that $Q(t) = Q_1(V(t)) \cap Q_2(V(t))$.

We will study contained and equivalent rewritings of tree patterns in $P\{/,[]\}$ using multiple views in the absence and presence of DTDs, with the intersection and union operations. Our main contributions are summarized below.

- We define (contained and equivalent) rewritings of a tree pattern using two different combinations of views, and show the relationship between these rewritings.

- We show the intersection of some tree patterns, if not empty, can be expressed as the union of tree patterns, and provide an efficient algorithm to translate the intersection into union.
- Based on the above, we provide algorithms for finding the maximal contained rewritings and equivalent rewritings using the intersection of views. We also show the effect of a non-recursive DTD on the rewritings.

The rest of the paper is organized as follows. Section 2 provides the background and notations. Section 3 presents the algorithm for reformulating intersections of tree patterns into union, and studies rewritings using the intersection of views. Section 4 defines rewritings using a different combination views, and compare them with rewritings using the intersection. Section 5 considers the effects of non-recursive DTDs. Section 6 surveys related work. Finally Section 7 concludes the paper.

2 Preliminaries

2.1 XML trees and tree patterns

Let Σ be an infinite set of tags. An XML tree is a tree such that every node is labeled with a tag in Σ . A *tree pattern* (TP) is a tree with a unique *distinguished node*, and with every node labeled with a tag in Σ , and every edge labeled with either $/$ or $//$. Such a TP corresponds to an XPath expression involving the child axis, descendant axis, and branching condition $[]$. A tree pattern has a tree representation. Fig.2 show several TPs, and the TP in Fig.2 (b) represents the XPath expression $a//x[b]/x[c]/x[d]$. Here, single and double lines represent $/$ -edges and $//$ -edges respectively. The distinguished node is indicated by a circle. Note: the TPs in our discussion correspond to the fragment $P^{\{//, []\}}$ defined in [8]. A subset of $P^{\{//, []\}}$, denoted $P^{\{/\}$, contains all TPs that has a single root-to-leaf path.

Let $\mathcal{T}$ be an XML tree or a TP, and v be a node in $\mathcal{T}$. We will use $N(\mathcal{T})$ to denote the set of all nodes in $\mathcal{T}$, $rt(\mathcal{T})$ to denote the root of $\mathcal{T}$, and $label(v)$ to denote the label of v . For any TP P , DN_P and DP_P denote, respectively, the distinguished node and *distinguished path* of P (i.e., the path from $rt(P)$ to DN_P).

A *matching* of a TP, P , in an XML tree, t , is a mapping δ from $N(P)$ to $N(t)$ satisfying the following conditions: (1) root-preserving, i.e., $\delta(rt(P)) = rt(t)$, (2) label-preserving, i.e., $\forall v \in N(P), label(v) = label(\delta(v))$, and (3) structure-preserving, i.e., for every edge (x, y) in P , if it is a $/$ -edge, then $\delta(y)$ is a child of $\delta(x)$; if it is a $//$ -edge, then $\delta(y)$ is a descendant of $\delta(x)$, i.e, there is a path from $\delta(x)$ to $\delta(y)$. Each matching δ produces a subtree of t rooted at $\delta(DN_P)$, denoted $sub_{\delta(DN_P)}^t$, which is called an *answer* to the TP. We use $P(t)$ to denote the *answer set* of P on t :

$$P(t) = \{sub_{\delta(DN_P)}^t \mid \delta \text{ is a matching of } P \text{ in } t\}.$$

Let T be a set of XML trees. We use $P(T)$ to denote the union of answer sets of P on the trees in T . That is, $P(T) = \bigcup_{t \in T} P(t)$.

2.2 Intersection and extension of TPs

Two tree patterns, P and Q , are said to be *comparable* if $label(rt(P)) = label(rt(Q))$ and $label(DN_P) = label(DN_Q)$. Let $P_1, \dots, P_n$ be comparable TPs. For any XML tree t , the *intersection* of $P_1, \dots, P_n$, denoted $P_1 \cap \dots \cap P_n$, returns $P_1(t) \cap \dots \cap P_n(t)$. The union of $P_1, \dots, P_n$, denoted $P_1 \cup \dots \cup P_n$, returns $P_1(t) \cup \dots \cup P_n(t)$.

