Analysis of a Multiresonant Forward Converter Based on Nonideal Coupling of the Transformer

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Abstract—There are many transformer applications where tight coupling is difficult to achieve. Therefore, an analysis of a resonant converter with a nonideal coupling is required. In this paper, a forward resonant converter with a coupling ranging from zero to one was analyzed. It was found that the traditional quasi-resonant converter becomes a multiresonant converter when the coupling is less than one. This is because the finite switching time of the rectifying diode reduces the input inductance of the transformer to effectively give a converter that uses two different inductances during any one switching period. It was also found that, in general, the converter has seven topological states. The design equations are derived analytically to provide a fundamental understanding of the converter. The peak voltage and current stress of the power switch as a function of the coupling were investigated. It was found that as the coupling is reduced, the peak voltage reduces and the peak current increases. A 48–5-V 20-W forward converter with a coupling of 0.9 was designed and tested. A coupling of 0.9 was chosen as it gives a duty cycle of 50%, has zero voltage switching for all loads, and has a peak switch voltage of 3.3 times the input voltage. The experiment verified the analysis and the practicality of a reduced coupling transformer, and the measured efficiency of the converter was 80%.

Index Terms—DC–DC power conversion, equations, resonant power conversion, transformers.

I. INTRODUCTION

A COMMON goal of transformer designers has been to maximize the coupling. A high coupling is generally achieved when the magnetic material has a large permeability, when the air gaps in the material are kept small, and the windings are in close proximity to the material. Although a coupling of close to one is physically possible, it is becoming a more difficult task as converter frequencies increase. This is mainly because high frequency power magnetics have a lower permeability and the relative number of turns is decreasing. Therefore, a more complete analysis of a converter that includes the effects of coupling is required.

For the traditional analysis of resonant [1] and multiresonant [2]–[4] forward converters it is often convenient to model the transformer magnetizing inductance as a current source. This is only accurate if the magnetizing inductance is much larger than the leakage inductance, that is, if the transformer coupling is close to unity. Another common analysis assumption is that the

rectifying and freewheeling diodes are ideal and their turn-on and turn-off times are instantaneous. This can be valid from a semiconductor point of view, but it is only accurate if the secondary leakage inductance is relatively small. This is because the rate of change of the rectifying diode current is governed by the secondary leakage inductance. Therefore, both of these assumptions cannot be used if the effects of transformer coupling are to be investigated.

Quasi-resonant converters use one inductor and one capacitor to achieve one resonant frequency [1]. The common multiresonant converter [2]–[4] uses one inductor and two capacitors to give more than one resonant frequency. This paper introduces the concept that a quasi-resonant forward converter actually becomes a multiresonant forward converter when the transformer coupling is less than one. This new converter uses one capacitor and effectively two inductors. The main advantage of this is that the nonideal coupling of a transformer can be exploited to give a very simple converter with a minimum parts count. The transformer can be less expensive and easier to construct, and the voltage stress on the switch can be reduced when compared to a quasi-resonant converter that has a wide load range. A typical quasi-resonant converter has a voltage stress that is proportional to the load range. For example, a fixed load can have a peak voltage as low as two times the input voltage, but a load range of 7 : 1 gives a peak voltage of eight times the input voltage [1]. A lower peak resonant voltage permits the use of a MOSFET with a lower on resistance or input capacitance, which is important for higher efficiency and simplified drive circuitry, respectively. This paper presents a multiresonant forward converter with a nonideal coupling and an experimental 48–5-V 20-W converter is designed and tested to confirm its practicality.

II. THEORY OF OPERATION

Fig. 1 shows a typical quasi-resonant forward converter where the transformer consists of two coupled inductors, the primary inductance (L_P) and the secondary inductance (L_S). Fig. 2(a) shows this transformer as a four terminal device. Fig. 2(b) shows an equivalent model of Fig. 2(a), derived from [5], which includes the transformer coupling (k). Fig. 2(c) shows the equivalent primary input impedance (Z_{INP}) when the rate of change of current flowing out of terminal c is constant. This occurs when the rectifying diode (D_R) is fully on with the constant load current, or fully off with zero current. Fig. 2(d) shows Z_{INP} when there is a short circuit between terminals c and d . This occurs when both D_R and the freewheeling diode (D_F) are conducting, such that their sum equals the load current. In this case, the voltage between

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Fig. 1. A quasi-resonant forward converter.

Fig. 2. A transformer as (a) two coupled inductors and (b) an equivalent model. The transformer input inductance when (c) D_R current is constant or zero and (d) D_R current is changing.

terminals c and d is clamped at 0 V by the diodes to give a virtual short circuit.

