Three Methods for Revising Hybrid Knowledge Bases

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Abstract. Contemporary approaches for the Semantic Web include *hybrid knowledge bases* that combine ontologies with rule-based languages. Despite a number of existing combination approaches, little attention has been given to change mechanisms for hybrid knowledge bases that can appropriately handle the dynamics of information on the Web. We present here three methods for *revising* hybrid knowledge bases in light of new information. We show by means of representation theorems that two of them fit properly into the classic belief change framework and that each of the two generalises the third method.

Keywords: Belief Revision, Hybrid Knowledge Bases, Logic Programs, Ontologies, Here-and-There Logic, Semantic Web

1 Introduction

In the last decade, ontologies [9] have become widely accepted as knowledge engineering artefacts in the Semantic Web for modelling domains of expertise. For example, SNOMED CT¹, the Gene Ontology², and OBO Foundry Ontologies³ represent standardised biomedical terminologies and are well-established within their field of application. They are usually specified in a fragment of first-order logic and can be viewed as first-order theories in restricted form. More recently, ontologies have been integrated with logic programs [4, 13] into *hybrid knowledge bases* to combine monotonic reasoning over the former component with nonmonotonic reasoning over the latter. A wide array of such integrations have been proposed (overviews can be found in [7, 12, 14]), which differ in the way ontologies and rules are combined. We concentrate here on hybrid knowledge bases under quantified here-and-there logic [3], because they place no restriction on the fragment of first-order logic or the class of logic programs to be used in the composition and capture several other integration approaches.

For hybrid knowledge bases to be successfully adopted in practical applications, it is crucial that they are not only static entities but also responsive to changes as required within the domain. It may be necessary to add new knowledge or retract some existing knowledge. We focus on this problem by framing

¹ <http://www.snomed.org/snomed-ct>

² <http://www.geneontology.org/>

³ <http://www.obofoundry.org/>

it within the traditional *belief change* framework [1, 8, 10], which provides sets of postulates that any rational change operator should satisfy. While the framework has been applied to a variety of knowledge representation formalisms (e.g., [2, 6, 23, 24]), a particular challenge in constructing appropriate belief change mechanisms for hybrid knowledge bases lies in their bi-component nature. Any change operator must be able to handle changes to the theory component, the logic program component, or both, depending on the new information acquired. When either component is changed, it may have an effect on the other component and cause the hybrid knowledge base to become inconsistent.

In the following, we address this gap by first adapting a partial meet revision operator from classical logics to hybrid knowledge bases. We then define a special case of partial meet revision, called protected revision, allowing to preferably revise only one component of the hybrid knowledge base. Finally, we present prioritised revision, a more fine-grained method, where a priority ordering over individual elements of the hybrid knowledge base determines the outcome. By evaluating our operators with respect to each other and the revision postulates, we find that partial meet revision and prioritised revision are interdefinable and comply with the belief change framework, whereas protected revision is a restricted version of partial meet revision.

2 Preliminaries

We first formally introduce hybrid knowledge bases under here-and-there logic and then briefly cover relevant belief change fundamentals.

2.1 Hybrid Knowledge Bases under Here-and-There Logic

Let $\mathcal{L} = (\mathcal{C}, \mathcal{P}_T \cup \mathcal{P}_P)$ be a function-free, first-order language, in which $\mathcal{C}$ is a set of constant symbols and $\mathcal{P}_T, \mathcal{P}_P$ are sets of predicate symbols such that $\mathcal{C}, \mathcal{P}_T, \mathcal{P}_P$ are pairwise disjoint. Let $\mathcal{L}$ further include the symbols ‘ $\wedge$ ’, ‘ $\vee$ ’, ‘ $\rightarrow$ ’, ‘ $\neg$ ’, ‘ $\forall$ ’, ‘ $\exists$ ’, the predicate symbol ‘ $\approx$ ’ for equality, a countably infinite set of variables, and the standard punctuation marks, as well as the symbols ‘ $;$ ’, ‘ $,$ ’, ‘*not*’, ‘ $\leftarrow$ ’, and ‘ $\perp$ ’. Atoms, formulas, closed formulas, and theories are defined in the usual way. A (*first-order*) *rule* r over $\mathcal{L}$ has the form

$$a_1; \dots; a_k; \text{not } b_1; \dots; \text{not } b_l \leftarrow c_1, \dots, c_m, \text{not } d_1, \dots, \text{not } d_n. \quad (1)$$

Here, all a_i, b_i, c_i, d_i are atoms from $\mathcal{L}$ and $k, l, m, n \geq 0$. If $m = n = 0$, then we omit ‘ $\leftarrow$ ’. A *hybrid knowledge base (HKB)* [3] $K = (K_T, K_P)$ over $\mathcal{L}$ consists of a classical first-order theory K_T over the language subset $\mathcal{L}_T = (\mathcal{C}, \mathcal{P}_T)$, and a set of rules of the form (1) over $\mathcal{L}$ that constitute a program K_P . We write $\mathcal{K}_{\mathcal{L}}$ for the class of all HKBs over $\mathcal{L}$.

A HKB can be used to combine specific constraints that hold in a domain, expressed by a program component K_P , with more general information, captured by a theory component K_T . For instance, a personal assistant system on a mobile phone may have in its HKB some general information about gender and marriage as well as some specific information about its user Chris, shown in Example 1.

Example 1. Let $\mathcal{C} = \{Chris, Alex\}$, $\mathcal{P}_T = \{male, female, married\}$, $\mathcal{P}_P = \emptyset$, and $K = (K_T, K_P)$ be a HKB over $(\mathcal{C}, \mathcal{P}_T \cup \mathcal{P}_P)$ such that

$$\begin{aligned} K_T &= \{ \forall x : male(x) \rightarrow \neg female(x), \\ &\quad \forall x, y : married(x, y) \rightarrow married(y, x), \\ &\quad \forall x, y : married(x, y) \wedge female(x) \rightarrow male(y) \}, \\ K_P &= \{ female(Chris) \}. \end{aligned} \quad \blacksquare$$

We extend the usual set operations to HKBs as follows. For any pairs of sets $K^1 = (K_T^1, K_P^1), \dots, K^n = (K_T^n, K_P^n)$, let $K^1 \odot \dots \odot K^n = (\{K_T^1 \odot \dots \odot K_T^n\}, \{K_P^1 \odot \dots \odot K_P^n\})$ for $\odot \in \{\cap, \cup\}$, $K^1 \setminus K^2 = (K_T^1 \setminus K_T^2, K_P^1 \setminus K_P^2)$, and for $i, j \in \{T, P\}$ with $i \neq j$:

$$\begin{aligned} K^1 &\subseteq K^2 \text{ iff } K_i^1 \subseteq K_i^2 \text{ and } K_j^1 \subseteq K_j^2, \\ K^1 &\subseteq_i K^2 \text{ iff } K_i^1 \subseteq K_i^2, \text{ and if } K_i^1 = K_i^2, \text{ then } K_j^1 \subseteq K_j^2, \text{ and} \\ K^1 &\subset_o K^2 \text{ iff } K^1 \subseteq_o K^2 \text{ and } K^2 \not\subseteq_o K^1 \text{ where } \subseteq_o \in \{\subseteq, \subseteq_T, \subseteq_P\}. \end{aligned}$$

For any $K \in \mathcal{K}_{\mathcal{L}}$, let $K|_T = (K_T, \emptyset)$, $K|_P = (\emptyset, K_P)$, and, for any $\mathbb{K} \subseteq 2^K$, $\mathbb{K}|_T = \{(\tau, \emptyset) \mid (\tau, \rho) \in \mathbb{K}\}$ and $\mathbb{K}|_P = \{(\emptyset, \rho) \mid (\tau, \rho) \in \mathbb{K}\}$.