Let P and Q be TPs such that $label(DN_P) = label(rt(Q))$. The *extension* of P using Q , denoted $Q \circ P$, is the TP obtained by merging DN_P with $rt(Q)$. The distinguished node of $Q \circ P$ is DN_Q . Fig.2 (d) shows the extension of V (Fig.2 (b)) using Q_1 (Fig.2 (c)). It is easy to see that, for any XML tree t , $(Q \circ P)(t) = Q(P(t))$, that is, $(Q \circ P)(t)$ is equivalent to the union of answer sets of Q on the subtrees in $P(t)$.

Let $P_1, \dots, P_n$ be comparable TPs, and Q be a TP such that $rt(Q)$ and DN_{V_i} have identical labels. We denote by $Q \circ (P_1 \cap \dots \cap P_n)$ the extension of $P_1 \cap \dots \cap P_n$ using Q , which returns, for any t , $Q(P_1(t) \cap \dots \cap P_n(t))$.

2.3 TP containment

Let P and Q be TPs. P is said to be *contained* in Q , denoted $P \subseteq Q$, if for every XML tree t , $P(t) \subseteq Q(t)$. It is shown in [1] that $P \subseteq Q$ iff there is a containment mapping from Q to P . Recall: A *containment mapping* (CM) from Q to P is a mapping δ from $N(Q)$ to $N(P)$ that is label-preserving, root-preserving as discussed in the last section, structure-preserving (which now means for any $/$ -edge (x, y) in Q , $(\delta(x), \delta(y))$ is a $/$ -edge in P , and for any $//$ -edge (x, y) , there is a path from $\delta(x)$ to $\delta(y)$ in P) and is output-preserving, which means $\delta(DN_Q) = DN_P$. A *homomorphism* from Q to P is similar to a CM, except there is no requirement of output-preserving.

The following lemma is proved in [13].

Lemma 1. For TPs $P_1, \dots, P_n, P \in P^{\{/, \emptyset\}}$, $P \subseteq P_1 \cup \dots \cup P_n$ iff there is $i \in [1, n]$ such that $P \subseteq P_i$.

3 TP rewriting using intersections of views

A *view* is an existing TP. We define rewritings using intersections of views as follows.

Definition 1. Let Q be a query, and $V_1, \dots, V_n$ be comparable views. If $V_1 \cap \dots \cap V_n$ is non-empty, and there exists Q' such that $label(rt(Q')) = label(DN_{V_1})$, and $Q' \circ (V_1 \cap \dots \cap V_n) \subseteq Q$, then we say $Q' \circ (V_1 \cap \dots \cap V_n)$ is a *contained rewriting* (CR) of Q using $V_1 \cap \dots \cap V_n$. If $Q' \circ (V_1 \cap \dots \cap V_n) = Q$, we say Q' is an *equivalent rewriting* (ER) using V . The maximal contained rewriting (MCR) of Q using $V_1 \cap \dots \cap V_n$, denoted $MCR(Q, V_1 \cap \dots \cap V_n)$, is the union of all CRs of Q using $V_1 \cap \dots \cap V_n$.

Note that when $n = 1$, the above definition reduces to those in [6] (CR and MCR) and [14] (ER).

3.1 Properties of intersections

In this section we identify conditions under which the intersection of TPs are not empty, and present algorithms to translate the intersection into the union of TPs. We present these results using two TPs. Generalization to more TPs is simple.

In many cases the intersection of two TPs P and Q is an *empty query*, that is, it always returns the empty set. For example, when $P = a/b/x$ and $Q = a/c/x$. The question arises: when is $P \cap Q$ a non-empty query? We have the follow answer to this question.

Lemma 2. $P \cap Q$ is non-empty iff there is a path P' in $P^{\{/\}} \}$ such that $P' \subseteq \text{DP}_P$, and $P' \subseteq \text{DP}_Q$.

The lemma is true because, when there is no DTD, every TP is non-empty, and $P \cap Q$ is non-empty iff $\text{DP}_P \cap \text{DP}_Q$ is non-empty.

Let's call the path P' in lemma 2 a *common distinguished path* of P and Q . If DP_P and DP_Q do not have $/$ -edges, then only when $\text{DP}_P = \text{DP}_Q$ there is a CDP of P and Q , which is DP_P itself. However, if DP_P and DP_Q do have $/$ -edges, then there may be (infinitely) many CDPs of P and Q . For example, if $P = a//x[b]//y$, $Q = a//x[c]//y$, then P and Q have the CDPs $a//x//y$, $a//x//x//y$, $a//x//x//x//y$ and so on. But there are only a finite number of CDPs that are of interest to us. These CDPs are annotated to form annotated CDPs.

