STRENGTH MODELLING OF LAMINATED VENEER LUMBER (LVL) BEAMS

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ABSTRACT: This paper develops a mechanical model to accurately predict the strength of Laminated Veneer Lumber (LVL) beams, and illustrates its applications to numerically predict the strength distribution of LVL beams manufactured from veneers rotary peeled from early to mid-rotation subtropical hardwood plantation logs. This resource is not traditionally used in the manufacturing of commercialised LVL beams. In the first part of the paper, the model is described, calibrated against experimental results performed on 8-ply LVL beams and then verified against experimental results performed on 13-ply LVL beams. Results show that the model is able to accurately reproduce the experimental results, both on flat and edge bending, with an average prediction-to-experiment ratio of 1.0 and a relatively low coefficient of variation of 0.10. A sound prediction of the non-linear behaviour of the beams before failure was also observed. In the second part of the paper illustrating the applications of the model, the mechanical properties of veneers analysed by the authors in a previous work are used as input values in the numerical model to predict the strength of six commercially available LVL beam sizes, manufactured from early to mid-rotation subtropical Gympie messmate (*Eucalyptus cloeziana*), spotted gum (*Corymbia citriodora*) and southern blue gum (*Eucalyptus globulus*) plantation veneers. The design strength (5th percentile) of the beams, obtained by Monte Carlo simulations, is reported and found to range from 32.3 MPa to 97.2 MPa, depending on the quality of the veneers used and the beam size. The LVL beams have design strengths comparable to, and in some cases up to 2.5 times higher than commercially available softwood LVL beams, making them attractive structural products. The

31 strength variability is also reported in the paper for developing probability-based limit state design
32 criteria in future studies.

33

34 **KEYWORDS:** Computational modelling, Veneer-based products, LVL beams, Hardwood
35 plantations, Thinning, Pulpwood

36 1 INTRODUCTION

37 Veneer Based Composite (VBC) structural products, such as plywood or Laminated Veneer
38 Lumber (LVL), are commonly used in timber constructions. Their advantages over sawn structural
39 timber lie in their higher capacities and more reliable mechanical properties, and capability of
40 spanning longer distances [1]. With the increasing number of tall timber buildings being built
41 worldwide, their use is expected to further grow. When new VBC products, such as those
42 manufactured from new timber species for instance, are introduced on the market, expensive
43 experimental tests must be carried out to determine their strength and stiffness distributions,
44 therefore ensuring reliable design properties.

45 If an accurate numerical model could be developed and combined with an established probabilistic
46 database of the mechanical properties of the individual veneers, it would represent a valuable tool in
47 predicting these distributions and optimising the use of the veneered resources. The model would
48 allow for instance to either cost-effectively select the type of veneers to be used in the manufacture
49 of structural products with targeted strength or predict the capacity of products manufactured from
50 new resources. The database on the individual veneers must probabilistically quantify their strength
51 from characteristics which can be and are frequently measured in line during manufacturing, such
52 as the veneer dynamic Modulus of Elasticity (MOE) and total knot area ratio (tKAR). Very little
53 research aiming at modelling the strength of VBC structural products parallel to the grain exists in
54 the literature and it mainly focuses on strand based composites [2-5]. Other numerical models have
55 been mainly developed for sawn timber [6-12] or glulam beams [13-15]. No model currently exists
56 to mechanically predict the strength of veneer-based structural products based on the veneer
57 properties.

58 This development of this numerical tool is currently undertaken by the authors to ultimately predict
59 the strength distributions of veneer-based structural products manufactured from veneers rotary
60 peeled from early to mid-rotation subtropical hardwood plantation logs. This resource represents a
61 new opportunity to the timber industry and not traditionally used in the manufacture of commercial

62 structural products. In this attempt, this paper (i) extends the numerical mechanical models
63 developed for Glulam [13] and sawn timber [7] beams to predict the strength of Laminated Veneer
64 Lumber (LVL) beams based on the known mechanical properties of the veneers, and (ii) illustrates
65 the applications of the model in the development of new products by numerically predicting the
66 strength distributions of LVL beams manufactured from the previously mentioned new feedstock.
67 An established database of the mechanical properties of the veneers [16] is used for purpose of the
68 latter item. Specifically, the numerical model, combined with the Monte Carlo simulation method,
69 is used to estimate the statistical characteristics of six commercially available LVL beam sizes,
70 manufactured from four grades of early to mid-rotation subtropical Gympie messmate (*Eucalyptus*
71 *cloeziana* - GMS), spotted gum (*Corymbia citriodora* - SPG) and southern blue gum (*Eucalyptus*
72 *globulus* - SBG) plantation veneers.

73 The paper is articulated around five main stages. First, a background of the overall research
74 undertaken is presented and the established database on the probabilistic mechanical properties of
75 the veneers of the three studied species used through the paper is summarised. Second, the
76 principles behind the numerical model are described and hypotheses made are justified. Third, the
77 model is calibrated against a first series of four-point bending tests performed on LVL beams for
78 which the dynamic MOE, tensile and compressive Modulus of Rupture (MOR) of each plies of the
79 beams were assessed. Fourth, the accuracy of the model is further validated against a second
80 independent series of four-point bending tests performed in McGavin, et al. [17]. Finally, using the
81 validated numerical model, the statistical characteristics of the strength of the six commercially
82 available LVL beam sizes are predicted and reported. The design strength (5th percentile) and the
83 statistical data of each investigated case are given and discussed.

84 The focus of this paper is on the strength modelling of the LVL beams. While the ability of the
85 model to predict the overall linear and non-linear behaviour of the beam up to failure is verified in
86 the aforementioned third stage, the use of the model to predict the static beam stiffness is outside
87 the scope of the paper. Indeed, the dynamic MOE of the veneers, measured in line during

88 manufacturing through a resonance method and used as model input herein, is different to the static
89 one. This effect is due to the viscoelastic nature of the wood [18, 19] and also likely from hot
90 pressing the veneers and gluing them during the manufacturing process. Unpublished tests
91 performed by the authors tend to show that these two effects result in the dynamic MOE of the
92 veneer being about 10% lower than its static one within a glued LVL panel. Therefore, while using
93 the veneer dynamic MOE in the model does not change the stress distribution in the beam, it does
94 not result in an accurate determination of the static bending stiffness needed for design purpose.
95 More investigations, outside the scope of this paper, need to be performed to quantify the actual
96 relationship between the static MOE of glued veneer-based products manufactured from the three
97 investigated species and their veneer dynamic MOE. Consequently, the model would in that case
98 also offer the possibility to accurately predict the static bending stiffness of LVL beams.