An $\mathcal{L}$ structure is a pair $\mathcal{I} = (U, I)$, where the universe $U = (\mathcal{D}, \sigma)$ consists of a non-empty domain $\mathcal{D}$ and a function σ from $\mathcal{C} \cup \mathcal{D}$ to $\mathcal{D}$ such that $\sigma(D) = D$ for all $D \in \mathcal{D}$, and an interpretation I is a set of variable-free atoms that can be constructed from $(\mathcal{C} \cup \mathcal{D}, \mathcal{P}_T \cup \mathcal{P}_P)$. For a tuple $t = (D_1, \dots, D_n)$, we define $\sigma(t) = (\sigma(D_1), \dots, \sigma(D_n))$. To obtain a ground instance of a rule in K_P , we replace each constant symbol C in the rule by $\sigma(C)$ and each variable in the rule by an element from $\mathcal{D}$. The set of all ground instances of rules in K_P is written $ground(K_P)$.

A $\mathbf{QHT}_{\subseteq}^s$ (quantified here-and-there logic with static domains and equality) structure with respect to $\mathcal{L}$ [18] (or simply $\mathbf{QHT}_{\subseteq}^s$ structure if $\mathcal{L}$ is clear from the context) is a triple $\mathcal{M} = (U, H, T)$ such that (U, H) and (U, T) are $\mathcal{L}$ structures with $H \subseteq T$. Let $\mathcal{QHT}$ be the set of all $\mathbf{QHT}_{\subseteq}^s$ structures over $\mathcal{L}$. For any set S of $\mathbf{QHT}_{\subseteq}^s$ structures, by $\bar{S}$ we denote the complement of S with respect to $\mathcal{QHT}$, that is, $\bar{S} = \mathcal{QHT} \setminus S$. We define the satisfaction relation for a $\mathbf{QHT}_{\subseteq}^s$ structure $\mathcal{M}$ recursively as follows. Let $p(t_1, \dots, t_n)$ be an atom and ϕ, ψ closed formulas built from $(\mathcal{C} \cup \mathcal{D}, \mathcal{P}_T \cup \mathcal{P}_P)$, x be a variable, and $w \in \{H, T\}$. Then

$$\begin{aligned} \mathcal{M} &\models_w p(t_1, \dots, t_n) \text{ iff } p(\sigma(t_1), \dots, \sigma(t_n)) \in w, \\ \mathcal{M} &\models_w \phi \wedge \psi \text{ iff } \mathcal{M} \models_w \phi \text{ and } \mathcal{M} \models_w \psi, \\ \mathcal{M} &\models_w \phi \vee \psi \text{ iff } \mathcal{M} \models_w \phi \text{ or } \mathcal{M} \models_w \psi, \\ \mathcal{M} &\models_T \phi \rightarrow \psi \text{ iff } \mathcal{M} \not\models_T \phi \text{ or } \mathcal{M} \models_T \psi, \\ \mathcal{M} &\models_H \phi \rightarrow \psi \text{ iff (i) } \mathcal{M} \models_T \phi \rightarrow \psi \text{ and (ii) } \mathcal{M} \not\models_H \phi \text{ or } \mathcal{M} \models_H \psi, \\ \mathcal{M} &\models_w \neg \phi \text{ iff } \mathcal{M} \not\models_T \phi, \\ \mathcal{M} &\models_T \forall x \phi(x) \text{ iff } \mathcal{M} \models_T \phi(D) \text{ for all } D \in \mathcal{D}, \\ \mathcal{M} &\models_H \forall x \phi(x) \text{ iff } \mathcal{M} \models_T \forall x \phi(x) \text{ and } \mathcal{M} \models_H \phi(D) \text{ for all } D \in \mathcal{D}, \end{aligned}$$

$$\begin{aligned}\mathcal{M} \models_w \exists x \phi(x) &\text{ iff } \mathcal{M} \models_w \phi(D) \text{ for some } D \in \mathcal{D}, \\ \mathcal{M} \models_w t_1 \approx t_2 &\text{ iff } \sigma(t_1) = \sigma(t_2).\end{aligned}$$

A $\mathbf{QHT}_{=}^s$ structure $\mathcal{M}$ is a $\mathbf{QHT}_{=}^s$ model of a closed formula ϕ , denoted $\mathcal{M} \models \phi$, iff $\mathcal{M} \models_w \phi$ for each $w \in \{H, T\}$. A $\mathbf{QHT}_{=}^s$ structure $\mathcal{M}$ is a:

- $\mathbf{QHT}_{=}^s$ model of a theory K_T iff it is a model of all formulas in K_T ;
- $\mathbf{QHT}_{=}^s$ model of a program K_P iff it is a $\mathbf{QHT}_{=}^s$ model of all rules in $\text{ground}(K_P)$ where each rule of the form (1) is interpreted as

$$c_1 \wedge \dots \wedge c_m \wedge \neg d_1 \wedge \dots \wedge \neg d_n \rightarrow a_1 \vee \dots \vee a_k \vee \neg b_1 \vee \dots \vee \neg b_l;$$

- $\mathbf{QHT}_{=}^s$ model of a HKB $K = (K_T, K_P)$ iff it is a $\mathbf{QHT}_{=}^s$ model of K_T and K_P .

The set of all $\mathbf{QHT}_{=}^s$ models of K is denoted by $QHT(K)$ and K is *satisfiable* iff $QHT(K) \neq \emptyset$.

Example 2. For K from Example 1, we obtain $QHT(K) = \{\mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3, \mathcal{M}_4, \mathcal{M}_5, \mathcal{M}_6, \mathcal{M}_7, \mathcal{M}_8\}$ such that, for $i = \{1, \dots, 8\}$, $U_i = (\{Chris, Alex\}, \sigma)$ for each $\mathcal{M}_i$, where σ is the identity function, and H_i, T_i contain the atoms marked with a H (here) or T (there), respectively, in Table 1. ■

Table 1. (H, T) -Elements for Models of K in Example 1

	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$	$\mathcal{M}_6$	$\mathcal{M}_7$	$\mathcal{M}_8$
$male(Chris)$								
$male(Alex)$	H,T	H,T	T	H,T	T			
$female(Chris)$	H,T	H,T	H,T	H,T	H,T	H,T	H,T	H,T
$female(Alex)$						H,T	T	
$married(Chris, Alex)$	H,T	T	T					
$married(Alex, Chris)$	H,T	T	T					

2.2 Belief Change

The *belief base framework* [8, 10, 19] of belief change defines *expansion*, *revision*, and *contraction* as the operations on a body of beliefs held by an agent, called a belief base, in light of some new information. In an expansion, new beliefs are simply added to an existing belief base, regardless of any inconsistencies that may arise. A revision operation also incorporates new beliefs into a belief base, but may discard some existing beliefs to ensure consistency of the outcome. During a contraction, some beliefs are removed from a belief base but no new beliefs are added. Particularly, and contrary to other frameworks, a belief base is

not required to be closed under logical consequence, which makes this framework suitable for consideration regarding belief change in HKBs.

