MODELLING THE EFFECTS OF POROUS MEDIA DEFORMATION ON THE PROPAGATION OF WATER TABLE WAVES IN A SANDY UNCONFINED AQUIFER

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Abstract

In this paper, the influence of porous media deformation on the dispersion of water table waves in an unconfined aquifer was examined using a numerical model which coupled Richards' unsaturated flow equation to the poro-elastic material model. The dispersion of the water table wave was quantified via the complex water table wave number which was extracted from the predicted amplitude and phase profiles. The motivation for the present study is the observed model-data discrepancy of Shoushtari et al. (2016). The observed wave dispersion (amplitude decay and phase lag) was unable to be reproduced with analytical solutions (including capillarity and vertical flow effects) or a numerical solution of Richards' equation (with and without hysteresis effects). The discrepancy inherent in the comparison is that there is more energy dissipation in the observations (faster decay rate and greater phase lag) than is predicted by any of the models. This paper examines whether deformation of the

porous media can explain the discrepancy. A sensitivity analysis was performed to establish the influence of main parameters in the poro-elastic model namely Young's modulus (E) and Poisson's ratio (ν). For a short oscillation period ($T = 16.4\text{s}$), the rate of increase in phase lag (imaginary part of the water table wave number, k_i) is sensitive to the chosen values of (E) and (ν) demonstrating an inverse relationship with both parameters. Changes in the amplitude decay rate (real part of the water table wave number, k_r) however were negligible. For a longer oscillation period ($T = 908.6\text{s}$), variations in the values of (E) and (ν) resulted in only small changes in both k_r and k_i . In both the short and long period cases however, the poro-elastic model is unable to reproduce the wave dispersion observed in the laboratory data. Hence porous media deformation is unable to explain the additional energy dissipation that is apparent in the existing sand flume data.

Keywords: Groundwater; Water table wave dispersion; Poro-elastic model; Richards' equation

1. Introduction

Oceanic forces such as waves and tides can influence the movement of water table waves in the aquifers. The propagation of water table waves plays an important role in many coastal processes like mixing seawater and freshwater (e.g. Li et al., 1999; Robinson et al., 2006; Xin et al., 2010) and also beach sediment transport (e.g. Elfrink and Baldock, 2002; Xin et al., 2010; Bakhtyar et al., 2011). The water table propagation can be described in terms of a complex water table wave number ($k = k_r + k_i i$) where k_r and k_i show the decay rate of water table wave amplitude and the rate of increase in phase lag respectively. Many studies have been done to extract theoretical water table wave dispersion relationship considering different physical influences such as vertical flows (non-hydrostatics pressure) (e.g. Nielsen et al., 1997), horizontal flows in the unsaturated zone (e.g. Kong et al., 2013) and capillary effects (Barry et al., 1996; Li et al., 2000). A summary of the existing analytical dispersion relationships can be found in Shoushtari et al. (2016).

Shoushtari et al. (2016) presented an extensive laboratory sand flume dataset on the propagation of groundwater waves in an unconfined sandy aquifer with a vertical boundary subject to simple harmonic forcing with a wide range of oscillation period from 10.7s to 909s. Their data showed a monotonic increase in both amplitude decay rate and rate of increase in phase lag of the water table waves with increasing oscillation frequency (increasing $\omega d / K_{sat}$) which was in contrast to existing theories which predict (1) zero phase lag or standing wave behaviour and (2) an asymptotic decay rate as the frequency increases. Shoushtari et al. (2016) considered possible influences like sand packing, measurement location, finite amplitude wave effects, unsaturated zone truncation and multiple wave mode effects but none of them can explain the observed discrepancy. They also compared laboratory data against numerical solutions of hysteresis and non-hysteresis Richards'

equation (Richards, 1931) and in both cases, the same qualitative behaviour as the analytic solutions described above was found.

