Nonlinearity, fuzziness and incommensurability in indicator-based assessments of vulnerability to climate change: a new mathematical framework

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Abstract

The earth's climate system is highly nonlinear and the vulnerability of a community to a climate hazard is no exception. While this fact is widely accepted, indicator-based vulnerability assessments (IBVA) hardly ever take such nonlinearities into account. This is mainly due to the fact that the majority of assessment studies use methods based on Multiple Attribute Utility Theory (MAUT) (e.g. simple additive weight or multiplicative aggregation) to aggregate indicators. These methods convert all indicators into a global utility function and produce only a linear, threshold-free scaling of the effects of an indicator on vulnerability. In a previous paper, we showed that outranking procedures developed in decision-making science offer a more theoretically-sound approach to aggregation because they allow the analyst to incorporate the incommensurability, fuzziness and uncertainty associated with indicators. In this paper, we develop a new mathematical framework for vulnerability in order to clearly identify various sources of nonlinearity and incommensurability in vulnerability assessments. We then propose a new outranking formulation which can accommodate both and can be used to conduct assessments at different scales. We do so by introducing the concept of *harm criterion* as a mediator between an indicator and the vulnerability it represents. The new assessment approach can aggregate a mix of indicators with various degrees of subjectivity and non-linearity, without converting them into a single utility function and without requiring them to be mutually compensating.

We illustrate the proposed approach by applying it to a simplified model of urban vulnerability to heat, focusing on the non-linear relationship between mortality and temperature above a 'comfort temperature', long evidenced in the epidemiological literature. We compare vulnerability rankings yielded by linear and non-linear characterizations of the relationship between temperature and mortality and find that the incorporation of non-linearity can have a significant impact on the rankings.

Key Words: vulnerability assessment, climate change, aggregation, nonlinear, outranking, mathematical framework

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1.0 Introduction

As global average temperatures increase and changes to the hydrological cycle become more evident, anthropogenic climate change presents significant threats to cities, economic sectors and infrastructure systems (IPCC, 2014). Climate risk assessment and climate change adaptation are becoming a central concern for policymakers, planners and engineers. Risk and vulnerability assessments aim at identifying

hotspots, understanding processes generating vulnerability and helping policymakers to prioritize, allocate resources and develop better adaptation planning.

Vulnerability assessment is a complex form of risk appraisal, which considers both bio-physical and socio-economic components of the environmental hazard. The most commonly used framework of vulnerability is the one proposed by the Intergovernmental Panel on Climate Change (IPCC) in its Third Assessment Report which recognises three dimensions of vulnerability, namely exposure, sensitivity and adaptive capacity, with the first generally reflecting, as the name indicates, the extent to which a socio-ecological system (SES) (e.g., locality, region, community, infrastructure system) is exposed to the hazard in question, the second its propensity to be damaged by that exposure and the third its ability to cope with, and recover from, that damage. In other words, while exposure focuses on the degree of contact between hazard and SES, sensitivity and adaptive capacity are concerned with the complex ramifications of the hazard in human societies (Eriksen and Kelly, 2007, Parry, 2007). However, this framework has come under criticism especially for the ambiguity of its concepts and its lack of specificity concerning the relationship between them (e.g., Ionescu et al., 2009) .

The literature on climate-change vulnerability assessment falls broadly into three categories. A number of papers over the last ten years have engaged with theoretical and semantic aspects of vulnerability in order to negotiate a multiplicity of definitions and some confusion surrounding the concept (e.g., Adger, 2006; Adger and Kelly, 1999; Cutter et al., 2003). This has led to a level of agreement about the need for precision in defining processes generating vulnerability, the importance of scale and the place-specific nature of assessments. For example, Füssel and Klein (2006), showed that vulnerability assessments have evolved over the years from an impact assessment focusing only on exposure and sensitivity of a system to a more complex form of evaluation which account for important place specific non-climatic factors and acknowledge the potential for adaptation measures at appropriate scale to reduce potential climate impacts.

A second, albeit small, set of studies proposed specific methodologies (as opposed to conceptual frameworks) to guide practitioners in conducting assessments (e.g., Füssel, 2007; Füssel and Klein, 2006; Luers et al., 2003; Tonmoy et al., 2014; El-Zein and Tonmoy, 2015) , while a third, and by far the largest, reports actual assessment studies (Hahn et al., 2009, Duriyapong and Nakhapakorn, 2011, Brenkert and Malone, 2005, Preston et al., 2008). To our knowledge, no paper, including methodological ones, has tackled specifically the various non-linearities present in relationships describing vulnerability, nor has there been a formal attempt at incorporating them in assessment studies.

Broadly, two approaches have been used in climate change quantitative impact studies in the literature (Tonmoy et al., 2014). Scenario-based analyses downscale predictions of Global Circulation Models (GCM) then combine them with mechanistic biophysical or biochemical models (e.g., hydrological, epidemiological, atmospheric) in order to derive GCM's implications at regional and local scales. The advantage of this approach is that it is usually based on robust climate science and sound understanding of the dynamics of the system in question and can represent threshold effects and nonlinearities. However, restrictions on the spatial resolution of GCMs and the complexity of incorporating the social, economic and institutional components of risk, limit the scope of this approach.

In the second approach, indicators offer an attractive and relatively simple way of quantifying different components of the risk, biophysical, institutional and socio-economic (Füssel, 2007, Hinkel, 2011). The challenge of indicator-based vulnerability assessments (IBVA) lies in identifying and selecting measurable indicators that can represent all significant processes generating vulnerability and then combining these indicators, using sound aggregation principles, in order to produce a proxy measure of vulnerability. While indicators can usually be identified with relative ease, the exact relationship they hold to vulnerability is either difficult or impossible to determine with precision. This relationship usually turns out to be more complex than the linear association that is assumed in most analyses. One partial way out of this impasse is to combine impacts studies simulating the biophysical components of the hazard with indicators representing its socio-economic and institutional components. However, another difficulty facing IBVA lies in developing aggregation principles that can take into account the different types of indicators (continuous, discrete and ordinal variables); different types of relationships between indicators and vulnerability (linear and non-linear, deterministic and stochastic, dichotomous and fuzzy); as well as different possible relationships of compensation and non-compensation *between* the indicators (El-Zein and Tonmoy, 2015). To date, the vast majority of the IBVA literature has used simple aggregation approaches whose validity is in serious doubt (Tonmoy et al., 2014, Tonmoy and El-Zein, 2013a, Tonmoy and El-Zein, 2013b). For example, most IBVA studies use Multiple Attribute Utility Theory (MAUT) based methods such as simple additive weight or multiplicative weight for aggregation of indicators which assume a linear relationship between indicators and vulnerability and complete compensation between indicators based on allocated weights.

This paper has two objectives:

- a) to present a new mathematical framework for vulnerability which allows us to clearly define different forms of nonlinearity and incommensurability in vulnerability assessments;
- b) to propose a new indicator-based approach which can accommodate non-linear relationships between an indicator and the vulnerability it represents, as well as different degrees of compensation between indicators (from total compensation to complete incommensurability).

We build on our previous work on vulnerability assessments in which we showed that outranking procedures first developed in decision-making science by Roy (1968) offer a more theoretically sound approach to aggregation than MAUT-based ones because they allow the analyst to incorporate incommensurability, uncertainty and multiple subjectivities in vulnerability models (El-Zein and Tonmoy, 2015). In this paper, we are not particularly concerned with semantic aspects of vulnerability, though we acknowledge their importance. Instead, we start from a definition of vulnerability (generally accepted in the literature and presented at the beginning of the next section) and abide by it throughout. First, we present the new vulnerability framework and definitions attached to it, and elicit the different forms of nonlinearity present in relationships of vulnerability. Second, based on this framework, we formulate a new outranking approach to aggregation which incorporates nonlinearity. Finally, we illustrate the new approach by applying it to a simplified model of vulnerability to heat stress and show that the incorporation of nonlinearity and partial compensation can have a significant impact on the ranking of vulnerabilities.

The goal of the paper is to propose a method that can be used by practitioners in assessment studies. Therefore, although we are proposing a mathematical formalism for the sake of precision and conceptual clarity, we will mostly use vulnerability terminology that is familiar to researchers in this field.