99

100 **2 INCENTIVE AND DATABASE ON RESOURCES USED**

101 **2.1 General**

102 Incentives to use juvenile hardwood plantation logs in the manufacture of VBC products is
103 observed in Australia [17, 20-22], but also in New-Zealand [23], South America [24] and Europe
104 [25]. These studies show that the use of juvenile hardwood logs results in attractive VBC structural
105 products, yet the number of tests was limited and further studies are required to accurately define
106 their mechanical performances. In this context, this paper results from a collaborative project
107 between universities and the Queensland government aiming at thoroughly quantifying the expected
108 strength of VBC products manufactured from logs sourced from hardwood plantations, either
109 primarily established for pulpwood production or for high quality solid wood markets. In the latter
110 type of plantation, only high quality trees are allowed to mature and the logs originate from the
111 lower quality trees (for instance, those that are crooked, smaller or have too many branches)
112 removed in an operation referred to as “thinning”. In the former type, the logs originate from the
113 plantations being clear cut with no thinning or pruning. In both cases, the trees are typically

114 harvested between 12 and 15 years after establishing the plantation, are about 15 to 30 cm in
115 diameter at breast height and referred to as “early to mid-rotation” trees. The manufacturing
116 industry is challenged with the use of these resources which display significant mechanical
117 properties variations, between and within trees. Because of the high proportion of defects in the
118 resources (knots, gum veins, grain deviation, etc.), the new structural products are expected to have
119 a reduced mean strength and larger variability in their mechanical properties than the conventional
120 LVL and plywood products manufactured from mature trees of the same species, to a degree
121 currently unknown.

122 In a first stage, a database on the mechanical properties of veneers recovered from early to mid-
123 rotation hardwood plantation logs of three species (GMS, SPG and SBG) was established in [16].
124 The database allows probabilistically predicting the compressive and tensile strength of the veneers
125 from their dynamic Modulus of Elasticity (MOE) and total knot area ratio (tKAR), which can be
126 measured in line during manufacturing. The trees used to establish the database and in this study,
127 were 10 to 12, 13 to 16 and 12 to 15 years old, and had an average diameter at breast height over
128 bark (DBHOB) of 20.6, 30.6 and 31.9 cm, for the SPG, SBG and GMS logs, respectively [26, 27].
129 This paper represents the next stage of the project where a numerical model is developed and its
130 potential is illustrated by determining the strength distribution of LVL beams manufactured from
131 the three species investigated in the overall project. As veneers recovered from these three species
132 are used through this paper to first calibrate and validate the model, and then illustrate its
133 applications, it is important to understand their mechanical properties (database). The main results
134 of the research in [16] is therefore summarised hereafter .

135

136 **2.2 Summary of mechanical properties of veneers used**

137 **2.2.1 General**

138 To establish the database on the mechanical properties of early to mid-rotation GMS, SPG and SBG
139 plantation veneer sheets in [16], compression and tension tests on veneers, parallel to the grain,

140 have been investigated by the authors. For each species, 90 veneer sheets were selected and a total
141 540×3 -ply LVL samples were manufactured with the three ply of each sample cut from the exact
142 same veneer sheet. This configuration aimed at determining the average properties of each veneer
143 sheet, as in veneer based composite applications, by including the effects of both hot pressing the
144 samples and the glue used during the manufacturing process. Nominal 2.5 mm veneer sheets were
145 used and the compression and tension samples were 630 mm long $\times$ 100 mm wide and 500 mm
146 long (gauge length) $\times$ 150 mm wide, respectively.

147 2.2.2 Veneer grades

148 Three different veneers grades are defined by the authors in [16] based on the expected distribution
149 of MOE encountered in a mill for the studied species [26]. The three grades, referred to as “Low”,
150 “Medium” and “High”, divide veneers into three bins of an equal number of veneers and mimic a
151 simple way an industrialist could divide the veneers using the veneer MOE value as the sole grading
152 indicator. As how the veneers are divided into bins varies with both the final structural products and
153 the specific strategy adopted by an industrialist, the three grades above were arbitrary chosen for
154 simplicity and are used in the present paper. The MOE cut-off value for each grade and each
155 investigated specie is given in Table 1.

156 2.2.3 Knot distribution

157 Knots or groups of knots create weak zones in timber structural member in which failure usually
158 initiates [28]. The total knot area ratio (tKAR), defined as the ratio of the sum of the projected area
159 of all knots within a timber piece of length $l_{ref} = 150$ mm to the total cross-sectional area of the
160 piece [29], is often used as an indicator to characterise the influence of knots on the timber
161 properties. This approach was used by the authors in [16] and the variation of the tKAR value along
162 the length of the 90 veneer sheets per species were analysed. Assuming that no correlation exists
163 between two consecutive tKAR values within a veneer sheet, the maximum tKAR value of a sheet
164 of length l was probabilistically modelled using a two-parameter Weibull [30] distribution for each
165 grade and species as,

$$P_f = 1 - e^{-\frac{l}{l_{ref}} \left(\frac{1 - tKAR}{m_{tKAR}} \right)^{k_{tKAR}}} \quad (1)$$

where P_f is the probability of having a knot area ratio greater than $tKAR$ in a l long sheet, m_{tKAR} is the scale parameter and k_{tKAR} the shape factor. As Weibull distributions are based on the weakest link theory, the value $1 - tKAR$ was used instead of $tKAR$ in Eq. (1). It is worth noting that for veneer with no defects, i.e. $tKAR = 0$, Eq. (1) leads to $1 - tKAR$ values greater than 1.0 and the actual maximum knot area ratio $tKAR$ of a sheet of length l is calculated from P_f in numerical analyses as,

$$\begin{cases} \text{if } P_f \leq 1 - e^{-\frac{l}{l_{ref}} \left(\frac{1}{m_{tKAR}} \right)^{k_{tKAR}}} & \text{then } tKAR = 1 - m_{tKAR} \left(-\frac{l_{ref}}{l} \ln(1 - P_f) \right)^{\frac{1}{k_{tKAR}}} \\ \text{else } tKAR = 0 \end{cases} \quad (2)$$

Table 2 gives the m_{tKAR} and k_{tKAR} reported in for all species and grades.

2.2.4 Stiffness-strength relationships

The relationship between the MOR (either in tension or compression) of the veneers and their MOE and maximum knot are ratio value ($tKAR$) was expressed as [16],

$$MOR = \alpha \cdot MOE^\beta (1 - \gamma \cdot tKAR) \quad (3)$$

Coefficients α , β and γ in Eq. (3) are summarised in Table 3 for each species and loading type.

The probabilistic variability in the MOR was expressed using two-parameter Weibull distributions, as commonly used to characterise the size effect and variability in timber material properties [31, 32]. Following a similar approach to Foschi, et al. [13], the distribution of the normalised strength, using the prediction in Eq. (3), is given as,

$$P_f = 1 - e^{-\frac{1}{V_{ref}} \int_V \left(\frac{\sigma}{M \alpha \cdot MOE^\beta (1 - \gamma tKAR)} \right)^k dV} \quad (4)$$

where P_f is probability of failure of a volume V at an applied stress σ , V_{ref} is the reference volume, M the scale parameter and k the volume shape factor [33] obtained from the tests performed at constant volume in [16]. Value of V_{ref} (test samples), M and k are given in Table 3 for all species

187 and loading type. In numerical analyses, the tensile and compressive MOR can then be
188 probabilistically expressed in terms of P_f by rearranging Eq. (4) and using the appropriate value in
189 Table 2 and Table 3 as,

$$190 \quad MOR = M\alpha \cdot MOE^\beta (1 - \gamma \cdot tKAR) \left[-\frac{V_{ref}}{V} \ln(1 - P_f) \right]^{1/k} \quad (5)$$

191

192 **3 NUMERICAL MODEL FOR LVL BEAMS**

193 **3.1 General**

194 As mentioned in the introduction, very little research aiming at modelling the strength of VBC
195 products parallel to the grain [2-5] and based on the actual strength of the individual veneers can be
196 found in the literature. Nevertheless, various models have been developed for sawn timber beams,
197 reinforced or not with other materials (see [6-12, 34] for instance), or for Glued-Laminated
198 (Glulam) beams (such as [13-15, 35, 36]).