In this paper, we test the hypothesis that the deformation of porous media due to periodic water table motion is the cause of the model-data discrepancy of Shoushtari et al. (2016). To examine the influence of porous media deformation on water table wave dispersion we develop a model that considers saturated-unsaturated water flow in elastic porous solids. Biot's poro-elastic theory (Biot, 1941, 1955, 1962) is adopted which combines Darcy's law with solid mechanics and describes the interaction between fluids and deformation in porous media. Numerous studies have been carried out to derive analytical (e.g. Madsen, 1978; Yamamoto et al., 1978; Okusa, 1985; Zienkiewicz et al., 1980; Yamamoto, 1981; Yamamoto and Schuckman, 1984; Mei and Foda, 1981; Rahman et al., 1994; Cha et al., 2002; Jeng and Cha, 2003) and numerical (e.g. Thomas, 1989, 1995; Jeng and Lin, 1996, 1997) solutions to Biot's theory. Biot's theory has been also used to analyse different physical systems such as wave-induced stresses and pore pressure in a porous seabed (e.g. Jeng and Hsu, 1996; Jeng, 2003) and wave-seabed-submarine structures interactions (e.g. Shabani et al. 2009; Zhang et al., 2011a,b). There has been no previous investigation of the influence of porous media deformation on water table wave propagation in an unconfined aquifer. This paper addresses this gap in knowledge and in doing so tests the hypothesis that the deformation of porous media due to periodic water table motion is the cause of the model-data discrepancy of Shoushtari et al. (2016).

The paper is organized as follows: Section 2 provides a brief description of the experimental data of Shoushtari et al. (2016) used for model-data comparison. Section 3 describes the numerical model and boundary conditions. In section 4 the model results have been compared with the laboratory data. Finally, Section 5 summarises the major findings and conclusions.

2. Experimental setup and procedures

In this section a brief outline of the sand flume experiments of Shoushtari et al. (2016) is presented for ease of reference.

Shoushtari et al. (2016) conducted a series of laboratory tests in the sand flume which is 9 m long, 1.5m high and 0.15m wide. A simple harmonic wave was applied across a vertical interface,

$$H_o = d + A \cos(\omega t) \quad (1)$$

where H_o [L] is the driving head, d [L] is the mean driving head, A [L] is the driving head amplitude and $\omega = 2\pi/T$ [T^{-1}] is the oscillation frequency and T [T] is the oscillation period. Table 1 summarises the driving head parameters used in this study noting that the two periods examined were selected in accordance with the shortest and longest periods examined in the experiments of Shoushtari et al. (2016).

Table 1- Driving head parameters

T (s)	ω (rad/s)	A (m)	d (m)	n (-)	K_{sat} (m/s)	$n\omega d/K_{sat}$ (-)
16.4	0.38	0.123	0.934	0.41	4.7×10^{-4}	243.63
908.6	0.01	0.149	0.843	0.41	4.7×10^{-4}	3.97

The flume had no-flow boundaries at the ‘landward’ end and at the bottom. The top of the flume was covered in loose plastic sheeting which allowed free connection of the aquifer with the surrounding atmospheric pressure but can be considered a no flow boundary in terms of moisture transport. The sand in the flume was well-sorted quartz sand whose properties were examined in detail by Nielsen and Perrochet (2000 a, b) (Table 2). Horizontal piezometers inside the sand at different location along the sand flume were used to measure the hydraulic head.

Table 2- Physical and hydraulic characteristics of the sand (Nielsen and Perrochet, 2000 a, b)

d_{50} (mm)	d_{90}/d_{10} (-)	K_{sat} (m/s)	θ_s (vol/vol)	θ_r (vol/vol)	α_d (1/m)	α_w (1/m)	β (-)	H_ψ (m)
0.20	1.83	4.7×10^{-4}	0.41	0.09	1.7	3.4	9	0.55

d_{50} , d_{90} and d_{10} are respectively the median and 90th and 10th percentiles of the

sieve curve analysis; K_{sat} , saturated hydraulic conductivity; θ_s

and θ_r saturated and residual moisture contents, respectively; α_d

and β are the best fit van Genuchten parameters for the first drying curve; α_w

is the adopted van Genuchten parameter for the first wetting curve; H_ψ , steady capillary

fringe thickness.

3. Numerical modeling

To consider the water flow in elastic porous solids, Biot's poro-elastic theory (Biot, 1941, 1955, 1962) is used. This theory is the combination of Darcy's law with solid mechanics and describes the interaction between fluids and deformation in porous media.

