HYPERBOLIC TIME-FREQUENCY POWER SPECTRUM

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Abstract

A new hyperbolic kernel and hyperbolic wavelet are presented along with their applications in signal detection and signal analysis. Comparisons are made between the hyperbolic kernel and highly-regarded Choi-Williams kernel. Applications of the hyperbolic kernel are outlined. Parallel computing is used to calculate the bispectrum and time-frequency power spectrum with encouraging results. Up-to-date applications of time-frequency power spectra are discussed.

1 INTRODUCTION

Time-frequency signal processing is a major branch of signal processing dealing with non-stationary signals whose power spectra vary with time [1, 2]. In time-frequency signal processing context, to represent a time-frequency power spectrum (TFPS), two axes are required: the frequency axis and the time axis. These axes represent changes of the signal power spectrum in frequency and time domain respectively which gives insight into the behaviour of such signals.

Physically, the TFPS is a joint distribution between time and frequency parameters which was derived by Wigner [1, 2] when studying quantum mechanics in 1932 using a unity or Wigner-Ville (WV) kernel. This work was later developed and generalised by the mathematician Leon Cohen [1, 2] so that all time-frequency distributions can be governed by his general equation. At the time, since the major work of the field was developed by mathematicians, the term "distribution" was widely used. In the 1950's, Rihaczek [3] found some applications by applying the time-frequency distribution to study varying power spectra in signal processing. Rihaczek proposed a new kernel and studied its effects on a number of signals and found that the time-frequency distribution is very effective in studying signals with varying power spectra and gave it a new name: time-frequency varying power spectrum. The general formula for the TFPS is given by Eq. (1)

$$P(t, \omega) = \frac{1}{4\pi^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} e^{-j\theta u - j\omega u} \cdot \Phi(\theta, \tau) \cdot R_{t,1}(t, \tau) \, du \, d\theta \, d\tau \quad (1)$$

where $\Phi(\theta, \tau)$ is a kernel with its parameters θ and τ (lagging parameter), $u = t - \frac{\tau}{2}$ and $R_{t,1}(t, \tau)$ the local auto-correlation function.

Although the TFPS can track changes in the power spectrum in both time and frequency domains, one of its major problems is the presence of cross terms in time-frequency plane. Cross terms originate from cross coupling among the individual signals in a multi-

component signal. These terms provide misleading information about the signal and thus need to be eliminated. The coupling of the individual signals themselves creates auto terms which need to be effectively supported. Although there are a number of better kernels than the Choi-Williams (CW) kernel in the literature, the CW kernel is chosen as a benchmark for comparison purposes because of its significance in generating new wavelets which will be discussed in Section 4.

This paper is organized as follows. Sections 2 and 3 propose a hyperbolic kernel and hyperbolic wavelet, and present some theoretical results. Section 4 discusses the first application of the hyperbolic kernel as a useful signal detector in time-frequency domain. Section 5 discusses the application of time-frequency wavelet power spectral technique for a variety of signals such as ECG, music, speech, chaotic Duffing oscillator, sinusoid and transients. Section 6 presents results on speedup factors (SFs) obtained when calculating the TFPS and bispectrum (BS) using parallel computing. Section 7 proposes a number of current applications of time-frequency signal processing. Section 8 concludes the main ideas that have been presented in the paper.

2 HYPERBOLIC KERNEL

The presence of cross terms in TFPSs motivates researchers to design new and effective kernels since the solution to cross-term elimination or "artifact" is a well-designed kernel. There have been very few kernels proposed in the literature, their forms and corresponding derived TFPSs can be found in [1, 2]. However, in 1989, Choi and Williams [4] proposed a new second-power exponential function which had been shown to be very effective in suppressing cross terms and supporting auto terms. Since then, the CW kernel has been widely used and found many applications in time-frequency signal processing. One disadvantage of the CW kernel is that it is a second-power exponential function which is hard to interpret and understand when the auto-correlation function and the signal are of high-power series or in complicated forms. This motivates the search for a simpler but effective kernel. In electrical engineering context, the first-power exponential functions are very common and popular. This function is a response of many useful first-order circuits. However, the use of the function alone is not effective as this has been done in the past and well reported in the literature [1, 2]. On the other hand, what other researchers have often forgotten is that two first-power exponential functions with opposite signed exponents can be combined to form a hyperbolic