For propositional logic, Hansson [11] defined a *partial meet (base) contraction operator* $-_\gamma$ for a belief base B and a sentence ϕ as $B -_\gamma \phi = \bigcap \gamma(B \perp \phi)$, where $B \perp \phi = \{B' \subseteq B \mid B' \not\models \phi \text{ and, for all } B'', B' \subset B'' \subseteq B \text{ implies } B'' \models \phi\}$ and γ is a selection function such that $\gamma(B \perp \phi) = \{B\}$ if $B \perp \phi = \emptyset$, and $\emptyset \neq \gamma(B \perp \phi) \subseteq B \perp \phi$ otherwise. He determined a set of four postulates that exactly characterises the class of partial meet contraction operators. He further defined a corresponding *partial meet (base) revision operator* $*_\gamma$ as $B *_\gamma \phi = (B -_\gamma \neg \phi) \cup \{\phi\}$ and showed that a set of five postulates exactly characterises the class of partial meet revision operators.

We now translate these five revision postulates to HKBs, so that we can later evaluate the rationality of our proposed operators. Let $+$ be an expansion operator for $K \in \mathcal{K}_\mathcal{L}$ such that for any $K' \in \mathcal{K}_\mathcal{L}$, $K + K' = K \cup K'$. The postulates are as follows, where we assume $K, K', K'' \in \mathcal{K}_\mathcal{L}$ and a revision operator $*$ to be a function from $\mathcal{K}_\mathcal{L} \times \mathcal{K}_\mathcal{L}$ to $\mathcal{K}_\mathcal{L}$.

- (*1) $K' \subseteq K * K'$
- (*2) $K * K' \subseteq K + K'$
- (*3) If $\kappa \subseteq K \setminus (K * K')$, then there exists a $\kappa' \in \mathcal{K}_\mathcal{L}$ such that $K * K' \subseteq \kappa' \subset K + K'$ and κ' is satisfiable but $\kappa' \cup \kappa$ is not satisfiable
- (*4) If it holds for all $\kappa \subseteq K$ that $\kappa + K'$ is satisfiable iff $\kappa + K''$ is satisfiable, then $K \cap (K * K') = K \cap (K * K'')$
- (*5) If K' is satisfiable, then $K * K'$ is satisfiable

3 Revision in Hybrid Knowledge Bases

In this section, we introduce three new methods for revising HKBs. We begin with an adaptation of *partial meet revision* (Section 3.1), then define a method that prioritises either component of a HKB in its entirety during a revision, called *protected revision* (Section 3.2), and finally propose a method that prioritises individual elements of a HKB, termed *prioritised revision* (Section 3.3).

3.1 Partial Meet Revision

As the foundation of our revision operations, we define the subsets of a HKB that are consistent with another HKB.

Definition 1 (Compatible Set). Let $K, K' \in \mathcal{K}_\mathcal{L}$ and $\subseteq_\circ \in \{\subseteq, \subseteq_T, \subseteq_P\}$. The set of compatible sets of K regarding K' is

$$\begin{aligned} \mathbb{K}_{K', \subseteq_\circ} = \{ \kappa = (\tau, \rho) \mid & \kappa \subseteq K, QHT(\kappa) \cap QHT(K') \neq \emptyset, \text{ and,} \\ & \text{for all } \kappa' = (\tau', \rho'), \kappa \subseteq_\circ \kappa' \subseteq K \text{ implies} \\ & QHT(\kappa') \cap QHT(K') = \emptyset \}. \end{aligned}$$

To construct our revision, we will further make use of a selection function that chooses from a set of compatible sets the most suitable ones. As we aim to employ our selection function for different types of compatible sets, we define it here with respect to an arbitrary set.

Definition 2 (Selection Function). A selection function γ for a set S is a function such that i) $\mathbb{S} \subseteq 2^S$, ii) $\gamma(\mathbb{S}) \subseteq \mathbb{S}$, and iii) if $\mathbb{S} \neq \emptyset$, then $\gamma(\mathbb{S}) \neq \emptyset$.

We are now ready to introduce a *partial meet revision* operator for HKBs as follows.

Definition 3 (Partial Meet Revision). Let $K \in \mathcal{K}_{\mathcal{L}}$ and γ be a selection function for K . A partial meet revision operator $*_{\gamma}$ for K is defined such that for any $K' \in \mathcal{K}_{\mathcal{L}}$:

$$K *_{\gamma} K' = \begin{cases} K + K' & \text{if } K' \text{ is not satisfiable,} \\ \bigcap \gamma(\mathbb{K}_{K', \subseteq}) + K' & \text{otherwise.} \end{cases}$$

Since one of the maxims for belief revision is to ensure primacy of new information [5], our revision operator $*_{\gamma}$ always includes the information expressed by the revising HKB K' in the outcome. In addition, the main characteristic that distinguishes a revision from an expansion is that the former aims at providing a consistent outcome. In the case that K' is itself not satisfiable, our revision operator returns the combination of K' and the initial HKB K in its entirety, since no removal of any part of K would lead to a satisfiable outcome. This behaviour adheres to the maxim of persistence of prior knowledge [5]. In all other cases, the revision operator returns the intersection of the selected compatible sets of K added to K' .

We obtain the following representation theorem, stating that our partial meet revision operator $*_{\gamma}$ is exactly characterised by the postulates (*1)–(*5).

Lemma 1. Let $K, K' \in \mathcal{K}_{\mathcal{L}}$ and γ be a selection function for K . If $\kappa \subseteq K \cap K'$, then $\kappa \subseteq \bigcap \gamma(\mathbb{K}_{K', \subseteq})$.

Theorem 1. An operator $*_{\gamma}$ is a partial meet revision operator for $K \in \mathcal{K}_{\mathcal{L}}$ determined by a selection function γ for K iff $*_{\gamma}$ satisfies (*1)–(*5).

Proof. We first show that a partial meet revision operator $*_{\gamma}$ for K determined by a given selection function γ for K satisfies (*1)–(*5).

(*1): Follows directly from Definition 3.

(*2): If K' is not satisfiable, then $K *_{\gamma} K' = K + K'$ by Definition 3. Otherwise, since $\bigcap \gamma(\mathbb{K}_{K', \subseteq}) \subseteq K$ we have $\bigcap \gamma(\mathbb{K}_{K', \subseteq}) + K' \subseteq K + K'$.

(*3): Let $\kappa \subseteq K$. Assume that for all κ' with $K *_{\gamma} K' \subseteq \kappa' \subset K + K'$ and κ' being satisfiable, it holds that $\kappa' \cup \kappa$ is satisfiable. In particular, for each $\kappa'' \in \mathbb{K}_{K', \subseteq}$ with $K *_{\gamma} K' \subseteq \kappa'' \cup K'$, this implies $\kappa'' \cup K' \cup \kappa$ is satisfiable. As each κ'' is $\subseteq$ -maximal, it follows that $\kappa \subseteq \kappa''$ and thus $\kappa \subseteq \bigcap \gamma(\mathbb{K}_{K', \subseteq})$. From Definition 3 we can then conclude $\kappa \not\subseteq K \setminus (K *_{\gamma} K')$.