3.1 Fluid flow

The Richards' equation (Richards, 1931) is used to estimate the flow field in the poro-elastic model as follow

$$\rho_f \left(\frac{C_m}{\rho_f g} + S_e S \right) \frac{\partial p_f}{\partial t} + \nabla \cdot \rho_f [-K_{sat} \kappa_r (\nabla p_f + \rho_f g \nabla z)] = -\rho_f \alpha_B \frac{\partial}{\partial t} \varepsilon_{vol} \quad (2)$$

where ρ_f [ML⁻³] is the fluid density, p_f [MT⁻²L⁻¹] is the fluid pore pressure, C_m [L⁻¹]

represents the specific moisture capacity, S_e [-] denotes the effective saturation, κ_r [-] is the

relative permeability. S [L⁻¹] is the storage coefficient, K_{sat} [LT⁻¹] is the saturated hydraulic

conductivity, g [LT^{-2}] is the acceleration of the gravity, z [L] is the elevation, $\partial \varepsilon_{\text{vol}} / \partial t$ [T^{-1}] is the rate of change in volumetric strain of the porous matrix.

The analytical formulas of van Genuchten (van Genuchten, 1980) are used to define the C_m , S_e and κ_r as follow

$$S_e = \frac{\theta - \theta_r}{\theta_s - \theta_r} = \begin{cases} \frac{1}{\left[1 + |\alpha \psi|^\beta\right]^m} & \psi < 0 \\ 1 & \psi \geq 0 \end{cases} \quad (3)$$

where ψ [L] is the pressure head, θ [-] is the volumetric moisture content, θ_s [-] and θ_r [-] are saturated and residual moisture contents respectively.

The relative permeability is the ratio of the unsaturated hydraulic conductivity relative to the saturated value and for the van Genuchten (1980) model is given by,

$$k_r = \begin{cases} S_e^l \left[1 - (1 - S_e^{1/m})^m\right]^2 & \psi < 0 \\ 1 & \psi \geq 0 \end{cases} \quad (4)$$

The specific moisture capacity is defined as,

$$C_m = \frac{d\theta}{dh} = \begin{cases} \frac{\alpha m}{1-m} (\theta_s - \theta_r) S_e^{1/m} (1 - S_e^{1/m})^m & \psi < 0 \\ 0 & \psi \geq 0 \end{cases} \quad (5)$$

where α [L^{-1}], β [-], $l = 0.5$ [-] and $m = 1 - 1/\beta$ [-] are empirical curve fitting parameters and $\psi = 0$ corresponds to the water table position.

The Biot-Willis coefficient α_B [-], which relates the volume of fluid expelled (or sucked into) a porous material element to the volumetric changes of the same element and it is defined in terms of the drained and solid bulk moduli as

$$\alpha_B = \frac{\partial p_m}{\partial p_f} \bigg|_{\varepsilon} = 1 - \frac{K_d}{K_s} \quad (6)$$

where p_m [MT⁻²L⁻¹] is the total mean pressure for the porous matrix-fluid system, ε [-] is the strain tensor, K_d and K_s [MT⁻²L⁻¹] are the drained and the solid bulk modulus respectively.

The drained bulk modulus (K_d) is always smaller than the solid bulk modulus (K_s), therefore Biot-Willis coefficient is always bounded to $\varepsilon_p \leq \alpha_B \leq 1$ (where ε_p [-] is the porosity). The α_B does not depend on the properties of the porous matrix. A soft porous matrix has a Biot-Willis coefficient close to 1 (since $K_d \ll K_s$), while for a stiff matrix, it is close to the porosity (since $K_d \approx (1 - \varepsilon_p)K_s$).

3.2 Porous matrix deformation

The governing equation for the poro-elastic material model is

$$-\nabla \cdot \sigma = \rho_{av} g = (\rho_f \varepsilon_p + \rho_d) g \quad (7)$$

where σ [MT⁻²L⁻¹] is the total stress tensor, ρ_{av} , ρ_f and ρ_d [ML⁻³] are the average, fluid and drained densities respectively. The total stress tensor σ , can be calculated by

$$\sigma = \mathbf{C} \varepsilon - \alpha_B p_f \mathbf{I} \quad (8)$$

where $\mathbf{C}$ [MT⁻²L⁻¹] is the elasticity matrix, ε [-] is the strain tensor and $\mathbf{I}$ is the identity matrix. For an isotropic porous material under plane strain condition which is common for 2D poro-elasticity problems (i.e. the normal strain to the xz-plane equals zero), this can be simplified to

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{zz} \\ \sigma_{xz} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & 1-2\nu \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{zz} \\ \varepsilon_{xz} \end{bmatrix} - \begin{bmatrix} \alpha_B p_f & 0 & 0 \\ 0 & \alpha_B p_f & 0 \\ 0 & 0 & \alpha_B p_f \end{bmatrix} \quad (9)$$

where E [$\text{MT}^{-2}\text{L}^{-1}$] is the Young's modulus and ν [-] is the Poisson's ratio for the drained porous matrix. The poro-elastic material model uses equation (7) to describe changes in the total stress tensor σ and porous matrix displacement $\mathbf{u}$ [L] due to boundary conditions and changes in pore pressure. The $\alpha_B p_f$ term is often described as the fluid-to structure coupling expression which counts the fluid pressure contribution.