199 In this paper, the models for sawn timber and Glulam beams described in Buchanan [7] and Foschi,
200 et al. [13], respectively, are adapted to LVL beams. These models are chosen for their simplicity to
201 implement without the need of using a finite element analysis (FEA) software, accuracy and
202 rational failure criteria, reflecting the actual behaviour of timber structural members for which the
203 bending MOR is higher than the tensile one [37].

204 In the following sections, the key mechanical principles behind the modelling of LVL beams,
205 namely appropriate stress-strain models, the incorporation of the size and load effects into the
206 model and the use of a realistic failure criteria, are explained first. In each subsection, when
207 appropriate, hypotheses based on literature are made and the rationale behind them is explained.
208 Second, the practical implementations of the aforementioned key principles into the numerical
209 model are described. The flowchart of the overall model is described at the end in Section
210 “Implementation”. The validity of the hypotheses made is confirmed when validating the numerical
211 model.

212

213 **3.2 *Stress-strain models***

214 3.2.1 *Stress-strain relationships*

215 Figure 1 shows a typical stress-strain relationship for wood loaded parallel to the grain [7], i.e.
216 brittle in tension and ductile in compression. In tension, various models have been adopted in the
217 literature from a perfect linear material with sudden brittle failure [7] to a perfect linear material
218 until peak stress followed by a rather large strain softening [9, 10, 12, 34]. In compression, adopted
219 models vary from purely linear [13] or perfect elastic-plastic stress-strain relationships ([38] as
220 reported in Buchanan [7]) to the model by Glos [39], which more realistically reproduces the actual
221 stress-strain behaviour shown in Figure 1.

222 By using a perfect linear material with sudden brittle failure in tension and a bilinear model in
223 compression (with a strain softening slope of 2% of the MOE), Buchanan [7] accurately predicted
224 experimental results on sawn timber beams. In this study, the same tensile behaviour is used and a
225 tri-linear stress-strain relationship in compression, closer to the Glos [39] model than the bi-linear
226 relationship in [7], is adopted. The tri-linear relationship is shown in Figure 1. The slope m_1 and
227 yielding point (proportional limit) y_1 in Figure 1 are based on the characteristic stress-strain
228 relationships measured from the compression test results performed by the authors in [16].
229 Measured characteristic values of $m_1 = 50\%$ and $y_1 = 1/1.2$ are used for the three investigated
230 species. For the strain softening branch, the slope m_2 in Figure 1 is estimated to best match the
231 experimental results in Section “Model calibration and validation” and a value of $m_2 = 7\%$ is used
232 adopted herein.

233 3.2.2 *Stress-strain distributions*

234 The Euler-Bernoulli beam theory is used in the model, with plane sections remaining plane.
235 Assuming LVL beam in pure bending, as shown in Figure 2 (a), the linear strain profile is shown in
236 Figure 2 (b) and the non-linear stress profile in Figure 2 (c-d) for flat and edge bending,
237 respectively. For each ply, the stress is calculated from the strain profile, the ply MOE and the

238 Hooke's law. Figure 2 shows the shift in neutral axis after yielding occurs in the compression zone.
239 Numerically, strain is increased in increment until failure, with the failure criteria developed in
240 Section "Failure criterion".

241

242 3.3 Size and load effects

243 Individual MOR values of each ply are needed to accurately model the stress-strain behaviour
244 (Figure 1) of the plies. As timber is known to be sensitive to both size and load configuration effects
245 [31], the MOR determined from tension and compression tests on individual veneers must be
246 corrected to take into account the actual dimensions of the plies within the LVL beam and bending
247 moment diagram. The weakest link theory [30] is well accepted to consider these effects in timber
248 structures in both tension and compression [31, 32] and used herein. The tensile or compressive
249 strength MOR_p of each ply of dimensions $L_p \times W_p \times H_p$ within the LVL beam is obtained from the
250 strength MOR_s of the samples of dimensions $L_s \times W_s \times H_s$ used to determine the material properties
251 of the veneers as,

$$252 \quad MOR_p = \left(\frac{L_s}{L_p} \right)^{1/k_{//}} \left(\frac{W_s}{W_p} \right)^{1/k_{\perp}} \left(\frac{H_s}{H_p} \right)^{1/k_{\perp}} MOR_s \quad (6)$$

253 where the grain is parallel to the longitudinal direction L , and $k_{//}$ and $k_{\perp}$ are the shape factors related
254 to the dimension(s) parallel and perpendicular to the grain, respectively. Timber structural members
255 usually have different shape factors $k_{//}$ and $k_{\perp}$ and their values are discussed in Subsection "Shape
256 factors" hereafter.

257 3.3.1 Ply dimensions to be considered

258 The dimensions $L_p \times W_p \times H_p$ of each ply within a LVL beam to be used in Eq. (6) depends on the
259 actual dimensions of the LVL beam $L_b \times W_b \times H_b$ and must also consider the actual stress
260 distribution in the beam [31]. Regarding the length, let's consider a beam loaded in four-point
261 bending, as shown in Figure 3, where L_1 is the distance between the supports and the points of
262 application of the loads and L_2 the distance between loads. The length of the ply in Eq. (6) is

calculated for an equivalent ply of length L_p and subjected to a pure axial stress σ_a to have the same probability of failure as the actual ply of length L_b and subjected to the axial stress distribution $\sigma(x)$ along its length resulting from the bending moment diagram shown in Figure 3 [31]. The Weibull weakest link theory states,

$$1 - e^{-L_p \left(\frac{\sigma_a}{m} \right)^{k_{//}}} = 1 - e^{-\int_{x=0}^{x=L_b} \left(\frac{\sigma(x)}{m} \right)^{k_{//}} dx} \quad (7)$$

Solving Eq. (7) gives [7],

$$L_p = L_2 + \frac{2L_1}{k_{//} + 1} \quad (8)$$

As the stress in the plies does not vary along the beam width, the width W_p of each ply in Eq. (6) is equal to W_b for flat bending and the ply thickness for edge bending.

For the height and while the stress in the plies varies with the beam height, this variation is not considered in Eq. (6) when determining the compressive MOR due to the ductile failure mode inducing the height of the plastic zone to vary within the beam. In tension, this effect is considered in the failure criteria, as detailed in Section “Failure criterion”. In Eq. (6), the height of each ply H_p is therefore taken in this study as the ply thickness for flat bending and H_b for edge bending.

In Eq. (6) and in reference to Section “Summary of mechanical properties of veneers used”, in this paper $L_s = 630 \text{ mm} \times W_s = 100 \text{ mm} \times H_s = 7.5 \text{ mm}$ for plies in compression and $L_s = 500 \text{ mm} \times W_s = 150 \text{ mm} \times H_s = 7.5 \text{ mm}$ for plies in tension.