2.0 Vulnerability to Climate Change: from Conceptual Framework to Assessment

2.1 Vulnerability Framework and Definitions

The aim of vulnerability assessment is to develop some measure, quantitative or qualitative, of the susceptibility to damage of, or damage likely to be inflicted on, the valued attribute of an SES, as a result of its exposure to one or more climate stresses. For the purpose of the discussion below, at any point in time, we denote damage by $D(t)$, vulnerability by $V(t)$ and the magnitude of the climate stress in question by $M(t)$, where t is time. It is reasonable to assume that, as the magnitude M of the climate stress increases, so does the damage D . In the remainder of the paper, we refer to $D(t)$, $V(t)$ and $M(t)$ for simplicity, as D , V and M , respectively, with the understanding that they are functions of time. We define vulnerability as the ratio of damage to magnitude:

$$D = VM \quad (1)$$

where D , V and M are positive numbers (for clarity, we represent D and M in italics and the slope connecting them, i.e. vulnerability, in bold-faced font, throughout). In some cases, V is largely independent of M and (1) simply reflects a linear relationship between D and M . For example, within a given range, the extent of physical damage inflicted on houses in a “do nothing” scenario may be roughly proportional to the level of sea rise causing it, i.e. V does not depend on M . In reality, such relationships are seldom linear. More often than not, D is a non-linear function of M . Rivers bursting their banks and sea waves breaching beach fortifications are examples in which a threshold effect generates a non-linear relationship between D and M . It is possible to represent such nonlinearity by introducing a dependence of V on M :

$$D = V(M)M \quad (2)$$

Hence, we can now speak about assessing vulnerability to a given *magnitude* of stress, i.e. developing some measure of $V(M)$ at a given M . We call such non-linearity (i.e. dependence of V on M) the *fundamental non-linearity* of the conceptual framework to distinguish it from other forms of non-linearities that we will be introducing below. Provided D is differentiable over M , we can generalise from equation (2) and define vulnerability as the marginal rate of damage relative to the magnitude of the stress:

$$V(M) = \frac{\partial D}{\partial M} \quad (2)$$

Using the IPCC (2007) well-known definition of vulnerability, it is also possible to write $V(M)$ as:

$$V(M) = f[E(M), S(M), A_c(M)] \quad (4)$$

where $E(M)$ is the degree of exposure of the valued attribute of the SES to the stress in question, $S(M)$ is its sensitivity to the stress and $A_c(M)$ is the adaptive capacity of the SES i.e. (its ability to reduce its exposure and/or sensitivity, or to recover from damage, through adaptation event(s)). Hence, according to equation (4), vulnerability is a (as yet unspecified) function of exposure, sensitivity and adaptive capacity. One of the criticisms levelled at the IPCC definition in the literature is that little is proffered about the nature of function f (Hinkel, 2011). One of the simplest possible incarnations of equation (4) is given by:

$$V(M) = \frac{E(M)S(M)}{A_c(M)} \quad (5)$$

The advantage of the multiplicative representation of vulnerability used in equation (5) is that it emphasises the compound effect likely to be generated by multiple sources of vulnerability. However, the equation suffers from an implicit assumption that a deficit in adaptive capacity can be fully compensated for by a decrease in exposure or sensitivity, and that an excess of exposure can be fully compensated for by an increase in adaptive capacity or a reduction in sensitivity, and so on. In reality, there may be conditions under which this is not possible. For example, it is widely recognized that beyond a threshold of sea level rise, the adaptive capacity of small-island states will be overcome and no increase in that capacity can protect the island residents and their built environments from devastating damage. This is the well-known problem of *non-compensation* or *incommensurability*. We call it here the *dimensional incommensurability* of the vulnerability framework (with reference to the 3 dimensions of vulnerability), to distinguish it from another case of incommensurability that we will introduce later. It is possible to cater for the problem of dimensional incommensurability through alternative definitions of vulnerability. For example:

$$V(M) = \text{Max}[E(M), S(M), A_c(M)^{-1}] \quad \text{where } E \neq 0 \quad (6)$$

or

$$V(M) = \text{Min}[E(M), S(M), A_c(M)^{-1}] \quad (7)$$

Under both equations (6) and (7), no compensation between the three dimensions of exposure, sensitivity and adaptive capacity is possible. Instead, in the case of equation (6), an SES is as vulnerable as its weakest dimension. Under equation (7), an SES is as resilient as its strongest dimension. We call the concepts represented by equations (6) and (7), “non-compensating strong vulnerability” and “non-compensating weak vulnerability”, respectively.

Yet another alternative would be to assess exposure, sensitivity and adaptive capacity, separately, based on which a qualitative evaluation of vulnerability is made *without* combining the three dimensions quantitatively. This would allow the analyst to make a judgment that lies somewhere between the two extremes of equations (6) and (7). In this case, vulnerability can be represented as a tensor in an $(\vec{e}, \vec{s}, \vec{a})$ space representing the three dimensions of exposure, sensitivity and adaptive capacity:

$$\vec{V}(M) = E(M)\vec{e} + S(M)\vec{s} - A_c(M)\vec{a} \quad (8)$$

We call this representation “general multi-dimensional vulnerability”. In reality, compensation is often possible *to some extent*; and it is in fact the possibility of (even partial) compensation that usually makes adaptation effective. The question therefore arises as to whether it is possible, in conducting vulnerability assessments, to incorporate incommensurability as well as limited and full compensations, as the case may be. The answer is positive as we will show later.

It is now possible to incorporate the effect of adaptation events, as opposed to adaptive capacity, in this framework. Adaptation can be conceived of as an event, or series of events, deliberate or non-deliberate, reactive or proactive, that can lead to a reduction in vulnerability. Note here that *adaptive capacity* is related to, but distinct from, *adaptation*: the former makes it more likely for the latter to take place at some point in the future. It is possible to represent the effect of adaptation as a modification of equation (2):

$$\bar{D} = [V(M) - A(M)]M \quad (9)$$

where $\bar{D}$ denotes damage with adaptation events taken into account, $A(M)$ represents the reduction in vulnerability brought about by adaptation and $V(M)$ is the vulnerability prior to this adaptation. Note that, consistent with our previous assumption, A depends on M . This is to say that the same set of adaptation events might lead to different degrees of vulnerability reduction, depending on the magnitude of the stress. While this might not always be the case, it is certainly a possibility that needs to be kept in mind. There can be of course other ways of incorporating the effects of adaptation events in equation (2), such as defining adaptation as a reduction in damage rather than vulnerability:

$$\bar{D} = V(M)M - A(M) \quad (10)$$

or representing the effect of adaptation by $a(M)$, a percentage reduction in both vulnerability and damage:

$$\bar{D} = \bar{V}(M)M \quad (11)$$

$$\bar{V}(M) = a(M)V(M) \quad (12)$$

where $\bar{V}(M)$ is vulnerability after adaptation events, $a(M)$ is an adaptation multiplier, that is a real number varying between 1 (no adaptation or unsuccessful adaptation) and 0 (perfect adaptation which eliminates all likelihood of damage). Formulations (9), (10) and (11)-(12) are conceptually similar, if not equivalent and, in this paper, we use equations (11)-(12). Going a step further, we assume that the effect of successful adaptation events can take the form of a reduction in exposure, a reduction in sensitivity and/or an increase in adaptive capacity. For example, the vulnerability of an elderly person to a heat-wave can be reduced by installing an air conditioning system in her flat (reduced exposure to heat), treating with drugs her existing cardiovascular condition (reduced sensitivity to heat) or setting up a heat warning system which offers her transport to a community shelter during a heat wave (increased adaptive capacity). Note here that, under this conceptual framework:

- a) the above three examples are adaptation events acting on different dimensions of vulnerability;
- b) the last example is an adaptation event leading to improved adaptive capacity of the subject;

- c) the resulting increase in adaptive capacity in this example, when activated during a heat wave, would reduce vulnerability by reducing exposure to heat (i.e., when the elderly person leaves her apartment and moves to a cool shelter).