Since D_R can only be fully on, fully off or partly on, then Fig. 2(c) and (d) describes the two possible equivalent input impedances of the transformer. Therefore, there will be two different values of inductance resonating with the effective resonant capacitance (C_R) during any one switching period. This gives rise to a multiresonant converter with two resonant frequencies of $1/\sqrt{L_P C_R}$ and $1/\sqrt{L_P(1-k^2)C_R}$, referred to as ω_a and ω_b , respectively. C_R can consist of an external capacitance in parallel with the output capacitance of the switch.

In general, there are seven states of operation for the converter. Fig. 3(a) shows the first five states for the voltage across the capacitor (V_{CR}), the current through the primary inductance and the current through the rectifying diode. Fig. 3(b) and (c) shows the options for states six and seven. Fig. 4 is a flowchart of the seven states. It indicates the condition to enter a new state and whether the state is nonresonant or resonant at ω_a or ω_b .

A steady-state description of the converter, with reference to Figs. 3 and 4, is as follows. In state 1 the switch is on, D_R is fully on, Z_{INP} equals L_P and L_P has a linearly increasing current (I_{LP}). The switch then turns off and the first resonant state, state 2, begins. D_R is still fully on so the resonant frequency is $1/\sqrt{L_P C_R}$. Another resonant state, state 3, begins when D_R starts to turn off. This occurs when the voltage across the switch (V_{CR}) equals the input voltage (V_I). Here, the resonant frequency is $1/\sqrt{L_P(1-k^2)C_R}$. The smaller equivalent inductance during this state lowers the peak resonant voltage when compared to a quasi-resonant converter. The next resonant state, state 4, begins when D_R is fully off, D_F is now fully on and supplying the load current (I_O). Since D_R is fully off the resonant frequency here is $1/\sqrt{L_P C_R}$. The peak resonant voltage occurs during this state. When V_{CR} lowers from its peak back down to V_I , D_R starts to conduct and a new resonant state, state 5, begins. Since D_R and D_F are both partly on, the resonant frequency is $1/\sqrt{L_P(1-k^2)C_R}$. This state continues until D_R

Fig. 3. V_{CR} , I_{LP} , and ID_R for (a) the first five states (maximum power in this case), (b) states 5, 6, 7 for option a, and (c) states 5, 6, 7 for option b.

is fully on or V_{CR} reaches 0 V. If D_R becomes fully on before V_{CR} reaches 0 V then the new state, state 6a, is resonant with $1/\sqrt{L_P C_R}$, otherwise the state, state 6b, is nonresonant. State 6a and 6b end when V_{CR} reaches 0 V or D_R reaches the load current (I_O), respectively. The final state, state 7, is when I_{LP} decreases linearly back to zero. Here, I_{LP} equals V_{CR} which equals zero, and state one begins again. If in state 5, V_{CR} , I_{LP} and the current through D_R , (ID_R), simultaneously reach 0 V, 0 A, and I_O A, respectively, then states 6 and 7 do not exist. The output voltage is regulated for lighter loads by decreasing the on time of the switch.

Fig. 4. Flowchart showing the sequence of states.

The turn-on and turn-off times of D_R each introduce a new state. This transforms the quasi-resonant converter into a multiresonant converter. As k becomes smaller, these two new states become more pronounced by occupying a larger percentage of the off time.

III. ANALYSIS

In general, the converter has seven states of operation. However, in this particular design, maximum power for the converter will require five states only, i.e., no states 6 or 7, and can be seen in Fig. 3(a). States 6 and 7 will then exist when the output power is decreased from its maximum, i.e., when the load is reduced. Designing for maximum power using either six or seven states is also possible and will be studied in a future paper.

To simplify the analysis in the Appendix, a 100% efficient converter was assumed. Some of the more common sources of power loss that were not included in the ideal analysis are copper ac/dc resistances, magnetic hysteresis and eddy currents, diode voltage drop, R_{ON} , and capacitor dielectrics.

The following equations for L_P , L_S , and C_R are derived in the Appendix and I_O refers to the maximum output current:

$$L_P = \frac{V_I^2 t_1}{V_O I_O} f_1(k) \quad (1)$$

$$L_S = \frac{V_O t_1}{I_O} f_2(k) \quad (2)$$

$$C_R = \frac{V_O I_O t_1}{V_I^2} f_3(k) \quad (3)$$

Fig. 5. Plots of f_1 , f_2 , and f_3 as universal functions of k .

where f_1 , f_2 , and f_3 are universal functions of k only, and are plotted in Fig. 5.

Equations for maximum output power (P_O) can be derived from these equations, from (1) and (3)

$$P_O = \frac{V_I^2}{Z_a} f_4(k) \quad (4)$$

where Z_a is the characteristic impedance $Z_a = \sqrt{L_P/C_R}$, $f_4 = \sqrt{f_1/f_3}$, and $P_O = V_O I_O$.