3.3.2 Shape factors

The shape factors $k_{//}$ and $k_{\perp}$ in Eq. (6) are usually best obtained by the slope method where samples from the same batch, from which one dimension varies, are tested [33]. Nevertheless, the shape factors reported in Section “Summary of mechanical properties of veneers” and Table 3 were obtained using the shape parameter method [33], i.e. from tests performed at constant volume, are unique and cannot be directly used in Eq. (6). While more investigations need to be performed, there is a strong evidence that the unique volume shape factor obtained from the shape parameter method is of the same order of magnitude as the shape factor $k_{//}$ related to the dimension parallel to

the grain and obtained using the slope method. Such evidence is outlined in Table 4 in which tension and compression tests gathered from the literature, where both volume and dimension shape factors were reported, are presented. In all studies but Madsen (1990), the value of the dimension shape factor $k_{//}$ is in the range of the volume shape factors with a maximum difference of 9% in Lam and Varoglu (1990). Based on this observation, an approach is taken in this study to use the volume shape factors reported in Table 3 as the dimension shape factor $k_{//}$, i.e. $k_{//} = \underline{k}$, where k is the value reported in Table 3 which is different for tension and compression. Despite sound scientific background needed to confirm the above, this approach gives accurate results when modelling LVL beams, as developed in Section “Model calibration and validation”.

The size effect in tension and compression of timber structural members is more pronounced in the direction perpendicular to the grain than in the one parallel to the grain. As outlined in Table 4, the shape factor $k_{\perp}$ is about 1.0 to 0.5 the value of $k_{//}$. The comparison between the ninety 150 mm wide and the thirty-nine 100 mm wide SPG samples tested by the authors in [16] and this paper (see Section “Calibration”), respectively, provides a shape factor $k_{\perp}$ of 3.5-4.0. This value is 0.47-0.54 times the shape factor k in Table 3 and a value of $k_{\perp} = 0.5 \times k_{//}$ is adopted herein for the studied species. No distinction is made between the shape factors in the radial and tangential perpendicular directions to the grain in this study.

305

3.4 Failure criterion

Various failure modes are experienced by timber beams with the strength typically reached when brittle failure occurs in the tension zone [7]. In extreme cases, when the tensile strength is significantly higher than the compressive one, the maximum bending moment is purely reached by the formation of a plastic hinge. All these cases must be considered in the model. The latter case is considered in the study with the non-linear compressive stress-strain curve shown in Figure 1 and the former needs to correctly consider the relationship between the bending stress distribution in the

plies and their tensile strength [37]. The numerical and mechanical aspects of this failure mode are discussed hereafter.

3.4.1 Tensile failure in flat bending

Let's consider a ply in tension in a LVL beam tested on flat. The non-uniform stress distribution through the ply thickness is shown in Figure 2 (c). Using the Weibull weakest link theory, in which the probability of failure of the non-uniformly stressed ply is equal to the one of the uniformly stressed ply in tension, tensile failure occurs when the stress at the ply bottom extreme fibre σ_{bottom} (see Figure 2 (c)) reaches the ply tensile strength MOR_p obtained from Eq. (6) multiplied by the factor $S_{bending}$ given as,

$$S_{bending} = \frac{\left(\left(\frac{\sigma_{top}}{\sigma_{bottom}} - 1 \right) (1 + k_{\perp}) \right)^{\frac{1}{k_{\perp}}}}{\left(\left(\frac{\sigma_{top}}{\sigma_{bottom}} \right)^{1+k_{\perp}} - 1 \right)} \quad (9)$$

where σ_{top} is the stress at the ply top extreme fibre.

3.4.2 Tensile failure in edge bending

The non-uniform tensile stress distribution in a ply on edge bending is shown in Figure 2 (d). The factor $S_{bending}$ is derived in [7] and expressed as,

$$S_{bending} = \left(\frac{1 + k_{\perp}}{D_{neutral}/H_b} \right)^{\frac{1}{k_{\perp}}} \quad (10)$$

where $D_{neutral}$ is the distance from the bottom of the beam to the neutral axis.

3.4.3 Limitation of tensile MOR

For small LVL beams, the tensile MOR calculated from Eq. (6, 9) or Eq. (6, 10) reaches unrealistic high values, while in quasi-brittle materials, when the volume $V \rightarrow 0$, the strength should be finite. Note that for increasing volume, after a transition phase, the weakest link theory applies [40]. For wood, the tensile MOR can be limited, to the extreme case, to the tensile capacity of the cell walls. Especially considering that while woods differ in their mechanical properties, the properties of the

cell walls are roughly similar for all types of wood [28]. The cell walls tensile strength is in the order of 200 to 400 MPa [28, 41, 42], i.e. about 1.5 to 2.5 times higher than the veneer tensile MOR found by the authors in [16]. For latewood *pinus sylvertris* L., Krauss, et al. [42] reported a ratio of about 2 between the wood and cell walls tensile strengths.

It was observed when calibrating the numerical model in Section “Model calibration and validation” that the tensile strength at the extreme fibre of a ply and obtained from Eq. (6, 9) or Eq. (6, 10) cannot be higher than twice MOR_s for the SPG species, with MOR_s is obtained in [16] at a nominal reference volume $V_{ref} = 500 \text{ mm} \times 150 \text{ mm} \times 7.5 \text{ mm}$ (dimensions of tensile samples, see Section “Summary of mechanical properties of veneers”).

Now, let’s assume there exists a minimum volume V_{min} for which the MOR is equal to the tensile strength of the cell walls, the limiting factor $LF_{Species}$ of any analysed species can then be deduced from the limitation factor $LF_{SPG} = 2.0$ of the SPG species and the weakest link theory given that,

$$\frac{MOR_{CellWall}}{MOR_{s,SPG}} = LF_{SPG} = \left(\frac{V_{ref}}{V_{min}} \right)^{\frac{1}{k_{SPG}}} \quad (11a) \quad \text{and} \quad \frac{MOR_{CellWall}}{MOR_{s,Species}} = LF_{Species} = \left(\frac{V_{ref}}{V_{min}} \right)^{\frac{1}{k_{Species}}} \quad (11b)$$

where $MOR_{CellWall}$ is the tensile MOR of the cell walls common to all species, $MOR_{s,SPG}$ and $MOR_{s,Species}$ are the tensile strength of the SPG and analysed species at the reference volume V_{ref} , respectively, and k_{SPG} and $k_{Species}$ are the shape factors of the SPG and analysed species, respectively. Combining Eq. (11a,b) gives,

$$LF_{Species} = (LF_{SPG})^{k_{SPG}/k_{Species}} \quad (12)$$

Using the volume shape factors of the species reported in Table 3, the limiting factors are found to be equal to 1.83 and 2.39 for the GMS and SBG species, respectively.

Figure 4 illustrates the limiting factors for the MOR, not considering the transition phase from small to large volume for which the weakest link theory applies [40].

358 3.5 Implementation

359 Figure 5 shows the overall flowchart of the model for both flat and edge bending taking into
360 account the model characteristics detailed in previous sections. In each step, the strain ε_{bottom} at the
361 beam bottom fibre is increased by a value $\Delta\varepsilon_{bottom}$ and the new position of the neutral axis is
362 iteratively calculated to satisfy the equilibrium of forces in the cross-section. The moment applied
363 to the beam is then calculated from this equilibrium. The curvature and associated beam deflection
364 at each step are obtained from classical beam theory. When tensile failure occurs in a ply for a beam
365 tested (i) on flat, the MOE of the ply is set to zero and a new equilibrium is calculated, and (ii) on
366 edge, as the tensile failure was observed to gradually occur near the extreme fibre of the ply on a
367 height of a few millimetres, the MOE of the ply is set to zero on an arbitrary height equal to twice
368 the ply thickness and a new equilibrium is then calculated. The algorithm is run until the moment in
369 the cross-section drops by more than 25% of its maximum value. At that stage, increase in the MOR
370 value due to load redistribution after failure of one veneer is more than unlikely.

