On joint determination of the number of states and the number of variables in Markov-switching models: A Monte Carlo study

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On joint determination of the number of states and the number of variables in Markov-switching models: A Monte Carlo study

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Abstract

In this paper, we examine the performance of two newly developed procedures that jointly select the number of states and variables in Markov-switching models by means of Monte Carlo simulations. They are Smith, Naik and Tsai (2006) and Psaradakis and Spagnolo (2006), respectively. The former develops Markov switching criterion (MSC) designed specifically for Markov-switching models, while the latter recommends the use of standard complexity-penalised information criteria (BIC, HQC, and AIC) in joint determination of the state dimension and the autoregressive order of Markov-switching models. The Monte Carlo evidence shows that BIC outperforms MSC while MSC and HQC are preferable over AIC.

JEL Classifications: C01; C13; C15; C50

Keywords: Information criteria; Markov-switching model; Monte Carlo

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1. Introduction

Markov-switching models have become increasingly popular in the modelling of economic or financial variables since the publication of Hamilton's (1989) influential paper. As shown in Timmermann (2000), these models are flexible enough to capture some well-known characteristics of economic or financial variables such as serial correlation, asymmetry, and fat tail behaviour.

To estimate Markov-switching models, a key problem that arises in applications is that the number of states is often unknown a priori. This leads to a breakdown of regularity conditions that prevents the use of the likelihood ratio test in the traditional framework of standard maximum likelihood theory. Another problem is that the order of autoregressive parameters is also unknown if the true model is believed to be a Markov-switching autoregressive model. The standard practice in applied econometrics is that these two problems are dealt with in a two-step procedure. Step One selects the order of autoregressive parameters using some kind of information criterion. Then Step Two determines the number of states accordingly. This procedure is problematic in the following sense: First, the efficacy of Step Two hinges upon the validity of the result of Step One. Second, in principle, one may start off with any one of the two problems first and then solve the remaining one later. But the result could be very different. For example, one may find the optimal autoregressive order being two in Step One and then the number of states being two in Step Two; if the procedure is reverse, it is likely that the number of states is two while the optimal autoregressive order is one. In this situation, one needs additional information as to which results one may count on.

Recently, Psaradakis and Spagnolo (2006) (hereafter called PS (2006)) propose a new procedure that gets around the limitations of standard practice. The new procedure combines two steps to one single step where an information criterion is used consistently in joint determination of the number of states and the autoregressive order. Based on simulation evidence, they show that complexity-penalised likelihood criteria like AIC, BIC, and HQC can accurately identify the correct model

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structure. One would like to know whether the same procedure could be used to Markov-switching static model where the problem of finding out the autoregressive order is replaced by the problem of selecting appropriate number of independent variables.

In a similar vein, Smith, Naik and Tsai (2006) (hereafter called SNT (2006)) derive a new information criterion called “Markov switching criterion” (MSC) that also simultaneously determines the state dimension and the number of variables in Markov-switching static models. Similar to AIC, BIC, and HQC, this criterion is also based on Kullback-Leibler divergence. Even though MSC is primarily designed for Markov-switching static models, they provide some (limited) simulation evidence that MSC also works well for some Markov-switching autoregressive models. In particular, only models with autoregressive order of two or four are considered in their simulation studies. It is interesting to see whether the same result can be carried over to other general settings.¹

The purpose of this paper is to compare whether MSC is better than standard information criteria such as AIC, BIC, and HQC using simulation studies. In particular, we deal with the following issues:

1. PS (2006) are concerned with Markov-switching autoregressive models only while SNT (2006)’s MSC is derived and based on Markov-switching static models. It is interesting to see whether PS (2006)’s procedure also works well in Markov-switching static model or not.
2. As far as the usefulness of AIC is concerned, there is apparently inconsistent result between SNT (2006) and PS (2006). SNT (2006) argue and present simulation evidence that AIC is misleading in the sense that it retains too many states and variables than the true model, resulting in reduction in accuracy of estimated parameters and precision of model forecasts. However, the results in PS (2006) reveal that AIC is the most successful criterion in terms of the

¹ In fact, Smith, Naik and Tsai (2006) made the following remark on page 560. “We encourage further research to investigate such comparisons using the proposed MSC.”

frequency of correct model identification, followed closely by BIC and HQC (p. 761). This paper sheds light on resolving this apparent conflict.

3. The primary focus of SNT (2006) is on the absolute performance of MSC even though it also discusses briefly the over-selection problem of using AIC. When it comes to applications, it is important to know about the relative performance of MSC over other standard information criteria. This paper aims to provide direct evidence on this issue.

The simulation results suggest that BIC performs well in Markov-switching models, regardless of whether the model is static or autoregressive. Consistent with SNT (2006), we find evidence that AIC retains too many states and variables than the true model and that MSC outperforms AIC in identifying the correct model structure. To our surprise, BIC is the best criterion in terms of the frequency of correct model identification while AIC is the least successful one. This result is in sharp contrast with that of PS (2006) and provides new evidence on the usefulness of BIC and MSC.

The reminder of the paper is organised as follows. Section 2 presents the class of Markov-switching models under consideration and discusses MSC and other standard information criteria to be examined. Section 3 presents the set up of Monte Carlo studies and their results. Section 4 concludes the paper.

2. Models and assumptions

This paper considers the Markov-switching models of the following form:²

$$y_t = v^{(s_t)} + \sum_{l=1}^p A_l^{(s_t)} y_{t-l} + \sigma^{(s_t)} e_t, \quad (1)$$

where e_t is independent and identically distributed random variables such that $E(e_t) = 0$ and

² Autoregressive model is used here for illustration purpose only. To get the equation for static model, just replace the set of lagged y_t variables by the same number of exogenous variables, x_t .

s_t are unobservable random variables that take values in the finite set $\{1, 2, \dots, r\}$ and are independent of e_t .

The random variables s_t (hereafter referred to as regime variables) are assumed to be a temporally homogeneous first-order Markov chain on $\{1, 2, \dots, r\}$ with transition matrix $P = p_{ij}$, i and $j \in \{1, 2, \dots, r\}$, where $p_{ij} = \text{Prob}(s_{t+1} = j | s_t = i)$. It is also assumed that s_t is periodic and irreducible. Notice that s_t may or may not be stationary. If the stationarity assumption is imposed, the above conditions guarantee a unique row-stochastic vector $\pi = (\pi_1, \pi_2, \dots, \pi_r)'$ such that $\pi P = \pi$ and $\pi_i = \text{Prob}(s_t = i) > 0$ for all $i \in \{1, 2, \dots, r\}$ and all t . Under these assumptions, the model could be referred as a r -regime p -order Markov-switching autoregressive model (or MS(r)-AR(p) model).

In this paper, it is assumed that both the autoregressive order, p , and the regime dimension, r , of the MS(r)-AR(p) model are unknown, so the interesting issue is to estimate both p and r on the basis of a finite segment $y_n = (y_1, y_2, \dots, y_T)$ of length T from Equation (1). This paper identifies both r and p by using the following four complexity-penalised likelihood criteria.

$$AIC = -2L + 2k; \quad (2)$$

$$BIC = -2L + k \log(T); \quad (3)$$

$$HQC = -2L + 2k \log(\log(T)); \quad (4)$$

$$MSC = -2L + \sum_{i=1}^r \frac{S_i(S_i + \lambda_i k)}{\delta_i S_i - \lambda_i k - 2}; \quad (5)$$

where L and T are the maximised log likelihood value and the total observations,

k is the number of estimated parameters in the case of AIC, BIC, and HQC; for Markov-switching static models, it is the number of exogenous variables, including an intercept term,

$$\delta_i = E[\pi_i^* / \hat{\pi}_i] \text{ and } \lambda_i = E[(\pi_i^* / \hat{\pi}_i)^2],$$

$\hat{\pi}_i$ is the i^{th} element of the principle eigenvector of $\hat{P}\hat{\pi} = \hat{\pi}$ from the estimation,

π_i^* is the i^{th} element of the principle eigenvector of $P^*\pi^* = \pi^*$ for the best estimation, and

S_i is the smoother probabilities at regime i .

