An efficient scaled boundary FEM model for wave interaction with a nonuniform porous cylinder

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SUMMARY

The scaled boundary finite-element method (SBFEM) by Tao, Song and Chakrabarti [Scaled boundary FEM solution of short-crested wave diffraction by a vertical cylinder, *Comput. Methods Appl. Mech. Engrg.* 197 (2007) 232-242], is only applicable for wave scattering problems by a structure of homogenous material. In this paper, the SBFEM is extended to deal with the interaction of water waves and porous offshore structure with an opening. The cylindrical structure is considered as a circular cylinder of anisotropic material in the form of variable porosity. A central feature of the newly-extended method is that the non-homogenous term caused by the complex configuration of the structure is processed by introducing a variable porous-effect parameter G . This leads to the final scaled boundary finite-element equation is still homogenous and can be solved in a similar manner. The modified SBFEM thus remains a semi-analytical fundamental-solution-less method. Numerical

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17 experiments in water wave interaction with a typical coastal/offshore structure - a cylinder with an
 18 opening attest to the efficacy and accuracy of the proposed approach. Copyright © 2008 John Wiley
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20 KEY WORDS: scaled boundary finite-element method; short-crested wave; wave diffraction;
 21 unbounded domain; nonuniform porous cylinder

22

23 1. INTRODUCTION

24 In order to reduce the direct wave impact and resonance from occurring, coastal and offshore
 25 structures are often constructed with protective porous layers. Due to its scientific and
 26 engineering significance, wave motion through a porous structure has attracted considerable
 27 attention. For many years, there have been widespread applications of porous structures in
 28 coastal and offshore engineering. For example, porous breakwaters are commonly constructed
 29 to protect coasts and harbours; The concentric porous cylinder system - Ekofisk gravity offshore
 30 structure was built with an porous exterior layer to protect interior column in the North Sea
 31 (see Figure 1). However, the understanding of the phenomenon is still far from complete.
 32 Among many issues, the ability of a coastal and ocean engineer to predict wave transmission
 33 and diffraction, so the wave elevation, wave forces and the stability of the structures, plays a
 34 central role in the protection of the structures [1].

35 [Figure 1 about here.]

36 Chwang [2] proposed a linearised porous-wavemaker theory to analyse small amplitude
 37 surface waves produced by horizontal oscillations of a porous vertical plate. The theory was

38 later applied to analyse surface waves generated by a piston-type porous wavemaker near the
 39 end of a semi-infinitely long channel of constant depth [3]. Several studies on the physical
 40 phenomenon of wave-trapping due to a porous plate or a concentric porous cylinder system
 41 using the porous-wavemaker theory were reported (e.g., [4, 5]).

42 Dalrymple et al. [6] studied the reflection and transmission of a wave train at an oblique angle
 43 of incidence by an infinitely long porous breakwater. Yu and Chwang [7] performed extensive
 44 study on the transmission characteristics of waves past a porous structure. The wave behaviour
 45 within the porous medium was also investigated. It was found that there is an optimum
 46 thickness for a porous structure beyond which any further increase of the thickness may not
 47 lead to an appreciable improvement in reducing its transmission and reflection characteristics.
 48 Yu and Chwang [8] employed the boundary integral method to study wave diffraction by a
 49 horizontal porous plate submerged at a distance below the free surface in a fluid of constant
 50 depth. Wang and Ren [9] also studied the performance of a flexible and porous breakwater.
 51 Additional related work can be found in the review article of Chwang and Chan [1].

52 One of the noticeable limitations in the previous studies is the two-dimensional plane wave
 53 assumption. Wind-generated waves in real oceans are much better represented by short-crested
 54 waves (3D) than by plane waves (2D) [10, 11]. Short-crested waves commonly arise from
 55 the oblique interaction of two travelling plane waves or intersecting swell waves, from the
 56 reflection of waves at non-normal incidence off a vertical seawall or a breakwater, as well as
 57 from diffraction about the surface boundaries of a structure of finite length. Such 3D waves
 58 are of paramount importance in coastal and offshore structure design.

59 Recently, a semi-analytical method, called scaled boundary finite-element method (SBFEM)
 60 for solving linear partial differential equations has been found successful applications to soil-

61 structure interaction problems. The SBFEM method was proposed by Song and Wolf [12]
 62 and systematically described by Wolf [13]. Combining the distinct advantages of the finite-
 63 element and boundary-element methods, only the structure boundary is discretised with
 64 surface finite-elements. This, in turn, transforms the governing partial differential equations to
 65 a set of ordinary differential equations, and solves them analytically. The method represents
 66 singularities and unbounded domains accurately and efficiently when compared to the complete
 67 FEM and requires no fundamental solution as needed by the BEM. Fewer elements are required
 68 to obtain very accurate results [13].

69 Only recently has the SBFEM been applied to wave diffraction in which the radiation
 70 condition at infinity is required to be satisfied by the scattered waves. Li et al. [14] solved
 71 the problem of plane wave diffraction by a vertical cylinder using SBFEM. Similar to the
 72 approach of Wolf [13] in obtaining a solution for soil-structure interaction, Li et al. [14] adopted
 73 an algebraic series to obtain the final solution. However, for low frequency waves, the series
 74 hardly converges to the exact solution.

75 Tao et al. [15] developed a modified SBFEM model and applied it to solve short-crested wave
 76 interaction with a circular cylinder. Instead of using a power series as proposed in the original
 77 SBFEM, Tao et al. [15] chose Hankel function as a base function to solve the 2D Helmholtz
 78 equation in the unbounded domain. As a direct consequence of such a modification, the radial
 79 differential equation is solved fully analytically in all frequency ranges. Without relying on any
 80 other numerical schemes, the semi-analytical model for the short-crested wave diffraction by a
 81 single circular cylinder is shown to reproduce the analytical solution for all physical properties
 82 including wave run-up, effective inertia and drag coefficients, and total force very accurately
 83 at very low computational cost.