Therefore, a straightforward approach for incorporating adaptation in the vulnerability framework, would be to apply an adaptation-event multiplier to each dimension of vulnerability:

$$\bar{E}(\mathbf{M}) = \mathbf{e}(\mathbf{M})\mathbf{E}(\mathbf{M}) \quad (13)$$

$$\bar{S}(\mathbf{M}) = \mathbf{s}(\mathbf{M})\mathbf{S}(\mathbf{M}) \quad (14)$$

$$\bar{A}_c(\mathbf{M}) = \mathbf{a}_c(\mathbf{M})\mathbf{A}_c(\mathbf{M}) \quad (15)$$

where $\bar{E}(\mathbf{M})$, $\bar{S}(\mathbf{M})$ and $\bar{A}_c(\mathbf{M})$ indicate exposure, sensitivity and adaptive capacity *after* a set of adaptation events (while the corresponding items without bars denote the entity *before* adaptation events); $\mathbf{e}(\mathbf{M})$ and $\mathbf{s}(\mathbf{M})$ are the percentage reductions in exposure and sensitivity as a result of adaptation and $\mathbf{a}_c(\mathbf{M})$ is the percentage increase in adaptive capacity as a result of adaptation. $\mathbf{e}(\mathbf{M})$ and $\mathbf{s}(\mathbf{M})$ vary between 0 (complete removal of vulnerability) and 1 (no effect on vulnerability). $\mathbf{a}_c(\mathbf{M}) \geq 1$ with $\mathbf{a}_c(\mathbf{M})=1$ indicating no effect on vulnerability. Equations (13) to (15) can also represent mal-adaptation, in the sense of events leading to an increase rather than a decrease in vulnerability. In such cases, $\mathbf{e}(\mathbf{M}) > 1$, $\mathbf{s}(\mathbf{M}) > 1$ and/or $\mathbf{a}_c(\mathbf{M}) < 1$. It is also conceivable that the same adaptation event might have conflicting effects on vulnerability (e.g., reduction of exposure and reduction in adaptive capacity).

Now, modifying for the effect of adaptation, equations (5) to (8) become, respectively:

$$\bar{V}(\mathbf{M}) = \frac{[\mathbf{e}(\mathbf{M})\mathbf{E}(\mathbf{M})] \cdot [\mathbf{s}(\mathbf{M})\mathbf{S}(\mathbf{M})]}{\mathbf{a}_c(\mathbf{M})\mathbf{A}_c(\mathbf{M})} \quad (16)$$

$$\bar{V}(\mathbf{M}) = \text{Max}\{\mathbf{e}(\mathbf{M})\mathbf{E}(\mathbf{M}), \mathbf{s}(\mathbf{M})\mathbf{S}(\mathbf{M}), [\mathbf{a}_c(\mathbf{M})\mathbf{A}_c(\mathbf{M})]^{-1}\} \quad (17)$$

$$\bar{V}(\mathbf{M}) = \text{Min}\{\mathbf{e}(\mathbf{M})\mathbf{E}(\mathbf{M}), \mathbf{s}(\mathbf{M})\mathbf{S}(\mathbf{M}), [\mathbf{a}_c(\mathbf{M})\mathbf{A}_c(\mathbf{M})]^{-1}\} \quad (18)$$

$$\vec{\bar{V}}(\mathbf{M}) = \mathbf{e}(\mathbf{M})\mathbf{E}(\mathbf{M})\vec{\mathbf{e}} + \mathbf{s}(\mathbf{M})\mathbf{S}(\mathbf{M})\vec{\mathbf{s}} - \mathbf{a}_c(\mathbf{M})\mathbf{A}_c(\mathbf{M})\vec{\mathbf{a}} \quad (19)$$

where $\bar{V}(\mathbf{M})$ is vulnerability *after* a given adaptation event or series of events. Comparing equations (5), (12) and (16), it is clear that:

$$\mathbf{a}(\mathbf{M}) = \frac{\mathbf{e}(\mathbf{M})\mathbf{s}(\mathbf{M})}{\mathbf{a}_c(\mathbf{M})} \quad (20)$$

Equation (11) together with one of equations (16), (17), (18), (19) and (20) are therefore a mathematical representation of vulnerability that takes into account the three dimensions of exposure, sensitivity and adaptive capacity, as well as the impact of adaptation events.

Next, we discuss how to make these equations operational, i.e. how to use them in order to generate a ranking of vulnerabilities.

3.0 Vulnerability Assessment: Nonlinearity, Fuzziness and the Harm Concept

Let $S_k = [S_1 \dots S_n]$ be a set of n comparable SESs which need to be ranked according to the vulnerability of a valued attribute (e.g., health, economic well-being, productivity etc.) to one or more climate hazards (e.g., increase in average temperatures, rise in sea level, increased frequency of flooding etc.), under a given dimension d of vulnerability. Indicator-based vulnerability assessments (IBVA) express the 3 dimensions of vulnerability as functions of measurable indicators. Hence, equation (8) applied to a specific socio-economic system (SES) k becomes:

$$\vec{V}_k(M) = f_{ek}(I_{e1k}, I_{e2k}, I_{e3k}, \dots) \vec{e} + f_{sk}(I_{s1k}, I_{s2k}, I_{s3k}, \dots) \vec{s} + f_{ck}(I_{c1k}, I_{c2k}, I_{c3k}, \dots) \vec{a} \quad (21)$$

where f_{dk} ($d=e, s$ or c) is a function expressing a given dimension of vulnerability (e : exposure; s : sensitivity; c : adaptive capacity) of SES k ($k=1, n$) in terms of a set of m_e indicators I_{dik} ($i=1, m_e$). In keeping with the comment made earlier about the *fundamental non-linearity*, f_{dk} may depend on the magnitude of the stress M . The choice of indicators is of course critical and can be challenging. Ideally, each indicator is chosen so as to represent a process generating vulnerability, based on intuitive or deductive reasoning. Typically, a mixed bag of indicators is used, one in which our degree of knowledge of the relationship between an indicator and the vulnerability it is representing, is *highly variable*. In fact, what we usually know about the relationship between indicators and vulnerability, as a minimum, can be summed up by:

$$f_{dk} \propto I_{d1k}, I_{d2k}, I_{d3k}, \dots \quad (22)$$

However, this doesn't take us very far. It simply reiterates the reason for which these indicators have been selected in the first place. A simple, if dangerous, way out of this impasse is to make the following assumption:

$$f_{dk} = \sum_{j=1}^{m_d} w_{dj} \bar{I}_{djk} \quad (23)$$

the bar on variable I_{djk} denotes normalised indicators; and w_{dj} is a weight for the j^{th} indicator. Normalisation can be conducted in a number of different ways; however, most commonly:

$$\bar{I}_{djk} = \frac{I_{djk} - I_{dj}^{\min}}{I_{dj}^{\max} - I_{dj}^{\min}} \quad (24)$$

where

$$I_{dj}^{\min} = \min_k I_{djk} \quad (25)$$

$$I_{dj}^{\max} = \max_k I_{djk} \quad (26)$$

This particular normalisation leads to a new variable which ranges between 0 and 1. Equation (23) essentially generates a function based on multi-attribute utility theory (MAUT) as a form of aggregation

of the indicators. This presents us immediately with two problems related to two assumptions underlying MAUT addition:

- i. all indicators are independent of each other (additive independence);
- ii. the analyst has a complete understanding of the system;
- iii. all indicators are commensurable with each other, i.e. a deficiency in one indicator can be made up for with an excess in any other indicator, with the exact rate of exchange between two indicators determined by the choice of respective weights;
- iv. vulnerability is a linear and monotonic function of indicators.

Note that points iii and iv are NOT necessary requirements of MAUT. However, they are almost always found in MAUT-based IBVA in the literature. Unfortunately, these assumptions rarely hold in reality in the context of IBVA, which renders the use of additive aggregation, such as the one described above, scientifically questionable. On the other hand, as discussed earlier, it is not usually possible to build function f (equation 30), as an alternative to additive MAUT. In this paper these problems, points i and ii, are called the fundamental uncertainty problem. On the other hand, the linear function is usually an oversimplified representation of a much more complex relationship which, in many cases, can be depicted through mechanistic modelling based on deductive arguments. Therefore, we call the problems brought up by points iii and iv above, *the indicator incommensurability* problem and *the deductive non-linearity* problem, respectively. It is possible to illustrate these problems by referring to a simple model in which average daily temperature T and average daily humidity H are used as indicators of exposure of a set of communities to heat stress. Epidemiological evidence from city-scale studies shows that, in many cases, an increase in each of these two indicators can lead to an increase in daily mortality. However, the interactions between temperature and humidity are complex and will not yield to an easy trade-off between the two as a MAUT-based aggregation would imply (indicator incommensurability). In addition, the relationship between mortality and temperature is highly non-linear with sharp increases in deaths observed beyond a threshold value of T , called comfort temperature (deductive nonlinearity).