Definition 2. Let P and Q be comparable TPs in $P^{\{/\}, \{\}} \}$. An annotated CDP (ACDP) of P and Q is a CDP P' of P and Q such that (1) every node in P' is annotated with either 1, or 2, or both, (2) there is a (unique) CM m_P from DP_P to P' such that every node in DP_P is mapped a node annotated with 1 (or both 1 and 2), and for every node v in P' annotated with 1 (or both 1 and 2), there is a unique node u in DP_P such that $m_P(u) = v$, (3) there is a (unique) CM m_Q from DP_Q to P' such that every node in DP_Q is mapped a node annotated with 2 (or both 1 and 2), and for every node v in P' annotated with 2 (or both 1 and 2), there is a unique node u in DP_Q such that $m_Q(u) = v$.

Intuitively, an ACDP of P and Q is a path in $P^{\{/\}} \}$ which contains exactly one “copy” of DP_P and one “copy” of DP_Q (the nodes annotated with 1 form a copy of DP_P , and the nodes annotated with 2 form a copy of DP_Q) and every node appears in at least one of the copies. Fig.3 shows all ACDPs of $P = a//x[b]//y$ and $Q = a//x[c]//y$.

Next we present an algorithm that finds all ACDPs of P and Q . Let $P_1 = \text{DP}_P$ and $P_2 = \text{DP}_Q$. We can find all ACDPs of P and Q by calling the function $\text{MERGE}(P_1, P_2)$. In the function, a *position* in P_j refers to either a node or a $/$ -edge in P_j . Each position in P_j is given a unique position number, with the position number of $\text{rt}(P_j)$ being 0, and each subsequent position's position number increases by 1. The function $\text{MERGE}(P_i, P_j)$ finds the ACDPs of P_i and P_j by “inserting” the nodes of P_i into P_j . To do so, it needs to find a position,

Fig. 3. ACDPs of $a//x[b]//y$ and $a//x[c]//y$

in P_j , for every node in P_i . There are three stages in the process. The first stage (lines 1-6) is to scan P_i top-down, marking each node v in P_i with a set of markings. Each marking is of the form $\{s : A(v, s)\}$ (except for $rt(P_i)$, which is marked $\{0\}$), where s is a position, and $A(v, s)$ is a set of positions. The meaning of this marking is that if $\text{parent}(v)$ is merged into P_j in position s , then the possible positions where v can be inserted in P_j are those in $A(v, s)$. The criterion to choose the positions in $A(v, s)$ is label-preservation and structure-preservation. The second stage (lines 8-10) removes the impossible positions from the markings, going from bottom-up. The only possible position for DN_{P_i} is the last position in P_j . Based on structure-preservation, if impossible positions for v are found and deleted, then impossible positions for $\text{parent}(v)$ may be found and deleted. In the last stage (lines 13-29), if every node has a possible position, we pick a position s_v for each node v in P_i according to the markings, and construct a ACDP. For each combination of positions (each combination has one position for every node in P_i) there is a ACDP constructed. Finally the function returns all ACDP constructed this way.

Example 3. Let $P_1 = a//x//y/y$ and $P_2 = a//x//x//y$. Fig.4 shows the process of running $\text{MERGE}(P_1, P_2)$.

First, we identify and label each position in P_2 . There are 7 positions labeled $0, 1, \dots, 6$, as shown in the figure. In stage 1, we mark each node in P_2 with its possible positions. The a -node is marked $\{0\}$. If the a -node is put in position 0, then the possible positions of the next node are 1,2,3,4,5. Therefore, the first x -node is marked with $(0 : \{1, 2, 3, 4, 5\})$. If the x -node is put to position 1, then the next node may be put to positions 1,3 or 5, to preserve label and structure of P_i . If the x -node is put to position 2, then the possible positions for its child are 3 and 5, ..., if the x -node is put to position 5, then its child must be put to position 5 (after the x -node). Therefore, the child of the x -node is marked with the markings as shown in the figure. Finally, the last node of P_1 must be put to the last position, position 6, in P_2 . Thus if its parent is put to position 1 or 3, then there are no possible positions for it. If its parent is put to position 5, then it can go to position 6. This explains the markings of the last node of P_1 . In stage 2, we remove the impossible positions in the markings. Because position 1 and 3 of the first y -node prohibits the second y -node to be put in position 6 (as indicated by the markings $(1, \{\})$ and $(3, \{\})$), we know