Expectation Maximisation algorithm is used to obtain maximum-likelihood estimates of the model parameters. To implement MSC, we closely follow SNT (2006) to set δ_i and λ_i to 1 and the total number of regimes (or r) since they show that the performance of the Monte Carlo simulation with this setting is reasonably satisfactory. Therefore, MSC is calculated as follows:

$$MSC = -2L + \sum_{i=1}^r \frac{S_i(S_i + rk)}{S_i - rk - 2}. \quad (6)$$

3. Simulation and results

3.1 Simulation setup

Here, the simulation settings and selection procedure are described as follows:

- (i) Markov-switching autoregressive model: the true model consists of two regimes (or $r = 2$) and one lag with an intercept term in each regime. The true regression coefficients are $v^{(s_t)} = (v^{(1)}, v^{(2)})$ and $A^{(s_t)} = (A^{(1)}, A^{(2)})$, where $v^{(s_t)} = (0, 1)$ and $A^{(s_t)} = (0.3, 0.9)$. The r -regime variable, s_t , is a Markov chain with transition probabilities $p_{11} = 0.6$ and $p_{22} = 0.4$ and the initial probabilities are set to 0.4 and 0.6. The total number of observations, T , is initially set to 200 and $e_t \sim N(0, 1)$ and $\sigma^{(s_t)} = (\sigma^{(1)}, \sigma^{(2)}) = (0.5, 0.5)$.
- (ii) Markov-switching autoregressive model with small/large coefficient: this paper considers variation from the setting in (i). First, this paper changes the autoregressive coefficient for both regimes to the same value: $v^{(s_t)} = (0, 1)$ and $A^{(s_t)} = (0.3, 0.3)$. Second, setting the intercept coefficient for both regimes to the same value: $v^{(s_t)} = (0, 0)$ and $A^{(s_t)} = (0.3, 0.9)$.
- (iii) Markov-switching autoregressive model with high/low transition probability: this paper also considers variations from the setting (i) and (ii) by changing the transition probability p_{11} from 0.6 to 0.9 and p_{22} from 0.4 to 0.1 and the initial probabilities from (0.4, 0.6) to (0.5, 0.5). To

examine the impact of persistence in $\{s_t\}$, the transition probabilities p_{11} and p_{22} are also set to 0.9 so that their sum is greater than one.³

- (iv) Markov-switching autoregressive model with small/large sample and high/low noise: the following variations from the setting in (i), (ii), (iii), and (iv) are considered. First, to examine the impact of sample size on performance, this paper conducts the above simulations using $T = 100, 400, \text{ and } 500$, respectively. Second, $\sigma^{(s_t)}$ will be set to $(1.0, 1.0)$ and $(0.5, 1.0)$ for all T to understand the effect of a change in noise level.
- (v) Markov-switching autoregressive model with regime-switch in noise level: setting (i) only allows for regime switch in either intercept term or regression coefficient. To consider the effect of switch in noise level, we set $\sigma^{(s_t)}$ to 0.5 if $s_t = \text{regime 1}$ and 1 if $s_t = \text{regime 2}$ for all T .
- (vi) Markov-switching static model: the paper also uses the same setting from the autoregressive model in (i) through (v) to examine the performance of the four information criteria applied to Equation (1) where l lagged y_t variables are replaced by the same number of exogenous variables, x_t . These variables are independent and identically distributed random variables with zero mean and unit variance.

The simulations proceed by first generating an artificial time series y_t (and exogenous variables, x_t , in the case of static model) of length $500+T$ according to the setting (i) and (v) and setting initial values to zero. The first 500 pseudo-data points then are discarded in order to eliminate start-up effects, while the remaining T points are used to determine both r and p by minimising the AIC, BIC, HQC, and MSC over r regimes and p lags ($r = 1, 2, 3; p = 1, 2, 3$). Since computations are very intensive, 1000 Monte Carlo replications are carried out for each setting to assess how often the information criteria select the true model.

³ We would like to thank an anonymous referee for suggesting this persistence scenario for us to examine.

3.2 Simulation results

3.2.1 Static Models Results

Tables 1 to 3 report the frequency of correct state and variable selection for Markov-switching static model with one exogenous variable for different sample size and noise level. Common to these three tables are the assumptions that the number of regimes is two and that initial and transition probabilities are kept constant at (0.4, 0.6) and (0.6, 0.4), respectively. Notice that Table 1 allows for change in regression coefficient induced by regime change only. In particular, if $s_t = \text{regime 1}$, then the regression coefficient is 0.3; if $s_t = \text{regime 2}$, then it is 0.9. Table 2 allows for change in intercept term the value of which can be either 0 or 1 depending on whether the regime variable is in state 1 or state 2. Table 3 considers both cases together.

Two main observations can be made before we get into the details of each of these tables. First, BIC consistently outperforms other information criteria regardless of sample size and noise level. The performance of MSC is generally comparable to that of HQC, while AIC seems to be the inferior one. Second, consistent with SNT (2006)'s observation, the frequency of correct states and variables identification for MSC is always higher than that for AIC and the latter has a tendency to retain too many states and variables.

Given that only change in regression coefficient is allowed, Table 1 suggests that BIC is the best and that AIC is always the worst. In a setting where the sample size is 200 with low noise level, for example, the frequency of incorrectly identifying the true model of BIC is 59 (out of 1000) while the relevant frequencies of AIC, HQC, and MSC are 274, 113, and 85, respectively. In addition, BIC has the lowest frequency of identifying the wrong number of variables. An increase in sample size generally leads to better performance. For example, with low noise level and increase in sample size from 100 to 500, the frequency of correct states and variables selection for BIC goes up from 920 to 988, while it also increases from 700 to 805 (AIC), 860 to 945 (HQC), and 893 to 955

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(MSC), respectively. When the noise level increases, however, it tends to have a negative impact on the performance. Allowing for a regime change in noise level implies a poor performance as it reduces the frequency of accurate model selection for all of the information criteria.

Table 2 exhibits a similar picture. BIC outperforms MSC in detecting regime change in intercept term. AIC still has the lowest frequency of correct model identification. As an example, if the sample size is 100 and the noise level is high, Table 2 indicates that the frequencies of true model selection for BIC, AIC, HQC, and MSC are 900, 701, 830, and 880, respectively. Regardless of the information criteria being considered, increasing sample size or reducing the noise level can help improve the performance, while allowing for change in variance cannot.

Table 3, where the true model allows for regime change in both intercept term and regression coefficient, basically reinforces observations from Tables 1 and 2. Compared to Table 1, however, Table 3 shows that the performance of all the information criteria is generally poorer. In the case of 400 observations with low noise level, for instance, the frequencies of correct states and variables selection for BIC, AIC, HQC, and MSC are 852, 722, 766, and 799 respectively in Table 3. These numbers are much lower than that in Table 1 because the corresponding numbers are 976, 759, 916, and 946, respectively. Despite this, BIC still stands out from other information criteria with AIC being the worst one.

Table 4 differs from Table 1 in that the transition probabilities change from (0.6, 0.4) to (0.9, 0.1). This change seems to make the results noisier but it does not have any significant impacts on the main conclusions. Table 5 shows that the change in initial probabilities from (0.4, 0.6) to (0.5, 0.5) generally leads to a lower performance as well. This paper also considers the case where the sum of diagonal probabilities of transition matrix is greater than one ($p_{11} + p_{22} > 1$), allowing for persistence in $\{s_t\}$. In this case, the probabilities of regime 1 and regime 2 (or p_{11} and p_{22}) are set to

0.9. This change generally lowers the frequency of the true model identification, as reported in Table 6. However, it does not alter the main conclusions.