One way of dealing with indicators incommensurability is to avoid building any utility function, i.e. avoid converting these indicators into compatible scales altogether. Instead, an alternative approach conducts pair-wise comparisons of SESs based on one indicator at a time, then ranks the vulnerabilities of these SESs, by following formal aggregation rules that elicit the balance of evidence from all pair-wise comparisons. This approach has been applied in multi-criteria decision-analysis (MCDA) over the last 40 years and has yielded a set of methods called outranking procedures (Roy, 1968, Hokkanen and Salminen, 1997, El Hanandeh and El-Zein, 2010). It is especially useful where a degree of subjectivity or multiple subjectivities are involved in the assessment process, which is the case in CCVA because expert judgement and stakeholder values are usually necessary inputs into the process (especially when it comes to sensitivity and adaptive capacity), alongside mechanistic modelling of the physical impact of the hazard on the SES. In a previous paper, we have shown that IBVA is analogous to MCDA and that

outranking procedures can be used to derive more scientifically valid forms of aggregation in IBVA (El-Zein and Tonmoy, 2015).

The question, therefore, from an analytical viewpoint, is how to build, based on a given indicator, a measure of the truth of the statements “a is more vulnerable than b”, “b is more vulnerable than a” or “a and b are equally vulnerable”, where a and b are two SESs. Outranking procedures start by recognising the *fuzziness* of the answer to the above question. Fuzziness can be described through the following illustrative example. Based on empirical evidence from social sciences, we often assume that the adaptive capacity of a community is partly reflected by its collective income and assets—the wealthier it is, the higher its adaptive capacity and the less vulnerable it is to the hazard in question. However, it is very difficult to characterise the exact relationship between wealth W and adaptive capacity A_c . On the other hand, it is reasonable to assume that small differences in wealth do not translate into differences in adaptive capacities A_c or vulnerabilities V . In other words, below a certain threshold of difference ΔW_1 (or more generically ΔI_1 for any indicator I) the corresponding ΔA_c (or ΔV) is negligible. Such a relationship can be represented by a discontinuous step function as shown in Figure 1a. Likewise, beyond a certain point ΔW_2 (or ΔI_2), an increase in wealth is no longer expected to yield an increase in adaptive capacity or vulnerability. Combining the two thresholds, a two-step function can be built as shown in Figure 1b. Fuzziness, in other words, is an intuitive form of non-linearity. We call it the *intuitive non-linearity* consistently with previous definitions of non-linearity. Conversely, a relationship in which $\Delta V/\Delta I$ is continuous is *intuitively-linear*.

Outranking procedures are especially powerful in representing fuzzy relationships. However, in their present form, they cannot accommodate other forms of nonlinearity, such as the deductive nonlinearity. This is an impediment to their application to IBVA, since as we have seen earlier, all three forms of non-linear relationships are present in these assessments. In what follows, we present a new outranking framework based on the concept of *harm*, which allows us to represent all of these non-linear relationships. We call it the Sydney Environmental Vulnerability Approach (SEVA).

Table 1 summarises the different forms of incommensurability and nonlinearity we have identified, as well as the ability of different frameworks to accommodate them.

4.0 The Sydney Environmental Vulnerability Approach (SEVA)

We begin by introducing *harm* as a concept mediating the relationship between the indicator and the vulnerability it represents. This was initially suggested by Hinkel (2011). *Harm* can be conceived of as a more concrete, less abstract form of vulnerability that is more amenable to quantification. In conventional IBVA studies, an indicator selected to represent vulnerability is usually taken to satisfy the following three conditions:

1. it represents a process generating vulnerability;
2. it holds a linear relationship with vulnerability;
3. it is either readily available or computable.

a. Step-function

b. Two-step function

Figure 1. Fuzziness: non-linear relationships between difference in vulnerabilities ΔV and difference in indicators ΔI

SEVA, we replace indicators with *harm criteria*. A harm criterion must satisfy the following conditions:

1. it must represent a process generating vulnerability;
2. it must hold either a linear relationship *or an intuitively non-linear relationship* with vulnerability;
3. it must be either readily available or computable; *computable harm criteria may be the output of deductively non-linear relationships or models whose input is a set of readily available indicators.*

Table 1. Summary of different forms of incommensurability and non-linearity in vulnerability assessments

Problem	Description	Assumptions Made in Different Frameworks		
		MAUT	Adapted ELECTRE III*	SEVA
Dimensional incommensurability	One dimension of vulnerability may not be convertible into another	Always fully convertible	Flexible convertibility	Flexible convertibility
Indicator incommensurability	One indicator of vulnerability may not be convertible into another, even within the same dimension	Always fully convertible	Flexible convertibility	Flexible convertibility
Fundamental non-linearity	Vulnerability V may depend on the magnitude of the hazard M	Linear and non-linear	Linear and non-linear	Linear and non-linear
Deductive non-linearity	A non-linear relationship between vulnerability V and indicator I may exist	Always linear	Always linear	Linear and non-linear
Intuitive non-linearity or fuzziness	A discontinuous relationship between change in vulnerability V and change in indicator I may exist	Always linear	Linear or non-linear	Linear or non-linear

*This refers to the adaptation of the ELECTRE III outranking procedure to vulnerability assessment presented by El-Zein and Tonmoy (2013) In

A harm criterion then, like an indicator, acts as a proxy for a process generating vulnerability (e.g., percent of population over 65 in a community as a proxy indicator of the increased vulnerability to heat with age; percent of people on lower income in a community as a proxy indicator of the increased vulnerability to flooding events with poverty). However, a harm criterion, as opposed to an indicator, allows us to achieve two key objectives in the process of building a vulnerability model:

- to relax the conditions concerning linearity;
- to separate deductive and intuitive non-linearity in order to better deal with both of them.

Figure 2 shows these features graphically, with Figure 2c highlighting the fact that, when the harm criterion is readily available (and no deductive non-linearity is present), there is no need for indicators. To illustrate how harm criteria can be used in vulnerability assessments, we present a simplified model aiming to rank the vulnerability to heat of a number of communities (e.g., neighbourhoods or city districts). The model, shown in Table 2, is *not* a complete and accurate representation of vulnerability to heat: it is only an illustrative one that we use to demonstrate key relationships between indicators, harm criteria and vulnerability. Harm criteria H_1 , H_2 and H_3 are usually readily available as primary data from demographic and population census databases. Hence, there is no need for indicators to help in computing them. Harm criterion H_4 , on the other hand, can be predicted by combining a measure of temperature increase generated by a climate model under a given scenario with an epidemiologically-based relationship that estimates the resulting number of excess deaths. The relationship between I_4 and H_4 is usually deductively non-linear: it is often represented by the so-called U-shaped or V-shaped curves (see Figure 3), whose parameters depend on climatic, demographic and socio-economic factors (e.g., McMichael, 2008). It is important to emphasise here that the construction of the model is guided by epistemological, as much as ontological considerations: deductive non-linearities emerge from the kind of data that is available to us, and not just from the nature of the relationship itself. For example, had we been assessing vulnerability at a certain point in the past and had a way of measuring H_4 directly, rather than predicting it through I_4 , there would be no need to consider the non-linearity in question.

Finally, the model assumes that H_1 and H_2 are intuitively-linear—they hold a linear relationship to vulnerability—while H_3 and H_4 are intuitively non-linear, following the two-step function in Figure 1b. That is to say that small differences in income and predicted excess mortality do NOT translate into differences in vulnerability (how *small* is small will be defined below); likewise, beyond a given threshold, bigger differences in income and excess mortality no longer translate into a bigger differences in vulnerability. This model will be used in the results section to illustrate how SEVA can be employed to conduct vulnerability assessments.