371

372 4 MODEL CALIBRATION AND VALIDATION

373 4.1 Calibration

374 4.1.1 Experimental program

375 Four 8-ply and one 7-ply 1,200 mm $\times$ 300 mm LVL panels were manufactured to calibrate and test
376 the accuracy of the numerical model. The overall experimental program is illustrated in Figure 6.
377 Specifically, in total thirty-nine 1,200 mm $\times$ 1,200 mm $\times$ 2.5 mm SPG veneer sheets were used and
378 each sheet was cut into four 1,200 mm $\times$ 300 mm strips, with the grain direction running along the
379 length of the strips. The longitudinal dynamic MOE of each strip was measured using the BING
380 software [43] as in [16].

381 The first strip out of each sheet was used to manufacture the five LVL panels and the three
382 remaining strips were glued together to form thirty-nine 3-ply LVL panels. The latter panels were
383 used to assess the tensile and compressive MOR of each ply of the five LVL panels following the

384 same protocol to the tests reported in [16], at the exception that the tension samples were 100 mm
385 wide instead of 150 mm. The same gluing procedure used to determine the mechanical properties of
386 the veneers [16] was followed to manufacture all LVL panels.

387 Various strategies were used to manufacture the five LVL panels:

- 388 • Panel 1 had 8 plies and was manufactured from randomly layered veneers of dynamic MOE
389 greater than 20 GPa.
- 390 • Panels 2 and 3 were similar to Panel 1 but from veneers of dynamic MOE between 16 GPa and
391 20 GPa, and less than 16 GPa, respectively.
- 392 • Panel 4 also had 8 plies and was manufactured from veneers with dynamic MOE (i) less than 16
393 GPa for the two middle layers, (ii) between 16 GPa to 20 GPa for each layer on each side of the
394 two middle layers and (iii) greater than 20 GPa for the two external layers on each face.
- 395 • Panel 5 had 7 plies and was manufactured from randomly selected veneers.

396 Out of each panel, six 400 mm × 95 mm and six 400 mm × 30 mm LVL samples were cut for
397 bending tests on flat and edge, respectively. A four-point bending configuration was used, with a
398 total span of 380 mm and 150 mm between the points of application of the loads. The load was
399 applied through rollers and the relative deformation $\delta_{relative}$ between the end of the rollers and the
400 mid-span at the bottom fibre of the beam was recorded. After performing the experiments, it was
401 noticed that the rollers experienced bending during the tests. Their relative stiffness to be taken into
402 account in the numerical model was estimated using Finite Element Analysis to be 20.1 kN/mm for
403 edge bending and, due to larger LVL samples minimising the bending deformation of the rollers,
404 50.7 kN/mm for flat bending. Figure 7 shows the test set-up. The tests were run in displacement
405 control in a 30 kN LLOYD universal testing machine to target failure in 3 to 5 minutes. The
406 bending modulus of rupture $MOR_{bending}$ of each LVL sample is calculated as,

407

$$MOR_{bending} = \frac{\frac{P}{2} L_1}{\frac{W_b H_b^2}{6}} \quad (13)$$

where L_I is the distance between the supports and the points of application of the load, P is the maximum applied load and W_b and H_b are measured width and height of the sample, respectively. The samples were kept at 20°C and 65% relative humidity until they reached moisture equilibrium before testing. Table 5 gives the average bending MOR for each panel.

4.1.2 Numerical procedure

The model was calibrated against the above bending tests. As reported in previous sections, the following parameters were observed to minimise the average error between the experimental tests and the numerical results:

- Strain softening branch slope in compression: $m_2 = 7\%$.
- Relationship between shape factors: $k_{\perp} = 0.5 \times k_{\parallel}$.
- Limitation factor of tensile MOR for SPG plies based on reference volume $V_{ref} = 500 \text{ mm} \times 150 \text{ mm} \times 7.5 \text{ mm}$: $LF_{SPG} = 2$.

The knot area ratios of the veneers were not measured and the tensile and compressive strength of the plies within the LVL beam (MOR_p in Eq. (6)) is further corrected to consider the different probability of having large strength reduction defects between the plies within the beam of equivalent length L_p (Eq. (8)) and the 3-ply LVL samples of length L_s used to determine the strength (MOR_s) of the veneers. This correction factor CF_{tKAR} is calculated using the Weibull weakest link theory as,

$$CF_{tKAR} = \frac{1 - \gamma \cdot tKAR_p}{1 - \gamma \cdot tKAR_s} = \left(\frac{L_s}{L_p} \right)^{1/k_{tKAR}} \quad (14)$$

where $tKAR_p$ and $tKAR_s$ are the values of the knot area ratio of the veneers within the LVL beam and the 3-ply LVL samples, respectively, and γ is defined in Eq. (3) and given in Table 3 for all species and loading types. k_{tKAR} is the knot area ratio shape factor reported in [16] and calculated following the methodology summarised in Section “Summary of mechanical properties of veneers” by using $1 - \gamma \cdot tKAR$ instead of $1 - tKAR$ in Eq. (1).

432 4.1.3 Numerical results

433 Table 5 gives the bending MOR resulting from the numerical model for all panels using the
434 measured dynamic MOE and tensile and compressive MOR of each ply as model input, as outlined
435 in the flowchart in Figure 5. Table 5 shows that the numerical model is able to accurately predict
436 the bending strength of all panels with an average and maximum error of 1.5% and 9% respectively.
437 Figure 8 plots both the recorded and numerical relative displacement of the beam against the
438 applied bending moment for Panels 1 and 3. Panels 2, 4 and 5 show similar curves to the ones
439 presented in Figure 8. In Figure 8, the numerical displacement is corrected to consider the stiffness
440 of the test rig. Figure 8 shows that the model is able to accurately replicate the overall non-linear
441 behaviour of the LVL samples in bending. The post failure trend observed on edge bending, due to
442 the load redistribution when the plies successively fail, are also captured by the model. However,
443 relative to the test results, the model results in similar and lower elastic bending stiffness, especially
444 for flat bending. This difference is likely attributed to an overestimation of the loading rig stiffness
445 and the difference between the veneer dynamic MOE used in the model and its actual static value
446 within the LVL beams. Nevertheless, fundamental to the scope of the study, the overall linear and
447 non-linear behaviour of the beams is well captured by the model, resulting in an accurate estimation
448 of the bending MOR.

449

450 4.2 Validation

451 4.2.1 Experimental program

452 Eight SPG and eight GMS 13-ply LVL 1,080 mm $\times$ 1,800 mm panels were manufactured in
453 McGavin, et al. [17] using four different construction strategies and 3.0 mm nominal thick veneers.
454 Out of each panel, four 1,050 mm $\times$ 95 mm and four 1,050 mm $\times$ 55 mm beams were cut for flat
455 and edge four-point bending tests, respectively. The tests were performed following the testing
456 recommendations in the Australian/New-Zealand standard AS/NZS 4357.2 (AS/NZS, 2006), i.e.
457 with a distance between supports of 18 times the beam dimension parallel to the loads and a

458 distance between loads of 1/3 the total span. The samples were stored at 20°C and 65% relative
459 humidity until moisture content equilibrium before testing.

460 Only the dynamic MOE of each veneer sheet was recorded before manufacturing the panels and the
461 actual MOR of each sheet was not assessed as in the previous calibration section.