Algorithm 1 MERGE(P_i, P_j)

```
1: let  $pos(rt(P_i)) = \{0\}$ ,  $S = \emptyset$ 
2: for every subsequent node  $v$  in  $P_i$  do
3:   for all  $s \in pos(\text{parent}(v))$  do
4:     let  $\text{position}(v, s) = \text{FIND\_POSITION}(v, P_i, s, P_j)$ 
5:     mark  $v$  with  $(s : \text{position}(v, s))$ 
6:   let  $pos(v) = \bigcup_{s \in pos(\text{parent}(v))} \text{position}(v, s)$ 
7:   if  $pos(v) = \emptyset$  then return  $\emptyset$ 
8: for each node  $v$  in  $P_i$  (starting from  $v = \text{DN}_{P_i}$ ) do
9:   while  $\exists$  marking  $(s : \text{position}(v, s))$  for  $v$  such that  $\text{position}(v, s) = \emptyset$  do
10:    delete  $s$  from  $pos(\text{parent}(v), s')$  for all  $s'$ 
11:   if  $pos(v) = \emptyset$  return  $\emptyset$ 
12: for each node  $v$  in  $P_i$  choose a position  $s_v$  from its markings. Initially,  $v = rt(P_i)$  and  $s_v = 0$ .
    For each subsequent node  $v$ , pick a position from  $pos(v, s_{\text{parent}(v)})$ 
13: for each combination of positions found above do
14:   let  $P' = P_j$ , annotate every node with  $j$ 
15:   for  $v$  in  $P_i$  do
16:     if position  $s_v$  points to node  $u$  in  $P'$  then
17:       annotate  $u$  with  $i$ .
18:       if  $\text{prt}(u)$  is annotated with  $i$  or  $i, j$  and  $(\text{parent}(v), v)$  is  $/$ -edge then
19:         change  $(\text{prt}(u), u)$  to  $/$ -edge
20:       else if position  $s_v$  represents  $//$ -edge  $(u_1, u_2)$  in  $P'$  then
21:         insert a  $\text{label}(v)$ -node  $u_0$  between  $u_1$  and  $u_2$ 
22:         if  $u_1$  is annotated with  $i$  or  $i, j$  then
23:           let edge  $(u_1, u_0)$  be of the same type as  $(\text{parent}(v), v)$ 
24:         else
25:           let edge  $(u_1, u_0)$  be of type  $//$ 
26:           let edge  $(u_0, u_2)$  be of type  $//$ ; let position  $s_v$  point to this edge  $(u_0, u_2)$ 
27: add  $P'$  to  $S$ 
28: return  $S$ 
```

positions 1 and 3 are impossible for the first y -node. Thus we delete them from its markings. In stage 3, for each combination of the positions, we construct an ACDP. A combination of position is made of a position for each node in P_1 . In this example, the combinations of positions are $(0,1,5,6)$, $(0,2,5,6)$, $\dots$, $(0,5,5,6)$. We use $(0,5,5,6)$ to explain the construction process. Initially $P' = P_2$ and every node in P' is annotated with 2. Position 0 points to the root of P_2 , therefore we annotate $rt(P_1)$ with 1. Position 5 points to a $//$ -edge, therefore we insert an x -node, x_0 , in this position and annotate this node with 1. The edge (x, x_0) and (x_0, y) are to be of type $//$ in this case. Now position 5 points to the edge (x_0, y) . Since the position for the next node is also 5, we insert another x -node, x_1 , between x_0 and y , and annotate x_1 with 1. The last node of P_1 has position 6, so we annotate the y -node in P_2 with 1. Since x_1 is annotated with 1, we can change the edge type of (x_1, y) to that of the corresponding edge in P_1 , in this case, $/$. The resulting P' is a ACDP: it is $a^{1,2}/x^2/x^2/x^1/y^1/y^{1,2}$, where the superscripts indicate the annotation. The ACDPs constructed using other combinations are: $a^{1,2}/x^1/x^2/x^2/y^1/y^{1,2}$, $a^{1,2}/x^{1,2}/x^2/y^1/y^{1,2}$, $a^{1,2}/x^2/x^1/x^2/y^1/y^{1,2}$, and $a^{1,2}/x^2/x^{1,2}/y^1/y^{1,2}$.