3.2.2 Autoregressive models results

For Tables 7 to 12, the data generating process is a two-regime Markov-switching autoregressive model of order one for different sample size, regression coefficient, and noise level. A set of assumptions similar to that of static model is also employed. In particular, Table 7 shows the benchmark case where the initial state probability of state 1 and state 2 are set to 0.4 and 0.6 respectively and the transition probability of moving from state 1 (2) to state 2 (1) is set to 0.4 (0.6). Notice that Table 7 allows for change in regression coefficient only. The case allowing for change in intercept term is presented in Table 8. Table 9 allows for both intercept term and regression coefficient to be regime-specific. Table 10 considers the case where there is a change in transition probabilities from (0.6, 0.4) to (0.9, 0.1) while Table 11 deals with a change in the initial probabilities. Table 12 shows the results for the case where $\{s_t\}$ is allowed to be persistent.

Allowing for change in regression coefficient only, Table 7 shows that BIC is the best among the four information criteria. For example, if the sample size is 100 and the noise level is low, the number of times that BIC correctly identifies the true model is 852 (out of 1000) while the relevant frequencies of AIC, HQC, and MSC are 696, 733, and 772, respectively. When the noise level increases, the performance becomes worse; however, this effect is not significant. BIC is still the best choice while MSC is as good as HQC. AIC comes last because it is too generous in retaining many regimes and variables. In addition, an increase in sample size does help improve the performance. For instance, the frequency of correct model identification for BIC, AIC, HQC, and MSC increases from 844, 600, 694, and 644 to 948, 706, 859, and 907 respectively when the sample size increases from 100 to 500 and the noise level is high.

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Comparison between Tables 1 and 7 reveals that MSC is more suitable for static model than for autoregressive model because the frequency of correctly identifying the true model is generally higher in static model. Consistent with SNT (2006)’s claim that MSC can also be used in autoregressive model, MSC still outperforms AIC and HQC.

Table 7 shows that the performance is always better than that in Table 8 when allowing for regime-change in variance, despite that its overall results are broadly similar to that of Table 8. With 500 observations and switch in noise level, for instance, the frequency of correctly selecting the true model for BIC, AIC, HQC, and MSC decreases from 986, 784, 921, and 945 as in Table 7 to 964, 750, 865, and 901 as in Table 8, respectively. Compared to Table 7, Table 9 shows that the performance of all the information criteria becomes slightly worse when both intercept term and regression coefficient are allowed to be regime switching. All of these tables clearly indicate that as far as performance is concerned, BIC always comes first, closely followed by MSC and HQC, and then AIC.

Change in transition probabilities from (0.6, 0.4) to (0.9, 0.1), as shown in Table 10, generally leads to a slightly improved performance. The comparison between Tables 7 and 11 reveals that change in the initial probabilities from (0.4, 0.6) to (0.1, 0.9) leads to a marginally deteriorated performance but it does not change main conclusions. In addition, persistence in $\{s_t\}$ generally lowers the frequency of selecting the true model as depicted in Table 12. However, the main conclusions are still the same.

4. Conclusions and practical recommendation

The simulations have shown that in the context of jointly determination of number of states and number of variables in Markov-switching models BIC has better performance over MSC, HQC, and

AIC. The performance of MSC is comparable to that of HQC while AIC is always inferior. We therefore recommend that BIC should be used in practical applications.

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Table 1: Frequency of selection of a MS(2) static model – Change in regression coefficient

The data generating process is the MS static model with two regimes and one exogenous variable shown as follows:

$$y_t = v^{(s_t)} + A^{(s_t)}x_t + \sigma^{(s_t)}e_t.$$

The initial probabilities are set to 0.4 and 0.6 with the transition probabilities as $p_{11} = 0.6$; $p_{22} = 0.4$. The true regression coefficients are $v^{(s_t)} = (v^{(1)}, v^{(2)})$ and $A^{(s_t)} = (A^{(1)}, A^{(2)})$, where $v^{(s_t)} = (0, 0)$ and $A^{(s_t)} = (0.3, 0.9)$. Notice that the total replication is 1000.

The first column shows the possible values that $\sigma^{(s_t)}$ can take. Notice that in the regime-specific case, $\sigma^{(s_t)}$ is set to 0.5 if s_t = regime 1 and 1 if s_t = regime 2. The second column shows the relevant information criterion being considered, while the third column indicates the number of regimes imposed on estimation. With sample size of 100, the fourth column depicts the frequency of selecting the correct number of variable (in this case, 1 variable) while the fifth column indicates the frequency of selecting the incorrect number of variables (in this case, 2 variables). Other columns show similar frequencies for different sample sizes.

Sigma	Information criteria	No. of regimes imposed	100 observations		200 observations		400 observations		500 observations	
			1 variable	2 variables	1 variable	2 variables	1 variable	2 variables	1 variable	2 variables
0.5	AIC	1	1	11	1	5	3	28	0	12
		2	700	247	726	210	759	88	805	107
		3	24	17	5	53	41	81	60	16
	BIC	1	4	1	6	1	3	1	1	1
		2	920	75	941	52	976	20	988	10
		3	0	0	0	0	0	0	0	0
	HQC	1	5	1	1	0	1	1	1	0
		2	860	133	887	109	916	75	945	48
		3	0	1	0	3	5	2	5	1
	MSC	1	1	11	2	19	1	0	1	0
		2	893	32	915	63	946	44	955	41
		3	23	40	0	1	8	1	3	0
	AIC	1	0	10	2	15	2	18	3	15
		2	697	258	710	250	753	199	801	81
		3	10	25	0	23	2	26	100	0
1.0	BIC	1	4	4	1	1	1	1	1	1
		2	911	81	934	64	967	31	984	11
		3	0	0	0	0	0	0	3	0
	HQC	1	1	23	1	24	1	13	2	17
		2	814	152	870	100	880	106	890	88
		3	10	0	0	5	0	0	3	0
	MSC	1	7	1	1	4	1	0	1	0
		2	883	57	904	75	928	70	938	1
		3	15	37	1	15	0	1	60	0
Regime-specific	AIC	1	1	10	2	18	1	3	1	5
		2	652	290	671	226	680	242	753	190
		3	0	47	3	80	8	66	3	48
	BIC	1	4	1	2	1	4	1	1	1
		2	884	108	937	60	953	42	962	36
		3	0	3	0	0	0	0	0	0
	HQC	1	1	25	1	1	1	0	2	0
		2	802	160	860	124	865	123	890	103
		3	0	12	2	12	1	10	1	4
	MSC	1	5	1	2	1	1	2	1	0
		2	856	59	885	67	924	65	927	68
		3	2	77	4	41	2	6	1	3

Table 2: Frequency of selection of a MS(2) static model – Change in intercept term

The true data generating process is the MS static model with two regimes and one exogenous variable, as follows:

$$y_i = v^{(s_i)} + A^{(s_i)} x_i + \sigma^{(s_i)} e_i.$$

The initial probabilities are set to 0.4 and 0.6 with the transition probabilities as $p_{11} = 0.6$; $p_{22} = 0.4$. The true regression coefficient coefficients are $v^{(s_i)} = (v^{(1)}, v^{(2)})$ and $A^{(s_i)} = (A^{(1)}, A^{(2)})$, where $v^{(s_i)} = (0, 1)$ and $A^{(s_i)} = (0.3, 0.3)$. Notice that the total replication is 1000.

The first column shows the possible values that $\sigma^{(s_i)}$ can take. Notice that in the regime-specific case, $\sigma^{(s_i)}$ is set to 0.5 if $s_i =$ regime 1 and 1 if $s_i =$ regime 2. The second column shows the relevant information criterion being considered, while the third column indicates the number of regimes imposed on estimation. With sample size of 100, the fourth column depicts the frequency of selecting the correct number of variable (in this case, 1 variable) while the fifth column indicates the frequency of selecting the incorrect number of variables (in this case, 2 variables). Other columns show similar frequencies for different sample sizes.