(*4): For all $\kappa \subseteq K$, let $\kappa + K'$ be satisfiable iff $\kappa + K''$ is satisfiable. Then $\mathbb{K}_{K', \subseteq} = \mathbb{K}_{K'', \subseteq}$ by Definition 1 and so $\bigcap \gamma(\mathbb{K}_{K', \subseteq}) = \bigcap \gamma(\mathbb{K}_{K'', \subseteq})$ as well as $K \cap \bigcap \gamma(\mathbb{K}_{K', \subseteq}) = K \cap \bigcap \gamma(\mathbb{K}_{K'', \subseteq})$. By Lemma 1 we obtain $(K \cap \bigcap \gamma(\mathbb{K}_{K', \subseteq})) \cup (K \cap K') = (K \cap \bigcap \gamma(\mathbb{K}_{K'', \subseteq})) \cup (K \cap K'')$. This means $K \cap (\bigcap \gamma(\mathbb{K}_{K', \subseteq}) \cup K') = K \cap (\bigcap \gamma(\mathbb{K}_{K'', \subseteq}) \cup K'')$. Thus, $K \cap (K *_\gamma K') = K \cap (K *_\gamma K'')$.

(*5): If K' is satisfiable, then, for any $\kappa \in \mathbb{K}_{K', \subseteq}$, $\kappa + K'$ is satisfiable, which implies $K *_\gamma K'$ is satisfiable.

We now show that any operator $\circ_\gamma$ for K satisfying (*1)–(*5) is a partial meet revision operator for K determined by some selection function for K . We first find a selection function γ for K . Let γ be such that (i) if $\mathbb{K}_{K', \subseteq} = \emptyset$, then $\gamma(\mathbb{K}_{K', \subseteq}) = \emptyset$ and (ii) $\gamma(\mathbb{K}_{K', \subseteq}) = \{\kappa \in \mathbb{K}_{K', \subseteq} \mid K \cap (K \circ_\gamma K') \subseteq \kappa\}$ otherwise.

We begin by showing that γ is a function. For any $K', K'' \in \mathcal{K}_L$, if $\mathbb{K}_{K', \subseteq} = \mathbb{K}_{K'', \subseteq}$, then $K \cap (K \circ_\gamma K') = K \cap (K \circ_\gamma K'')$ by (*4). This means $\gamma(\mathbb{K}_{K', \subseteq}) = \gamma(\mathbb{K}_{K'', \subseteq})$ according to our definition of γ .

We next show that γ is a selection function. Clearly, $\gamma(\mathbb{K}_{K', \subseteq}) \subseteq \mathbb{K}_{K', \subseteq}$ by our definition of γ . If $\mathbb{K}_{K', \subseteq} \neq \emptyset$, then K' is satisfiable by Definition 1 and thus $K \circ_\gamma K'$ is satisfiable by (*5). Since $K' \subseteq K \circ_\gamma K'$ by (*1) and $K \circ_\gamma K' \subseteq K \cup K'$ by (*2), it follows that $(K \cap (K \circ_\gamma K')) \cup K'$ is satisfiable. This means that there exists $\kappa \in \mathbb{K}_{K', \subseteq}$ such that $K \cap (K \circ_\gamma K') \subseteq \kappa$. From our definition of γ we therefore obtain that $\gamma(\mathbb{K}_{K', \subseteq}) \neq \emptyset$.

Finally, we show that $\circ_\gamma$ is a partial meet revision operator for K , that is, $K \circ_\gamma K' = K \cup K'$ if K' is not satisfiable and $K \circ_\gamma K' = \bigcap \gamma(\mathbb{K}_{K', \subseteq}) \cup K'$ otherwise. Consider first the limiting case that K' is not satisfiable. If $\kappa \subseteq K \setminus (K \circ_\gamma K')$, then there exists κ' such that $K \circ_\gamma K' \subseteq \kappa' \subset K \cup K'$ and κ' is satisfiable but $\kappa' \cup \kappa$ is not satisfiable by (*3). This is a contradiction since $K' \subseteq \kappa'$ by (*1). Therefore, it holds for all $\kappa \subseteq K$ that $\kappa \subseteq K \circ_\gamma K'$, that is, $K \subseteq K \circ_\gamma K'$. Since $K' \subseteq K \circ_\gamma K'$ by (*1) and $K \circ_\gamma K' \subseteq K \cup K'$ by (*2), we can conclude $K \circ_\gamma K' = K \cup K'$.

Assume now that K' is satisfiable. Let $\kappa \subseteq K \setminus (K \circ_\gamma K')$. If $\mathbb{K}_{K', \subseteq} = \emptyset$, then it follows from (*1) and (*3) that $K \circ_\gamma K' = K'$. Since $\gamma(\mathbb{K}_{K', \subseteq}) = \emptyset$ by our definition of γ , we thus have $K \circ_\gamma K' = K' = \bigcap \gamma(\mathbb{K}_{K', \subseteq}) \cup K'$. If $\mathbb{K}_{K', \subseteq} \neq \emptyset$, then it follows directly from our definition of γ that $K \cap (K \circ_\gamma K') \subseteq \bigcap \gamma(\mathbb{K}_{K', \subseteq})$. From (*1) and (*2) we then obtain $K \circ_\gamma K' \subseteq \bigcap \gamma(\mathbb{K}_{K', \subseteq}) \cup K'$. To show the converse inclusion, first assume the case that $K \cup K'$ is satisfiable. This implies that for any $\kappa' \subseteq K \cup K'$ it holds that κ' is satisfiable. Applying (*3), we obtain $K \setminus (K \circ_\gamma K') = \emptyset$ and thus $K \subseteq K \circ_\gamma K'$. From (*1) and (*2) it follows that $K \circ_\gamma K' = K \cup K'$. Moreover, due to the assumption that $K \cup K'$ is satisfiable and Definition 1, we have $\mathbb{K}_{K', \subseteq} = \{K\}$. By our definition of γ , we obtain $\gamma(\mathbb{K}_{K', \subseteq}) = \{K\}$ and thus $\bigcap \gamma(\mathbb{K}_{K', \subseteq}) = K$ and can conclude $K \circ_\gamma K' = \bigcap \gamma(\mathbb{K}_{K', \subseteq}) \cup K'$. Lastly, assume the case that $K \cup K'$ is not satisfiable. We will show that $\kappa \not\subseteq K \circ_\gamma K'$ implies $\kappa \not\subseteq \bigcap \gamma(\mathbb{K}_{K', \subseteq}) \cup K'$. If $\kappa \not\subseteq K$, then $\kappa \not\subseteq (K \circ_\gamma K') \setminus K'$ by (*1) and (*2) and $\kappa \not\subseteq \bigcap \gamma(\mathbb{K}_{K', \subseteq})$ by Definition 1. Since $\kappa \not\subseteq K \circ_\gamma K'$ implies $\kappa \not\subseteq K'$ by (*1), it follows that $\kappa \not\subseteq ((K \circ_\gamma K') \setminus K') \cup K' = K \circ_\gamma K'$ and $\kappa \subseteq \bigcap \gamma(\mathbb{K}_{K', \subseteq}) \cup K'$. Now assume $\kappa \subseteq K \setminus (K \circ_\gamma K')$. According to (*3), then there exists κ' such that $K \circ_\gamma K' \subseteq \kappa' \subset K \cup K'$ and κ' is satisfiable but $\kappa' \cup \kappa$ is not satisfiable.