Number of ACDPs. Let n and m be the number of edges in P_i and P_j respectively, and $f(m, n)$ be the worst-case number of ACDPs of P_i and P_j (which occurs when all nodes in P_i and P_j have the same label, and all edges are $//$ -edges). The following theorem can be proved using induction on m and n .

Algorithm 2 FIND_POSITION(v, P_i, s, P_j)

```

1: if  $s$  is node  $u$  in  $P_j$  then
2:   if ( $\text{parent}(v, v)$  is  $//$ -edge then
3:     let  $A$  consist of all  $//$ -edges after  $u$  and all nodes labeled  $\text{label}(v)$  after  $u$ .
4:   else if ( $\text{parent}(v, v)$  is  $/$ -edge then
5:     if  $u$  has child  $u'$  then
6:       if  $(u, u')$  is  $//$ -edge then add this  $//$ -edge to  $A$ 
7:       if  $\text{label}(u') = \text{label}(v)$  then add  $u'$  to  $A$ 
8:   if  $s$  is  $//$ -edge  $(u_1, u_2)$  in  $P_j$  then
9:     add  $s$  to  $A$ 
10:  if ( $\text{parent}(v, v)$  is  $//$ -edge then
11:    add all  $//$ -edges after  $u_2$  and all nodes labeled  $\text{label}(v)$  after  $u_1$  to  $A$ 
12:  if ( $\text{parent}(v, v)$  is  $/$ -edge and  $\text{label}(u_2) = \text{label}(v)$  then add  $u_2$  to  $A$ 
13: if  $v$  is the last node in  $P_i$  then
14:   delete from  $A$  all positions except that of  $\text{DN}_{P_j}$ 
15: return  $A$ 

```

Fig. 4. Finding ADCPs of P_1 and P_2

Theorem 1. $f(m, n) = f(n, m)$, which can be calculated recursively as follows:

$$\begin{aligned}
 f(m, n) &= f(m-1, n) + 2(f(m-1, n-1) \\
 &\quad + f(m-1, n-2) + \dots + f(m-1, 1)).
 \end{aligned}$$

For example, $f(m, 1) = 1$, $f(2, 2) = 3$, $f(3, 2) = 5$, $f(3, 3) = 13$, $f(4, 3) = 25$, $f(4, 4) = 63$ and so on. Thus $f(m, n)$ grows exponentially in general. However, in most practical cases, the number of ADCPs is much smaller than $f(m, n)$.

Complexity. Algorithm MERGE(P_i, P_j) runs in $O(|P_i| \times |P_j|^2)$, where $|P_i|$ is the number of nodes in P_i : function FIND_POSITION runs in $O(|P_j|)$, the top-down scan visits each node in P_i once, and for each node in P_i , the function FIND_POSITION is called at most $2|P_j|$ times. The bottom-up scan and the construction of ADCPs can be done in $O(|P_i| \times |P_j|)$.

Let P be a TP in $P^{\{/, \emptyset\}}$. For every node v in P , we use P_v to denote the subpattern of P rooted at v . Let v be a node in DP_P , and u be the child of v on DP_P (if u exists). We call the subpattern obtained by removing P_u from P_v the *branching subtree at v* . For example, in Fig.5 (a), the branching subtrees are indicated by the dotted oval. Next we define *merged TPs (MTPs)* of P_1 and P_2 .

Definition 3. Let P_1 and P_2 be comparable TPs, and P' be an ADCP of P_1 and P_2 . Let δ_i be the unique CM from P_i to P' that maps every node in P_i to a node in P' annotated with i . The merged TP (MTP) of P_1 and P_2 wrt to P' is the

Fig. 5. TPs P_1 , P_2 and the MTP of P_1, P_2

TP obtained as follows: for every node v in P_i , add the branching subtree at v_i under $\delta_i(v_i)$.

Fig.5 (c) shows the only MTP of the TPs in Figures 5 (a) and (b).
The following theorem is straightforward.

Theorem 2. Let P_1 and P_2 be TPs in $P^{\{/, \sqcup\}}$. The union of all MTPs of P_1 and P_2 is equivalent to $P_1 \cap P_2$.