Figure 2. The *harm* concept mediating the relationship between indicator and vulnerability in the presence of deductive non-linearity

It is now possible to express equation (21) in terms of harm criteria rather than indicators:

$$\vec{V}_k(\mathbf{M}) = f_{ek}(H_{e1k}, H_{e2k}, H_{e3k}, \dots) \vec{e} + f_{sk}(H_{s1k}, H_{s2k}, H_{s3k}, \dots) \vec{s} + f_{ck}(H_{c1k}, H_{c2k}, H_{c3k}, \dots) \vec{a} \quad (27)$$

where H_{dik} is either given or can be computed as a nonlinear, non-monotonic function of a set of indicators $I_{d1k}, I_{d2k}, I_{d3k}, \dots$ or the outcome of a complex mechanistic model, which may contain thresholds and tipping points, with a set of indicators as input:

$$H_{dik} = f_{dik}(I_{d1k}, I_{d2k}, I_{d3k}, \dots) \quad (28)$$

Hence, introducing harm allows, not only for non-linear relationships to be represented, but also for multiple interactions between indicators.

Starting from equation (27), which we now take as fully representing the vulnerability of the system, we conduct aggregation and ranking of vulnerabilities, as follows. We assume, without loss of generality, that the higher the value of the harm criterion, the more vulnerable the SES. (However, it is possible to use positive counterparts of *harm* such as *resource* or *benefit*, and opposites of vulnerability such as *immunity* or *resilience*. A list of antonyms for the concepts used in the framework is given in Table 3). Also, for simplicity of notation, we drop the dimension index d and use H_{ik} instead of H_{dik} , and so on, with the understanding that any H_{ik} corresponds to a specific dimension of vulnerability.

Figure 3. Idealised U-shaped relationship between daily mortality and average temperature taken from the literature on the epidemiology of heat: $M_{\min}$ is minimum daily mortality; $I_{\min 1}$ and $I_{\min 2}$ define a range for minimum-mortality temperature; β_{cold} and β_{hot} are cold and hot slopes (Curriero et al., 2002, McMichael et al., 2008, El-Zein et al., 2004)

For each pair of SESs, we can define three different categories of relative vulnerability (representing the two-step function shown in Figure 1b):

1. b is indifferent to a according to harm criterion H_i if and only if $|H_{ib} - H_{ia}| \leq q_i$, where $q_i \geq 0$ is the *relative vulnerability indifference threshold for harm criterion H_i* ;
2. b is strictly more vulnerable than a according to harm criterion H_i if and only if $H_{ib} - H_{ia} \geq p_i$, where $p_i \geq 0$ is the *relative vulnerability threshold for harm criterion H_i* ($p_i \geq q_i$);
3. b is weakly or proportionately more vulnerable than a according to harm criterion H_i if and only if $q_i < H_{ib} - H_{ia} < p_i$.

Note that q_i and p_i correspond to ΔI_1 and ΔI_2 in Figure 1a, respectively. The set of rules 1 to 3 express the fuzziness of the relationship between harm and vulnerability. In order to cater for incommensurability or partial compensation between harm criteria, a fourth rule is introduced as follows:

4. b is at least as vulnerable as a , if there exists one criterion for which $H_{ib} - H_{ia} > v_i$, *regardless of the performances of a and b on all other harm criteria*, where $v_i \geq p_i$ is called *dominance threshold for harm criterion I_i* .

Table 2a. Hypothetical 4-harm criteria model of vulnerability to heat stress (NA: Not Applicable; A: Available; C: Computable)

Harm Criterion	Description	Dir ^a	Process	Deductive Non-Linearity?	Predictive Indicator	Relationship	Intuitive Non-Linearity?	Compensation	References
H₁ (A)	% of population > 65 years of age	↑	older population is at higher risk of death	No	NA	NA	No	Total	(Curriero et al., 2002)
H₂ (A)	% of built-up land cover	↑	urban heat-island (UHI) effect leads to more deaths	No	NA	NA	No	Total	(Harlan et al., 2006, Vaneckova et al., 2010)
H₃ (A)	median monthly household income in \$	↓	lack of access to adaptive resources leads to more deaths	No	NA	NA	Yes	Partial	(Reid et al., 2009, Johnson and Wilson, 2009)
H₄ (C)	daily mortality counts as a temperature-dependent variable	↑	higher temperature leads to more deaths	Yes	I_4^b	$H_4 = -\beta_{cold}(I_4 - I_{min1}) + M_{min}$ for $I_4 \leq I_{min1}$ $H_4 = M_{min}$ for $I_{min1} \leq I_4 \leq I_{min2}$ $H_4 = \beta_{hot}(I_4 - I_{min2}) + M_{min}$ for $I_{min1} \leq I_4$ (see note c below)	Yes	Partial	(Curriero et al., 2002, McMichael et al., 2008, El-Zein et al., 2004)

a :Dir= Direction: ↑ (↓) indicates that vulnerability increases (decreases) with increasing harm.

b: I_4 is average daily temperature.

c: entities M_{min} , I_{min1} , I_{min2} , β_{cold} and β_{hot} are defined in Figure 3.

Table 2b. Adopted values for hypothetical 4-harm criteria model of vulnerability to heat stress

	SES1	SES2	SES3	SES4
H1	35%	29%	38%	25%
H2	43%	48%	52%	38%
H3	\$875	\$800	\$1000	\$955
H4-L	87	50	125	113
H4-DNL	100	107	125	113
I4	30°C	28°C	34°C	31°C

L= Linear, DNL= Deductively Non-Linear

Hence, v_i sets a limit beyond which a disparity in the values of harm criterion for 2 SESs is so great, that the resulting difference in vulnerability cannot be compensated for by reverse disparity in another harm criterion. In other words, compensation is either partial or completely absent.

We refer to q_i , p_i and v_i collectively as *thresholds of difference* to emphasize that they provide a reference to disparities between harm criteria and not to the harm criteria themselves. Note that, under some circumstances, it is more convenient to define p_i , q_i and v_i as percentages, whereby the relationships shown above can be written in terms of the percent difference $(H_{ib} - H_{ia})/H_{ia}$ rather than the difference $H_{ib} - H_{ia}$. It is also possible to define v_i as a dominance threshold relative to the value of the criterion rather than the difference in values, in which case rule 4 can be stated as:

5. b is at least as vulnerable as a , if there exists one criterion for which $H_{ib} > v_i$ and $H_{ia} \leq v_i$, regardless of the performances of a and b on all other harm criteria, where $v_i \geq p_i$ is called *dominance threshold* for harm criterion i .

In this paper, we will use only the original definitions shown in rules 1 to 4.

Table 3. Vulnerability concepts and antonyms*

Negative	Positive
Vulnerability	Immunity or Resilience
<i>Harm</i>	<i>Resource or Benefit</i>
Exposure	Protection or Inexposure
Sensitivity	Insensitivity or Indifference
Adaptive Deficiency	Adaptive Capacity

*harm (benefit) acts as an analytical intermediary between vulnerability (immunity) and its three dimensions of exposure (protection), sensitivity (insensitivity) and adaptive deficiency (adaptive capacity)

Converting the vulnerability matrix into a ranking of SESs according to their vulnerabilities to a climate change hazard can be made using any one of a number of available outranking procedures. In what follows, we use the ELECTRE-III outranking method (Roy, 1968, El-Zein and Tonmoy, 2015) and present here a summary of its various steps. For more details on the rationale for this choice, the reader is referred to El-Zein and Tonmoy (2015).

A concordance matrix for each harm criterion H_i is defined by:

$$c_i(a, b) = \begin{cases} 0 & \text{if } H_{ib} - H_{ia} \geq p_i \\ \frac{p_i - (H_{ib} - H_{ia})}{p_i - q_i} & \text{if } q_i < H_{ib} - H_{ia} < p_i \\ 1 & \text{if } H_{ib} - H_{ia} \leq q_i \end{cases} \quad (29)$$

$c_i(a, b)$ is a measure of the truth of the statement that “ a is at least as vulnerable as b according to harm criterion H_i ”. The discordance matrix for each harm criterion H_i is defined by:

$$d_i(a, b) = \begin{cases} 0 & \text{if } H_{ib} - H_{ia} \leq p_i \\ \frac{(H_{ib} - H_{ia}) - p_i}{v_i - p_i} & \text{if } p_i < H_{ib} - H_{ia} < v_i \\ 1 & \text{if } H_{ib} - H_{ia} \geq v_i \end{cases} \quad (30)$$

The statement “a is at least as vulnerable as b” (denoted $a \geq b$) is true provided:

- i. a “majority” of harm criteria supports it (concordance principle); and
- ii. no single harm criterion vetoes it (discordance principle).