462 4.2.2 Numerical procedure

463 The numerical model was run following the flowchart in Figure 5. For each ply, the following
464 values were randomly determined using the measured dynamic MOE and the Weibull distributions
465 summarised in Section “Summary of mechanical properties of veneers”: (i) a tKAR value from Eq.
466 (2) and the ply equivalent length L_p , and (ii) a tensile and compressive strength from Eq. (5) and the
467 previous tKAR value. Per panel, 1,000 Monte Carlo simulations were run to estimate the expected
468 distribution of the bending MOR of the tests performed in McGavin, et al. [17].

469 4.2.3 Numerical results

470 Table 6 gives the ratios of the simulated bending MOR at the 5th, 50th and 95th percentile to the
471 measured average MOR of each tested panel. At the 50th percentile, the average ratio of all panels
472 ranges from 0.93 (SPG on edge) to 1.17 (GMS on flat). The latter ratio of 1.17 is biased towards
473 Panels 6 and 8 for which high ratios of 1.58 and 1.61, respectively, are obtained. These low
474 measured bending MOR relative to the prediction are likely due to weak veneers at the extreme
475 tensile fibre, with non-negligible diving grain angles, that govern the strength on flat. Removing
476 Panels 6 and 8 from the calculation, the average ratio of the 50th percentile prediction to the
477 measured MOR of the GMS panels tested on flat drops to as low as 1.03. For GMS Panels 6 and 8,
478 the measured MOR on flat correspond to MOR values found at the 0.5th to 1st percentile in the
479 simulations, respectively.

480 Disregarding GMS Panels 6 and 8 on flat, the minimum and maximum ratio of the 50th percentile
481 prediction to the measured MOR are 0.82 for SPG Panel 7 on flat and 1.22 for GMS Panel 7 on flat,
482 respectively. While the numerical model seems to slightly underestimate and overestimate the MOR
483 for the SPG and GMS species, respectively, it accurately matches the experimental results overall

484 with an average ratio of the 50th percentile prediction to the measured MOR for all panels
485 (excluding GMS Panels 6 and 8 on flat) of 1.00 with a relatively low coefficient of variation (CoV)
486 of 11.1 %.

487

488 **5 PARAMETRIC STUDY**

489 **5.1 General**

490 The expected bending strength distribution of six commercially available LVL beam sizes,
491 manufactured from early to mid-rotation GMS, SPG and SBG hardwood plantation veneers, are
492 investigated using the validated numerical model. The beams are manufactured from 3.0 mm thick
493 veneers and are (i) 15-ply (45 mm thick) of height of 200 mm, 300 mm and 400 mm, and (ii) 21-ply
494 (63 mm thick) of height of 240 mm, 400 mm and 600 mm. Four different construction strategies are
495 investigated:

- 496 • Low (L): the beams are solely manufactured from the “Low” grade veneers (refer to Section
497 “Summary of mechanical properties of veneers” for grade definition and [16] for MOE cut-off
498 for each species and grade).
- 499 • Medium (M): the beams are solely manufactured from the “Medium” grade veneers.
- 500 • High (H): the beams are solely manufactured from the “High” grade veneers.
- 501 • All (A): the beams are manufactured with an equal number of veneers from each grade, using the
502 “Low” for the middle veneers, the “Medium” for the adjacent veneers and the “High” for the
503 external veneers.

504 The expected distribution of MOE encountered for the studied veneered resources in a mill [26] and
505 the strength distribution of the veneers summarised in Section “Summary of mechanical properties
506 of veneers” are used as model input following the flowchart in Figure 5. Per studied configuration,
507 10,000 simulations are run. Four-point bending tests are simulated on both flat and edge following
508 the recommendations in the Australian/New-Zealand standard AS/NZS 4357.2 (AS/NZS, 2006).

509 The uncertainty in the numerical model in predicting the actual bending strength of the LVL beams
510 is also taken into account in this parametric study. The average predicted to actual strength ratio
511 (Mean/Nominal = 1.00) and CoV (= 0.10), derived from the tests reported in Sections “Calibration”
512 and “Validation” (at the exception of GMS Panels 6 and 8), are used with an assumed normal
513 distribution (see Figure 5).

514

515 **5.2 Limitations**

516 The veneers are assumed to be as long as the LVL beams in the study and the size effects are
517 consequently calculated considering the entire length of the beam, as in both Section “Size and load
518 effects”. The effect on the bending MOR of joints between veneers will need to be investigated in
519 further studies.

520

521 **5.3 Results**

522 Figure 9 gives the strength distribution of the 200×45 LVL beams on both flat and edge bending for
523 all strategies and species. The simulations for all remaining beams show a similar trend to the ones
524 presented in Figure 9. As expected the MOR variability in edge bending is less than on flat bending.
525 Also edge bending generally gives higher bending MOR than flat bending for the same
526 configuration. Using all the veneered resources in the construction strategy allows reaching bending
527 MOR, between the “Medium” and “High” construction strategies on flat bending and very similar
528 to the “Medium” strategies on edge bending.

529 The 3-parameter Weibull distribution for which the cumulative distribution function $F(MOR_{bending})$
530 is expressed in the form,

$$531 \quad F(MOR_{bending}) = 1 - e^{-\left(\frac{MOR_{bending} - c}{b}\right)^a} \quad (15)$$

532 provides the best fit for all strength distributions, with a low Kolmogorov-Smirnov test result
533 typically less than 0.04. The bending design MOR, defined as the 5th percentile MOR, and the

parameters a , b and c in Eq. (15) are reported in Table 7 and Table 8, respectively. Table 8 is given as a reference for further studies. The mean and standard deviation MOR are not reported herein and can be deduced from parameters a , b and c .

Softwood LVL beams available in Australia [45, 46] have design bending MOR of about 42 to 44 MPa and 38 MPa for 200 mm and 600 mm deep beams, respectively, on edge and 42 MPa on flat. Within the limitation of the model, design bending MOR, superior to the previous commercialised softwood LVL beams, are reported in Table 7. Only the SPG beams manufactured from low grade veneers have similar or lower bending MOR to the ones of the commercialised softwood beams. For all analysed species and configurations, the LVL beams manufactured from the high grade veneers have design bending MOR 1.5 to 2.5 greater than the commercialised softwood beams. On edge bending, using all veneer grades in the manufacturing gives design bending MOR ranging from 48.1 MPa for the 600×63 SPG beams to 79.2 MPa for the 200×45 GMS beams. For all flat and edge studied cases, the highest design values of 83.9 and 97.2 MPa, respectively, are both found for the 200×45 GMS beams.

In general, the SPG and GMS beams have the lowest and highest design bending MOR of all three analysed species.

6 CONCLUSION

A numerical mechanical model to predict the strength of LVL beams on both flat and edge bending has been developed in this paper. This model extends the previous works on Glulam by Foschi, et al. [13] and sawn timber by Buchanan [7] beams to LVL beams. The model was calibrated and validated against experimental results and observed to accurately reproduce the test results with an mean predicted to experimental strength ratio of 1.00 and low coefficient of variation of 0.10.