This means that there exists $\kappa'' \in \mathbb{K}_{K', \subseteq}$ such that $K \cap \kappa' \subseteq \kappa''$ and $\kappa \not\subseteq \kappa''$. Since $K \cap (K \circ_\gamma K') \subseteq K \cap \kappa' \subseteq \kappa''$, we obtain from our definition of γ that $\kappa'' \in \gamma(\mathbb{K}_{K', \subseteq})$. We can thus conclude from $\kappa \not\subseteq \kappa''$ that $\kappa \not\subseteq \bigcap \gamma(\mathbb{K}_{K', \subseteq})$. $\square$

3.2 Protected Revision

As a HKB consists of two components, it may be desirable to emphasise one component over the other during a revision operation. We can achieve this by a *protected revision*, which keeps the theory (or program) component of the initial HKB intact as much as possible whenever the theory (or program) component of the revising HKB is compatible with the initial HKB.

Definition 4 (Protected Revision). Let $K \in \mathcal{K}_{\mathcal{L}}$ and γ be a selection function for K . A protected revision operator $*_{\gamma, \subseteq_\circ}$ for K is defined such that for any $K' \in \mathcal{K}_{\mathcal{L}}$:

$$K *_{\gamma, \subseteq_\circ} K' = \begin{cases} K + K' & \text{if } K' \text{ is not satisfiable,} \\ \bigcap \gamma(\mathbb{K}_{K', \subseteq_\circ}) + K & \text{otherwise,} \end{cases}$$

where $\subseteq_\circ = \subseteq_T$ if $K + K'|_T$ is satisfiable, $\subseteq_\circ = \subseteq_P$ if $K + K'|_P$ is satisfiable, and $\subseteq_\circ = \subseteq$ otherwise.

In contrast to our partial meet revision operator, the protected revision operator $*_{\gamma, \subseteq_\circ}$ relies on selecting only among those compatible sets that have a maximal theory (or program) component, whenever $K + K'|_T$ ($K + K'|_P$, respectively) is satisfiable. The two revision operators coincide whenever neither $K + K'|_T$ nor $K + K'|_P$ is satisfiable. Obviously, the parameter $\subseteq_\circ$ could also be fixed to $\subseteq_T$ (or $\subseteq_P$) in settings where the theory (program, respectively) component is required to have absolute priority over the other. We illustrate the operation of protected revision in the following example.

Example 3. Consider again K from Example 1. Suppose the user Chris now states the command: “Call my wife Alex.” This input by the user contains new information, i.e., Alex is female and Chris is married to Alex, to be added to K . Formally, let $K' = (K'_T, K'_P) \in \mathcal{K}_{\mathcal{L}}$ with $K'_T = \emptyset$ and $K'_P = \{female(Alex), married(Chris, Alex)\}$. Since $K'_T = \emptyset$, $K + K'|_T$ is satisfiable and we obtain $\mathbb{K}_{K', \subseteq_T} = \{(K_T, \emptyset)\}$. Thus, $K *_{\gamma, \subseteq_T} K' = (K_T, K'_P)$. $\blacksquare$

The next proposition establishes that partial meet revision is a generalisation of protected revision.

Proposition 1. Let $K, K' \in \mathcal{K}_{\mathcal{L}}$. For any selection function γ for K , there exists a selection function γ' for K such that $K *_{\gamma, \subseteq_\circ} K' = K *_{\gamma'} K'$.

Proof. Let $K, K' \in \mathcal{K}_{\mathcal{L}}$ and γ be a selection function for K . In the limiting case that K' is not satisfiable, it holds by Definitions 3 and 4 that $K *_{\gamma, \subseteq_\circ} K' = K + K' = K *_{\gamma} K'$ for any selection function γ' . Otherwise, since $\mathbb{K}_{K', \subseteq_\circ} \subseteq \mathbb{K}_{K', \subseteq}$ for $\subseteq_\circ \in \{\subseteq, \subseteq_T, \subseteq_P\}$ by Definition 1, we can choose a selection function γ' for K such that $\gamma'(\mathbb{K}_{K', \subseteq}) = \gamma(\mathbb{K}_{K', \subseteq_\circ})$. From Definitions 3 and 4 it then follows that $K *_{\gamma, \subseteq_\circ} K' = K *_{\gamma'} K'$. $\square$

The other direction of the proposition does not hold because the use of the parameter $\subseteq_o$ in Definition 4 restricts the compatible sets to be included in $\mathbb{K}_{K', \subseteq_o}$. Due to this restriction, protected revision also does not satisfy (*4). If we consider the weaker postulate (*4c) below, then we can show that our protected revision operator $*_{\gamma, \subseteq_o}$ satisfies (*1)–(*3), (*4c), and (*5).

(*4c) If it holds for all $\kappa \subseteq K$ that $\kappa + K'|_T$ is satisfiable iff $\kappa + K''|_T$ is satisfiable and $\kappa + K'|_P$ is satisfiable iff $\kappa + K''|_P$ is satisfiable, then $K \cap (K * K') = K \cap (K * K'')$

Theorem 2. *Let $K \in \mathcal{K}_{\mathcal{L}}$ and γ be a selection function for K . If $*_{\gamma, \subseteq_o}$ is a protected revision operator for K , then $*_{\gamma, \subseteq_o}$ satisfies (*1)–(*3), (*4c), and (*5).*

Proof. (*1)–(*3) and (*5): Follow analogously to the proofs for Theorem 1.

(*4c): Assume that for all $\kappa \subseteq K$, it holds that $\kappa + K'|_T$ is satisfiable iff $\kappa + K''|_T$ is satisfiable and $\kappa + K'|_P$ is satisfiable iff $\kappa + K''|_P$ is satisfiable. We proceed by cases.

Case 1: $K + K'|_T$ is satisfiable. This implies $K + K''|_T$ is satisfiable by the assumption and thus $\mathbb{K}_{K'|_T, \subseteq_o} = K = \mathbb{K}_{K''|_T, \subseteq_o}$ by Definition 1. In particular, $\mathbb{K}_{K'|_T, \subseteq_T} = K = \mathbb{K}_{K''|_T, \subseteq_T}$ as Definition 4 requires $\subseteq_o = \subseteq_T$. Since $\kappa + K'|_P$ is satisfiable iff $\kappa + K''|_P$ is satisfiable for all $\kappa \subseteq K$ by the assumption, we obtain $\mathbb{K}_{K', \subseteq_T} = \mathbb{K}_{K'', \subseteq_T}$ by Definition 1. Then $\bigcap \gamma(\mathbb{K}_{K', \subseteq_T}) = \bigcap \gamma(\mathbb{K}_{K'', \subseteq_T})$ as well as $K \cap \bigcap \gamma(\mathbb{K}_{K', \subseteq_T}) = K \cap \bigcap \gamma(\mathbb{K}_{K'', \subseteq_T})$. By Lemma 1 we obtain $(K \cap \bigcap \gamma(\mathbb{K}_{K', \subseteq_T})) \cup (K \cap K') = (K \cap \bigcap \gamma(\mathbb{K}_{K'', \subseteq_T})) \cup (K \cap K'')$. This means $K \cap (\bigcap \gamma(\mathbb{K}_{K', \subseteq_T}) \cup K') = K \cap (\bigcap \gamma(\mathbb{K}_{K'', \subseteq_T}) \cup K'')$. Thus, $K \cap (K *_{\gamma, \subseteq_T} K') = K \cap (K *_{\gamma, \subseteq_T} K'')$.