84 However, the methodology proposed by Tao et al. [15] is only suitable for a closed circular
 85 cylinder. If a cylinder is not closed, i.e., has openings, the final scaled boundary finite-element
 86 equation (SBFEE) becomes non-homogeneous, rising much difficulty in the solution procedure.
 87 The goal of this paper is to solve this physical problem in the same efficient way as presented
 88 in [15]. By introducing a porous-effect parameter G (Chwang's parameter), the structure
 89 is treated as a nonuniform porous cylinder. We will show that the final SBFEE derived
 90 is still homogeneous and can be solved in a similar way as proposed in our previous work
 91 [15]. The modified SBFEM thus remains a semi-analytical fundamental-solution-less method.
 92 The newly-developed approach is then applied to solve short-crested wave interaction with a
 93 cylindrical structure with variable porosity in the circumferential direction. Convergence tests
 94 are given to demonstrate the accuracy and efficiency of the numerical model. Only a few finite
 95 elements discretised on the circumference of the cylinder are shown to be sufficient to obtain
 96 accurate results. Detailed numerical results on wave forces and surface elevations over a broad
 97 range of incident wave parameters and structure configuration including the cylinder radius,
 98 structural porosity, opening angle and location on the wave elevation and forces are examined
 99 in detail.

100 2. THEORETICAL FORMULATION OF THE BVP

101 We consider a monochromatic short-crested wave train propagating in the direction of the
 102 positive x axis. A vertical porous cylindrical structure extends from the sea bottom to above
 103 the free surface of the ocean along z axis. The origin is placed at the centre of the cylinder on the
 104 mean water surface (Figure 2). The fluid domain is divided into two regions, an interior region
 105 Ω_1 and a region outside the cylinder Ω_2 . The following notation have been used in the paper:

106 Φ_j = total velocity potential, Φ^I = velocity potential of incident wave, Φ^S = velocity potential
 107 of scattered wave, k = total wave number, k_x = wave number in x direction, k_y = wave number
 108 in y direction, ω = wave frequency, h = water depth, A = amplitude of incident wave, a =
 109 cylinder radius, t = time, ρ = mass density of water, and g = gravitational acceleration. The
 110 subscripts j ($j = 1, 2$) denote the physical parameters in the region Ω_j ($j = 1, 2$).

111 [Figure 2 about here.]

112 Assuming the fluid to be inviscid, incompressible and the flow to be irrotational, the fluid
 113 motion can be described by a velocity potential Φ_j satisfying the Laplace equation

$$\nabla^2 \Phi_j = 0 \quad \text{in } \Omega_j, \quad (1)$$

114 subject to the combined free surface boundary condition

$$\Phi_{j,tt} + g\Phi_{j,z} = 0 \quad \text{at } z = 0, \quad (2)$$

115 and the bottom condition

$$\Phi_{j,z} = 0 \quad \text{at } z = -h, \quad (3)$$

116 where the comma in the subscript designates partial derivative with respect to the variable
 117 following the comma.

118 The velocity potentials can be decomposed by separating the vertical variable z and the
 119 time t from each component as

$$\Phi_j(x, y, z, t) = \phi_j(x, y)Z(z)e^{-i\omega t} \quad \text{in } \Omega_j, \quad (4)$$

$$\Phi_2^I(x, y, z, t) = \phi_2^I(x, y)Z(z)e^{-i\omega t} \quad \text{in } \Omega_2, \quad (5)$$

$$\Phi_2^S(x, y, z, t) = \phi_2^S(x, y)Z(z)e^{-i\omega t} \quad \text{in } \Omega_2, \quad (6)$$

120 where

$$Z(z) = \frac{\cosh k(z + h)}{\cosh kh}. \quad (7)$$

121 This procedure leads to the sea bottom condition being automatically satisfied, while the
122 linearised free surface boundary condition is satisfied using the following dispersion relationship

$$\omega^2 = gk \tanh kh. \quad (8)$$

123 The total velocity potential in the outer region Ω_2 is given as a linear sum of the scattered
124 wave potential and the incident wave potential,

$$\Phi_2 = \Phi_2^I + \Phi_2^S, \quad \phi_2 = \phi_2^I + \phi_2^S \quad \text{in } \Omega_2. \quad (9)$$

125 Taylor [16] showed that the fluid flow passing through the porous boundary can be essentially
126 assumed to obey Darcy's law if the boundary is made of fine pores. Hence, the porous flow
127 velocity is linearly proportional to the pressure difference between the two sides of the porous
128 boundary, and the boundary condition on porous cylinder can be expressed as [2]

$$\phi_{1,n} = -\phi_{2,n} = iG(\theta)k(\phi_1 - \phi_2) \quad \text{on } r = a, \quad (10)$$

where $G(\theta)$ is a measure of the porous effect, named as Chwang's parameter. For a cylinder of nonuniform porosity, $G(\theta)$ varies along the circumferential direction. For special cases, $G = 0, \infty$ represent a cylinder with a partial solid wall and a partial opening respectively [2].

The BVP becomes two-dimensional at the free surface. The function $\phi_2^S(x, y)$ in the unbounded domain Ω_2 is governed by the Helmholtz equation with the boundary condition at the interface of fluid and porous cylinder, and the radiation condition at infinity, namely, the Sommerfeld condition as follows:

$$\nabla^2 \phi_2^S + k^2 \phi_2^S = 0 \quad \text{in } \Omega_2, \quad (11)$$

$$\phi_{2,n}^S = -iG(\theta)k(\phi_1 - \phi_2^S - \phi_2^I) - \phi_{2,n}^I \quad \text{on } r = a, \quad (12)$$

$$\lim_{kr \rightarrow \infty} (kr)^{1/2} (\phi_{2,r}^S - ik\phi_2^S) = 0 \quad \text{in } \Omega_2, \quad (13)$$

where r is the radial axis, and $i = \sqrt{-1}$.