Using an established data of the veneer mechanical properties from characteristics which can be measured in line during manufacturing, the applications of the model were then illustrated. The strength distribution of six commercially available LVL beam sizes, manufactured from early to

560 mid-rotation Gympie messmate, spotted gum and southern blue gum subtropical hardwood
561 plantation veneers was estimated. Various construction strategies for each beam were analysed by
562 either solely using the low, medium and high quality veneers in the manufacturing process or all
563 qualities. 3-parameters Weibull distributions were found to accurately match the strength
564 distributions of all analysed cases. Despite the high number of defects in the veneered resources, the
565 design bending strength of hardwood LVL beams manufactured from early to mid-rotation
566 hardwood plantation logs were found to be superior to commercialised softwood LVL beams. For
567 all construction strategies, it ranges between 37.0 MPa to 83.9 MPa on flat and between 38.7 MPa
568 to 97.2 MPa on edge. The products therefore represent a viable market for the studied small
569 diameter logs.

570

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576

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Table 1: MOE cut-off values for each grade and investigated species in Gilbert, et al. [16]

Species	Low grade (MPa)	Medium grade (MPa)	High grade (MPa)
SPG	MOE < 14,000	14,000 ≤ MOE < 17,900	MOE ≥ 17,900
SBG	MOE < 14,100	14,100 ≤ MOE < 18,000	MOE ≥ 18,000
GMS	MOE < 14,600	14,600 ≤ MOE < 18,200	MOE ≥ 18,200

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Table 2: Two-parameter Weibull distribution scale parameter m_{tKAR} and shape factor k_{tKAR} for predicting the knot area ratios (1-tKAR) for all grades of the three investigated species in Gilbert, et al. [16]

Species	Low grade		Medium grade		High grade	
	m_{tKAR}	k_{tKAR}	m_{tKAR}	k_{tKAR}	m_{tKAR}	k_{tKAR}
SPG	0.885	10.43	0.939	13.35	0.981	17.74
SBG	0.948	22.67	0.974	25.39	0.980	43.13
GMS	0.977	30.02	0.991	46.99	0.998	63.86

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Table 3: Coefficients α , β , γ and two-parameter Weibull distribution scale parameter M , shape factor k and reference volume V_{ref} for predicting the MOR of the three investigated species in Gilbert, et al. [16]

Species	Compressive tests					Tension tests				
	α	β	γ	k	$M(V_{ref}/V)^{1/k}$	α	β	γ	k	$M(V_{ref}/V)^{1/k}$
SPG	0.490	0.506	0.437	15.60	1.035	0.025	0.842	1.110	7.38	1.068
SBG	0.810	0.437	0.246	11.74	1.039	0.117	0.694	1.575	6.47	1.063
GMS	0.680	0.474	0.388	16.09	1.031	0.013	0.896	1.211	8.48	1.050

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Table 4: Shape factors in tension and compression reported in the literature

Reference	Species	Timber type	Grade	Volume shape factor	Dimension shape factor $k_{//}$	Dimension shape factor $k_{\perp}$	Factor calculated at
<i>Tension</i>							
Clouston, et al. [47]	Eastern pine	PSL	--	10.5-11.0 ⁽¹⁾	10.65	--	Average strength
		LVL	--	11.6-12.1 ⁽¹⁾	11.76	--	
Takeda, et al. [48]	Japanese larch	Lumber	High	3.29-3.97	3.73	--	50th percentile
			Low	3.16-3.40	3.29	--	
Lam, et al. [49]	Spruce-pine-fir	Lumber	Select structural	6.67-9.99	10.89 ⁽¹⁾	--	50th percentile
			Nº 2	4.29-5.34	4.96 ⁽¹⁾	--	
Madsen [50]	Spruce-pine-fir	Lumber	Standard and better	2.91 ⁽¹⁾	8.45	--	50th percentile
			All grades	2.77	7.69	--	
Zhou, et al. [51]	Chinese Larch	Lumber	Visual grade I _c to IV _c	--	4.35-4.76 ⁽²⁾	3.03-3.45	5th percentile
ASTM D1900-16 [52]	--	Lumber	--	--	7.14	3.45-∞	--
Barrett, et al. [53]	D Fir-Larch, Hem-Fir, Spruce-pine-fir	Lumber	All grades	--	5.88 ⁽³⁾⁽⁴⁾	4.76 ⁽⁴⁾	Average strength
<i>Compression</i>							
Madsen [50]	Spruce-pine-fir	Lumber	Standard and better	6,26 ⁽¹⁾	12.2	--	50th percentile
ASTM D1900-16 [52]	--	Lumber	--	--	∞	7.69-∞	--
Zhou, et aas/nzsl. (2015)	Chinese Larch	Lumber	Visual grade I _c to IV _c	--	--	5-8.33	5th percentile
Barrett, et al. (1995)	D Fir-Larch, Hem-Fir, Spruce-pine-fir	Lumber	All grades	--	10 ⁽⁶⁾	9.09	Average strength

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⁽¹⁾: Calculated from figures reported in the paper
⁽²⁾: Based on bending test results. As both bending and tension tests have brittle tensile failure, they produce similar length effects [53]
⁽³⁾: Based on other studies, including Madsen [50]
⁽⁴⁾: Average for all analysed species

Table 5: Numerical model calibration: Experimental and numerical bending MOR results

	Experiment ⁽¹⁾ (MPa)	Flat Model (MPa)	Ratio Model/Exp.	Experiment ⁽¹⁾ (MPa)	Edge Model (MPa)	Ratio Model/Exp.
Panel 1	155.3	162.7	1.05	138.0	142.6	1.03
Panel 2	128.3	123.8	0.96	128.4	122.3	0.95
Panel 3	112.6	102.7	0.91	98.4	92.4	0.94
Panel 4	151.2	138.0	0.91	124.7	121.5	0.97
Panel 5	151.6	151.2	1.00	135.6	146.4	1.08
Mean			0.97			1.00
CoV (%)			6.0			5.9

⁽¹⁾: Average experimental bending strength based on six tests, except for Panels 1 and 3 (five tests on flat) and Panel 4 (two tests on flat and four tests on edge)

Table 6: Numerical model validation: Ratios of numerical simulation bending MOR at 5th, 50th and 95th percentile to experimental MOR in McGavin, et al. [26]

	SPG						GMS					
	5 th /Exp. ⁽¹⁾		50 th /Exp. ⁽¹⁾		95 th /Exp. ⁽¹⁾		5 th /Exp. ⁽¹⁾		50 th /Exp. ⁽¹⁾		95 th /Exp. ⁽¹⁾	
	Flat	Edge	Flat	Edge	Flat	Edge	Flat	Edge	Flat	Edge	Flat	Edge
Panel 1	0.71	0.98	0.89	1.06	0.99	1.12	0.74	0.95	0.92	1.02	1.01	1.07
Panel 2	0.73	0.88	0.92	0.95	1.01	1.01	0.78	1.00	0.94	1.07	1.03	1.13
Panel 3	0.72	0.78	1.00	0.86	1.16	0.94	0.78	1.11	0.97	1.21	1.12	1.30
Panel 4	0.71	0.79	1.01	0.88	1.17	0.96	0.83	0.86	1.11	0.94	1.32	1.01
Panel 5	0.70	0.79	0.89	0.85	0.99	0.92	0.83	0.97	1.03	1.06	1.17	1.14
Panel 6	0.82	0.88	1.06	0.96	1.19	1.03	1.25	1.07	1.58	1.18	1.75	1.27
Panel 7	0.66	0.85	0.82	0.92	0.94	0.99	1.07	1.03	1.22	1.11	1.34	1.18
Panel 8	0.91	0.90	1.16	0.97	1.35	1.04	1.31	1.08	1.61	1.16	1.85	1.24
Mean	0.74	0.86	0.97	0.93	1.10	1.00	0.95	1.01	1.17	1.09	1.32	1.17
CoV (%)	11.1	8.1	11.2	7.3	12.6	6.6	24.1	8.0	23.8	8.3	24.1	8.5