For small deformation with the plane strain condition, the normal strains ($\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz}$) and shear strains ($\varepsilon_{xy}, \varepsilon_{yz}, \varepsilon_{xz}$) relate to displacements as

$$\varepsilon_{xx} = \frac{\partial u}{\partial x}, \quad \varepsilon_{zz} = \frac{\partial v}{\partial z}, \quad \varepsilon_{xz} = \varepsilon_{zx} = \frac{1}{2} \left(\frac{\partial u}{\partial z} + \frac{\partial v}{\partial x} \right), \quad \varepsilon_{xy} = \varepsilon_{yx} = \varepsilon_{yz} = \varepsilon_{zy} = \varepsilon_{zz} = 0 \quad (10)$$

where u and v [L] are the displacement components in x and z direction respectively.

Equations (9) and (10) are substituted into equation (7) which is then solved simultaneously with the time-dependent flow model (equation (2)) through the time rate of change in strain ($\partial \varepsilon_{vol} / \partial t$). In this paper, the finite element method is applied using the COMSOL 4.3b (COMSOL, 2013) software package.

3.3 Model domain and boundary conditions

The model domain and boundary conditions of the sand flume using in the poro-elastic model is shown in Figure 1.

Figure 1- The model domain and boundary conditions using in the poro-elastic model

The no-flow boundary condition in the fluid flow model is given by,

$$\mathbf{n} \cdot K_{sat} \nabla p_f = 0 \quad (11)$$

where $\mathbf{n}$ is the vector normal to the boundary.

The Cauchy boundary condition has been used to implement the periodic seepage face boundary in the fluid flow model. The Cauchy boundary condition can be expressed by,

$$\mathbf{n} \cdot \rho_f K_{sat} \nabla p_f = \rho_f R_b \left[\left(\frac{p_b - p_f}{\rho_f g} \right) + (z_b - z) \right] \quad (12)$$

where p_b [$\text{MT}^{-2}\text{L}^{-1}$] and z_b [L] are the pressure and elevation of the distant fluid source, respectively and R_b [T^{-1}] is the conductance of the material between the source and the model domain.

By applying appropriate logical statements, the Cauchy boundary condition can be switched between a Dirichlet boundary condition and a Neumann boundary condition. For full details of implement of the seepage face boundary condition in the numerical model, the reader is referred to Shoushtari et al. (2015a) and Chui and Freyberg (2009).

The boundary conditions for the porous matrix deformation model are a series of constraints on the displacement which are including:

- Free constraint at the surface (the upper surface is free to move in horizontal and vertical directions)
- Roller constraint at the bottom, left and right hand sides (the displacement is zero in the direction perpendicular (normal) to the boundary, but it is free to move in the tangential direction).

3.4 Model parameters

The coefficients and parameters used in the poro-elastic model are summarized in Table 3. Depending on the degree of compaction in the sand, Young's modulus (E) and Poisson's ratio (ν) range between $10 < E(\text{MPa}) < 69$ and $0.25 < \nu < 0.4$ respectively. Hence a sensitivity analysis was conducted (see Section 4) to examine the influence of these model parameters on the water table wave propagation.

Table 3- Parameters used in the poro-elastic model

Parameter	Symbol	Value
Fluid density	ρ_f	1000 (kg/m ³)
Drained density of porous layer	ρ_d	2750 (kg/m ³)
Dynamic viscosity of water	μ	0.001 (Pa.s)
Drained Young's modulus for loose sand	E	10-28 (MPa)
Drained Young's modulus for dense sand	E	35-69 (MPa)
Drained Poisson's ratio for sandy soil	ν	0.25-0.4
Biot-Willis coefficient	α_B	1
Saturated volume fraction for sand	θ_s	0.41
Residual volume fraction for sand	θ_r	0.09
van Genuchten parameters	α	1.7 (1/m)
van Genuchten parameters	β	9
van Genuchten parameters	l	0.5
Saturated hydraulic conductivity	K_{sat}	4.7×10^{-4} (m/s)

4. Results and Discussion

To analyse the water table wave propagation, the water table wave number has been calculated from a linear regression of water table wave amplitude and phase profiles which were extracted using harmonic analysis on the simulated pore pressure time series (e.g. Nielsen, 1990; Cartwright et al., 2003).