The function $\phi_1(x, y)$ in the bounded domain Ω_1 is governed by the Helmholtz equation with the boundary condition at the interface of fluid and porous cylinder at $r = a$:

$$\nabla^2 \phi_1 + k^2 \phi_1 = 0 \quad \text{in } \Omega_1, \quad (14)$$

$$\phi_{1,n} = iG(\theta)k(\phi_1 - \phi_2^S - \phi_2^I) \quad \text{on } r = a. \quad (15)$$

140 The velocity potential of the linear short-crested incident wave travelling principally in the
 141 positive x direction is given by the real part of [17]

$$\Phi_I = -\frac{igA}{\omega} Z(z) e^{i(k_x x - \omega t)} \cos(k_y y) \quad \text{in } \Omega_2. \quad (16)$$

142 Equations (11)-(15) constitute two sets of the governing equation and boundary conditions
 143 for the diffraction of short-crested waves by a vertical porous cylindrical structure,
 144 corresponding to boundary value problems in a bounded domain and an unbounded domain
 145 respectively. After obtaining ϕ_2^S , Φ_2 and Φ_1 by solving the above boundary-value problems,
 146 the velocity, free surface elevation and the dynamic pressure can be calculated respectively
 147 from

$$\mathbf{v}_j = \nabla \Phi_j, \quad (17)$$

$$\eta_j = \frac{i\omega}{g} \phi_j, \quad (18)$$

$$p_j = -\rho \Phi_{j,t}. \quad (19)$$

148 3. SCALED BOUNDARY TRANSFORMATION

149 In this section, ϕ_1 and ϕ_2^S will both be denoted as ϕ for brevity, and the region Ω_j will be
 150 denoted as Ω . If the velocity boundary is defined by Γ_v , we have

$$\phi_{,n} = \bar{v}_n, \quad \text{on } \Gamma_v, \quad (20)$$

151 where the overbar denotes a prescribed value.

152 The finite-element method requires the weighted residuals of the governing equation to
 153 be zero. Hence Equations (11), (14) and (20) are multiplied by a weighting function w and
 154 integrated over the flow domain and the boundary. Performing integration by parts, the
 155 resulting equation becomes

$$\int_{\Omega} \nabla^T w \nabla \phi d\Omega - \int_{\Omega} w k^2 \phi d\Omega - \oint_{\Gamma} w \bar{v}_n d\Gamma = 0. \quad (21)$$

156 SBFEM defines the domain Ω by scaling a single piecewise-smooth curve S relative to a
 157 scaling centre (x_0, y_0) , which is chosen at the cylinder centre in this case (see Figure 3). The
 158 circumferential coordinate s is anticlockwise along the curve S and the normalised radial
 159 coordinate ξ is a scaling factor, defined as 1 at curve S and 0 at the scaling centre. The whole
 160 solution domain Ω is in the range of $\xi_0 \leq \xi \leq \xi_1$ and $s_0 \leq s \leq s_1$. The two straight sections
 161 $s = s_0$ and $s = s_1$ are called side-faces. They coincide, if the curve S is closed. For bounded
 162 domain, $\xi_0 = 0$ and $\xi_1 = 1$; whereas, for unbounded domain, $\xi_0 = 1$ and $\xi_1 = \infty$. Therefore
 163 the Cartesian coordinates are transformed to the scaled boundary coordinate ξ and s with the
 164 scaling equations

$$x = x_0 + \xi x_s(s), \quad y = y_0 + \xi y_s(s). \quad (22)$$

165 [Figure 3 about here.]

166 By employing SBFEM, an approximate solution of ϕ is sought as

$$\phi_A(\xi, s) = \mathbf{N}(s) \mathbf{a}(\xi), \quad (23)$$

167 where $\mathbf{N}(s)$ is the shape function, the vector $\mathbf{a}(\xi)$ is analogous to the nodal values same as
 168 in FEM. The radial function $a_j(\xi)$ represents the variation of the wave potential in the radial
 169 axis ξ at each node j , and the shape function $\mathbf{N}(s)$ interpolates between the nodal potential
 170 values in the circumferential axis s .

Performing the scaled boundary transformation [15], we have

$$\mathbf{E}_0 \xi^2 \mathbf{a}(\xi)_{,\xi\xi} + (\mathbf{E}_0 + \mathbf{E}_1^T - \mathbf{E}_1) \xi \mathbf{a}(\xi)_{,\xi} - \mathbf{E}_2 \mathbf{a}(\xi) + k^2 \xi^2 \mathbf{M}_0 \mathbf{a}(\xi) = \xi \mathbf{F}_s(\xi), \quad (24)$$

subject to the boundary conditions

$$\mathbf{E}_0 \xi_0 \mathbf{a}(\xi_0)_{,\xi} + \mathbf{E}_1^T \mathbf{a}(\xi_0) = - \int_S \mathbf{N}(s)^T (\bar{v}_n(\xi_0, s)) \xi_0 ds, \quad (25)$$

$$\mathbf{E}_0 \xi_1 \mathbf{a}(\xi_1)_{,\xi} + \mathbf{E}_1^T \mathbf{a}(\xi_1) = \int_S \mathbf{N}(s)^T (\bar{v}_n(\xi_1, s)) \xi_1 ds, \quad (26)$$

171 where

$$\mathbf{E}_0 = \int_S \mathbf{B}_1(s)^T \mathbf{B}_1(s) |J| ds, \quad (27)$$

$$\mathbf{E}_1 = \int_S \mathbf{B}_2(s)^T \mathbf{B}_1(s) |J| ds, \quad (28)$$

$$\mathbf{E}_2 = \int_S \mathbf{B}_2(s)^T \mathbf{B}_2(s) |J| ds, \quad (29)$$

$$\mathbf{M}_0 = \int_S \mathbf{N}(s)^T \mathbf{N}(s) |J| ds, \quad (30)$$

$$\mathbf{F}_s(\xi) = \mathbf{N}(s_0)^T (-\bar{v}_n(\xi, s_0)) |J(s_0)| + \mathbf{N}(s_1)^T (-\bar{v}_n(\xi, s_1)) |J(s_1)|, \quad (31)$$

$$\mathbf{B}_1(s) = \mathbf{b}_1(s) \mathbf{N}(s), \quad \mathbf{B}_2(s) = \mathbf{b}_2(s) \mathbf{N}(s)_{,s}, \quad (32)$$

$$\mathbf{b}_1(s) = \frac{1}{|J|} \begin{Bmatrix} y_s(s)_{,s} \\ -x_s(s)_{,s} \end{Bmatrix}, \quad \mathbf{b}_2(s) = \frac{1}{|J|} \begin{Bmatrix} -y_s(s) \\ x_s(s) \end{Bmatrix}, \quad (33)$$