Note also that the subpatterns of the MTPs rooted at the distinguished nodes are all identical.

3.2 Finding MCRs using $V_1 \cap V_2$

Let Q be the query and V_1 and V_2 be comparable views. We assume $V_1 \cap V_2$ is not empty, and $V_1 \not\subseteq V_2$, $V_2 \not\subseteq V_1$. Suppose $V_1 \cap V_2$ is equivalent to $V'_1 \cup \dots \cup V'_k$. Clearly $Q' \circ (V_1 \cap V_2) \subseteq Q$ if and only if $Q' \circ V'_i \subseteq Q$ for all $i \in [1, k]$. In other words, Q' is a CR of Q using $V_1 \cap V_2$ iff it is a CR of Q using V'_i for all $i \in [1, k]$. Therefore, to find the MCR of Q using $V_1 \cap V_2$, we can find the MCR of Q using each V'_i and intersect them. That is,

$$\text{MCR}(Q, V_1 \cap V_2) = \bigcap_{i=1}^k \text{MCR}(Q, V'_i).$$

Fig. 6. Finding MCR using intersection

Example 4. Consider the views V_1 , V_2 , V'_1 , V'_2 and the query Q in Fig.6. $V_1 \cap V_2 = V'_1 \cup V'_2$. We find the MCR of Q using V'_1 , which is y/z , and the MCR of Q using V'_2 , which is also y/z . Thus y/z is the MCR of Q using $V_1 \cap V_2$.