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sech(·) function. This function has a similar shape to that of the CW kernel and satisfies admissibility constraints. Therefore, it is a valid kernel and can be used for TFPSs to suppress cross terms and support auto terms. In addition, other higher-power functions of the hyperbolic sech(·) kernel: $\Phi(\theta, \tau) = [\text{sech}(\cdot)]^n$, where $n = 1, 2, 3, \dots$ are also valid kernel functions. However, simulations showed that the even powers of the hyperbolic sech(·) kernels are not effective since they are unbound and therefore have an infinite volume under the surface of their weighting functions. Thus, only the odd powers of the hyperbolic kernels, i.e. $\text{sech}(\cdot)$, $[\text{sech}(\cdot)]^3$, $[\text{sech}(\cdot)]^5, \dots$ are useful. In this paper, we restrict analysis to the first-order hyperbolic kernel $\text{sech}(\cdot)$.

Comparisons of the hyperbolic and CW kernels have been extensively reported in [5] and they will not be repeated here. From [5], it can be suggested that the hyperbolic kernel is more effective in cross-term suppression than the CW kernel but less useful in auto-term support. At this point, there exists a trade-off between cross-term suppression and auto-term support. In practice, one often encounters noise when processing different types of signals and therefore, noise robustness is important in this context. The effectiveness of the hyperbolic and CW kernels was compared in the presence of noise in [5] using a sum of two chirp signals. The results were encouraging and showed that the hyperbolic kernel is more noise robust than the CW kernel. Thus, the trade-off that we encountered earlier can be restated as the more cross-term effective a kernel is, the more noise robust it is but offers less auto-term support. Some simulation results are repeated here for clarity and completeness.

Figure 1: The CW TFPS of a sum of two chirp signals in 3dB noise

Figure 2: The HTFPS of chirp signals in 3dB noise.

As can be seen in Figure 1 and Figure 2, the hyperbolic TFPS has weaker auto-term arms and smaller cross terms in the middle region between the two arms. However, it is the opposite for the CW TFPS which is noisier than the hyperbolic TFPS. Thus, it can be concluded that the hyperbolic kernel is more noise robust than the CW kernel.

3 HYPERBOLIC WAVELET

Wavelet transform is one area of time-frequency signal processing that has been very popular in video coding, video compression and image processing [1, 2]. In the field of wavelet transform, a Mexican-hat wavelet, which has been widely known due to its familiar bell shape, has found many applications. The Mexican-hat wavelet is generated by taking the second derivative function of a Gaussian pulse. It is interesting that the Gaussian pulse is identical to the CW kernel used in TFPS context. In other words, there is a strong link between kernels in TFPSs and wavelet functions. However, this important relationship has not been reported in the literature. The significant contribution of this relationship is that it can be used to diversify the TFPS and wavelet areas, i.e. if a new kernel is found, correspondingly, a new wavelet is found and vice versa, provided that the wavelet function satisfies a zero-mean admissibility constraint. Since the CW kernel can be used to generate the Mexican-hat wavelet, the first-order hyperbolic kernel can similarly be used to generate its own wavelet function. General formulas of the CW and hyperbolic wavelet functions are given in Eqs. (2) and (3) respectively [6]

$$\psi_{CW}(\theta) = \frac{2}{\sigma} \cdot \exp\left(-\tau^2/\sigma\right) \cdot (-1 + 2\tau^2/\sigma), \quad (2)$$

$$\psi_{H\beta}(\theta) = -n\beta^2 \cdot [\text{sech}(\beta\theta)]^n \cdot \{n - (n+1) \cdot [\text{sech}(\beta\theta)]^2\}. \quad (3)$$

Detailed investigations along with comparisons can be found in [6] and will not be repeated here. From [6], it can be suggested that the hyperbolic wavelet is more effective than the CW wavelet by having a finer scale resolution. This allows detailed investigations of a signal wavelet power spectrum over a low-scale range and also minor changes can be detected easier by using the hyperbolic wavelet than the CW wavelet. However, the trade-off in this case is that the hyperbolic wavelet has a smaller total number of scales than that of the CW wavelet which limits its ability in analysing signals with high-frequency power spectra such as transient signals and ECG.