4.1 Model sensitivity analysis

Figure 2 and Figure 3 show the predicted water table wave number components for $T = 16.4$ s and $T = 908.6$ s respectively and their dependence on Young's modulus (E) and Poisson's ratio (ν).

For the short period ($T = 16.4s$), the real part of water table wave number $k_r d$ (Figure 2 (a)), is seen to be insensitive to E and ν with a maximum of $k_r d = 1.248$ obtained for $E = 10$ (MPa) and $\nu = 0.25$ and a minimum of $k_r d = 1.21$ for $E = 69$ (MPa) and $\nu = 0.40$, i.e. less than 4% changes in the value of the $k_r d$. In terms of the imaginary part of water table wave number $k_i d$ (Figure 2 (b)), the maximum value is 0.396 for $E = 10$ (MPa) and $\nu = 0.25$ and a minimum value of 0.092 for $E = 69$ (MPa) and $\nu = 0.40$, i.e. 76% reduction, indicates that the $k_i d$ (the rate of increase in phase lag) is sensitive to E and ν .

Figure 2- Water table wave number components for $T = 16.4s$ extracting from poro-elastic model with different values of Young's modulus (E) and Poisson's ratio (ν) of the sand. (a) the real part of the water table wave number ($k_r d$) for $\nu = 0.25$ (○) and $\nu = 0.40$ (●); (b) the imaginary part of the water table wave number ($k_i d$) for $\nu = 0.25$ (△) and $\nu = 0.40$ (▲).

For the long period ($T = 908.6s$, Figure 3), both real and imaginary parts of water table wave number are essentially insensitive to E and ν . The maximum value for $k_r d$ is 0.929 for

$E = 35$ (MPa) and $\nu = 0.25$ which is reduced by 2% to a minimum value of 0.908 for $E = 10$ (MPa) and $\nu = 0.25$. In terms of $k_r d$, the maximum value is 0.276 for $E = 35$ (MPa) and $\nu = 0.25$ and the minimum value is 0.250 which indicates 9% a reduction.

Figure 3- Water table wave number components for $T = 908.6s$ extracting from poro-elastic model with different values of Young's modulus (E) and Poisson's ratio (ν) of the sand. (a) the real part of the water table wave number ($k_r d$) for $\nu = 0.25$ ($\circ$) and $\nu = 0.40$ ($\bullet$); (b) the imaginary part of the water table wave number ($k_i d$) for $\nu = 0.25$ (Δ) and $\nu = 0.40$ ($\blacktriangle$).

The results of the maximum total displacement ($\sqrt{u^2 + v^2}$ where u and v are the displacement components in x and z direction respectively) are summarized in Table 4. As it can be seen in this table, there is a trend that the deformation increases with decreasing Young's modulus (E) and decreasing Poisson's ratio (ν). However, with the exception of the $E = 10$ (MPa) results, the deformation is less than 1mm in all cases. This negligible response of the porous media is likely due the fact that the water table fluctuations in the experimental data are not large enough to induce a significant deformation; hence, overall the

water table wave numbers are generally insensitive to Young's modulus (E) and Poisson's ratio (ν) due to the negligible deformation of the sand body.

In comparing the trends seen in Table 4 with the wave numbers shown in Figures 2 and 3 an interesting relationship is found which, whilst perhaps not important under the present experimental parameters (since maximum deformation is $< 1\text{mm}$ in almost all cases), may become important under more energetic fluctuations. Figure 2 shows a clear trend of increasing wave numbers with decreasing Young's modulus (E) which correlates with the predicted increase in deformation with decreasing Young's modulus (E) for $T = 16.4\text{s}$ (cf. Table 4). The increase in maximum displacement for $T = 908.6\text{s}$ is very similar to that of the $T = 16.4\text{s}$ results however Figure 3 shows no obvious trend with E . This indicates that, if the influence of porous media deformation on water table wave dispersion was to become more significant under more energetic fluctuations, this would be more likely to occur at shorter oscillation periods than longer ones.