The concordance principle can be measured by the following concordance index:

$$c(a, b) = \frac{1}{\sum_{i=1}^m w_i} \sum_{i=1}^m w_i c_i(a, b) \quad (31)$$

where w_i is a measure of the relative importance of harm criterion H_i , applied to the pair-wise comparisons, rather than a weight in a global utility function. An outranking matrix combines the concordance and discordance principles in order to quantify the degree to which $a \geq b$ is true. It is given by:

$$S(a, b) = \begin{cases} c(a, b) & \text{if } d_i(a, b) \leq c(a, b) \quad \forall i = 1, m \\ c(a, b) \prod_{i \in I_v(a, b)} \frac{[1 - d_i(a, b)]}{[1 - c(a, b)]} & \text{otherwise} \end{cases} \quad (32)$$

where $I_v(a, b)$ is the set of harm criteria for which $d_i(a, b) > c(a, b)$

If the discordance $d_i(a, b)$ is 1 for any criterion, there is no confidence that a outranks b, and therefore, $S(a, b) = 0$.

The most vulnerable SES is the one that outranks the largest number of SESs and is outranked by least. Hence, $S(a, b)$ is used next to build two partial, descending and ascending pre-orders DP and AP respectively. These partial pre-orders are established using the indifference, preference or incomparability relation for each pair of SES found through equations 29 to 32. Last, a final ranking is given by $DP \cap AP$ (El Hanandeh and El-Zein, 2010).

6.0 Significance of Thresholds of Difference

Table 4 presents definitions for ‘thresholds of difference’. These definitions can be used in establishing values of ‘thresholds of differences’ in a practical assessment exercises. These thresholds dictate the extent of intuitive nonlinearity and compensation and Table 5 shows eight different combinations of non-linearity and partial compensation, based on q_i , p_i and v_i . In addition, the thresholds of difference determine the rules by which aggregation is conducted in SEVA. As stated earlier, one of SEVA’s strengths is that it does not require the conversion of harm criteria into a common scale. It’s important to remember that the vulnerability matrix typically carries a collection of harm criteria of very different nature and precision: continuous, discrete, ordinal and binary; climatic, physical, demographic and socio-economic. Clearly, no deductive arguments can be developed to guide such an aggregation: this is a reflection of the fact that vulnerability is a social construct, influenced by physical risk but not determined by it. Hence, the way harm criteria are combined to generate rankings ought to be highly influenced by normative judgements, as well as ‘hard’ data reflecting exposure to risk.

It is precisely through the thresholds of difference that such value judgements are systematically elicited in SEVA as part of the data collection stage. For example, two measures of vulnerability in comparing two districts in a city may be the dollar value of likely damage to coastal infrastructures reflecting

exposure to risk and percent of population under 12 years of age as a proxy for sensitivity or adaptive capacity. If one district is likely to sustain \$30,000 less damage than another but has 20% more children population, how do we determine which one is more vulnerable? In SEVA, through the thresholds of difference, the question is divided into 3 parts:

1. What is the importance of dollar value of likely damage relative to percent of children population (reflected by relative importance factor w_i)?
2. How significant is a difference of \$30,000 (reflected by q_i and p_i)?
3. Is there a threshold of difference in values of damage beyond which the difference in vulnerability between the two districts is considered decisive, regardless of their performances on other harm criteria (reflected by v_i)?

Table 4. Definitions of thresholds of difference and relative importance factors

Types of thresholds of difference	Notation	Working definition
Indifference threshold	q_i	All other harm criteria being equal, what is the difference in values of harm criterion H_i for two SESs below which the vulnerabilities of the two systems are the same?
Relative vulnerability threshold	p_i	All other harm criteria being equal, what is the difference in values of harm criterion H_i for two SESs above which one system is strictly more vulnerable than the other?
Dominance threshold	v_i	What is the difference in values of harm criterion H_i for two SESs a and b ($H_{ib} - H_{ia}$) above which a cannot be more vulnerable than b, regardless of the performances of a and b on other harm criteria (no compensation)?*
Relative importance factor	w_i	In determining whether a 'majority' of harm criteria supports the statement that one SES is at least as vulnerable as another, what is the strength of a 'vote' by harm criterion H_i relative to a reference harm criterion?

*this definition assumes that vulnerability increases with increasing harm criterion and would need to be suitably adjusted if the reverse is true ("... above which b cannot be more vulnerable than a, regardless ...").

Two observations can be made here. First, thresholds of difference and relative importance factors must emanate from knowledge of the community under study and its values, and have to be determined through a consultative process involving experts and stakeholders. Second, the smaller the geographical scale under study, the more likely for the processes generating vulnerability to be well-defined and for the thresholds of difference and relative importance factors to be faithful reflections of the community and its values. In other words, our framework formalises an observation about vulnerability assessment that has already been made in the literature, namely that indicator-based assessments are more likely to be meaningful at smaller rather than larger scales (Fussel, 2007; Hinkel, 2011).

Figure 4 shows the overall architecture of SEVA while Figure 5 depicts all the different steps in building a vulnerability model using the SEVA framework.

Figure 4. Overall architecture of SEVA. Here, n is the number of SESs and m is the number of harm criteria in the analysis.

Table 5. Thresholds of difference for various combinations of deductive nonlinearity, intuitive non-linearity and partial compensation (in all cases $q_i \leq p_i \leq v_i$ *)

	Intuitively Linear and Fully-Compensating	Intuitively Non-Linear		
		Full Compensation	Full or Partial Compensation	Full, Partial or No Compensation
Deductively Linear	Type 1 H_{ik} $q_i = 0$ $\max(H_{ik} - H_{ij}) \ (k=1, n; j=1, n) \leq p_i \leq v_i$	Type 2 H_{ik} $q_i \geq \min H_{ik} - H_{ij} \ (k=1, n; j=1, n)$ $\max(H_{ik} - H_{ij}) \ (k=1, n; j=1, n) \leq p_i \leq v_i$	Type 3 H_{ik} $q_i \geq 0$ $p_i < \max(H_{ik} - H_{ij}) \ (k=1, n; j=1, n) \leq v_i$	Type 4 H_{ik} $q_i \geq 0$ $p_i \geq 0$ $v_i \leq \max(H_{ik} - H_{ij}) \ (k=1, n; j=1, n)$
Deductively Non-Linear	Type 5 $H_i = f(l_1, l_2, \dots)$ $q_i = 0$ $\max(H_{ik} - H_{ij}) \ (k=1, n; j=1, n) \leq p_i \leq v_i$	Type 6 $H_i = f(l_1, l_2, \dots)$ $q_i \geq \min H_{ik} - H_{ij} \ (k=1, n; j=1, n)$ $\max(H_{ik} - H_{ij}) \ (k=1, n; j=1, n) \leq p_i \leq v_i$	Type 7 $H_i = f(l_1, l_2, \dots)$ $q_i \geq 0$ $p_i < \max(H_{ik} - H_{ij}) \ (k=1, n; j=1, n) \leq v_i$	Type 8 $H_i = f(l_1, l_2, \dots)$ $q_i \geq 0$ $p_i \geq 0$ $p_i < v_i < \max(H_{ik} - H_{ij}) \ (k=1, n; j=1, n)$

* see Table 4 for definition of p, q and v. In all these equations j and k refer to SESs. As an example, H_{ij} and H_{ik} refer to the value of i^{th} harm criteria of SES_j and SES_k respectively.

Figure 5. SEVA Step-by-Step Vulnerability Ranking Procedure

*available harm criteria are ones whose values for all the SEEs under consideration can be found from available databases, or as output from climate or other models, without further computation.