Case 2: $K + K'|_P$ is satisfiable. Follows symmetrically to Case 1.

Case 3: Neither $K + K'|_T$ nor $K + K'|_P$ is satisfiable. This implies neither $K + K''|_T$ nor $K + K''|_P$ is satisfiable by the assumption as well as $\subseteq_o = \subseteq$ by Definition 4. Thus, $*_{\gamma, \subseteq_o} = *_{\gamma}$ and the proof reduces to the one for satisfaction of (*4) in Theorem 1. $\square$

At this point it becomes interesting to investigate how protected revision relates to a “stepwise” application of partial meet revision, that is, $K|_P *_{\gamma} (K|_T *_{\gamma} K')$ or $K|_T *_{\gamma} (K|_P *_{\gamma} K')$. It turns out that the result of protected revision $K *_{\gamma, \subseteq_T} K'$ is a subset of the result of stepwise revision $K|_P *_{\gamma} (K|_T *_{\gamma} K')$, provided that the selection function chooses similar sets in each respective operation, more specifically, the same theory-maximal compatible sets for protected revision as for the first step of stepwise revision and only those compatible sets for the second step of stepwise revision that include the program component of a selected compatible set for protected revision (analogously for $K|_T *_{\gamma} (K|_P *_{\gamma} K')$ and $K *_{\gamma, \subseteq_P} K'$). Before we show these results formally in Propositions 2 and 3, we introduce some useful notation. For any $K, K' \in \mathcal{K}_{\mathcal{L}}$, let

$$\begin{aligned} \mathbb{T}_{K'} &= \{ (\tau, \emptyset) \mid \tau \subseteq K_T, QHT((\tau, \emptyset)) \cap QHT(K') \neq \emptyset, \text{ and, for all } \tau', \\ &\quad \tau \subseteq \tau' \subseteq K_T \text{ implies } QHT((\tau', \emptyset)) \cap QHT(K') = \emptyset \}, \\ \mathbb{P}_{K'} &= \{ (\emptyset, \rho) \mid \rho \subseteq K_P, QHT((\emptyset, \rho)) \cap QHT(K') \neq \emptyset, \text{ and, for all } \rho', \end{aligned}$$

$$\rho \subset \rho' \subseteq K_P \text{ implies } QHT((\emptyset, \rho')) \cap QHT(K') = \emptyset \}.$$

Proposition 2. *Let $K, K' \in \mathcal{K}_{\mathcal{L}}$ and $K + K'|_T$ be satisfiable. There exists a selection function γ for K such that $K *_{\gamma, \subseteq T} K' \subseteq K|_P *_{\gamma} (K|_T *_{\gamma} K')$.*

Proof. Let $K, K' \in \mathcal{K}_{\mathcal{L}}$ and $K + K'|_T$ be satisfiable. Assume that γ is a selection function for K such that $\gamma(\mathbb{T}_{K'}) = \gamma(\mathbb{K}_{K', \subseteq T})|_T$ and $\gamma(\mathbb{P}_{K|_T *_{\gamma} K'}) = \{(\emptyset, \rho) \mid (\emptyset, \rho') \in \gamma(\mathbb{K}_{K', \subseteq T})|_P \text{ and } (\emptyset, \rho') \subseteq (\emptyset, \rho)\}$. If K' is not satisfiable, then it follows directly from Definitions 3 and 4 that $K *_{\gamma, \subseteq T} K' = K + K' = K|_P *_{\gamma} (K|_T *_{\gamma} K')$. Otherwise, it follows from Definition 1 and our definition of $\mathbb{T}_{K'}$ that $\mathbb{T}_{K'} = \mathbb{K}_{K', \subseteq T}|_T$. Due to the assumption, this implies $\bigcap \gamma(\mathbb{T}_{K'}) = \bigcap \gamma(\mathbb{K}_{K', \subseteq T})|_T$. For each $(\tau, \emptyset) \in \gamma(\mathbb{K}_{K', \subseteq T})|_T$, since $\bigcap \gamma(\mathbb{T}_{K'}) \subseteq (\tau, \emptyset)$ it holds that $QHT((\tau, \emptyset)) \cap QHT(K') \subseteq QHT(\bigcap \gamma(\mathbb{T}_{K'})) \cap QHT(K')$. Thus, for each $(\emptyset, \rho) \in \gamma(\mathbb{K}_{K', \subseteq T})|_P$ there exists a $(\emptyset, \rho') \in \mathbb{P}_{K|_T *_{\gamma} K'}$ such that $(\emptyset, \rho) \subseteq (\emptyset, \rho')$. From the assumption we then obtain $\bigcap \gamma(\mathbb{K}_{K', \subseteq T})|_P \subseteq \bigcap \gamma(\mathbb{P}_{K|_T *_{\gamma} K'})$, which implies $K *_{\gamma, \subseteq T} K' \subseteq K|_P *_{\gamma} (K|_T *_{\gamma} K')$ by Definitions 3 and 4. $\square$

Proposition 3. *Let $K, K' \in \mathcal{K}_{\mathcal{L}}$ and $K + K'|_P$ be satisfiable. There exists a selection function γ for K such that $K *_{\gamma, \subseteq P} K' \subseteq K|_T *_{\gamma} (K|_P *_{\gamma} K')$.*

Proof. Let $K, K' \in \mathcal{K}_{\mathcal{L}}$ and $K + K'|_P$ be satisfiable. Assume that γ is a selection function for K such that $\gamma(\mathbb{P}_{K'}) = \gamma(\mathbb{K}_{K', \subseteq P})|_P$ and $\gamma(\mathbb{T}_{K|_P *_{\gamma} K'}) = \{(\tau, \emptyset) \mid (\tau', \emptyset) \in \gamma(\mathbb{K}_{K', \subseteq P})|_T \text{ and } (\tau', \emptyset) \subseteq (\tau, \emptyset)\}$. The proof then follows symmetrically to the proof of Proposition 2. $\square$

For completeness, we also show here the relationship between stepwise revision and partial meet revision in Propositions 4 and 5, namely, that the result of stepwise revision is a subset of the result of partial meet revision, given that the selection function behaves similarly in each operation. Particularly, this is the case for $K|_P *_{\gamma} (K|_T *_{\gamma} K')$ whenever the theory part of K retained in the first step of stepwise revision is included in each selected compatible set for partial meet revision and the set of selected compatible sets for the second step of stepwise revision is the same as the set of program parts in the selected compatible sets for partial meet revision (analogously for $K|_T *_{\gamma} (K|_P *_{\gamma} K')$).