Since the analysis assumed a 100% efficient converter then P_O must equal the input power (P_I). Z_I is the input impedance of the converter, from (4)

$$P_I = \frac{V_I^2}{Z_I} \quad (5)$$

where

$$Z_I = \frac{Z_a}{f_4(k)}.$$

Therefore, the input impedance of the converter is simply the characteristic impedance multiplied by some function of k . It is interesting to note that $f_4(k)$ equals one when k equals 0.9. Since $f_4(k)$ is proportional to k then Z_I is inversely proportional to k . Therefore, k effectively increases and decreases Z_I when k is less than or greater than 0.9, respectively.

The voltage ratio (V_O/V_I) can also be derived, from (1) and (2)

$$\frac{V_O}{V_I} = \sqrt{\frac{L_S}{L_P}} f_5(k) \quad (6)$$

where $f_5 = \sqrt{f_1/f_2}$.

Therefore, the voltage ratio is simply the turns ratio multiplied by some function of k . Since $f_5(k)$ is proportional to k then the voltage ratio is proportional to k . It is interesting to note that $f_5(k)$ equals one when k equals one. This is not physically possible since a k of one also implies a duty cycle of one. Therefore, the voltage ratio will always be less than the turns ratio, as in the original nonresonant forward converter [6]. In

Fig. 6. The peak switch current and voltage as a function of k .

this case, the voltage ratio equals the turns ratio multiplied by the duty cycle as in (7)

$$\frac{V_O}{V_I} = D \sqrt{\frac{L_S}{L_P}}. \quad (7)$$

It is interesting to do a comparison of the new multiresonant forward converter with the original nonresonant one. It is stated in the Appendix that the time for each state can be expressed as functions of the on time and k . Therefore, the duty cycle (D), which is a function of the state times, can be expressed purely as a function of k . When this is done it is found that it equals exactly twice $f_1(k)$. From (6)

$$\frac{V_O}{V_I} = D \sqrt{\frac{L_S}{L_P}} f_6(k) \quad (8)$$

where

$$f_6 = \frac{1}{\sqrt{4f_1f_2}}.$$

Therefore, the new multiresonant voltage ratio is the same as the original one, but multiplied by some function of k . $f_6(k)$ equals one when k equals one and decreases to 0.707 when k equals zero. Therefore, the new voltage ratio will always be less than the original one.

It appears then that the multiresonant converter presented in this paper has the same equations as the traditional nonresonant converters except that they are all multiplied by universal functions of k .

IV. EXPERIMENTAL INVESTIGATION

To investigate the effects of k , the circuit of Fig. 1 was designed for a 48–5-V 20-W converter with an on time of 0.5 μ s. The theoretical peak switch voltage and peak switch current were found as a function of k and are shown in Fig. 6. It can be seen that a tradeoff between voltage stress and current stress exists, minimum current means maximum voltage, and vice versa. An optimum value of k would depend solely on

the application of the converter, and the relative effects for the whole system must be addressed. For example, lowering k can lower the voltage stress on a MOSFET which in turn permits the use of a MOSFET with a lower on resistance (R_{ON}). This would reduce the I^2R loss. However, according to Fig. 6, the peak current stress is increased. This would increase the I^2R loss. Other tradeoffs are also present, if k is reduced then the required value of L_P is reduced and that of L_S and C_R are increased. Therefore, if the reduction in parasitic resistance of L_P is more substantial than the increase from L_S and C_R then the total I^2R loss would be lower.

From Fig. 6, it can be seen that if k is chosen to be approximately at the knee of the peak voltage curve, then at the expense of a slight increase in peak current, the peak voltage can be dramatically reduced. A k of 0.9 was chosen to practically demonstrate the new multiresonant technique because it is near the knee and also corresponds to a duty cycle of 50%. The duty cycle equals 50% when f_1 equals 0.25, from Fig. 5 this occurs when k is 0.9.

Equations (1)–(3) were used to design a 48–5-V 20-W converter. L_P , L_S , and C_R were found to be 14.4 μ H, 0.7 μ H, and 1.2 nF, respectively. The analytical expressions for the waveforms from the Appendix were programmed in MATLAB and compared to a lossless circuit in PSPICE and an accurate match was found. The circuit was then built and all waveforms matched the theory accurately except that V_O was only 4.2 V. The on time could be increased from 0.5 μ s to increase V_O to 5 V, but this increased the duty cycle and peak waveforms by approximately 50%. It was then found that the main reason for V_O being 4.2 V instead of 5 V was because of the diode voltage drop. This voltage drop needed to be included in the analysis to improve the accuracy and hence the practicality of the design equations. A method to include the voltage drop in the analysis without compromising its simplicity, is to add the forward voltage, in this case approximately 1 V, to that required at the output while retaining the original output current. Therefore, a 48–5-V 20-W requirement means that a 48–6-V 24-W design should be implemented. This method increases the peak current

Fig. 7. Measured drain-to-source voltage and the primary current for an output of 20 W.

slightly and the peak voltage remains the same. Fig. 6 is actually generated from a 48–6-V 24-W design.