Table 4. The summary of the maximum total displacement

ν (-)	E (MPa)	Max total displacement (mm)	
		$T = 16.4$ (s)	$T = 908.6$ (s)
0.25	10	2.59	2.72
	28	0.93	0.97
	35	0.74	0.78
	69	0.37	0.39
0.4	10	1.45	1.52
	28	0.52	0.54
	35	0.42	0.44
	69	0.21	0.22

4.2 Comparison with existing laboratory data and non-elastic model results

The comparisons of the present poro-elastic model results with the existing laboratory data and non-elastic (hysteretic and non-hysteretic) Richards' equation model results of Shoushtari et al. (2016) are presented in Figure 4 and Figure 5 and are also summarised in Table 5. The non-hysteretic Richards' equation model employed the van Genuchten (1980) parameters $\alpha = 1.7 \text{ (m}^{-1}\text{)}$ and $\beta = 9$ and the hysteretic model used $\alpha_w = 3.4 \text{ (m}^{-1}\text{)}$, $\alpha_d = 1.7 \text{ (m}^{-1}\text{)}$ $\beta = 9$. Full details of non-hysteretic and hysteretic Richards' model can be found in Shoushtari et al. (2015b).

In terms of $k_r d$ for the short period ($T = 16.4\text{s}$, Figure 4), the poro-elastic model results (solid and open circles with an average of $k_r d = 1.218$), are very close to non-hysteretic Richards' model (plus sign with $k_r d = 1.151$) which is almost 20% less than laboratory data (asterisk with $k_r d = 1.499$). The hysteretic Richards' model (cross sign with $k_r d = 1.426$) predicts $k_r d$ better than the other two models (poro-elastic and non-hysteretic model) with only less than 5% underestimation respect to laboratory data.

In terms of $k_i d$ for the short period (Figure 4), the non-hysteretic Richards' model results (plus sign with $k_i d = 0.101$) is similar to the poro-elastic model with $E = 69 \text{ (MPa)}$ and $\nu = 0.40$ (open circle with $k_i d = 0.099$) which is 87% less than laboratory data (asterisk with $k_i d = 0.751$). The result of hysteretic Richards' model (cross sign with $k_i d = 0.160$) is close to poro-elastic model with $E = 35 \text{ (MPa)}$ and $\nu = 0.25$ (open circles with $k_i d = 0.170$) which is 78% less than the laboratory data. The closest result to the laboratory data is obtained from poro-elastic model with $E = 10 \text{ (MPa)}$ and $\nu = 0.25$ (open circles with $k_i d = 0.395$), but this is still 47% less than the laboratory data.

Figure 4- Model-data comparison of the water table wave number components for $T = 16.4s$. Circles show the poro-elastic model results for different values of Young's modulus (E) for $\nu = 0.25$ ($\circ$) and $\nu = 0.40$ ($\bullet$). (+) denotes the non-hysteretic Richards' model with $\alpha = 1.7$ (m^{-1}) and $\beta = 9$; (x) shows the hysteretic Richards's model with $\alpha_w = 3.4$ (m^{-1}), $\alpha_d = 1.7$ (m^{-1}) and $\beta = 9$; (*) denotes the laboratory data of Shoushtari et al. (2016).

For the long period ($T = 908.6s$, Figure 5), the values of $k_r d$ and $k_i d$ predicted by the poro-elastic model (solid and open circles, average values of 0.919 and 0.263 respectively), are very close to the non-hysteretic Richards' model with $k_r d = 0.870$ and $k_i d = 0.231$ as compared to the laboratory data (0.471 and 0.295 for $k_r d$ and $k_i d$ respectively). The closest result to the laboratory data has been obtained with the hysteretic Richards' model which underestimates $k_r d$ and $k_i d$ as 17% and 24% less than laboratory data respectively.

Figure 5- Model-data comparison of the water table wave number components for $T = 908.6s$. Circles show the poro-elastic model results for different values of Young's modulus (E) for $\nu = 0.25$ ($\circ$) and $\nu = 0.40$ ($\bullet$). (+) denotes the non-hysteretic Richards' model with $\alpha = 1.7$ (m^{-1}) and $\beta = 9$; ($\times$) shows the hysteretic Richards's model with $\alpha_w = 3.4$ (m^{-1}), $\alpha_d = 1.7$ (m^{-1}) and $\beta = 9$; (*) denotes the laboratory data of Shoushtari et al. (2016).