4 HYPERBOLIC SIGNAL DETECTOR

The main problem that needs to be solved is to detect a non-stationary signal in the presence of a stationary, zero-mean white noise with variance σ_{noise} . This problem was dealt with in 1984 [7] by using Moyal's formula for the unity WV kernel. Moyal's formula for WV kernel has been extended so that it is also applicable for non-unity kernels. Lengthy details of the derivation of Moyal's formula for non-unity kernels are not given here but can be found in [8]. After applying the formula to the hyperbolic and CW kernels, the SNRs of the two detectors are obtained as functions of β and σ which are

control parameters of the two kernels respectively. The final comparison is presented in Figure 3.

Figure 3: SNR comparison of the hyperbolic and CW detectors

As can be seen, the hyperbolic detector performs better than the CW detector by roughly 20% with β in the range of $3 \leq \beta \leq 20$. Outside that range, the CW detector performs slightly better than the hyperbolic detector. In general, the hyperbolic detector performs better than the WV and the cross-correlator detectors by about 41% [8] in terms of SNR.

5 SIGNAL ANALYSIS USING A WAVELET POWER SPECTRAL TECHNIQUE

Signal analysis has been done by several researchers using the wavelet bispectral technique [9] in which the wavelet BS is calculated at different times and frequencies in the time-frequency plane. The main problem of this method is that the wavelet BS has four main axes that need to be displayed at the same time which makes the process complicated and difficult to use. However, with an effective wavelet function, a wavelet power spectral technique can be employed. The hyperbolic wavelet has been shown to be more effective than the CW wavelet and has a finer scale (frequency) resolution which allows detection of fine changes over low-scale regions in the time-frequency plane. This motivates the use of the hyperbolic wavelet to analyse a number of signals including sinusoid, ECG, music, speech, transient, Duffing oscillator and so on. Some of the results are presented here, more details on this topic can be found in [10]. The hyperbolic wavelet power spectra (HWPSs) of Duffing Period 1 and Duffing chaotic waveforms are given in Figure 4 and Figure 5 respectively. To observe periodicity or chaos from these graphs, it is important to note the patterns of the energy distribution of the signal over the whole time and frequency domains. From Figure 4, it is easy to note that the signal energy distribution in time and frequency is uniform which strongly suggests that the signal is periodic. From Figure 5, it can be seen that the energy is distributed over a larger scale range and it is no longer uniformly distributed. This indicates a chaotic signal. Even though the hyperbolic wavelet power spectral technique provides better understanding about the behaviour of signals, Fourier transform is also employed for signals which contain high frequencies such as music and transients [10]. This can be identified as one major disadvantage of the hyperbolic wavelet power spectral technique due to having a very fine scale resolution.

Figure 4: HWPS of Duffing oscillator Period 1 output

Figure 5: HWPS of Duffing oscillator chaotic output

6 PARALLEL COMPUTATION OF THE TFPS AND BS

The BS has been calculated in parallel in [11-13] with encouraging results. The TFPS has also been calculated using a supercomputer [14-19] with linear SFs. The main idea in using parallel computing is to split the largest task in a serial program into smaller tasks (coarse-grained parallel method) which will be concurrently executed by independent processors. The disadvantages of parallel computing are cost and parallel overhead which might lower its efficiency if the data sets are not sufficiently large. The speedup factors and parallel efficiency of the BS and TFPS calculation processes are presented in Figures 7 and 8 respectively.

Figure 6: Measured SF of the BS calculation process.

Figure 7: Measured SF of the TRPS calculation process

7 APPLICATIONS

Followings are the most up-to-date applications of TFPSs

1. *Weather prediction*: calculations of the TFPS of a weather pattern data allow forecasters to exactly locate *where* and *when* a particular event occurs [1];
2. *Telecommunication*: to instantaneously monitor and predict the Internet bandwidth usage of a telecommunication network;
3. *Biomedical signal processing*: design of an optimum kernel to process biomedical signals such as ECG and EEG;
4. *Commercial*: computer chips can be programmed to solely process biomedical signals by calculating their TFPSs and WPSs. From that, the patient conditions can be monitored and predicted by doctors.

8 CONCLUSIONS

The hyperbolic time-frequency and wavelet power spectra have shown to be useful in signal detection and signal analysis. The hyperbolic kernel has been shown to be more effective in cross-term suppression than the Choi-Williams kernel. The hyperbolic wavelet has been shown to possess a finer scale resolution than the Choi-Williams and Morlet wavelets. An important link among kernels and wavelets has been introduced in which a new wavelet can be found from a new kernel and vice versa. It has also been shown that the time-frequency power spectrum and bispectrum are suitable candidates for parallel computing.

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