Using (1)–(3) for a 48–6-V 24-W converter the circuit of Fig. 1 was redesigned and built using an IRF620 as the MOSFET. C_R was a 1.3-nF capacitor in parallel with the IRF620. L_P was 12.0 μ H, L_S was 0.85 μ H, and k was 0.9. The primary and secondary windings on a toroid were separated until a coupling of 0.9 was achieved. Lower coupling transformers can also be built by adding tuning inductors in series with the primary and secondary.

Fig. 7 shows the measured drain-to-source voltage (V_{DS}), which is the same as V_{CR} , and the primary current (I_{LP}). The practical and theoretical comparison of peak voltage, peak current and duty cycle matched exactly at 158 V, 2 A, and 50%, respectively. However, the on time was approximately 10% greater than predicted, and therefore, the switching frequency was 10% lower than predicted. It should be noted that the rise and fall times of the MOSFET gate voltage were each approximately 50 ns which leads to inaccuracies between the theoretical and practical on time. The on time can be increased or decreased slightly to compensate for inaccuracies and parasitics in the circuit. The efficiency was measured at 80%.

Many converters require the output voltage to be regulated from full load to zero load. Regulation for lighter loads is achieved by decreasing the on time. ZVS might not always be achieved because reducing the on time also reduces the current, and hence energy, in the primary and secondary inductances prior to switching. A simple program utilizing the equations from the Appendix was implemented to investigate whether ZVS and output voltage regulation can be obtained for lighter loads. This was done by reducing the on time for each load until ZVS was just achieved. This gives the minimum output voltage where ZVS is still achieved. Fig. 8 shows this minimum voltage for couplings of 0.9, 0.8, 0.7, and 0.6. The figure shows that ZVS, or near ZVS, can be achieved for all loads when coupling is 0.9. Also observed from the figure is that ZVS, or near ZVS, for lighter loads becomes impossible as coupling decreases below 0.8. This does not necessarily mean that the output cannot be regulated, but the efficiency would decrease. It should be noted that the theoretical curves near zero load

Fig. 8. Minimum output voltage that achieves ZVS for lighter loads.

Fig. 9. Measured drain-to-source voltage and the primary current for an output of 2.5 mW.

may not be achievable because these require an on time which is practically too small.

ZVS and an output voltage of 5 V was obtainable for all loads. Fig. 9 shows the measured V_{DS} and I_{LP} for an output power as low as 2.5 mW.

V. CONCLUSION

Converter frequencies are increasing to facilitate size reduction and improved efficiency. As a result, obtaining near ideal transformer coupling is becoming increasingly difficult. A power supply that benefits from leakage inductances is required. In analyzing a forward converter with nonideal coupling, it was recognized that the magnetizing inductance should not be modeled with a current source, nor should it be assumed that the rectifying diode is switched instantaneously. When coupling is included in the analysis it is found that a quasi-resonant converter becomes a multiresonant converter which generally has seven states. The converter becomes multiresonant because there are effectively two inductors and one capacitor in any one switching period as opposed to the more common one inductor and two capacitors. The benefits of this are that the nonidealities of the transformer can be exploited and the peak voltage stress on the power switch can be reduced

at the expense of a slight increase in the peak current. An experimental 48–5-V 20-W forward converter was designed and tested. The converter verified that by reducing the coupling of the transformer to 0.9 a practical and efficient converter with a realistic peak voltage and current can be designed.

APPENDIX

This Appendix derives the first order design equations for the converter. The analysis assumes there are no resistances in the circuit, i.e., 100% efficient, and that the output filter is ideal and there are no diode voltage drops. The design inputs and outputs are as follows.

1) Design Inputs:

V_I Input voltage.

V_O Output voltage.

I_O Output current.

k Transformer coupling.

t_1 On time of the switch.

2) Design Outputs—The Subscript n Refers to the Relevant State:

L_P Value of the primary inductance.

L_S Value of the secondary inductance.

C_R Value of the resonant capacitor.

I_{Ln0} Initial current of L_P for each state.

V_{Cn0} Initial voltage on C_R for each state.

I_{Ln} Current through L_P for each state.

V_{Cn} Voltage across C_R for each state.

t_n Time for each state.