Sigma	Information criteria	No. of regimes imposed	100 observations		200 observations		400 observations		500 observations	
			1 variable	2 variables	1 variable	2 variables	1 variable	2 variables	1 variable	2 variables
0.5	AIC	1	0	41	1	31	1	17	1	7
		2	714	162	790	26	822	40	840	72
		3	73	10	152	0	40	80	54	26
	BIC	1	1	0	1	0	1	0	1	0
		2	930	61	954	42	980	18	990	9
		3	7	1	3	0	1	0	0	0
	HQC	1	1	3	1	9	1	5	1	7
		2	820	133	840	109	870	55	890	60
		3	41	2	41	0	60	9	30	12
	MSC	1	6	5	1	0	1	0	1	0
		2	915	32	934	15	950	9	978	0
		3	34	8	49	1	39	1	21	0
1.0	AIC	1	2	20	0	15	0	2	0	6
		2	701	215	714	148	734	130	787	141
		3	56	6	80	43	79	55	32	34
	BIC	1	0	0	0	0	1	0	0	0
		2	900	100	959	41	977	20	984	14
		3	0	0	0	0	2	0	2	0
	HQC	1	4	5	1	8	0	7	2	3
		2	830	153	850	132	890	57	912	66
		3	8	0	7	2	40	6	16	1
	MSC	1	1	5	1	1	0	0	0	0
		2	880	24	900	83	911	41	917	46
		3	69	21	14	1	47	1	36	1
Regime-specific	AIC	1	0	34	11	18	8	11	0	6
		2	701	104	713	142	730	102	790	119
		3	157	4	56	60	53	96	26	59
	BIC	1	2	0	0	0	1	0	2	0
		2	902	32	918	30	950	19	968	20
		3	64	0	52	0	30	0	9	1
	HQC	1	10	4	8	7	4	6	0	11
		2	800	85	840	85	856	83	865	42
		3	100	1	50	10	45	6	66	16
	MSC	1	3	2	1	6	0	1	0	0
		2	832	45	854	70	890	32	908	28
		3	89	29	37	32	75	2	59	5

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Table 3: Frequency of selection of a MS(2) static model – Changes in intercept term and regression coefficient

The true data generating process is the MS static model with two regimes and one exogenous variable, as follows:

$$y_i = v^{(s_i)} + A^{(s_i)}x_i + \sigma^{(s_i)}e_i.$$

The initial probabilities are set to 0.4 and 0.6 with the transition probabilities as $p_{11} = 0.6$; $p_{22} = 0.4$. The true regression coefficients are $v^{(s_i)} = (v^{(1)}, v^{(2)})$ and $A^{(s_i)} = (A^{(1)}, A^{(2)})$, where $v^{(s_i)} = (0, 1)$ and $A^{(s_i)} = (0.3, 0.9)$. Notice that the total replication is 1000.

The first column shows the possible values that $\sigma^{(s_i)}$ can take. Notice that in the regime-specific case, $\sigma^{(s_i)}$ is set to 0.5 if s_i = regime 1 and 1 if s_i = regime 2. The second column shows the relevant information criterion being considered, while the third column indicates the number of regimes imposed on estimation. With sample size of 100, the fourth column depicts the frequency of selecting the correct number of variable (in this case, 1 variable) while the fifth column indicates the frequency of selecting the incorrect number of variables (in this case, 2 variables). Other columns show similar frequencies for different sample sizes.

Sigma	Information criteria	No. of regimes imposed	100 observations		200 observations		400 observations		500 observations	
			1 variable	2 variables	1 variable	2 variables	1 variable	2 variables	1 variable	2 variables
0.5	AIC	1	3	24	5	20	0	7	2	3
		2	601	226	660	264	722	205	792	181
		3	136	10	10	41	49	17	11	11
	BIC	1	6	4	5	2	0	0	1	0
		2	752	236	785	208	852	134	916	69
		3	2	0	0	0	14	0	13	1
	HQC	1	0	3	0	7	0	3	1	1
		2	665	301	733	241	766	123	805	82
		3	28	3	8	11	85	23	65	46
	MSC	1	6	6	2	1	0	0	0	1
		2	745	130	755	215	799	111	838	100
		3	83	30	12	15	87	3	60	1
1.0	AIC	1	0	15	3	15	0	5	4	16
		2	519	253	570	242	650	147	792	114
		3	180	33	150	20	108	90	24	50
	BIC	1	6	3	1	0	3	1	9	1
		2	667	320	711	287	834	160	906	81
		3	3	1	1	0	1	1	2	1
	HQC	1	9	8	1	21	4	0	0	0
		2	630	302	656	202	770	181	820	132
		3	37	14	78	42	17	28	39	9
	MSC	1	2	7	3	1	0	0	0	0
		2	631	294	640	260	707	175	801	17
		3	11	55	46	50	104	14	181	1
Regime-specific	AIC	1	11	29	0	37	6	10	0	4
		2	558	310	629	236	705	159	754	130
		3	64	28	96	2	28	92	38	74
	BIC	1	8	1	6	4	2	0	1	0
		2	772	216	838	151	887	110	989	7
		3	3	0	1	0	1	0	3	0
	HQC	1	7	4	0	1	1	2	0	3
		2	633	290	722	136	800	171	809	152
		3	62	4	125	16	17	9	32	4
	MSC	1	5	5	2	7	0	0	0	0
		2	687	170	771	165	800	157	803	128
		3	73	60	21	34	42	1	69	0

Table 4: Frequency of selection of a MS(2) static model – Change in transition probabilities

The true data generating process is the MS static model with two regimes and one exogenous variable, as follows:

$$y_i = v^{(s_i)} + A^{(s_i)} x_i + \sigma^{(s_i)} e_i.$$

The initial probabilities are set to 0.4 and 0.6 with the transition probabilities as $p_{11} = 0.9$; $p_{22} = 0.1$. The true regression coefficients are $v^{(s_i)} = (v^{(1)}, v^{(2)})$ and $A^{(s_i)} = (A^{(1)}, A^{(2)})$, where $v^{(s_i)} = (0, 0)$ and $A^{(s_i)} = (0.3, 0.9)$. Notice that the total replication is 1000.

The first column shows the possible values that $\sigma^{(s_i)}$ can take. Notice that in the regime-specific case, $\sigma^{(s_i)}$ is set to 0.5 if $s_i = \text{regime 1}$ and 1 if $s_i = \text{regime 2}$. The second column shows the relevant information criterion being considered, while the third column indicates the number of regimes imposed on estimation. With sample size of 100, the fourth column depicts the frequency of selecting the correct number of variable (in this case, 1 variable) while the fifth column indicates the frequency of selecting the incorrect number of variables (in this case, 2 variables). Other columns show similar frequencies for different sample sizes.

Sigma	Information criteria	No. of regimes imposed	100 observations		200 observations		400 observations		500 observations	
			1 variable	2 variables	1 variable	2 variables	1 variable	2 variables	1 variable	2 variables
0.5	AIC	1	3	6	0	13	5	5	0	5
		2	711	250	764	151	792	178	807	154
		3	0	30	22	50	1	19	0	34
	BIC	1	5	2	1	0	0	0	0	0
		2	842	151	870	129	979	21	988	12
		3	0	0	0	0	0	0	0	0
	HQC	1	1	5	4	4	4	3	0	0
		2	740	250	802	174	815	173	842	154
		3	0	4	5	11	5	0	0	4
	MSC	1	2	9	2	5	0	1	0	0
		2	829	120	840	92	947	48	977	22
		3	10	30	29	32	0	4	0	1
1.0	AIC	1	3	4	5	7	0	4	0	9
		2	690	281	750	228	815	176	841	140
		3	0	22	0	10	0	5	0	10
	BIC	1	4	5	7	3	6	4	0	0
		2	826	165	856	134	870	120	885	115
		3	0	0	0	0	0	0	0	0
	HQC	1	4	2	3	4	3	6	0	0
		2	730	264	791	201	830	161	844	156
		3	0	0	0	1	0	0	0	0
	MSC	1	6	2	5	5	6	4	0	0
		2	799	138	836	147	850	139	859	141
		3	12	43	0	7	0	1	0	0
Regime-specific	AIC	1	2	4	3	6	3	6	0	8
		2	682	291	747	200	761	135	842	70
		3	12	9	2	42	29	66	10	70
	BIC	1	1	3	2	1	1	0	0	0
		2	856	135	943	54	950	42	974	26
		3	1	4	0	0	5	2	0	0
	HQC	1	8	1	9	0	2	0	1	0
		2	790	170	800	190	881	66	907	81
		3	11	20	0	1	21	30	4	7
	MSC	1	2	3	2	1	1	2	0	0
		2	850	100	907	48	911	56	938	56
		3	13	32	12	30	14	16	4	2

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Table 5: Frequency of selection of a MS(2) static model – Change in initial probabilities

The true data generating process is the MS static model with two regimes and one exogenous variable, as follows:

$$y_i = v^{(s_i)} + A^{(s_i)} x_i + \sigma^{(s_i)} e_i.$$

The initial probabilities are set to 0.5 and 0.5 with the transition probabilities as $p_{11} = 0.6$; $p_{22} = 0.4$. The true regression coefficients are $v^{(s_i)} = (v^{(1)}, v^{(2)})$ and $A^{(s_i)} = (A^{(1)}, A^{(2)})$, where $v^{(s_i)} = (0, 0)$ and $A^{(s_i)} = (0.3, 0.9)$. Notice that the total replication is 1000.