Table 5- Summary of water table wave number components

	ν (-)	E (MPa)	k_r (-)	k_i (-)	$k_r d$ (m)	$k_i d$ (m)
$T = 16.4$ (s) ($n\omega d/K_{sat} = 243.63$)						
Poro-elastic model	0.25	10	1.337	0.423	1.248	0.395
		28	1.301	0.290	1.215	0.195
		35	1.296	0.183	1.210	0.170
		69	1.300	0.132	1.214	0.123
	0.40	10	1.312	0.249	1.225	0.232
		28	1.297	0.144	1.212	0.134
		35	1.296	0.130	1.210	0.121
		69	1.296	0.106	1.210	0.099
Non-hysteretic, rigid media Richards' model ($\alpha = 1.7$ (m ⁻¹) , $\beta = 9$)	-	-	1.232	0.108	1.151	0.101
Hysteretic, rigid media Richards' model ($\alpha_w = 3.4$ (m ⁻¹), $\alpha_d = 1.7$ (m ⁻¹), $\beta = 9$)	-	-	1.527	0.171	1.426	0.160
Laboratory data	-	-	1.604	0.804	1.499	0.751
$T = 908.6$ (s) ($n\omega d/K_{sat} = 3.97$)						
Poro-elastic model	0.25	10	1.077	0.296	0.908	0.250
		28	1.100	0.326	0.927	0.275
		35	1.102	0.328	0.929	0.276
		69	1.095	0.315	0.923	0.266
	0.40	10	1.086	0.312	0.915	0.263
		28	1.094	0.315	0.922	0.265
		35	1.084	0.298	0.914	0.251
		69	1.086	0.307	0.916	0.259
Non-hysteretic, rigid media Richards' model ($\alpha = 1.7$ (m ⁻¹) , $\beta = 9$)	-	-	1.032	0.274	0.870	0.231
Hysteretic, rigid media Richards' model ($\alpha_w = 3.4$ (m ⁻¹), $\alpha_d = 1.7$ (m ⁻¹), $\beta = 9$)	-	-	0.464	0.266	0.391	0.224
Laboratory data	-	-	0.558	0.350	0.471	0.295

5. Conclusion

This paper has disproved the hypothesis that porous media deformation can explain the observed model-data discrepancy of Shoushtari et al. (2016) in which the experimental data showed large amounts water table wave dispersion that their models were unable to predict.

The present study used a numerical model to solve Biot's poro-elastic equation coupled to Richards' saturated-unsaturated flow equation to examine the influence of porous media deformation on the water table wave dispersion which was quantified using the complex water table wave number.

The poro-elastic model results were found to be generally insensitive to Young's modulus (E) and Poisson's ratio (ν) of the sand with maximum displacements generally less than 1mm with the exception of the $E = 10$ (MPa) results which indicate maximum displacements > 2 mm. This is likely due to the fact that the experimental water table fluctuations are not energetic enough to induce a significant deformation response.

A sensitivity analysis shows that the only model result which was sensitive to these parameters was the imaginary part of the water table wave number ($k_i d$) for the short period oscillation ($T = 16.4$ s). In this case there was a 76% change over the range of acceptable values for E and ν . Correspondingly, the real part of the water table wave number ($k_r d$) only changed about 4% for the short period and for the long period ($T = 908.6$ s), $k_r d$ and $k_i d$ varied by only 2% and 9% respectively.

The poro-elastic model results were compared with both the laboratory data and rigid medium model results of Shoushtari et al. (2016). For the short period ($T = 16.4$ s), the difference in predicted $k_r d$ between the poro-elastic and rigid medium model results was

negligible. However, both model predictions were almost 20% less than the laboratory data. Decreasing the values of E and ν in the poro-elastic model resulted in some small improvement in the prediction of $k_i d$ compared to the rigid media results although it was still 47% less than the laboratory data. For the long period ($T = 908.6s$), the results of poro-elastic and rigid media models were almost same for both $k_r d$ and $k_i d$ with both models over predicting $k_r d$ by 94% relative to the laboratory data.

Whilst the poro-elastic model was unable to reproduce the experimental results, the model results suggest that, if porous media deformation was to become more significant under more energetic fluctuations, the influence of the deformation on water table wave dispersion is likely to be more significant at shorter oscillation periods than longer ones.

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