172 and $|J|$ is the Jacobian at the boundary

$$|J| = x_s(s)y_s(s)_{,s} - y_s(s)x_s(s)_{,s}. \quad (34)$$

Equation (24) is the so-called scaled boundary finite-element equation, which is non-homogeneous because of the term $\mathbf{F}_s(\xi)$ arising from the structure configuration, e.g., cylinder with an opening. In the present approach, by introducing the variable porous parameter, the cylindrical structure can then be treated as a closed cylinder of variable porosity. Thus, the side-faces in the scaled boundary finite element coordinate system coincide. Since the flow across the side-faces is equal and opposite, the term $\mathbf{F}_s(\xi)$ vanishes. Therefore, the final governing equation, Equation (24), is a *homogeneous* second-order ordinary differential equation in terms of matrix.

Boundary conditions, Equations (12) and (13) or (15), are weakened in the form of Equations (25) and (26) respectively, indicating the relationship between the integrated nodal flow on the boundary and the velocity potentials of the nodes. For the wave diffraction problem in the unbounded region Ω_2 , $\xi_0 = 1$ on the boundary of porous cylinder and $\xi_1 = +\infty$ at infinity. For the boundary-value problem in the bounded region Ω_1 , $\xi_0 = 0$ and $\xi_1 = 1$.

4. SOLUTION PROCEDURE

For a circular cylinder, we have

$$x_s(s) = a \cos(s/a), \quad y_s(s) = a \sin(s/a). \quad (35)$$

From Equations (22), (33), (34), (32) and (27)-(30), $x_s(s)_{,s}$, $y_s(s)_{,s}$, $\mathbf{b}_1(s)$, $\mathbf{b}_2(s)$, $|J|$, $\mathbf{B}_1(s)$, $\mathbf{B}_2(s)$, $\mathbf{E}_0$, $\mathbf{E}_1$, $\mathbf{E}_2$, and $\mathbf{M}_0$ can be calculated accordingly. The following relationships hold:

$$\mathbf{E}_1 = 0 \cdot \mathbf{I}, \quad \mathbf{E}_0^{-1} \mathbf{M}_0 = a^2 \mathbf{I}, \quad (36)$$

$$\mathbf{E}_0 = \frac{1}{a} \int_S \mathbf{N}(s)^T \mathbf{N}(s) ds, \quad (37)$$

where $\mathbf{I}$ is the identity matrix of rank m .

Using Equation (36), pre-multiplying both sides of Equation (24) by $\mathbf{E}_0^{-1}$ and simplifying,

we have

$$\zeta^2 \mathbf{a}(\zeta)_{,\zeta\zeta} + \zeta \mathbf{a}(\zeta)_{,\zeta} - \mathbf{E}_0^{-1} \mathbf{E}_2 \mathbf{a}(\zeta) + \zeta^2 \mathbf{a}(\zeta) = 0, \quad (38)$$

where

$$\zeta = ka\xi. \quad (39)$$

4.1. Solution for unbounded domain Ω_2

Equation (38) is the matrix form of Bessel's differential equation. Considering the Sommerfeld radiation condition Equation (13), here we select $H_{r_j}(\zeta) \mathbf{T}_j$ as a base solution of Equation (38) in the unbounded region Ω_2 .

The solution for $\mathbf{a}_2(\zeta)$ is then expressed in the series form:

$$\mathbf{a}_2(\zeta) = \sum_{j=1}^m c_j H_{r_j}(\zeta) \mathbf{T}_j = \mathbf{T} \mathbf{H}(\zeta) \mathbf{C}, \quad (40)$$

in which $\mathbf{T}_j$ are vectors of rank m , c_j are coefficients, $H_{r_j}(\zeta)$ are the Hankel functions of the first kind, and

$$\mathbf{T} = [\mathbf{T}_1, \mathbf{T}_2, \dots, \mathbf{T}_m], \quad (41)$$

$$\mathbf{H}(\xi) = \text{diag}[H_{r_1}(ka\xi), H_{r_2}(ka\xi), \dots, H_{r_m}(ka\xi)], \quad (42)$$

$$\mathbf{C} = [c_1, c_2, \dots, c_m]^T, \quad (43)$$

201 where “diag” denotes a diagonal matrix with the elements in the square brackets on the main
202 diagonal.

203 Substituting Equation (40) into Equation (38), and using the following properties of Hankel
204 function

$$\zeta^2 H_{r_j}''(\zeta) = -\zeta^2 H_{r_j}(\zeta) + \zeta H_{r_j+1}(\zeta) - r_j H_{r_j}(\zeta) + r_j^2 H_{r_j}(\zeta), \quad (44)$$

$$\zeta H_{r_j}'(\zeta) = -\zeta H_{r_j+1}(\zeta) + r_j H_{r_j}(\zeta), \quad (45)$$

205 where the prime and the double prime denote the first and second derivatives with respect to
206 the argument ζ respectively, we have

$$\sum_{j=1}^m (\mathbf{E}_0^{-1} \mathbf{E}_2 - r_j^2 \mathbf{I}) \mathbf{T}_j \cdot c_j H_{r_j}(\zeta) = 0. \quad (46)$$

207 For any $c_j H_{r_j}(\zeta)$, Equation (46) yields

$$(\mathbf{E}_0^{-1} \mathbf{E}_2 - r_j^2 \mathbf{I}) \mathbf{T}_j = 0. \quad (47)$$

208 Let λ_j be the eigenvalues of $\mathbf{E}_0^{-1} \mathbf{E}_2$, then $r_j = \sqrt{\lambda_j}$, and $\mathbf{T}_j$ are the eigenvectors of $\mathbf{E}_0^{-1} \mathbf{E}_2$.