Each state of the circuit of Fig. 1 can be viewed on the primary side as an equivalent series LC circuit, this can be seen in Fig. 10. The relevant values for L_n and C_n will depend on the state, as per Figs. 3 and 4. They are given in Table I along with the known initial conditions. A value of infinity for C_n indicates that the switch is on and the state is nonresonant. A value of $L_P(1-k^2)$ for L_n indicates that the current through D_R is changing as previously described in the theory of operation Fig. 2.

The general equations of Fig. 10 are [6]

$$I_{Ln} = \left(\frac{V_I - V_{Cn0}}{Z_n} \right) \sin(\omega_n t) + I_{Ln0} \cos(\omega_n t) \quad (A1a)$$

$$V_{Cn} = V_I + (V_{Cn0} - V_I) \cos(\omega_n t) + I_{Ln0} Z_n \sin(\omega_n t) \quad (A1b)$$

where

$$Z_n = \sqrt{\frac{L_n}{C_n}} \quad (A1c)$$

$$\omega_n = \frac{1}{\sqrt{L_n C_n}}. \quad (A1d)$$

Since Z_n and ω_n each have only two possible values, then Z_a , Z_b , ω_a , and ω_b will be defined as

$$Z_a = \sqrt{\frac{L_P}{C_R}} \quad (A2a)$$

Fig. 10. A series LC circuit with a switch.

TABLE I
INDUCTANCE AND CAPACITANCE VALUES
WITH THEIR KNOWN INITIAL CONDITIONS FOR EACH STATE

State (n)	L_n	I_{Ln0}	C_n	V_{Cn0}
1	L_P	0	∞	0
2	L_P	I_{L20}	C_R	0
3	$L_P(1-k^2)$	I_{L30}	C_R	V_I
4	L_P	I_{L40}	C_R	V_{C40}
5	$L_P(1-k^2)$	I_{L50}	C_R	V_I
6a	L_P	I_{L6a0}	C_R	V_{C6a0}
6b	$L_P(1-k^2)$	I_{L6b0}	∞	0
7a	L_P	I_{L7a0}	∞	0
7b	L_P	I_{L7b0}	∞	0

$$Z_b = \sqrt{\frac{L_P(1-k^2)}{C_R}} \quad (A2b)$$

$$\omega_a = \frac{1}{\sqrt{L_P C_R}} \quad (A2c)$$

$$\omega_b = \frac{1}{\sqrt{L_P(1-k^2)C_R}}. \quad (A2d)$$

3) Equations for Current and Voltage with Initial Conditions: The following rules will be used in the analysis and referred to where applicable:

$$\pm A \cos(\phi) + B \sin(\phi) = \pm \sqrt{A^2 + B^2} \cos \left(\phi - \tan^{-1} \left(\frac{B}{A} \right) \right) \quad (R1)$$

$$\cos(\sin^{-1}(\alpha)) = \sin(\cos^{-1}(\alpha)) = \sqrt{1 - \alpha^2} \quad (R2)$$

$$\tan^{-1} \frac{A}{B} + \tan^{-1} \frac{B}{A} = \frac{\pi}{2} \quad (R3)$$

$$\sin(\tan^{-1}(\alpha)) = \frac{\alpha}{\sqrt{1 + \alpha^2}} \quad (R4)$$

$$\cos(\tan^{-1}(\alpha)) = \frac{1}{\sqrt{1 + \alpha^2}}. \quad (R5)$$

The following equations for I_{Ln} and V_{Cn} are derived by substituting the relevant values from Table I into (A1) and (A2):

$$I_{L1} = \frac{V_I}{L_P} t \quad (\text{A3})$$

$$V_{C1} = 0 \quad (\text{A4})$$

$$I_{L2} = \frac{V_I}{Z_a} \sin(\omega_a t) + I_{L20} \cos(\omega_a t) \quad (\text{A5})$$

$$V_{C2} = V_I - V_I \cos(\omega_a t) + I_{L20} Z_a \sin(\omega_a t) \quad (\text{A6})$$

$$I_{L3} = I_{L30} \cos(\omega_b t) \quad (\text{A7})$$

$$V_{C3} = V_I + I_{L30} Z_b \sin(\omega_b t) \quad (\text{A8})$$

$$I_{L4} = \left(\frac{V_I - V_{C40}}{Z_a} \right) \sin(\omega_a t) + I_{L40} \cos(\omega_a t) \quad (\text{A9})$$

$$V_{C4} = V_I + (V_{C40} - V_I) \cos(\omega_a t) + I_{L40} Z_a \sin(\omega_a t) \quad (\text{A10})$$

$$I_{L5} = I_{L50} \cos(\omega_b t) \quad (\text{A11})$$

$$V_{C5} = V_I + I_{L50} Z_b \sin(\omega_b t) \quad (\text{A12})$$

$$I_{L6a} = \left(\frac{V_I - V_{C6a0}}{Z_a} \right) \sin(\omega_a t) + I_{L6a0} \cos(\omega_a t) \quad (\text{A13})$$