7.0 An illustrative Example: Assessment of Vulnerability to Heat Stress using SEVA

Table 6 shows the results of SEVA analyses conducted on the model shown in Table 2. The SESs in this case are four hypothetical city districts of similar scales. We compare vulnerabilities of the well-being of residents of these districts to the effects of temperature, assuming they are adequately reflected by the four harm criteria shown in Table 2. Starting from a scenario where all relationships are linear and full compensation between harm criteria is available (case 1), we generate new scenarios (cases 2 to 5) by gradually introducing different forms of intuitive and deductive non-linearity and partial compensation. We do this in order to illustrate the effects of different nonlinearities on the rankings of vulnerabilities. Case 5 represents the full non-linear model as presented in Table 2. In each case, we also show the thresholds of difference used for each harm criterion. In all analyses, we assume that all four harm criteria have a relative importance factor of one. We conduct a SEVA analysis for each case and generate rankings for the four SESs in each of the five cases. Although not shown here, in all cases, we have also conducted a large number of additional analyses to assess the sensitivity of the ranking outcomes to small changes ($\pm 5\%$) in harm criteria scores, thresholds and relative importance factors. Results generated by SEVA have been found to be robust.

It can be seen in Table 6 that, in **case 1**, SES3 is the most vulnerable district because it performs worst on all but one of the 4 harm criteria (median income). SES4 is found to be least vulnerable primarily because of its low percentages of over 65 and a built-up land cover. SES2 is more vulnerable than SES1 according to harm criteria 1 and 4, but vice versa judging by harm criteria 2 and 3. The analysis ranks them equally. In **case 2**, the introduction of an indifference threshold of $q_3 = \$90$ for harm criterion 3 cancels the mean income advantage of SES4 over SES1 because the difference in median income between the two is \$80, less than \$90, now considered insignificant as an indicator of difference in vulnerability. Likewise, the advantage of SES1 over SES2 derived from harm criterion 3 is now inconsequential because the difference is \$75. Hence, the overall effect of introducing the indifference threshold for H_3 is that SES1 and SES4 are now ranked equally in third place. **Case 3** shows the effect of introducing partial compensation for H_3 , with $p_3 = \$100$ and $v_3 = \$110$. Hence, the advantage that SES3 carries over SES2 in terms of median income has become decisive: SES3 can no longer be more vulnerable than SES1, regardless of its performance on other harm criteria. Note that the newly-introduced dominance threshold also affects the comparison between SES2 and SES4, with SES4 carrying a decisive income advantage. However, in this case, SES2 had already been found to be more vulnerable than SES4 in case 1. Overall, in case 3, SES1 is found to be most vulnerable and SES4 least.

The non-linear relationship between mortality and temperature is introduced into the model in **case 4**. It is clear that a linear relationship between temperature and mortality underestimates mortality for SES1 and SES2. Introducing the non-linear relationship hence leads to poorer performance of both SES1 and SES2 on harm criterion H_4 , relative to case 1, with average daily mortality increasing from 87 and 50 to 100 and 107, respectively. In other words, SES2 is much worse off in case 4, compared to case 1, with its average mortality increasing 67 while that of SES1 increases by 13. The introduction of this non-linearity does not change first and last ranks but it does push SES3 down to third rank. In **case 5**, all nonlinearities and degrees of compensation are present. SES2 is found to be the most vulnerable district and SES4 the least. Note that the introduction of a relative vulnerability threshold p_4 for H_4 , affects the

analysis by capping the extent to which a difference in values of the harm criterion translates for two SESs translates into a difference in vulnerability.

Next, we assess the effects on the analysis outcomes of a 100% change in the relative importance factors. To this end, we conduct four additional analyses for case 5, changing one relative importance factor from 1 or 2, each time, while keeping all the others constant at 1. We find that all rankings remain the same in all four additional analyses and, therefore, do not show the results here.

The illustrative example has demonstrated the following:

1. the proposed SEVA framework allows a combination of non-linear and partial compensation effects to be incorporated into vulnerability assessments;
2. non-linearity and partial compensation do have an effect on the rankings of vulnerability;
3. the SEVA assessment appears to be robust to small changes in scores and thresholds and to small and up to 100% changes in relative importance factors.

It is clear from the above discussion therefore that SEVA offers analytical added-value in climate-change vulnerability assessments, relative to more conventional approaches based on multiple-attribute utility theory (MAUT).

Conclusions

Assessments of vulnerability to climate change can help in developing adaptation policies. In particular, sound resource allocation may require the ranking of different socio-ecological systems according to the vulnerability of a valued attribute to a climate-related stress. Given the multi-dimensional nature of vulnerability as a construct reflecting physical and socio-economic components of risk, assessments inevitably require aggregation of data from different knowledge domains. In this paper, we have developed a new mathematical framework which achieves two objectives:

- a. It allows us to define, with precision, different forms of non-linearity and degrees of compensation between indicators that obtain in vulnerability assessments.
- b. It incorporates a methodology, based on outranking methods, to rank the vulnerabilities of a set of comparable socio-ecological systems while taking into account a heterogeneous set of indicators which are not necessarily commensurate with each other.

The framework is flexible enough to accommodate a wide range of data types and, as stated earlier, rankings generated by the framework are robust. A useful extension of this work is the application of the framework to a real-life vulnerability assessment exercise. This would allow us to assess the ease with which the framework can be implemented, especially in relation to establishing thresholds of difference.

Table 6. Five sample scenarios and effects of nonlinearity and degrees of compensation on rankings using SEVA (rankings: 1:most vulnerable; 4:least vulnerable; all thresholds of difference correspond to harm criteria not indicators; cases 2, 3 and 4 are modifications of case 1 with the change highlighted in bold characters; in all intuitively linear relationship $q_i=0$ and $p_i=\max(H_{ik}-H_{ij})$, $k=1,n$; $j=1,n$)

Cases	Description	Harm	Units	Indicator	Dir ¹	Type ²	q_i	p_i	v_i	w_i	SES1	SES2	SES3	SES4
Case 1	All harm criteria are intuitively and deductively linear and fully-compensating ³	H ₁	%	N/A	↑	1	0	8	N/A	1	35%	29%	38%	25%
		H ₂	%	N/A	↑	1	0	14	N/A	1	43%	48%	52%	38%
		H ₃	\$	N/A	↓	1	0	200	N/A	1	\$875	\$800	\$1000	\$955
		H ₄	counts	I ₄	↑	1	0	75	N/A	1	87	50	125	113
						Vulnerability Ranking					2	2	1	4
Case 2	Intuitive non-linearity present only for H ₃ with full compensation between all harm criteria	H ₁	%	N/A	↑	1	0	8	N/A	1	35%	29%	38%	25%
		H ₂	%	N/A	↑	1	0	14	N/A	1	43%	48%	52%	38%
		H₃	\$	N/A	↓	2	90	200	N/A	1	\$875	\$800	\$1000	\$955
		H ₄	counts	I ₄	↑	1	0	75	N/A	1	87	50	125	113
						Vulnerability Ranking					3	2	1	3
Case 3	Intuitive non-linearity and partial compensation present only for H ₃	H ₁	%	N/A	↑	1	0	8	N/A	1	35%	29%	38%	25%
		H ₂	%	N/A	↑	1	0	14	N/A	1	43%	48%	52%	38%
		H₃	\$	N/A	↓	4	90	100	110	1	\$875	\$800	\$1000	\$955
		H ₄	counts	I ₄	↑	1	0	75	N/A	1	87	50	125	113
						Vulnerability Ranking					1	2	2	4
Case 4	Deductive non-linearity introduced for H ₄	H ₁	%	N/A	↑	1	0	8	N/A	1	35%	29%	38%	25%
		H ₂	%	N/A	↑	1	0	14	N/A	1	43%	48%	52%	38%
		H ₃	\$	N/A	↓	1	0	200	N/A	1	\$875	\$800	\$1000	\$955
		H₄	counts	I₄	↑	5	0	25	N/A	1	100	107	125	113
						Vulnerability Ranking					3	2	1	4
Case 5	Full model of Table 2 with all relevant non-linearities and degrees of compensation present	H ₁	%	N/A	↑	1	0	8	N/A	1	35%	29%	38%	25%
		H ₂	%	N/A	↑	1	0	14	N/A	1	43%	48%	52%	38%
		H₃	\$	N/A	↓	4	90	100	110	1	\$875	\$800	\$1000	\$955
		H₄	counts	I₄	↑	8	15	20	25	1	100	107	125	113
						Vulnerability Ranking					2	1	3	4

¹Dir= Direction: ↑ (↓) indicates that vulnerability increases (decreases) with increasing harm.