209 Since the Sommerfeld radiation condition (13) has been satisfied by the Hankel functions,
210 we now only consider the body boundary condition (25) of the circular cylinder

$$\mathbf{E}_0 k a \sum_{j=1}^m c_j H'_{r_j}(ka) \mathbf{T}_j = - \left[\int_S \mathbf{N}(s)^T \mathbf{N}(s) ds \right] \bar{\mathbf{v}}_{2n}^S, \quad (48)$$

where $\bar{\mathbf{v}}_{2n}^S$ is the vector of nodal normal velocity of scattered wave in region Ω_2 on the body boundary.

4.2. Solution for bounded domain Ω_1

Similar approach is applied to the region Ω_1 . Assume

$$\mathbf{a}_1(\zeta) = \sum_{j=1}^m c_j^1 J_{r_j}(\zeta) \mathbf{T}_j = \mathbf{T} \mathbf{J}(\zeta) \mathbf{C}^1, \quad (49)$$

where c_j^1 are coefficients, $J_{r_j}(\zeta)$ are the Bessel functions of the first kind, and

$$\mathbf{C}^1 = [c_1^1, c_2^1, \dots, c_m^1]^T, \quad (50)$$

$$\mathbf{J}(\xi) = \text{diag}[J_{r_1}(ka\xi), J_{r_2}(ka\xi), \dots, J_{r_m}(ka\xi)]. \quad (51)$$

$$(52)$$

Again if λ_j are the eigenvalues of $\mathbf{E}_0^{-1} \mathbf{E}_2$, then $r_j = \sqrt{\lambda_j}$, and $\mathbf{T}$ is the eigenvector of $\mathbf{E}_0^{-1} \mathbf{E}_2$.

Applying boundary condition on the porous cylinder Equation (26), we have

$$\mathbf{E}_0 k a \mathbf{T} \mathbf{J}'_a \mathbf{C}^1 = \left[\int_S \mathbf{N}(s)^T \mathbf{N}(s) ds \right] \bar{\mathbf{v}}_{1n}, \quad (53)$$

where $\bar{\mathbf{v}}_{1n}$ is the vector of nodal total normal velocity in region Ω_1 on the body boundary of porous cylinder.

Combining Equations (10), (20), (23), (37), (40), (48), (49) and (53), and noting

$$\bar{\mathbf{v}}_{2n}^I + \bar{\mathbf{v}}_{2n}^S = \bar{\mathbf{v}}_{2n} = -\bar{\mathbf{v}}_{1n}, \quad (54)$$

221 where $\bar{\mathbf{v}}_{2n}^I$ is the vector of nodal normal velocity of incident wave in the region Ω_2 on the body
 222 boundary of the porous cylinder, $\mathbf{a}_1(\xi)$ and $\mathbf{a}_2(\xi)$ are solved as

$$\mathbf{a}_1(\xi) = \mathbf{T}\mathbf{J}(\xi)\mathbf{W}^{-1}(\mathbf{T}^{-1}\bar{\mathbf{a}}_2^I + \mathbf{H}_{ah}\mathbf{T}^{-1}\bar{\mathbf{v}}_{2n}^I/k), \quad (55)$$

$$\mathbf{a}_2(\xi) = \mathbf{T}\mathbf{H}_h(\xi)(\mathbf{J}'_a\mathbf{W}^{-1}\mathbf{T}^{-1}\bar{\mathbf{a}}_2^I + \mathbf{H}_{ah}^{-1}\mathbf{V}\mathbf{W}^{-1}\mathbf{H}_{ah}\mathbf{T}^{-1}\bar{\mathbf{v}}_{2n}^I/k), \quad (56)$$

223 where

$$\mathbf{V} = i\mathbf{T}^{-1}\mathbf{G}^{-1}\mathbf{T}\mathbf{J}'_a + \mathbf{J}_a, \quad (57)$$

$$\mathbf{W} = \mathbf{V} - \mathbf{H}_{ah}\mathbf{J}'_a, \quad (58)$$

$$\mathbf{J}_a = \text{diag}[J_{r_1}(ka), J_{r_2}(ka), \dots, J_{r_m}(ka)], \quad (59)$$

$$\mathbf{J}'_a = \text{diag}[J'_{r_1}(ka), J'_{r_2}(ka), \dots, J'_{r_m}(ka)], \quad (60)$$

$$\mathbf{H}_h(\xi) = \text{diag}[H_{r_1}(ka\xi)/H'_{r_1}(ka), H_{r_2}(ka\xi)/H'_{r_2}(ka), \dots, H_{r_m}(ka\xi)/H'_{r_m}(ka)], \quad (61)$$

$$\mathbf{H}_{ah} = \text{diag}[H_{r_1}(ka)/H'_{r_1}(ka), H_{r_2}(ka)/H'_{r_2}(ka), \dots, H_{r_m}(ka)/H'_{r_m}(ka)], \quad (62)$$

224 and $\mathbf{G}$ is a diagonal matrix where the value on the diagonal designates the porosity G in the
 225 corresponding elements.

Using Equations (4), (5), (16), (20) and (23), $\bar{\mathbf{v}}_{2n}^I$ and $\bar{\mathbf{a}}_2^I$ can be easily determined on the porous cylinder boundary. From Equations (6), (9), (23), (55) and (56), the approximation of velocity potential in both region Ω_1 and region Ω_2 can be obtained.