$$V_{C6a} = V_I + (V_{C6a0} - V_I) \cos(\omega_a t) + I_{L6a0} Z_a \sin(\omega_a t) \quad (\text{A14})$$

$$I_{L6b} = \frac{V_I}{L_P(1-k^2)} t + I_{L6b0} \quad (\text{A15})$$

$$V_{C6b} = 0 \quad (\text{A16})$$

$$I_{L7a} = \frac{V_I}{L_P} t + I_{L7a0} \quad (\text{A17})$$

$$V_{C7a} = 0 \quad (\text{A18})$$

Fig. 11. Equivalent secondary side circuit during states 3 and 5.

$$I_{L7b} = \frac{V_I}{L_P} t + I_{L7b0} \quad (\text{A19})$$

$$V_{C7b} = 0. \quad (\text{A20})$$

4) *Equations for Time with Initial Conditions:* The time for a state t_n is found by equating either I_{Ln} or V_{Cn} to $I_{L(n+1)0}$ or $V_{C(n+1)0}$, respectively, and then solving for t .

Time 1— t_1 : Although t_1 is an input parameter, its relationship to L_P is still required, from (A3)

$$t_1 = \frac{I_{L20}}{V_I} L_P. \quad (\text{A21})$$

Time 2— t_2 : From (A6) and (R1)

$$t_2 = \frac{1}{\omega_a} \tan^{-1} \left(\frac{V_I}{I_{L20} Z_a} \right). \quad (\text{A22})$$

Time 3— t_3 : t_3 is the time taken for ID_{R3} to go from I_O to zero. To find t_3 an equation for ID_{R3} has to be found. t_3 is then found by equating ID_{R3} to zero. The equivalent transformer model, when referred to the secondary, can also be derived from [5] and it is the same as Fig. 2(b), but replacing L_P with L_S . The voltage between terminals **a** and **b** (V_{ab}) will now be the primary voltage (V_{Pn}) multiplied by the turns ratio, subscript n is the state number

$$V_{ab} = V_{Pn} \sqrt{\frac{L_S}{L_P}}. \quad (\text{A23})$$

Fig. 11 shows the equivalent circuit to solve for ID_{R3} and hence t_3 . Since D_F is starting to conduct at the beginning of t_3 then both diodes are on and the voltage between terminals **c** and **d** is clamped at zero, i.e., short circuit. At the beginning of t_3 , ID_R equals I_O so the secondary leakage inductance, $L_S(1-k)$, has an initial condition of I_O . Since at the beginning of t_3 , V_P equals zero and I_{L30} is at a peak and constant, there will be no initial current through the reflected inductance, $L_S(1-k)$. Therefore, the secondary magnetizing inductance must also have an initial condition of I_O . From (A8), Fig. 1, and Fig. 11

$$ID_{R3} = \frac{-I_{L30} Z_b k}{\omega_b L_S(1-k^2)} \sqrt{\frac{L_S}{L_P}} (1 - \cos(\omega_b t)) + I_O. \quad (\text{A24})$$

From (A24)

$$t_3 = \frac{1}{\omega_b} \left(\cos^{-1} \left(1 - \frac{I_O}{k I_{L30}} \sqrt{\frac{L_S}{L_P}} \right) \right). \quad (\text{A25})$$

Time 4— t_4 : From (A10) and (R1)

$$t_4 = \frac{1}{\omega_a} \left(\frac{\pi}{2} + \tan^{-1} \left(\frac{I_{L40} Z_a}{V_{C40} - V_I} \right) \right). \quad (\text{A26})$$

Time 5a— t_{5a} : t_{5a} is the time taken for ID_R to go from zero to I_O . t_{5a} is found the same way as t_3 except that the initial currents in the secondary leakage and magnetizing inductances are zero and V_{Pn} equals V_{P5} . t_{5a} is then found by setting ID_R equal to I_O . This gives

$$t_{5a} = \frac{1}{\omega_b} \left(\cos^{-1} \left(1 + \frac{I_O}{k I_{L50}} \sqrt{\frac{L_S}{L_P}} \right) \right). \quad (\text{A27})$$

Time 5b— t_{5b} : From (A12)

$$t_{5b} = \frac{1}{\omega_b} \sin^{-1} \left(\frac{-V_I}{I_{L50} Z_b} \right). \quad (\text{A28})$$

Time 6a— t_{6a} : From (A14) and (R1)

$$t_{6a} = \frac{1}{\omega_a} \left(\cos^{-1} \left(\frac{-V_I}{\sqrt{(V_{C6a0} - V_I)^2 + (I_{L6a0} Z_a)^2}} \right) - \frac{\pi}{2} - \tan^{-1} \left(\frac{V_{C6a0} - V_I}{I_{L6a0} Z_a} \right) \right). \quad (\text{A29})$$