Proposition 4. *Let $K, K' \in \mathcal{K}_{\mathcal{L}}$. There exists a selection function γ for K such that $K|_P *_{\gamma} (K|_T *_{\gamma} K') \subseteq K *_{\gamma} K'$.*

Proof. Let $K, K' \in \mathcal{K}_{\mathcal{L}}$. Assume that γ is a selection function for K such that, for any $(\tau, \rho) \in \gamma(\mathbb{K}_{K', \subseteq})$, $\bigcap \gamma(\mathbb{T}_{K'}) \subseteq (\tau, \emptyset)$ and $\gamma(\mathbb{P}_{K|_T *_{\gamma} K'}) = \gamma(\mathbb{K}_{K', \subseteq})|_P$. If K' is not satisfiable, then it follows directly from Definition 3 that $K|_P *_{\gamma} (K|_T *_{\gamma} K') = K + K' = K *_{\gamma} K'$. Otherwise, for any $(\emptyset, \rho') \in \mathbb{P}_{K|_T *_{\gamma} K'}$, there exists a $(\tau'', \rho'') \in \mathbb{K}_{K', \subseteq}$ such that $(\emptyset, \rho') = (\emptyset, \rho'')$ by Definition 1 and our definition of $\mathbb{P}_{K'}$ and $(K|_T *_{\gamma} K') \setminus K' = \bigcap \gamma(\mathbb{T}_{K'}) \subseteq (\tau'', \emptyset)$ by Definitions 1 and 3. Due to the assumption, it then follows that $\bigcap \gamma(\mathbb{K}_{K', \subseteq})|_P = \bigcap \gamma(\mathbb{P}_{K|_T *_{\gamma} K'})$ and $\bigcap \gamma(\mathbb{T}_{K'}) \subseteq \bigcap \gamma(\mathbb{K}_{K', \subseteq})|_T$. This implies $K|_P *_{\gamma} (K|_T *_{\gamma} K') \subseteq K *_{\gamma} K'$ by Definition 3. $\square$

Proposition 5. *Let $K, K' \in \mathcal{K}_{\mathcal{L}}$. There exists a selection function γ for K such that $K|_T *_{\gamma} (K|_P *_{\gamma} K') \subseteq K *_{\gamma} K'$.*

Proof. Let $K, K' \in \mathcal{K}_{\mathcal{L}}$. Assume that γ is a selection function for K such that, for any $(\tau, \rho) \in \gamma(\mathbb{K}_{K', \subseteq})$, $\bigcap \gamma(\mathbb{P}_{K'}) \subseteq (\emptyset, \rho)$ and $\gamma(\mathbb{T}_{K|P *_{\gamma} K'}) = \gamma(\mathbb{K}_{K', \subseteq})|_T$. The proof then follows symmetrically to the proof of Proposition 4. $\square$

3.3 Prioritised Revision

While protected revision allows us to emphasise entire components of a HKB during a revision, its flexibility is somewhat limited. Specifically, in Example 3 above we saw that the theory component of K was fully preserved due to the structure of the new information K' . Intuitively, the revision outcome states that the unisex name Chris was incorrectly identified or entered as female. Yet, what if Chris is actually female and she married Kate after the law changed to allow same-sex marriage? In that case, we need to have a more fine-grained method than protected revision to allow the theory component to be changed. We therefore define *prioritised revision* now, based on an idea by Nebel [17].

Definition 5 (Priority Level). *Let $K \in \mathcal{K}_{\mathcal{L}}$ and $\leq$ be a total preorder on $K_T \cup K_P$. A priority level of K is*

$$\lambda = \{(\tau, \rho) \subseteq K \mid e_1 \leq e_2 \text{ and } e_2 \leq e_1 \text{ for all } e_1, e_2 \in \tau \cup \rho\}.$$

We call $\leq$ defined as above a *prioritisation* of K . Furthermore, for all $e_1 \in \lambda_1$ and $e_2 \in \lambda_2$, we write $\lambda_1 \leq \lambda_2$ iff $e_1 \leq e_2$, and $\lambda_1 < \lambda_2$ iff $e_1 \leq e_2$ and $e_2 \not\leq e_1$. A prioritisation partitions K into a hierarchical sequence of levels and places each formula or rule of K in a particular level. We modify our definition of compatible sets to include priority levels.

Definition 6 (Prioritised Compatible Set). *Let $K, K' \in \mathcal{K}_{\mathcal{L}}$ and $\leq$ be a prioritisation of K . The set of prioritised compatible sets of K regarding K' is*

$$\mathbb{K}_{K', \leq} = \{(\tau, \rho) \mid (\tau, \rho) = \bigcup_{\lambda \subseteq K} (\tau_{\lambda}, \rho_{\lambda})\},$$

such that for all $\lambda \subseteq K$:

- a) $(\tau_{\lambda}, \rho_{\lambda}) \subseteq \lambda$,
- b) $QHT(\bigcup_{\lambda < \lambda'} (\tau_{\lambda'}, \rho_{\lambda'})) \cap QHT(K') \neq \emptyset$, and
- c) for all $(\tau'_{\lambda}, \rho'_{\lambda}) \subseteq \lambda$, $(\tau_{\lambda}, \rho_{\lambda}) \subset (\tau'_{\lambda}, \rho'_{\lambda})$ implies $QHT(\bigcup_{\lambda < \lambda'} (\tau_{\lambda'}, \rho_{\lambda'})) \cap QHT((\tau'_{\lambda}, \rho'_{\lambda})) \cap QHT(K') = \emptyset$.

Each prioritised compatible set is composed of a maximal subset of the highest level that is compatible with K' , a maximal subset of the next highest level that is compatible with K' and the subsets formed previously in higher levels, and so on. With these concepts established, we can now define a prioritised revision operator as follows.

Definition 7 (Prioritised Revision). Let $K \in \mathcal{K}_{\mathcal{L}}$ and $\leq$ be a prioritisation of K . A prioritised revision operator $*_{\leq}$ for K is defined such that for any $K' \in \mathcal{K}_{\mathcal{L}}$:

$$K *_{\leq} K' = \begin{cases} K + K' & \text{if } K' \text{ is not satisfiable,} \\ \bigcap \mathbb{K}_{K', \leq} + K' & \text{otherwise.} \end{cases}$$

Whenever K' is satisfiable, our prioritised revision operator $*_{\leq}$ returns those elements of K that are common to all prioritised compatible sets together with K' . The following example illustrates the operation of prioritised revision and shows that it can indeed return the desired result with respect to our running example, that is, appropriately represent a change in marriage legislation.

Example 4. Consider again K from Example 1 and K' from Example 3. Let $\leq$ be a prioritisation of K such that assertion of the user's gender is given higher priority than prohibition of same-sex marriage. Formally, let $\lambda_1 = (\{\forall x, y : \text{married}(x, y) \wedge \text{female}(x) \rightarrow \text{male}(y)\}, \emptyset)$, $\lambda_2 = (\emptyset, \{\text{female}(\text{Chris})\})$, $\lambda_3 = K \setminus (\lambda_1 \cup \lambda_2)$, and $\lambda_1 < \lambda_2 < \lambda_3$. It then follows that $\mathbb{K}_{K', \leq} = \{\lambda_2 \cup \lambda_3\}$ and thus $K *_{\leq} K' = \lambda_2 \cup \lambda_3 \cup K'$. $\blacksquare$

Comparing our prioritised revision operator to our partial meet revision operator, we find that they coincide, as stated in Propositions 6 and 7.

Proposition 6. Let $K, K' \in \mathcal{K}_{\mathcal{L}}$. For any selection function γ for K , there exists a prioritisation $\leq$ of K such that $K *_{\gamma} K' = K *_{\leq} K'$.