The first column shows the possible values that $\sigma^{(s_i)}$ can take. Notice that in the regime-specific case, $\sigma^{(s_i)}$ is set to 0.5 if s_i = regime 1 and 1 if s_i = regime 2. The second column shows the relevant information criterion being considered, while the third column indicates the number of regimes imposed on estimation. With sample size of 100, the fourth column depicts the frequency of selecting the correct number of variable (in this case, 1 variable) while the fifth column indicates the frequency of selecting the incorrect number of variables (in this case, 2 variables). Other columns show similar frequencies for different sample sizes.

Sigma	Information criteria	No. of regimes imposed	100 observations		200 observations		400 observations		500 observations	
			1 variable	2 variables	1 variable	2 variables	1 variable	2 variables	1 variable	2 variables
0.5	AIC	1	4	8	5	4	2	4	1	2
		2	682	217	694	221	737	158	796	158
		3	25	64	2	74	8	91	2	41
	BIC	1	5	2	2	0	1	0	0	0
		2	850	143	884	114	917	82	976	24
		3	0	0	0	0	0	0	0	0
	HQC	1	3	0	4	1	2	5	2	0
		2	750	232	769	213	820	167	921	69
		3	5	10	0	13	1	5	0	8
	MSC	1	6	9	8	2	0	0	0	0
		2	800	80	854	84	865	134	948	50
		3	25	80	2	50	1	0	0	2
1.0	AIC	1	2	34	4	37	0	7	0	8
		2	633	280	655	261	719	237	773	205
		3	3	48	21	22	11	26	0	14
	BIC	1	3	0	2	1	2	0	0	0
		2	826	170	854	143	866	132	969	31
		3	0	1	0	0	0	0	0	0
	HQC	1	5	1	2	8	1	7	1	1
		2	733	252	754	232	812	179	892	105
		3	0	9	0	4	0	1	0	1
	MSC	1	1	5	0	9	0	0	0	0
		2	750	183	800	181	854	145	921	79
		3	8	53	0	10	0	1	0	0
Regime-specific	AIC	1	2	9	3	4	0	7	2	2
		2	786	150	803	160	817	152	823	110
		3	1	52	0	30	3	21	2	61
	BIC	1	5	4	6	4	3	2	1	0
		2	913	78	946	43	953	42	960	39
		3	0	0	0	1	0	0	0	0
	HQC	1	9	1	4	5	3	1	0	0
		2	816	168	850	140	871	124	883	108
		3	0	6	0	1	0	1	1	8
	MSC	1	3	7	1	6	0	0	0	0
		2	832	38	849	130	903	96	911	78
		3	13	107	1	13	0	1	0	11

Table 6: Frequency of selection of a MS(2) static model – Persistence in $\{s_t\}$

The true data generating process is the MS static model with two regimes and one exogenous variable, as follows:

$$y_t = v^{(s_t)} + A^{(s_t)}x_t + \sigma^{(s_t)}e_t.$$

The initial probabilities are set to 0.4 and 0.6 with the transition probabilities as $p_{11} = 0.9$; $p_{22} = 0.9$. The true regression coefficients are $v^{(s_t)} = (v^{(1)}, v^{(2)})$ and $A^{(s_t)} = (A^{(1)}, A^{(2)})$, where $v^{(s_t)} = (0, 0)$ and $A^{(s_t)} = (0.3, 0.9)$. Notice that the total replication is 1000.

The first column shows the possible values that $\sigma^{(s_t)}$ can take. Notice that in the regime-specific case, $\sigma^{(s_t)}$ is set to 0.5 if $s_t = \text{regime 1}$ and 1 if $s_t = \text{regime 2}$. The second column shows the relevant information criterion being considered, while the third column indicates the number of regimes imposed on estimation. With sample size of 100, the fourth column depicts the frequency of selecting the correct number of variable (in this case, 1 variable) while the fifth column indicates the frequency of selecting the incorrect number of variables (in this case, 2 variables). Other columns show similar frequencies for different sample sizes.

Sigma	Information criteria	No. of regimes imposed	100 observations		200 observations		400 observations		500 observations	
			1 variable	2 variables	1 variable	2 variables	1 variable	2 variables	1 variable	2 variables
0.5	AIC	1	0	9	2	5	1	5	0	0
		2	663	294	720	232	770	193	822	154
		3	14	20	0	41	0	31	0	24
	BIC	1	9	1	7	3	5	4	0	0
		2	811	175	963	27	976	15	992	8
		3	1	3	0	0	0	0	0	0
	HQC	1	4	2	0	4	0	1	0	0
		2	787	162	899	94	918	77	936	63
		3	6	39	0	3	0	4	0	1
	MSC	1	2	4	0	5	0	3	0	0
		2	771	110	812	121	920	44	949	47
		3	20	93	60	2	30	3	0	4
1.0	AIC	1	2	20	9	20	3	16	1	3
		2	651	256	716	220	760	200	775	187
		3	1	70	6	29	1	20	4	30
	BIC	1	6	2	8	0	1	0	0	0
		2	802	190	940	52	968	31	978	22
		3	0	0	0	0	0	0	0	0
	HQC	1	7	9	8	5	8	2	7	2
		2	739	238	885	101	900	90	920	67
		3	1	6	0	1	0	0	1	3
	MSC	1	1	17	4	4	0	0	0	0
		2	762	150	806	146	909	69	939	53
		3	10	60	15	25	10	12	2	6
Regime-specific	AIC	1	3	15	9	12	5	9	0	0
		2	780	160	807	130	825	130	849	88
		3	2	40	1	41	18	13	10	53
	BIC	1	4	1	5	0	1	0	1	0
		2	885	110	901	94	910	85	951	48
		3	0	0	0	0	0	4	0	0
	HQC	1	9	1	3	7	0	5	0	0
		2	800	190	805	181	810	153	893	97
		3	0	0	2	2	2	30	0	10
	MSC	1	6	4	6	2	0	1	0	2
		2	860	20	901	47	910	72	915	80
		3	19	91	13	31	3	14	1	2

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Table 7: Frequency of selection of a MS(2)-AR(1) autoregressive model – Change in autoregressive coefficient

The true data generating process is the MS-AR model with two regimes and one lag, as follows:

$$y_t = v^{(s_t)} + A^{(s_t)}y_{t-1} + \sigma^{(s_t)}e_t.$$

The initial probabilities are set to 0.4 and 0.6 with the transition probabilities as $p_{11} = 0.6$; $p_{22} = 0.4$. The true regression coefficients are $v^{(s_t)} = (v^{(1)}, v^{(2)})$ and $A^{(s_t)} = (A^{(1)}, A^{(2)})$, where $v^{(s_t)} = (0, 0)'$ and $A^{(s_t)} = (0.3, 0.9)'$. Notice that the total replication is 1000.

The first column shows the possible values that $\sigma^{(s_t)}$ can take. Notice that in the regime-specific case, $\sigma^{(s_t)}$ is set to 0.5 if s_t = regime 1 and 1 if s_t = regime 2. The second column shows the relevant information criterion being considered, while the third column indicates the number of regimes imposed on estimation. With sample size of 100, the fourth column depicts the frequency of selecting the correct lag length (in this case, 1 lag) while the fifth and sixth columns indicate the frequency of selecting the incorrect lag lengths (in this case, 2 and 3 lags). Other columns show similar frequencies for different sample sizes.