Time 6b— t_{6b} : State 6b occurs because V_{C5} reaches zero before ID_R reaches I_O . t_{6b} is the time taken for ID_{R6b} to go from ID_{R6b0} to I_O . In state 6b, V_{P6b} equals V_I and the voltage across the primary magnetizing inductance (V_{PM}) equals $\frac{kV_I}{k+1}$. Therefore, the voltage across the secondary magnetizing inductance (V_{SM}) is, from (Fig. 11)

$$V_{SM} = \frac{kV_I}{k+1} \sqrt{\frac{L_S}{L_P}}. \quad (\text{A30})$$

Since both diodes are on, then V_{SM} also falls across the secondary leakage inductance, $L_S(1-k)$. $\therefore$ from (A30)

$$ID_{R6b} = \frac{kV_I}{L_S(1-k^2)} \sqrt{\frac{L_S}{L_P}} t + ID_{R6b0} \quad (\text{A31})$$

where ID_{R6b0} equals ID_{R5} at the end of t_{5b} .

From (A12), (A28), (R2), Fig. 1, and Fig. 11

$$ID_{R6b0} = \frac{-k I_{L50} Z_b}{\omega_b L_S(1-k^2)} \sqrt{\frac{L_S}{L_P}} \left(1 - \sqrt{1 - \left(\frac{V_I}{I_{L50} Z_b} \right)^2} \right). \quad (\text{A32})$$

From (A31) and (A32)

$$t_{6b} = \frac{I_O L_S(1-k^2)}{kV_I} \sqrt{\frac{L_P}{L_S}} + \frac{I_{L50} L_P(1-k^2)}{V_I} \cdot \left(1 - \sqrt{1 - \left(\frac{V_I}{I_{L50} Z_b} \right)^2} \right). \quad (\text{A33})$$

Time 7a— t_{7a} : t_{7a} is the time taken for I_{L7} to reach zero. From (A17)

$$t_{7a} = \frac{-I_{L7a0} L_P}{V_I}. \quad (\text{A34})$$

Time 7b— t_{7b} : From (A19)

$$t_{7b} = \frac{-I_{L7b0} L_P}{V_I}. \quad (\text{A35})$$

5) Initial Conditions: If the initial conditions I_{Ln0} and V_{Cn0} cannot be found from Figs. 3 and 4 then they are found by substituting t_{n-1} into either I_{Ln-1} or V_{Cn-1} . From Figs. 3 and 4

$$\begin{aligned} I_{L10} &= 0 & V_{C10} &= 0 & V_{C20} &= 0 & V_{C30} &= V_I \\ V_{C50} &= V_I & V_{C6b0} &= 0 & V_{C7a0} &= 0. \end{aligned} \quad (\text{A36})$$

From (A3)

$$I_{L20} = \frac{V_I}{L_P} t_1. \quad (\text{A37})$$

From (A5), (A22), and (R1)

$$I_{L30} = \sqrt{I_{L20}^2 + \left(\frac{V_I}{Z_a} \right)^2}. \quad (\text{A38})$$

From (A7) and (A25)

$$I_{L40} = I_{L30} - \frac{I_O}{k} \sqrt{\frac{L_S}{L_P}}. \quad (\text{A39})$$

From (A8), (A25), (A39), and (R2)

$$V_{C40} = V_I + Z_b \sqrt{I_{L30}^2 - I_{L40}^2}. \quad (\text{A40})$$

From (A9), (A25), (A26), (R1), and (R3)

$$I_{L50} = -\sqrt{I_{L40}^2 + \left(\frac{V_I - V_{C40}}{Z_a} \right)^2}. \quad (\text{A41})$$

From (A11) and (A27)

$$I_{L6a0} = I_{L50} + \frac{I_O}{k} \sqrt{\frac{L_S}{L_P}}. \quad (\text{A42})$$

From (A27) and (R2)

$$V_{C6a0} = V_I - Z_b \sqrt{I_{L50}^2 - I_{L6a0}^2}. \quad (\text{A43})$$

From (A11), (A12), (A28), and (R2)

$$I_{L6b0} = -\sqrt{I_{L50}^2 - \left(\frac{V_I}{Z_b} \right)^2}. \quad (\text{A44})$$

From (A29) and (R2)

$$I_{L7a0} = -\sqrt{I_{L6a0}^2 + V_{C6a0}^2 - 2V_I V_{C6a0}}. \quad (\text{A45})$$

From (A13), (A33), and (A15)

$$I_{L7b0} = \frac{I_O}{k(1+k)} \sqrt{\frac{L_S}{L_P}} + \frac{I_{L50} - I_{L6b0}}{(1+k)} + I_{L6b0}. \quad (\text{A46})$$

6) Calculating L_P , L_S , and C_R : In this section, expressions for L_P , L_S , and C_R will be calculated such that maximum power is delivered in five states as seen in Fig. 3(a). It will be seen that there are three unique equations for I_{L50} and one for

V_O . These four equations will contain four unknowns L_P , L_S , C_R , and I_{L50} .