All the other physical properties of engineering interest including velocity, surface elevation, and pressure can now be determined based on the velocity potentials by Equations (17)-(19). The total force per unit length on the cylinder in the u ($u = x, y$) direction is then calculated as:

$$\frac{dF_u}{dz} = -a \int_0^{2\pi} p \cdot \cos(\theta) d\theta = 2\pi a P_u(k_x, k_y, k, a) \cdot \rho g A \cdot Z(z) e^{-i\omega t}, \quad (63)$$

where the function $P_u(k_x, k_y, k, a)$ is the dimensionless parameter of dF_u/dz without the term $\rho g A \cdot Z(z) e^{-i\omega t}$.

The function $P_u(k_x, k_y, k, a)$ determines the first-order total horizontal force in u direction on the cylinder, F_u , which can be obtained by integrating Equations (63) with respect to z ,

$$F_u = \int_{-h}^0 \frac{dF_u}{dz} dz = 2\pi a P_u(k_x, k_y, k, a) \cdot \rho g A e^{-i\omega t} \cdot \tanh(kh)/k. \quad (64)$$

The total moment about an axis parallel to the y -axis passing through the bottom of the cylinder is

$$M_y = \int_{-h}^0 (z + h) \frac{dF_x}{dz} dz = 2\pi a P_x(k_x, k_y, k, a) \cdot \rho g A e^{-i\omega t} \cdot f(kh)/k^2, \quad (65)$$

$$M_x = - \int_{-h}^0 (z + h) \frac{dF_y}{dz} dz = 2\pi a P_y(k_x, k_y, k, a) \cdot \rho g A e^{-i\omega t} \cdot f(kh)/k^2, \quad (66)$$

where

$$f(kh) = [kh \tanh(kh) + \operatorname{sech}(kh) - 1]. \quad (67)$$

It can be concluded from Equations (64)-(66) that only the functions $P_x(k_x, k_y, k, a)$ and $P_y(k_x, k_y, k, a)$ need to be discussed.

5. VALIDATION OF SBFEM MODEL

As a first step to ensure the convergence of the solution to be found, it is essential to examine the detailed relationship between the number of elements required on the cylinder surface and ka . The scaled boundary FEM model described in the previous section is then further validated with theoretical solutions [18]. For the physical problem with symmetric properties, only half the cylinder needs to be discretised. In this paper, the cylinder circumference is discretised with three-node quadratic elements (see Figure 4).

[Figure 4 about here.]

Figure 5 shows the convergence of the surface elevation at the origin and wave forces in relation to ka for a cylinder with uniform porosity ($G_0 = 1$). It is clearly seen in Figure 5 that even 2 elements lead to very accurate results of elevations for ka up to 1.8, while 4 and 6 elements are required to produce valid solutions in the range of $ka \leq 5.2$ and $ka \leq 8.4$, respectively. As can be seen in Figure 5, the convergence zone of ka for wave force calculation is notably broader.

[Figure 5 about here.]

Figure 6 shows surface elevation at the origin (left) and wave forces (right) vs. nondimensional radius ka for a cylinder with nonuniform porosity ($G_0 = 1$ and $G_1 = 10$) along

the circumferential direction. Three types of the configuration are considered, i.e., the region of the partial porosity G_1 lies in (a) $[\pi/2, 3\pi/2]$; (b) $[3\pi/4, 5\pi/4]$; (c) $[7\pi/8, 9\pi/8]$ respectively. Again, only half of the cylinder surface needs to be discretised. As shown in Figs. 6(a) and 6(d), even 4 elements are seen to produce very accurate results of surface elevation and wave force for ka up to 4. A general trend that more elements are required for increasing ka are clearly seen in the figures. For both surface elevation and wave force calculations, Figure 6 shows that the difference between the present SBFEM solutions with 8 elements and analytical solutions is indistinguishable for ka up to 10 for all three configurations presented.

[Figure 6 about here.]

For the physical problems of non-symmetric configurations, whole cylinder needs to be discretised along the circumferential direction, thus the number of required elements doubles. Figure 7 shows the convergence of elevation and wave forces for a cylinder with a partial opening in $[3\pi/4, \pi]$ interaction with the incoming short-crested waves with equal wave numbers in two perpendicular directions. The two different porosities are $G_1 = +\infty$ in $[3\pi/4, \pi]$ and $G_0 = 1$ in the rest of the cylinder. It can be seen in Figure 7 that 16 elements (corresponding to 8 elements in the half cylinder) is adequate to achieve relatively high accuracy for calculation of both elevation and wave forces in the range of $ka \leq 10$. For elevation, 8 elements (corresponding to 4 elements in the half cylinder) exhibit excellent agreement with the analytical solutions until $ka = 5.3$. Since the physical problem is no longer symmetric to the x axis, wave force in y direction comes forth with rather smaller magnitude compared to the inline force.

[Figure 7 about here.]

6. NUMERICAL RESULTS AND DISCUSSION

6.1. Surface elevation

6.1.1. *Influence of ka* The Equi-amplitude (left) and Equi-phase (right) contours for incident short-crested wave with the wave number $k_x = k_y = \sqrt{2}/2 \text{ m}^{-1}$, interaction with a porous cylinder ($G_0 = 1$) with a partial opening at $7/8\pi < \theta < 9/8\pi$, for $a = 2, 3, 4 \text{ m}$ are shown in Figure 8. The thick lines in phase contours represent changes from π to $-\pi$. It can be seen clearly that the wave patterns inside the porous cylinder becomes more complex as a increases, with increases in density of the both amplitude and phase contours. This is mainly due to more physical space for the waves transmitted into the interior region to develop. Amphidromic points come forth when the bounded area is adequately large to form them.

[Figure 8 about here.]

6.1.2. *Influence of G_0* The Equi-amplitude (left) and Equi-phase (right) contours for incident short-crested wave ($k_x = k_y = \sqrt{2}/2 \text{ m}^{-1}$) interaction with a cylinder ($a = 4 \text{ m}$) of three different porosities $G_0 = 0.5, 1, 10$, with a partial opening at $7/8\pi < \theta < 9/8\pi$, are shown in Figure 9. As can be seen from the figure, the wave pattern is more complex as the porosity G_0 is smaller due to the stronger interaction of the waves reflected by the internal surface of the cylinder. As porosity G_0 increases, the effect of diffraction weakens and the cylinder becomes transparent gradually, leading to a wave pattern similar to that of a progressive wave.