Sigma	Information Criteria	No. of regimes imposed	100 observations			200 observations			400 observations			500 observations		
			AR(1)	AR(2)	AR(3)	AR(1)	AR(2)	AR(3)	AR(1)	AR(2)	AR(3)	AR(1)	AR(2)	AR(3)
0.5	AIC	1	38	64	83	4	77	72	33	42	73	46	48	46
		2	696	24	80	712	35	83	735	52	60	740	59	47
		3	5	2	8	7	5	5	3	1	1	4	5	5
	BIC	1	56	24	14	19	1	0	21	0	0	17	0	0
		2	852	37	13	934	39	7	946	27	6	959	20	4
		3	1	1	2	0	0	0	0	0	0	0	0	0
	HQC	1	49	53	45	20	18	15	8	11	3	3	11	3
		2	733	45	37	842	51	45	877	66	35	893	53	36
		3	8	7	23	2	3	4	0	0	0	1	0	0
	MSC	1	31	11	29	38	21	2	0	0	0	0	0	0
		2	772	85	63	838	32	49	876	36	88	914	42	43
		3	7	2	0	5	8	7	0	0	0	0	1	0
1.0	AIC	1	67	71	100	83	72	82	47	79	66	57	43	82
		2	600	56	84	634	14	101	651	22	127	706	50	50
		3	6	7	9	3	9	2	6	2	0	0	8	4
	BIC	1	41	38	17	8	11	2	10	10	0	14	0	0
		2	844	47	10	929	32	18	942	28	10	948	33	5
		3	1	1	1	0	0	0	0	0	0	0	0	0
	HQC	1	43	72	61	71	18	12	51	13	8	38	13	2
		2	694	64	53	762	86	45	806	80	40	859	52	35
		3	4	9	0	0	2	4	2	0	0	1	0	0
	MSC	1	43	1	11	4	0	1	0	0	0	0	0	0
		2	644	185	103	716	190	72	803	155	42	907	38	53
		3	6	4	3	6	2	9	0	0	0	0	2	0
Regime-specific	AIC	1	69	39	31	47	10	14	9	2	3	4	0	0
		2	637	74	140	700	86	140	760	87	118	784	81	114
		3	1	1	8	1	2	0	5	8	8	2	9	6
	BIC	1	40	29	15	23	17	3	30	8	0	2	8	0
		2	855	44	14	869	51	36	954	8	0	986	4	0
		3	1	0	2	1	0	0	0	0	0	0	0	0
	HQC	1	7	61	26	34	30	24	39	3	1	12	2	0
		2	739	75	81	813	52	37	880	60	8	921	54	6
		3	5	5	1	4	2	4	4	4	1	2	3	0
	MSC	1	77	7	24	10	1	2	0	0	1	0	0	0
		2	751	27	105	859	26	90	883	101	10	945	10	45
		3	3	5	1	5	6	1	0	0	5	0	0	0

Table 8: Frequency of selection of a MS(2)-AR(1) autoregressive model – Change in intercept term

The true data generating process is the MS-AR model with two regimes and one lag, as follows:

$$y_t = v^{(s_t)} + A^{(s_t)} y_{t-1} + \sigma^{(s_t)} e_t.$$

The initial probabilities are set to 0.4 and 0.6 with the transition probabilities as $p_{11} = 0.6$; $p_{22} = 0.4$. The true regression coefficients are $v^{(s_t)} = (v^{(1)}, v^{(2)})$ and $A^{(s_t)} = (A^{(1)}, A^{(2)})$, where $v^{(s_t)} = (0, 1)'$ and $A^{(s_t)} = (0.3, 0.3)'$. Notice that the total replication is 1000.

The first column shows the possible values that $\sigma^{(s_t)}$ can take. Notice that in the regime-specific case, $\sigma^{(s_t)}$ is set to 0.5 if $s_t =$ regime 1 and 1 if $s_t =$ regime 2. The second column shows the relevant information criterion being considered, while the third column indicates the number of regimes imposed on estimation. With sample size of 100, the fourth column depicts the frequency of selecting the correct lag length (in this case, 1 lag) while the fifth and sixth columns indicate the frequency of selecting the incorrect lag lengths (in this case, 2 and 3 lags). Other columns show similar frequencies for different sample sizes.

Sigma	Information Criteria	No. of regimes imposed	100 observations			200 observations			400 observations			500 observations		
			AR(1)	AR(2)	AR(3)	AR(1)	AR(2)	AR(3)	AR(1)	AR(2)	AR(3)	AR(1)	AR(2)	AR(3)
0.5	AIC	1	9	61	45	3	55	43	3	42	41	3	31	41
		2	677	73	128	680	94	105	705	106	88	756	39	127
		3	6	1	0	9	4	7	9	0	6	0	2	1
	BIC	1	35	10	0	11	23	0	22	10	0	39	0	0
		2	900	44	11	924	37	5	935	25	8	945	15	1
		3	0	0	0	0	0	0	0	0	0	0	0	0
	HQC	1	86	21	29	58	17	2	21	1	11	6	20	5
		2	720	89	36	802	94	23	853	72	41	885	57	27
		3	4	8	7	0	2	2	0	1	0	0	0	0
	MSC	1	8	3	10	1	0	0	0	0	0	0	0	0
		2	723	132	114	792	100	103	794	109	96	845	132	23
		3	3	1	6	1	0	3	0	1	0	0	0	0
1.0	AIC	1	37	76	57	34	77	69	0	0	0	60	15	8
		2	536	141	139	544	148	119	634	116	230	669	61	170
		3	9	4	1	0	1	8	5	9	6	0	8	9
	BIC	1	39	3	4	7	10	9	0	0	0	0	0	0
		2	884	55	15	912	47	14	920	55	25	936	25	39
		3	0	0	0	1	0	0	0	0	0	0	0	0
	HQC	1	65	14	29	9	38	30	10	3	1	10	3	1
		2	734	89	58	765	84	60	866	78	42	886	78	22
		3	1	7	3	5	4	5	0	0	0	0	0	0
	MSC	1	40	5	10	39	4	10	0	0	0	0	0	0
		2	672	126	127	722	113	90	858	112	30	888	110	1
		3	8	4	8	6	7	9	0	0	0	0	0	1
Regime-specific	AIC	1	38	8	7	0	1	2	0	0	0	46	21	27
		2	614	126	196	668	110	205	698	143	145	750	40	105
		3	1	4	6	6	1	7	8	3	3	0	7	4
	BIC	1	89	37	11	51	15	3	4	0	0	0	0	0
		2	833	23	5	864	17	50	941	55	0	964	34	2
		3	1	1	0	0	0	0	0	0	0	0	0	0
	HQC	1	14	32	14	39	3	6	0	0	0	3	2	0
		2	737	102	85	805	104	25	821	158	16	865	104	26
		3	3	9	4	8	2	8	5	0	0	0	0	0
	MSC	1	24	2	11	6	0	0	0	0	0	0	0	0
		2	706	110	138	745	153	78	790	150	60	901	10	89
		3	6	0	3	3	6	9	0	0	0	0	0	0

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Table 9: Frequency of selection of a MS(2)-AR(1) autoregressive model – Changes in intercept term and autoregressive coefficient

The true data generating process is the MS-AR model with two regimes and one lag, as follows:

$$y_t = v^{(s_t)} + A^{(s_t)} y_{t-1} + \sigma^{(s_t)} e_t.$$

The initial probabilities are set to 0.4 and 0.6 with the transition probabilities as $p_{11} = 0.6$; $p_{22} = 0.4$. The true regression coefficients are $v^{(s_t)} = (v^{(1)}, v^{(2)})$ and $A^{(s_t)} = (A^{(1)}, A^{(2)})$, where $v^{(s_t)} = (0, 1)'$ and $A^{(s_t)} = (0.3, 0.9)'$. Notice that the total replication is 1000.