Algorithm 3 Finding equivalent rewriting

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1: for  $i = 1$  to  $n$  do
2:   if exists node  $v$  such that  $\text{DP}_Q^v$  is isomorphic to  $p_i$  then
3:     if  $\text{sub}_Q^v \circ V'_i = Q$  then
4:       if  $\forall j \in [1, k], \text{sub}_Q^v \circ V'_j \subseteq Q$  then
5:         return  $\text{sub}_Q^v$ 

```

3.3 Finding ERs using $V_1 \cap V_2$

Suppose Q has an ER Q' using $V_1 \cap V_2$, and $V_1 \cap V_2 = V'_1 \cup \dots \cup V'_k$, that is, $Q' \circ (V'_1 \cup \dots \cup V'_k) = Q$. By Lemma 1, there exists $i \in [1, k]$, such that $Q' \circ V'_i = Q$. Hence for all $j \in [1, k]$, $Q' \circ V'_j \subseteq Q' \circ V'_i$, and thus there is a CM from $Q' \circ V_i$ to $Q' \circ V_j$, so the length of $\text{DP}_{V'_i}$ cannot be longer than that of $\text{DP}_{V'_j}$. Furthermore, $\text{DP}_{Q' \circ V'_i}$ is isomorphic to DP_Q , and the subpattern of Q rooted at the node that corresponds to $\text{DN}_{V'_i}$, is an ER of Q using V'_i (by Lemma 4.8 of [14]).

Using the properties above, we provide a heuristic algorithm, Algorithm 3, for finding ERs using $V_1 \cap V_2$. In the algorithm, we assume $p_1, \dots, p_n$ ($n \leq k$) are the shortest ACDPs of V_1 and V_2 , and the corresponding MTPs are $V'_1, \dots, V'_n$. For any node v on DP_Q , the path from $rt(Q)$ to v is denoted DP_Q^v , and the subpattern rooted at v is denoted sub_Q^v . The basic idea of the algorithm is as follows. For each shortest ACDP p_i , we first check whether it is isomorphic to some DP_Q^v , if not, there is no ER using V'_i ; if yes, we further check whether sub_Q^v is an ER using V'_i . If yes, we further check whether the extension of all other MTPs of V_1 and V_2 with sub_Q^v are contained in V_i . If yes, sub_Q^v is returned as an ER of Q using $V_1 \cap V_2$.

Let u be a node on the distinguished path of V . We use $V^{(u)}$ to denote the pattern obtained from V by removing all descendants of u and making u the distinguished node, and $\bar{V}$ to denote $V^{(\text{DN}_V)}$. It is easy to prove the following theorem.

Theorem 3. (1) Suppose $V_1 \cap V_2 = V'_1 \cup \dots \cup V'_n$. If Q has an ER using $V_1 \cap V_2$, then there is $i \in [1, n]$ such that (i) V'_i has the shortest distinguished path among $V'_1, \dots, V'_n$, (ii) for all $j \in [1, n]$, there exists u_j in $\text{DP}_{V'_j}$ such that $V_j^{(u_j)} \subseteq \bar{V}'_i$. (2) Algorithm 3 finds the ER if it exists.

Example 5. (1) Consider the views in Fig.6. Since $V_1 \cap V_2 = V'_1 \cup V'_2$, $\text{DP}_{V'_1}$ is shorter than $\text{DP}_{V'_2}$, and there is no node u on the distinguished path of V'_2 such that $V_2^{(u)} \subseteq \bar{V}'_1$, we know there is no ER of Q using $V_1 \cap V_2$ for any Q .

(2) Consider the two views V_1 and V_2 in Fig.7. There are two MTPs V'_1 and V'_2 , and $V'_2 \subseteq V'_1$. Therefore, $V_1 \cap V_2 = V'_1$. For any TP Q , Q' is an ER of Q using $V_1 \cap V_2$ iff Q' is an ER using V'_1 .

³ Note that although there may be many ACDPs of V_1 and V_2 , usually there are few shortest ones.

Fig. 7. Example for illustrating ER using intersection

4 Rewriting using other combinations of views

We now define a second type of rewritings of Q using multiple views that does not require the views to be comparable.

Definition 4. Let $V_1, \dots, V_n$ be some views with identical root label (possibly $V_i = V_j$ for some i, j), and Q be a query. If there are TPs $Q_1, \dots, Q_n$ such that $\bigcap_{i=1}^n (Q_i \circ V_i) \subseteq Q$, and $\bigcap_{i=1}^n (Q_i \circ V_i)$ is non-empty, then we say $\langle Q_1, \dots, Q_n \rangle$ is a contained rewriting of Q using $\langle V_1, \dots, V_n \rangle$. If $\bigcap_{i=1}^n (Q_i \circ V_i) \supseteq Q$ also holds, we call the CR an equivalent rewriting.

Intuitively, if there is a contained rewriting $\bigcap_{i=1}^n (Q_i \circ V_i)$, then to partially answer Q over t , we can evaluate Q_i over $V_i(t)$ and then find the intersection $\bigcap_{i=1}^n Q_i(V_i(t))$. Note that, when $n = 1$, the definition reduces to that in [6] (CR and MCR) and [14].

4.1 Relationship between rewritings using $\langle V_1, V_2 \rangle$ and rewritings using $V_1 \cap V_2$

First, it is easy to prove the following lemma.

Lemma 3. Let V_1 and V_2 be comparable views. Then $Q \circ (V_1 \cap V_2) \subseteq (Q \circ V_1) \cap (Q \circ V_2)$.

However, generally $Q \circ (V_1 \cap V_2) \not\supseteq (Q \circ V_1) \cap (Q \circ V_2)$. Consider $V_1 = a//b/x$ and $V_2 = a//c/x$, and $Q = x//z$. Clearly $V_1 \cap V_2 = \emptyset$, hence $Q \circ (V_1 \cap V_2) = \emptyset$. However, $(Q \circ V_1) \cap (Q \circ V_2) \neq \emptyset$. This example also shows that sometimes even if $V_1 \cap V_2$ is empty, it is still possible to have CRs of Q using $\langle V_1, V_2 \rangle$, although there are clearly no CRs of Q using $V_1 \cap V_2$.

The next lemma identifies some special cases where $Q \circ (V_1 \cap V_2) = (Q \circ V_1) \cap (Q \circ V_2)$.