[Figure 9 about here.]

6.1.3. *Influence of opening angle* Figure 10 shows the Equi-amplitude (left) and Equi-phase (right) contours for short-crested wave ($k_x = k_y = \sqrt{2}/2 \text{ m}^{-1}$) interaction with a porous

301 cylinder ($a = 4$ m and $G_0 = 1$) with a partial opening centred at $\beta = \pi$ for three different
 302 opening angles $\alpha = \pi/4, \pi/2, \pi$. The amplitudes in the inline directions are seen varying rapidly
 303 whilst the transverse values change slowly for these three opening angles. It appears that the
 304 opening angle has rather limited effect on the Equi-phase contour patterns.

305 [Figure 10 about here.]

306 *6.1.4. Influence of opening location* The Equi-amplitude (left) and Equi-phase (right)
 307 contours for short-crested wave ($k_x = k_y = \sqrt{2}/2$ m⁻¹) interaction with a cylinder ($a = 4$
 308 m and $G_0 = 1$) with a partial opening ($\alpha = \pi/4$) at three different locations along the
 309 cylinder circumference direction, $\beta = \pi, \pi/2, 0$ respectively, are shown in Figure 11. It is seen
 310 that, surface elevation in the interior region reduces as the opening location shift away from
 311 the incident wave direction, indicating more effective protection. Such an effect of increased
 312 protection by rearranging openings away from incident wave direction is expected to be more
 313 profound for a cylinder with low porosity.

314 [Figure 11 about here.]

315 6.2. Wave forces

316 *6.2.1. Influence of G_0* The influence of ka on wave forces experienced by a cylinder with a
 317 partial opening at $\beta = \pi$ and opening angle $\alpha = \pi/4$ is also investigated for the same wave
 318 condition, $k_x = k_y = \sqrt{2}/2$ m⁻¹. Three cases of different porosities $G_0 = 0.5, 1, 10$ respectively
 319 are calculated and the results are presented in Figure 12. For different G_0 , a similar pattern
 320 of wave forces oscillating with ka is clearly observed. The porosity effect parameter is seen
 321 to have significant impact on the wave forces imposed on the cylinder, i.e., $|P_x|$ is markedly

322 lower as G_0 increases indicating increased wave transmission. As ka increases, peak and trough
 323 attenuation is clearly evident for different G_0 .

324 [Figure 12 about here.]

325 *6.2.2. Influence of opening angle* The variations of the wave forces on a cylinder with a
 326 partial opening at $\beta = \pi$ by the incident short-crested wave of equal wave numbers ($k_x = k_y$)
 327 vs. ka are shown in Figure 13 for $G_0 = 1$. The results plotted in Figure 13 are for three
 328 different values of opening angle $\alpha = \pi/4, \pi/2, \pi$ respectively. It can be seen that the wave
 329 forces fluctuate and peak attenuation is evident. Significant reduction in wave force fluctuation
 330 is clearly seen as the opening angle increases. For a cylinder of porosity $G_0 = 1$ with a opening
 331 of half circle ($\alpha = \pi$), Figure 13 shows that the wave forces rather smoothly decrease with ka
 332 after a initial rapid increase to a peak value.

333 [Figure 13 about here.]

334 *6.2.3. Influence of opening location* Figure 14 shows wave forces on a porous cylinder
 335 ($G_0 = 1$) with a partial opening ($\alpha = \pi/4$) at five different locations $\beta = 0, \pi/4, \pi/2, 3\pi/4, \pi$
 336 respectively. Again, the incident short-crested wave is of equal wave numbers, i.e., $k_x = k_y$. It
 337 can be seen clearly that the opening location is very sensitive at low ka to the inline forces but
 338 much less influence as $ka \geq 2$. As shown in Figure 14, trough attenuation is no longer seen as
 339 ka increases while peak attenuation is found for all β values. The magnitude of the transverse
 340 wave forces shown in Figure 14 is rather small and becomes virtually zero for $\beta = 0$ and π
 341 since the corresponding configurations being symmetric to x direction.

342 [Figure 14 about here.]

343 It is worthnoting that, the computational times (recorded on a 2GHz Pentium IV PC and
344 MATLAB 7.1) of the present SBFEM solutions are very small. For all cases presented in
345 this paper, accurate results are obtained in less than 3 seconds, a clear demonstration that
346 it significantly outperforms any current finite-element or boundary-element method for wave
347 diffraction problems. Such computational efficiency and accuracy ensure great potential of
348 direct application of the present model to many engineering problems especially in ocean
349 engineering.

350 7. CONCLUSIONS

351 The versatility of the newly developed semi-analytical SBFEM is demonstrated in this paper in
352 considering the interaction of short-crested waves with a cylindrical structure with an opening.
353 By introducing a porous-effect parameter, the final scaled boundary finite-element equation
354 has been successfully transformed from non-homogeneous to homogeneous, and further solved
355 in the frame work of the SBFEM. With no additional approximations being introduced,
356 the SBFEM thus remains a semi-analytical fundamental-solution-less approach, but capable
357 of dealing with much more complex structural configurations. Only the body boundary is
358 discretised with surface finite-elements. Excellent computational efficiency and accuracy of the
359 SBFEM model have been demonstrated, as the governing equations are solved analytically
360 in the radial direction. The influence of structural configuration including cylinder radius,
361 porosity, opening angle and opening location are studied and the results in terms of the wave
362 forces and surface elevations are presented. The results presented here should be found useful
363 in the design of coastal and ocean structures.