The first column shows the possible values that $\sigma^{(s_t)}$ can take. Notice that in the regime-specific case, $\sigma^{(s_t)}$ is set to 0.5 if s_t = regime 1 and 1 if s_t = regime 2. The second column shows the relevant information criterion being considered, while the third column indicates the number of regimes imposed on estimation. With sample size of 100, the fourth column depicts the frequency of selecting the correct lag length (in this case, 1 lag) while the fifth and sixth columns indicate the frequency of selecting the incorrect lag lengths (in this case, 2 and 3 lags). Other columns show similar frequencies for different sample sizes.

Sigma	Information Criteria	No. of regimes imposed	100 observations			200 observations			400 observations			500 observations		
			AR(1)	AR(2)	AR(3)	AR(1)	AR(2)	AR(3)	AR(1)	AR(2)	AR(3)	AR(1)	AR(2)	AR(3)
0.5	AIC	1	2	0	0	7	2	0	0	0	0	0	0	69
		2	614	217	155	653	174	151	694	157	130	725	141	51
		3	5	2	5	9	2	2	8	9	2	8	6	0
	BIC	1	111	6	4	44	0	0	0	0	0	0	0	1
		2	835	37	6	942	11	1	988	12	0	994	5	0
		3	1	0	0	2	0	0	0	0	0	0	0	0
	HQC	1	18	4	3	15	2	0	0	0	0	0	0	0
		2	799	103	61	893	63	16	947	42	7	945	44	8
		3	3	4	5	3	5	3	2	1	1	3	0	0
	MSC	1	10	0	1	8	0	0	0	0	0	0	0	0
		2	759	165	61	845	120	8	831	97	70	909	41	50
		3	3	0	1	1	2	16	0	1	1	0	0	0
1.0	AIC	1	95	17	13	20	5	5	27	122	68	1	1	0
		2	572	129	154	655	207	95	676	45	60	712	169	105
		3	7	5	8	5	0	8	1	1	0	2	3	7
	BIC	1	95	37	18	116	18	15	94	12	0	32	0	0
		2	821	19	8	845	6	0	879	14	1	960	8	0
		3	0	1	1	0	0	0	0	0	0	0	0	0
	HQC	1	48	43	19	43	20	9	10	1	0	2	0	0
		2	719	85	72	821	73	20	909	57	13	923	54	15
		3	1	8	5	9	4	1	8	1	1	5	1	0
	MSC	1	18	4	4	12	0	0	10	0	0	0	0	0
		2	812	51	99	870	51	59	888	50	50	898	51	51
		3	3	5	4	0	3	5	1	1	0	0	0	0
Regime-specific	AIC	1	2	1	0	0	0	0	0	0	0	0	0	0
		2	609	112	260	623	189	168	682	171	130	718	147	125
		3	6	6	4	9	9	2	9	2	6	2	8	0
	BIC	1	72	7	2	20	0	0	10	0	0	0	0	0
		2	875	35	5	958	18	3	972	17	1	993	7	0
		3	2	1	1	1	0	0	0	0	0	0	0	0
	HQC	1	15	2	1	0	0	0	0	0	0	0	0	0
		2	790	113	65	906	62	21	936	45	11	940	50	7
		3	3	9	2	4	3	4	8	0	0	2	1	0
	MSC	1	29	2	4	10	0	0	0	0	0	0	0	0
		2	732	129	86	802	101	75	902	20	77	940	10	50
		3	1	9	8	2	3	7	1	0	0	0	0	0

Table 10: Frequency of selection of a MS(2)-AR(1) autoregressive model – Change in transition probabilities

The true data generating process is the MS-AR model with two regimes and one lag, as follows:

$$y_t = v^{(s_t)} + A^{(s_t)} y_{t-1} + \sigma^{(s_t)} e_t.$$

The initial probabilities are set to 0.4 and 0.6 with the transition probabilities as $p_{11} = 0.9$; $p_{22} = 0.1$. The true regression coefficients are $v^{(s_t)} = (v^{(1)}, v^{(2)})$ and $A^{(s_t)} = (A^{(1)}, A^{(2)})$, where $v^{(s_t)} = (0, 0)'$ and $A^{(s_t)} = (0.3, 0.9)'$. Notice that the total replication is 1000.

The first column shows the possible values that $\sigma^{(s_t)}$ can take. Notice that in the regime-specific case, $\sigma^{(s_t)}$ is set to 0.5 if $s_t = \text{regime 1}$ and 1 if $s_t = \text{regime 2}$. The second column shows the relevant information criterion being considered, while the third column indicates the number of regimes imposed on estimation. With sample size of 100, the fourth column depicts the frequency of selecting the correct lag length (in this case, 1 lag) while the fifth and sixth columns indicate the frequency of selecting the incorrect lag lengths (in this case, 2 and 3 lags). Other columns show similar frequencies for different sample sizes.

Sigma	Information Criteria	No. of regimes imposed	100 observations			200 observations			400 observations			500 observations		
			AR(1)	AR(2)	AR(3)	AR(1)	AR(2)	AR(3)	AR(1)	AR(2)	AR(3)	AR(1)	AR(2)	AR(3)
0.5	AIC	1	19	64	94	43	61	72	77	40	42	73	43	29
		2	674	52	93	705	51	60	761	35	35	790	43	10
		3	0	1	3	0	7	1	4	0	6	1	4	7
	BIC	1	56	0	5	6	1	8	1	0	0	0	0	0
		2	888	43	8	941	38	6	959	36	4	963	31	6
		3	0	0	0	0	0	0	0	0	0	0	0	0
	HQC	1	40	20	39	27	8	6	15	2	1	8	3	1
		2	758	89	47	842	81	31	868	72	41	878	78	32
		3	2	5	0	1	3	1	1	0	0	0	0	0
	MSC	1	53	8	8	6	0	1	1	0	0	0	0	0
		2	848	20	56	886	31	52	890	54	52	921	44	35
		3	2	2	3	2	8	14	0	0	3	0	0	0
1.0	AIC	1	16	70	76	69	80	80	62	46	47	65	29	37
		2	607	44	179	640	103	18	663	47	125	691	60	94
		3	2	4	2	0	4	6	0	3	7	7	8	9
	BIC	1	46	5	2	15	15	4	0	1	0	2	0	0
		2	873	65	9	925	37	4	937	29	33	959	25	14
		3	0	0	0	0	0	0	0	0	0	0	0	0
	HQC	1	81	20	21	53	52	29	1	1	0	14	1	0
		2	718	95	54	762	73	22	855	90	45	869	87	29
		3	3	5	3	2	1	6	4	4	0	0	0	0
	MSC	1	47	1	16	7	2	2	0	0	0	0	0	0
		2	754	162	7	814	102	55	858	77	64	911	80	9
		3	2	3	8	2	7	9	0	1	0	0	0	0
Regime-specific	AIC	1	17	43	52	18	50	31	30	20	21	8	25	10
		2	669	70	131	703	64	125	766	74	77	788	65	89
		3	8	6	4	5	0	4	3	4	5	9	0	6
	BIC	1	26	26	27	34	19	8	10	5	7	1	8	5
		2	850	52	12	887	33	15	954	20	4	963	20	3
		3	3	1	3	0	1	3	0	0	0	0	0	0
	HQC	1	9	58	71	31	40	48	20	38	14	8	30	9
		2	733	53	64	787	50	29	842	42	26	870	54	21
		3	2	9	1	9	5	1	5	4	9	3	2	3
	MSC	1	82	17	26	39	4	13	14	2	1	4	1	0
		2	783	26	52	819	30	86	896	54	22	939	44	7
		3	6	4	4	2	1	6	3	6	2	1	4	0

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Table 11: Frequency of selection of a MS(2)-AR(1) autoregressive model – Change in initial probabilities

The true data generating process is the MS-AR model with two regimes and one lag, as follows:

$$y_t = v^{(s_t)} + A^{(s_t)}y_{t-1} + \sigma^{(s_t)}e_t.$$

The initial probabilities are set to 0.1 and 0.9 with the transition probabilities as $p_{11} = 0.6$; $p_{22} = 0.4$. The true regression coefficients are $v^{(s_t)} = (v^{(1)}, v^{(2)})$ and $A^{(s_t)} = (A^{(1)}, A^{(2)})$, where $v^{(s_t)} = (0, 0)'$ and $A^{(s_t)} = (0.3, 0.9)'$. Notice that the total replication is 1000.