²Type refers to the different relationships shown in Table 5.

References

- ADGER, W. N. 2006. Vulnerability. *Global Environmental Change*, 16, 268-281.
- ADGER, W. N. & KELLY, P. M. 1999. Social vulnerability to climate change and the architecture of entitlements. *Mitigation and Adaptation Strategies for Global Change*, 4, 253-266.
- BRENNERT, A. L. & MALONE, E. L. 2005. Modeling vulnerability and resilience to climate change: A case study of India and Indian states. *Climatic Change*, 72, 57-102.
- CURRIERO, F. C., HEINER, K. S., SAMET, J. M., ZEGER, S. L., STRUG, L. & PATZ, J. A. 2002. Temperature and mortality in 11 cities of the eastern United States. *American Journal of Epidemiology*, 155, 80-87.
- CUTTER, S. L., BORUFF, B. J. & SHIRLEY, W. L. 2003. Social Vulnerability to Environmental Hazards. *Social Science Quarterly*, 84, 242-261.
- DURIYAPONG, F. N., K. 2011. Coastal vulnerability assessment: A case study of Samut Sakhon coastal zone. *Songklanakarin Journal of Science and Technology*, 33, 469-476.
- EL HANANDEH, A. & EL-ZEIN, A. 2010. The development and application of multi-criteria decision-making tool with consideration of uncertainty: The selection of a management strategy for the biodegradable fraction in the municipal solid waste. *Bioresource Technology*, 101, 555-561.
- EL-ZEIN, A., TEWTEL-SALEM, M. & NEHME, G. 2004. A time-series analysis of mortality and air temperature in Greater Beirut. *Science of the Total Environment*, 330, 71-80.
- EL-ZEIN, A. & TONMOY, F. N. 2015. Assessment of vulnerability to climate change using a multi-criteria outranking approach with application to heat stress in Sydney. *Ecological Indicators*, 48, 207-217.
- ERIKSEN, S. & KELLY, P. 2007. Developing Credible Vulnerability Indicators for Climate Adaptation Policy Assessment. *Mitigation and Adaptation Strategies for Global Change*, 12, 495-524.
- FÜSSEL, H.-M. 2007. Vulnerability: A generally applicable conceptual framework for climate change research. *Global Environmental Change*, 17, 155-167.
- FÜSSEL, H.-M. & KLEIN, R. 2006. Climate Change Vulnerability Assessments: An Evolution of Conceptual Thinking. *Climatic Change*, 75, 301-329.
- HARLAN, S. L., BRAZEL, A. J., PRASHAD, L., STEFANOV, W. L. & LARSEN, L. 2006. Neighborhood microclimates and vulnerability to heat stress. *Social Science & Medicine*, 63, 2847-2863.
- HINKEL, J. 2011. "Indicators of vulnerability and adaptive capacity": Towards a clarification of the science-policy interface. *Global Environmental Change*, 21, 198-208.
- HOKKANEN, J. & SALMINEN, P. 1997. ELECTRE III and IV Decision Aids in an Environmental Problem. *Journal of Multi-Criteria Decision Analysis*, 6, 215-226.
- IONESCU, C., KLEIN, R. J. T., HINKEL, J., KAVI KUMAR, K. & KLEIN, R. 2009. Towards a formal framework of vulnerability to climate change. *Environmental Modeling and Assessment*, 14, 1-16.
- IPCC 2014. *Climate Change 2014: Impacts, Adaptation, and Vulnerability. Part A: Global and Sectoral Aspects. Contribution of Working Group II to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change*, Cambridge, United Kingdom and New York, NY, USA, Cambridge University Press.
- IPCC, M. L. P., O.F. CANZIANI, J.P. PALUTIKOF, P.J. VAN DER LINDEN, C.E. HANSON (EDS.), 2007. *Climate Change 2007: Impacts, Adaptation and Vulnerability. Contribution of Working Group II to the Fourth Assessment Report of the Intergovernmental Panel on Climate Change*, Cambridge University Press, Cambridge, UK.
- JOHNSON, D. P. & WILSON, J. S. 2009. The socio-spatial dynamics of extreme urban heat events: The case of heat-related deaths in Philadelphia. *Applied Geography*, 29, 419-434.
- LUERS, A. L., LOBELL, D. B., SKLAR, L. S., ADDAMS, C. L. & MATSON, P. A. 2003. A method for quantifying vulnerability, applied to the agricultural system of the Yaqui Valley, Mexico. *Global Environmental Change*, 13, 255-267.

- MCMICHAEL, A. J., WILKINSON, P., KOVATS, R. S., PATTENDEN, S., HAJAT, S., ARMSTRONG, B., VAJANAPOOM, N., NICIU, E. M., MAHOMED, H., KINGKEOW, C., KOSNIK, M., O'NEILL, M. S., ROMIEU, I., RAMIREZ-AGUILAR, M., BARRETO, M. L., GOUVEIA, N. & NIKIFOROV, B. 2008. International study of temperature, heat and urban mortality: the 'ISOTHURM' project. *International Journal of Epidemiology*, 37, 1121-1131.
- PARRY, M. L., O.F. CANZIANI, J.P. PALUTIKOF, P.J. VAN DER LINDEN AND C.E. HANSON 2007. *Climate Change 2007: Impacts, Adaptation and Vulnerability. Contribution of Working Group II to the Fourth Assessment Report of the Intergovernmental Panel on Climate Change*, Cambridge, UK, Cambridge University Press
- PRESTON, B., SMITH, T. F., BROOKE, C., GORDDARD, R., MEASHAM, T. G., WITHYCOMBE, G., BEVERIDGE, B., MORRISON, C., MCLNNES, K. & ABBS, D. 2008. Mapping Climate Change Vulnerability in Sydney Coastal Council Group. Report Available at "<http://www.csiro.au/resources/SydneyClimateChangeCoastalVulnerability.html>" [19th September 2011].
- REID, C. E., O'NEILL, M. S., GRONLUND, C. J., BRINES, S. J., BROWN, D. G., DIEZ-ROUX, A. V. & SCHWARTZ, J. 2009. Mapping Community Determinants of Heat Vulnerability. *Environ Health Perspect*, 117.
- ROY, B. 1968. Classement et choix en présence de points de vue multiples (la méthode ELECTRE)'. *RIRO*, 8, 57-75.
- TONMOY, F. N. & EL-ZEIN, A. 2013a. Assessment of Vulnerability to Climate Change Using Indicators: Methodological Challenges. In: DINCER, I., COLPAN, C. O. & KADIOGLU, F. (eds.) *Causes, Impacts and Solutions to Global Warming*. Springer New York.
- TONMOY, F. N. & EL-ZEIN, A. Vulnerability of infrastructure to Sea Level Rise: A combined outranking and system-dynamics approach. European Safety and Reliability (ESREL-2013), 2013b. CRC Press, pp 509. ISBN: 978-1-315-81559-6, 2407-2414.
- TONMOY, F. N., EL-ZEIN, A., GHETTI, I. & MASSIE, R. Vulnerability to Sea Level Rise Of 8 Beaches In Shoalhaven: A New Multi-Dimensional Assessment Methodology. 21st NSW Coastal Conference, NSW Coastal Management-Coming of Age? November, 2012, Kiama, Australia., 2012.
- TONMOY, F. N., EL - ZEIN, A. & HINKEL, J. 2014. Assessment of vulnerability to climate change using indicators: a meta - analysis of the literature. *Wiley Interdisciplinary Reviews: Climate Change*, 5, 775-792.
- VANECKOVA, P., BEGGS, P. J. & JACOBSON, C. R. 2010. Spatial analysis of heat-related mortality among the elderly between 1993 and 2004 in Sydney, Australia. *Social Science & Medicine*, 70, 293-304.