Proof. Let $K, K' \in \mathcal{K}_{\mathcal{L}}$ and γ be a selection function for K that determines the outcome of $K *_{\gamma} K'$. By $S = \bigcap \gamma(\mathbb{K}_{K', \subseteq})$ we denote the part of K that is preserved in the partial meet revision and by $S' = K \setminus S$ the part of K that is discarded. We can then create a prioritisation $\leq$ of K as follows. In the limiting case that $S = \emptyset$, let $\leq$ consist of one level $\lambda_1 = S'$ only. From Definitions 1 and 6 it then follows that $\mathbb{K}_{K', \subseteq} = \mathbb{K}_{K', \leq}$. Since $\gamma(\mathbb{K}_{K', \subseteq}) \subseteq \mathbb{K}_{K', \subseteq}$ by Definition 2 and $\bigcap \gamma(\mathbb{K}_{K', \subseteq}) = \emptyset$ by the assumption, we obtain $\bigcap \mathbb{K}_{K', \leq} = \emptyset$ and consequently $K *_{\gamma} K' = K *_{\leq} K'$. Otherwise, let $\leq$ consist of exactly two levels λ_1 and λ_2 such that $\lambda_1 = S'$, $\lambda_2 = S$, and $\lambda_1 < \lambda_2$. We now show that $\bigcap \mathbb{K}_{K', \leq} = S$. For any $\kappa \in \mathbb{K}_{K', \leq}$, it follows from Definition 6 and our choice of $\leq$ that $S \subseteq \kappa$, which implies $S \subseteq \bigcap \mathbb{K}_{K', \leq}$ by Definition 7. Assume that there exists a $\kappa' \subseteq S'$ such that $\kappa' \subseteq \bigcap \mathbb{K}_{K', \leq}$. Then for each compatible set $\kappa'' \in \mathbb{K}_{K', \subseteq}$ with $S \subseteq \kappa''$ it would hold that $\kappa' \subseteq \kappa''$ because κ'' is maximal by Definition 1. Yet, this would imply $\kappa' \subseteq \bigcap \gamma(\mathbb{K}_{K', \subseteq})$ and thus $\kappa' \subseteq S$, a contradiction. $\square$

Proposition 7. Let $K, K' \in \mathcal{K}_{\mathcal{L}}$. For any prioritisation $\leq$ of K , there exists a selection function γ for K such that $K *_{\leq} K' = K *_{\gamma} K'$.

Proof. Let $K, K' \in \mathcal{K}_{\mathcal{L}}$ and $\leq$ be a prioritisation of K that determines the outcome of $K *_{\gamma} K'$. In the trivial case that K' is not satisfiable, we have $K *_{\leq} K' = K + K' = K *_{\gamma} K'$. Let K' be satisfiable in the following. We first show

that for any prioritised compatible set $\kappa_1 \in \mathbb{K}_{K', \leq}$, there exists a compatible set $\kappa_2 \in \mathbb{K}_{K', \subseteq}$ such that $\kappa_1 = \kappa_2$. It follows directly from Definitions 1 and 6 that $\kappa_1 \subseteq \kappa_2$. Assume that $\kappa_1 \subset \kappa_2$. From Definition 6 it would then follow that $QHT(\kappa_1) \cap QHT(\kappa_2 \setminus \kappa_1) \cap QHT(K') = \emptyset$. Yet, this is a contradiction since $QHT(\kappa_2) \cap QHT(K') \neq \emptyset$ by Definition 1 and κ_1 is required to be maximal by Definition 6. Thus, for any $\kappa_1 \in \mathbb{K}_{K', \leq}$ it holds that $\kappa_1 \in \mathbb{K}_{K', \subseteq}$, that is, $\mathbb{K}_{K', \leq} \subseteq \mathbb{K}_{K', \subseteq}$. We can now set γ as a selection function for K such that $\gamma(\mathbb{K}_{K', \subseteq}) = \mathbb{K}_{K', \leq}$ and consequently obtain $K *_{\leq} K' = K *_{\gamma} K'$. $\square$

Since prioritised revision and partial meet revision coincide, it also holds that prioritised revision generalises protected revision and further that prioritised revision is exactly characterised by the postulates (*1)–(*5).

Corollary 1. *Let $K, K' \in \mathcal{K}_{\mathcal{L}}$. For any selection function γ for K , there exists a prioritisation $\leq$ of K such that $K *_{\gamma, \subseteq} K' = K *_{\leq} K'$.*

Proof. Follows directly from Propositions 1 and 6. $\square$

Theorem 3. *Let $K \in \mathcal{K}_{\mathcal{L}}$. An operator $*_{\leq}$ is a prioritised revision operator for K determined by a prioritisation $\leq$ of K iff $*_{\leq}$ satisfies (*1)–(*5).*

Proof. Follows directly from Theorem 1 and Propositions 6 and 7. $\square$

4 Related Work

Closest related work concerns itself with belief update, a relative of belief revision, in hybrid MKNF knowledge bases [21], which combine first-order theories and logic programs in a different semantic composition to the one considered here. That approach takes the combination of an initial HKB and an updating HKB and splits it into hierarchical levels, similar to the process of logic program stratification, such that updating information is given priority. It then applies given ontology update and rule update semantics to the respective components to evaluate the models of the updated HKB. In contrast to our prioritised revision, the update method does not allow arbitrary combinations of formulas and rules on the same level. In addition, since the update operation preserves all existing information and relies on the chosen semantics to determine a satisfiable outcome, the initial HKB grows with each operation and may contain duplicate elements to express information that was first asserted, then contradicted, and later asserted again.

Multi-context systems allow reasoning over two or more knowledge bases, specified in some monotonic or nonmonotonic logic, by connecting them via nonmonotonic bridge rules. An adaptation of Dalal's [5] and Satoh's [20] classic belief revision operators to multi-context systems [22] computes the outcome of revising one multi-context system by another as the set of those models of the revising multi-context system that are closest to the ones of the initial multi-context system, where closeness is determined by Dalal's or Satoh's notion of

distance. The adaptation to multi-context systems exhibits the same properties as the classic construction with respect to the belief revision postulates.

To assist with the task of rule authoring in HKBs, inductive logic programming can be used to learn new rules from the existing rules and formulas in a HKB [15]. As any induced rule is consistent with existing knowledge and no existing knowledge is removed, this process can be compared to screened revision [16] where the entire existing HKB is protected.

5 Conclusion

In this study, we proposed three operators to revise hybrid knowledge bases, namely, a general partial meet revision operator that does not assume any ordering of the elements of a hybrid knowledge base, a protected revision operator that emphasises the theory or program component of a hybrid knowledge base depending on the structure of information in the revising hybrid knowledge base, and lastly a prioritised revision operator that computes the outcome based on a pre-defined priority ordering among the elements in a hybrid knowledge base. We demonstrated the interdefinability between our operators, provided representation theorems for partial meet revision and prioritised revision with respect to the belief revision postulates, and showed that partial meet revision as well as prioritised revision are generalisations of protected revision.

It is straightforward to define corresponding partial meet, protected, and prioritised *contraction* operators, by using the complement of $QHT(K')$ in the definitions of compatible sets. Properties and representation theorems analogous to revision can then be obtained, after translating the classic contraction postulates in a similar manner as the ones for revision.

While our operators return a revised hybrid knowledge base, they do not specify how a selection function or prioritisation changes during the operation. In future work, we aim to investigate such an extension to our approach.

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