The design uses zero voltage switching and zero current switching at the maximum output power as a boundary condition. Therefore, when V_{C5} equals zero then I_{L5} equals zero and ID_R equals I_O .

7) *Equations for I_{L50}* : To make V_{C5} equal zero when I_{L5} equals zero means that the minimum of V_{C5} equals zero. This means that the peak ac voltage plus the dc voltage has to equal zero. It should be noted that if the output power is increased beyond the design limit then ZVS will no longer be possible. From (A12)

$$I_{L50} = \frac{-V_I}{Z_b}. \quad (\text{A47})$$

The boundary condition for maximum power is that the peak ac voltage equals the dc voltage. Therefore, the time for state 5 is actually one quarter of the period of the appropriate resonant frequency. The resonant frequency of state 5 is ω_b . That is,

$$t_5 = \frac{\pi}{2\omega_b}. \quad (\text{A48})$$

ID_R equals I_O when V_{C5} equals zero is also a boundary condition at maximum power. Therefore, since t_{5a} is the time for ID_R to go from zero to I_O then t_{5a} must also be one quarter of the period. From (A27) and (A48)

$$I_{L50} = \frac{-I_O}{k} \sqrt{\frac{L_S}{L_P}}. \quad (\text{A49})$$

From (A37) to (A41), see (A50), given at the bottom of the page.

8) *Equation for V_O* : V_O is the dc component, or average, of the voltage across $D_F(V_{DF})$. When ID_R is zero or changing, states 3–5, then V_{DF} is zero. When ID_R equals I_O and is constant, states 1 and 2, V_{DF} does not equal zero. There is no voltage drop across the secondary leakage inductance when ID_R is constant

$$\begin{aligned} V_{DFn} &= V_{SMn} & (n = 1, 2) \\ V_{DFn} &= 0 & (n = 3, 4, 5) \end{aligned} \quad (\text{A51})$$

$$V_{SMn} = kV_{PMn} \sqrt{\frac{L_S}{L_P}} \quad (\text{A52})$$

$$V_{PMn} = V_I - V_{Cn}. \quad (\text{A53})$$

From (A51) to (A53)

$$V_{DFn} = k(V_I - V_{Cn}) \sqrt{\frac{L_S}{L_P}} \quad (n = 1, 2)$$

$$V_{DFn} = 0 \quad (n = 3, 4, 5). \quad (\text{A54})$$

The dc component is then

$$V_O = \frac{\sum_{n=1 \text{ to } 5} \int_0^{t_n} V_{DFn} dt}{\sum_{n=1 \text{ to } 5} t_n}. \quad (\text{A55})$$

From (A2a), (A2c), (A3), (A4), (A6), (A54), (A55), (R4), and (R5)

$$V_O = \frac{kV_I \sqrt{L_S C_R + \frac{L_S}{L_P} t_1^2}}{t_1 + t_2 + t_3 + t_4 + t_5} \quad (\text{A56})$$

where t_2 – t_5 can be expressed as functions of t_1 and k .

9) *Solution of the Four Equations*: The three equations for I_{L50} (A47), (A49), and (A50) and the one equation for V_O (A56) can be solved by substitution. This yields the equations for L_P , L_S , and C_R

$$L_P = \frac{V_I^2 t_1}{V_O I_O} f_1(k) \quad (\text{A57})$$

$$L_S = \frac{V_O t_1}{I_O} f_2(k) \quad (\text{A58})$$

$$C_R = \frac{V_O I_O t_1}{V_I^2} f_3(k) \quad (\text{A59})$$

where f_1 , f_2 , and f_3 are only functions of k , i.e., do not depend on any other parameters, and are plotted in Fig. 5 in the analysis section. Equations for input power, output power, and voltage ratio are derived in the analysis section.

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$$I_{L50} = -\sqrt{\left(\frac{V_I t_1}{L_P}\right)^2 + \left(\frac{V_I}{Z_a}\right)^2 - 2kI_O \sqrt{\frac{L_S}{L_P}} \sqrt{\left(\frac{V_I t_1}{L_P}\right)^2 + \left(\frac{V_I}{Z_a}\right)^2}} + I_O^2 \frac{L_S}{L_P} \quad (\text{A50})$$

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