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364

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404

List of Figures

405	1	Ekofisk Gravity Structure (Courtesy ConocoPhillips).	26
406	2	Definition sketch of wave interaction with a porous cylindrical structure	27
407	3	The coordinate definition of SBFEM	28
408	4	Scaled boundary finite element mesh for a porous cylindrical structure	29
409	5	Surface elevation at the origin and wave forces vs. ka for short-crested wave	
410		$(k_x = k_y)$ interaction with a porous cylinder of uniform porosity $G_0 = 1$	30
411	6	Surface elevation at the origin (left) and wave forces (right) vs. ka for short-	
412		crested wave $(k_x = k_y)$ interaction with a porous cylinder of nonuniform porosity	
413		$G_0 = 1$ and $G_1 = 10$	31
414	7	Surface elevation at the origin and wave forces vs. ka for short-crested wave	
415		$(k_x = k_y)$ interaction with a porous cylinder ($G_0 = 1$) with a partial opening at	
416		$[3\pi/4, \pi]$	32
417	8	Equi-amplitude (left) and Equi-phase (right) contours for short-crested wave	
418		$(k_x = k_y = \sqrt{2}/2 \text{ m}^{-1})$ interaction with a porous cylinder ($G_0 = 1$) with a	
419		partial opening at $7/8\pi < \theta < 9/8\pi$ ($a = 2, 3, 4 \text{ m}$).	33
420	9	Equi-amplitude (left) and Equi-phase (right) contours for short-crested wave	
421		$(k_x = k_y = \sqrt{2}/2 \text{ m}^{-1})$ interaction with a porous cylinder ($G_0 = 0.5, 1, 10$) with	
422		a partial opening at $7/8\pi < \theta < 9/8\pi$ ($a = 4 \text{ m}$).	34
423	10	Equi-amplitude (left) and Equi-phase (right) contours for short-crested wave	
424		$(k_x = k_y = \sqrt{2}/2 \text{ m}^{-1})$ interaction with a porous cylinder ($G_0 = 1$) with a	
425		partial opening ($\alpha = \pi/4, \pi/2, \pi$) centred at $\beta = \pi$ ($a = 4 \text{ m}$).	35
426	11	Equi-amplitude (left) and Equi-phase (right) contours for short-crested wave	
427		$(k_x = k_y = \sqrt{2}/2 \text{ m}^{-1})$ interaction with a porous cylinder ($G_0 = 1$) with a	
428		partial opening ($\alpha = \pi/4$) centred at $\beta = \pi, \pi/2, 0$ ($a = 4 \text{ m}$).	36
429	12	Wave forces vs. ka for a cylinder with a partial opening at $\beta = \pi$ and opening	
430		angle $\alpha = \pi/4$ ($G_0 = 0.5, 1, 10$).	37
431	13	Wave force vs. ka for a porous cylinder ($G_0 = 1$) with a partial opening at	
432		$\beta = \pi$ (opening angle $\alpha = \pi/4, \pi/2, \pi$).	38
433	14	Wave force vs. ka for a porous cylinder ($G_0 = 1$) with a partial opening	
434		$(\alpha = \pi/4)$ at $\beta = 0, \pi/4, \pi/2, 3\pi/4, \pi$	39

Figure 1. Ekofisk Gravity Structure (Courtesy ConocoPhillips).

Figure 2. Definition sketch of wave interaction with a porous cylindrical structure

Figure 3. The coordinate definition of SBFEM

Figure 4. Scaled boundary finite element mesh for a porous cylindrical structure

Figure 5. Surface elevation at the origin and wave forces vs. ka for short-crested wave ($k_x = k_y$) interaction with a porous cylinder of uniform porosity $G_0 = 1$.

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Figure 6. Surface elevation at the origin (left) and wave forces (right) vs. ka for short-crested wave ($k_x = k_y$) interaction with a porous cylinder of nonuniform porosity $G_0 = 1$ and $G_1 = 10$.

Figure 7. Surface elevation at the origin and wave forces vs. ka for short-crested wave ($k_x = k_y$) interaction with a porous cylinder ($G_0 = 1$) with a partial opening at $[3\pi/4, \pi]$.

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Figure 8. Equi-amplitude (left) and Equi-phase (right) contours for short-crested wave ($k_x = k_y = \sqrt{2}/2 \text{ m}^{-1}$) interaction with a porous cylinder ($G_0 = 1$) with a partial opening at $7/8\pi < \theta < 9/8\pi$ ($a = 2, 3, 4 \text{ m}$).

FIGURE 9

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Figure 9. Equi-amplitude (left) and Equi-phase (right) contours for short-crested wave ($k_x = k_y = \sqrt{2}/2 \text{ m}^{-1}$) interaction with a porous cylinder ($G_0 = 0.5, 1, 10$) with a partial opening at $7/8\pi < \theta < 9/8\pi$ ($a = 4 \text{ m}$).

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Figure 10. Equi-amplitude (left) and Equi-phase (right) contours for short-crested wave ($k_x = k_y = \sqrt{2}/2 \text{ m}^{-1}$) interaction with a porous cylinder ($G_0 = 1$) with a partial opening ($\alpha = \pi/4, \pi/2, \pi$) centred at $\beta = \pi$ ($a = 4 \text{ m}$).

Contours

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Figure 11. Equi-amplitude (left) and Equi-phase (right) contours for short-crested wave ($k_x = k_y = \sqrt{2}/2 \text{ m}^{-1}$) interaction with a porous cylinder ($G_0 = 1$) with a partial opening ($\alpha = \pi/4$) centred at $\beta = \pi, \pi/2, 0$ ($a = 4 \text{ m}$).

Figure 12. Wave forces vs. ka for a cylinder with a partial opening at $\beta = \pi$ and opening angle $\alpha = \pi/4$ ($G_0 = 0.5, 1, 10$).

Figure 13. Wave force vs. ka for a porous cylinder ($G_0 = 1$) with a partial opening at $\beta = \pi$ (opening angle $\alpha = \pi/4, \pi/2, \pi$).

Figure 14. Wave force vs. ka for a porous cylinder ($G_0 = 1$) with a partial opening ($\alpha = \pi/4$) at $\beta = 0, \pi/4, \pi/2, 3\pi/4, \pi$.