The first column shows the possible values that $\sigma^{(s_t)}$ can take. Notice that in the regime-specific case, $\sigma^{(s_t)}$ is set to 0.5 if s_t = regime 1 and 1 if s_t = regime 2. The second column shows the relevant information criterion being considered, while the third column indicates the number of regimes imposed on estimation. With sample size of 100, the fourth column depicts the frequency of selecting the correct lag length (in this case, 1 lag) while the fifth and sixth columns indicate the frequency of selecting the incorrect lag lengths (in this case, 2 and 3 lags). Other columns show similar frequencies for different sample sizes.

Sigma	Information Criteria	No. of regimes imposed	100 observations			200 observations			400 observations			500 observations		
			AR(1)	AR(2)	AR(3)	AR(1)	AR(2)	AR(3)	AR(1)	AR(2)	AR(3)	AR(1)	AR(2)	AR(3)
0.5	AIC	1	95	61	40	74	98	49	27	94	58	48	40	50
		2	680	41	62	691	21	57	770	6	26	807	16	32
		3	7	8	6	2	3	5	9	8	2	2	3	2
	BIC	1	80	4	1	15	0	1	19	0	0	10	0	0
		2	915	0	0	920	52	12	930	37	14	946	38	6
		3	0	0	0	0	0	0	0	0	0	0	0	0
	HQC	1	56	36	38	75	15	12	4	13	6	7	12	1
		2	703	95	52	758	92	39	862	84	28	873	77	30
		3	5	8	7	2	3	4	3	0	0	0	0	0
	MSC	1	43	4	13	5	0	3	0	0	0	0	0	0
		2	713	150	69	754	163	70	828	150	20	842	145	13
		3	5	2	1	3	0	2	1	0	1	0	0	0
1.0	AIC	1	7	75	55	65	83	54	6	94	65	20	80	58
		2	610	115	119	637	36	116	680	8	127	799	12	8
		3	8	8	3	1	2	6	6	8	6	6	9	8
	BIC	1	14	29	11	15	1	0	18	0	0	8	0	0
		2	879	44	20	929	43	12	948	30	4	960	28	4
		3	1	1	1	0	0	0	0	0	0	0	0	0
	HQC	1	41	57	62	80	15	7	27	12	6	9	9	1
		2	695	79	58	769	85	36	839	86	30	878	73	29
		3	2	1	5	2	3	3	0	0	0	0	0	1
	MSC	1	41	3	15	8	0	3	0	0	0	0	0	0
		2	734	136	55	807	77	91	891	58	51	902	57	41
		3	9	3	4	1	6	7	0	0	0	0	0	0
Regime-specific	AIC	1	17	41	41	50	13	9	3	1	4	0	0	0
		2	618	154	114	715	91	103	775	59	146	806	60	123
		3	2	6	7	3	7	9	8	4	0	4	0	7
	BIC	1	26	25	15	58	9	4	12	13	0	0	3	1
		2	874	41	14	885	39	5	965	7	2	989	6	1
		3	1	2	2	0	0	0	0	1	0	0	0	0
	HQC	1	40	43	33	53	31	10	51	6	3	6	2	1
		2	760	83	28	793	56	36	858	63	9	923	49	15
		3	7	0	6	7	9	5	4	3	3	4	0	0
	MSC	1	70	12	25	8	2	1	0	0	0	0	0	0
		2	741	50	84	850	33	88	856	127	15	952	24	22
		3	8	5	5	7	2	9	0	0	2	0	1	1

Table 12: Frequency of selection of a MS(2)-AR(1) autoregressive model - Persistence in $\{s_t\}$

The true data generating process is the MS-AR model with two regimes and one lag, as follows:

$$y_t = v^{(s_t)} + A^{(s_t)} y_{t-1} + \sigma^{(s_t)} e_t.$$

The initial probabilities are set to 0.4 and 0.6 with the transition probabilities as $p_{11} = 0.9$; $p_{22} = 0.9$. The true regression coefficients are $v^{(s_t)} = (v^{(1)}, v^{(2)})$ and $A^{(s_t)} = (A^{(1)}, A^{(2)})$, where $v^{(s_t)} = (0, 0)'$ and $A^{(s_t)} = (0.3, 0.9)'$. Notice that the total replication is 1000.

The first column shows the possible values that $\sigma^{(s_t)}$ can take. Notice that in the regime-specific case, $\sigma^{(s_t)}$ is set to 0.5 if $s_t = \text{regime 1}$ and 1 if $s_t = \text{regime 2}$. The second column shows the relevant information criterion being considered, while the third column indicates the number of regimes imposed on estimation. With sample size of 100, the fourth column depicts the frequency of selecting the correct lag length (in this case, 1 lag) while the fifth and sixth columns indicate the frequency of selecting the incorrect lag lengths (in this case, 2 and 3 lags). Other columns show similar frequencies for different sample sizes.

Sigma	Information Criteria	No. of regimes imposed	100 observations			200 observations			400 observations			500 observations		
			AR(1)	AR(2)	AR(3)	AR(1)	AR(2)	AR(3)	AR(1)	AR(2)	AR(3)	AR(1)	AR(2)	AR(3)
0.5	AIC	1	12	81	64	53	2	83	5	40	97	43	28	57
		2	643	45	147	693	59	104	700	56	81	728	60	64
		3	0	6	2	1	3	2	4	8	9	9	8	3
	BIC	1	70	3	7	32	3	3	14	1	4	9	7	2
		2	838	67	14	911	51	0	951	30	0	961	21	0
		3	0	0	1	0	0	0	0	0	0	0	0	0
	HQC	1	51	54	33	18	4	5	43	7	8	27	9	9
		2	760	41	50	823	109	35	832	73	34	861	73	21
		3	8	0	3	2	1	3	0	1	2	0	0	0
	MSC	1	25	87	53	65	3	0	1	41	9	3	4	2
		2	758	45	14	810	40	73	851	60	35	890	20	80
		3	6	8	4	1	3	5	0	1	2	0	0	1
1.0	AIC	1	30	86	44	45	13	76	65	35	1	57	28	82
		2	575	116	136	601	98	148	668	46	169	684	49	85
		3	7	1	5	4	6	9	4	9	3	8	7	0
	BIC	1	74	9	1	4	15	5	11	18	10	5	20	3
		2	840	52	23	935	41	0	941	20	0	951	20	1
		3	0	0	1	0	0	0	0	0	0	0	0	0
	HQC	1	96	10	54	19	26	2	8	26	8	9	28	1
		2	759	25	36	803	108	37	827	90	41	859	72	31
		3	7	4	9	1	3	1	0	0	0	0	0	0
	MSC	1	43	1	22	8	9	4	0	6	4	2	7	1
		2	751	58	114	794	60	111	810	80	100	850	130	10
		3	8	2	1	1	7	6	0	0	0	0	0	0
Regime-specific	AIC	1	33	70	53	54	59	41	11	28	24	99	38	26
		2	611	21	195	666	33	130	685	51	189	703	17	107
		3	6	8	3	4	6	7	3	4	5	8	2	0
	BIC	1	69	47	16	68	34	5	91	8	0	41	3	4
		2	789	39	32	842	21	22	859	26	16	932	17	1
		3	0	2	6	1	1	6	0	0	0	1	1	0
	HQC	1	3	76	36	21	50	19	54	37	24	67	26	13
		2	689	76	102	758	63	77	807	40	29	831	38	17
		3	6	5	7	1	5	6	3	2	4	4	2	2
	MSC	1	50	75	17	90	6	15	6	3	5	5	2	6
		2	666	80	105	781	34	63	809	40	123	838	30	110
		3	1	2	4	2	5	4	2	7	5	1	2	6