Lemma 4. Let V_1 and V_2 be comparable views. If one of the following conditions holds, then $Q \circ (V_1 \cap V_2) = (Q \circ V_1) \cap (Q \circ V_2)$.

- (1) $DP_{V_1} = DP_{V_2}$, and all edges in DP_{V_1} are $/$ -edges.
- (2) Every edge in DP_Q is a $/$ -edge.
- (3) $V_1 \subseteq V_2$ or $V_2 \subseteq V_1$.

Using the above lemmas, we can prove the following theorem:

Theorem 4. *Let V_1, V_2 be comparable views. For any query Q , if there is a CR of Q using $V_1 \cap V_2$, then there is a CR of Q using $\langle V_1, V_2 \rangle$.*

Note that the above theorem does not say there is CR using $\langle V_1, V_2 \rangle$ which contains the CR using $V_1 \cap V_2$. The next example shows that it is possible for Q to have a CR using $V_1 \cap V_2$, and this rewriting is not contained in any CRs of Q using $\langle V_1, V_2 \rangle$.

Example 6. Let $V_1 = a//x//z$, $V_2 = a//y/z$, and $Q = a//x//y/z/c$. It can be verified that $V_1 \cap V_2 = a//x//y/z$. Now let $Q' = z/c$, then $Q' \circ (V_1 \cap V_2) = Q$. Thus Q' is an ER of Q using $V_1 \cap V_2$.

If there are Q_1 and Q_2 such that $(Q_1 \circ V_1) \cap (Q_2 \circ V_2) \subseteq Q$, then DP_{Q_1} must be z/c or $z//c$, and DP_{Q_2} must be z/c or $z//c$, because $rt(Q_i)$ must be labeled z and DN_{Q_i} must be labeled c . One can verify that if $\text{DP}_{Q_2} = z//c$, then $(Q_1 \circ V_1) \cap (Q_2 \circ V_2)$ is not contained in Q , and if $\text{DP}_{Q_2} = z/c$, then $(Q_1 \circ V_1) \cap (Q_2 \circ V_2)$ is not equivalent to Q either.

5 The presence of non-recursive DTDs

In the following, we assume every TP P is satisfiable under a non-recursive DTD G , that is, there is an XML tree t which conforms to G , and $P(t) \neq \emptyset$.

In the presence of G , no label in an XML tree can appear in a path more than once. Thus any TP that is satisfiable under G cannot have a path that contains two or more nodes with the same label. Therefore, for any comparable views V_1 and V_2 , there is at most one ACDP of V_1 and V_2 , and at most one MTP of V_1 and V_2 . In other words, $V_1 \cap V_2$ is equivalent to a single TP V' under G . Therefore, to find the MCR or ER of Q using $V_1 \cap V_2$, we only need find the MCR or ER of Q using V' , and this can be done using the method of [6]. Furthermore, we can prove (see full version of this paper) the following theorem, which implies that, in the presence of G , Q' is a CR using $V_1 \cap V_2$ iff $\langle Q', Q' \rangle$ is a CR of Q using $\langle V_1, V_2 \rangle$.

Theorem 5. *Let V_1 and V_2 be comparable views, and $\text{label}(rt(Q')) = \text{label}(\text{DN}_{V_1})$. In the presence of G , $Q' \circ (V_1 \cap V_2)$ is equivalent to $(Q' \circ V_1) \cap (Q' \circ V_2)$.*

6 More related work

Besides the recent works discussed in Section 1, several other papers dealt with tree pattern query rewriting. In particular, [10] studied the problem of query answerability using views for general XPath queries, that is, given Q and $V_1, \dots, V_n$, whether there are $Q_1, \dots, Q_n$ such that $Q_1 \circ V_1 \cup \dots \cup Q_n \circ V_n = Q$. [3] addressed the problem of answering XPath queries using a single materialized view where, for the view, a combination of node references, typed data values, and full paths may be stored. However, the way in which a query is answered using the view

is different from ours (and those in Section 1): one can follow node references to go to the original document, so the original XML tree cannot be discarded. [2] studied a different type of equivalent rewriting using multiple views in the presence of *structural summaries* and integrity constraints: the answer sets of the views are nodes rather than subtrees, and the answers to the new query are obtained by combining answers to the views through a number of algebraic operations. [11] studied *correct rewritings* of TPs, using a single view, which can be seen as a special form of contained rewritings. [4] attempted to speed-up the finding of MCRs using single views by combining the views into a single tree. [12] studied *equivalently answering XPath* queries using multiple views based on the assumption that the Dewey codes are stored in the materialized views so that the common ancestors of nodes in different views can be found. Our work is clearly different from all of the above.

7 Conclusion

We studied the problem of rewriting TP queries using multiple views for the class $P\{/, \square\}$, and defined rewritings using two different combinations of views. We studied the relationship between the two types of rewritings and presented efficient algorithms to reformulate the intersection of TPs into a union of TPs, as well as algorithms for finding the MCRs and ERs using intersections of views. Our definitions and algorithms enable us to make better use of the views in order to answer a query.

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