⁽¹⁾: Average experimental bending strength based on four tests

Table 7: Flat and edge bending parametric study results – design bending MOR (5th percentile of simulations)

Beam	Grade ⁽¹⁾	SPG		SBG		GMS	
		Flat (MPa)	Edge (MPa)	Flat (MPa)	Edge (MPa)	Flat (MPa)	Edge (MPa)
200×45	L	43.2	52.2	53.1	61.0	47.6	58.6
	M	62.6	73.8	68.4	74.2	67.9	81.1
	H	81.6	92.6	81.7	84.2	83.9	97.2
	A	75.2	73.0	75.7	73.2	77.6	79.2
300×45	L	39.3	43.9	47.1	52.2	43.6	51.1
	M	57.4	63.4	61.6	65.1	61.3	71.7
	H	74.2	80.8	74.5	75.0	76.7	87.1
	A	68.4	63.0	67.9	64.3	71.5	70.3
400×45	L	37.0	38.7	43.7	46.5	40.7	46.3
	M	53.1	56.6	56.6	58.6	57.9	65.9
	H	68.5	73.5	68.4	69.0	72.3	80.7
	A	63.7	56.4	63.4	58.9	66.2	64.6
240×63	L	41.7	48.7	48.8	56.7	45.3	55.4
	M	60.9	69.3	63.7	70.2	65.0	77.0
	H	76.4	86.8	75.5	80.2	79.8	92.8
	A	69.1	68.5	68.8	69.8	71.6	75.9
400×63	L	37.4	39.1	43.0	46.8	40.5	46.8
	M	54.8	57.1	55.9	59.0	58.3	66.1
	H	68.7	73.6	65.9	68.9	71.9	80.5
	A	61.6	56.8	60.4	58.7	64.8	64.7
600×63	L	33.7	32.3	38.2	39.4	37.1	40.4
	M	49.9	48.4	50.7	50.7	54.1	58.1
	H	62.5	63.5	59.8	60.5	65.9	71.8
	A	56.1	48.1	54.0	50.5	59.3	56.7

⁽¹⁾: L = Low MOE veneers, M = Medium MOE veneers, H = High MOE veneers and A = All veneers

Table 8: Flat and edge bending parametric study results – 3-Parameters Weibull *a*, *b*, *c* coefficients

Beam	Grade ⁽¹⁾	SPG		SBG		GMS	
		<i>a/b</i> (MPa)/ <i>c</i>	<i>a/b</i> (MPa)/ <i>c</i>	<i>a/b</i> (MPa)/ <i>c</i>	<i>a/b</i> (MPa)/ <i>c</i>	<i>a/b</i> (MPa)/ <i>c</i>	<i>a/b</i> (MPa)/ <i>c</i>
		Flat	Edge	Flat	Edge	Flat	Edge
200×45	L	4.7/55.7/12.4	3.9/29.9/37.4	5.1/60.0/18.6	3.9/31.4/45.5	5.1/55.3/14.5	3.8/32.0/42.9
	M	5.1/72.7/20.8	4.0/38.9/54.2	6.0/78.5/19.1	4.3/40.0/53.1	5.0/66.4/29.2	3.8/39.7/61.9
	H	5.5/90.2/26.6	4.1/48.0/67.7	5.6/78.6/34.3	4.2/43.9/61.5	5.1/79.3/37.3	3.9/47.4/74.3
	A	5.6/83.1/24.6	4.0/39.3/53.3	6.0/78.9/25.1	4.4/41.6/50.7	5.3/76.3/31.8	4.6/47.4/52.9
300×45	L	4.5/49.0/12.8	4.2/28.6/28.6	5.3/57.8/12.9	3.9/27.9/38.4	4.4/44.4/19.5	3.6/26.6/38.9
	M	4.7/62.3/23.0	4.1/34.6/45.8	5.6/70.0/18.9	4.1/33.7/48.1	5.2/64.7/23.1	3.8/36.2/54.2
	H	4.8/74.5/32.1	4.2/43.9/58.0	5.4/72.5/30.5	4.5/42.2/51.8	5.1/74.0/33.8	4.5/50.0/59.6
	A	5.0/71.0/27.6	4.0/34.5/45.6	5.9/72.8/22.2	3.9/32.9/48.2	4.9/66.4/33.5	4.4/40.7/48.3
400×45	L	4.3/43.1/14.6	3.6/22.3/28.1	5.3/55.1/10.7	4.1/27.0/32.4	4.3/40.6/19.4	3.8/26.0/33.7
	M	4.7/59.7/20.1	4.0/30.4/41.6	5.9/69.4/13.3	4.2/31.9/42.1	4.7/55.7/27.0	4.3/36.5/46.5
	H	5.4/78.9/21.2	3.9/38.0/54.8	5.4/70.3/26.1	4.0/34.5/51.7	5.3/73.8/27.9	4.0/41.0/60.3
	A	5.2/70.7/22.2	4.3/33.3/38.8	6.0/70.9/18.5	4.5/33.6/40.3	5.3/69.7/24.4	4.3/36.1/45.7
240×63	L	4.3/39.6/20.3	4.5/30.8/31.6	4.7/46.8/22.2	4.2/31.7/40.2	4.5/39.4/23.4	4.2/31.8/38.7
	M	3.9/43.8/38.8	4.4/38.9/48.2	4.1/47.4/39.7	4.2/36.4/51.3	3.8/42.1/44.3	4.0/38.5/57.6
	H	5.1/70.2/34.1	4.2/47.0/62.4	4.8/61.6/40.3	4.3/42.7/57.6	5.0/67.1/39.6	4.0/45.5/70.4
	A	4.7/62.4/33.7	4.1/36.7/49.8	4.9/58.9/35.3	3.7/32.1/55.0	4.6/59.6/38.2	4.4/42.4/52.6
400×63	L	4.3/34.5/18.8	3.8/21.3/28.7	4.5/39.0/21.6	4.4/27.6/31.7	4.8/38.6/17.9	4.3/27.7/31.9
	M	4.8/47.6/26.5	3.8/28.6/43.2	4.0/41.2/35.2	4.3/32.1/41.8	4.5/44.6/33.4	4.4/36.3/46.6
	H	4.3/54.1/39.1	4.2/39.4/52.9	5.7/66.2/24.2	5.1/43.3/42.9	4.6/55.3/40.5	4.0/40.9/60.2
	A	5.3/64.2/22.1	4.1/31.1/40.6	4.9/53.8/29.8	4.5/33.6/40.3	4.4/51.4/36.9	4.3/36.3/45.3
600×63	L	4.3/31.5/16.8	3.9/18.8/22.8	5.0/39.6/14.9	3.6/19.5/30.4	4.6/33.0/18.5	4.2/23.5/28.1
	M	3.8/34.3/33.1	4.3/27.9/33.4	3.6/33.8/35.0	4.2/27.1/36.7	3.9/35.2/36.3	4.4/32.7/40.5
	H	4.5/51.1/34.0	4.7/38.8/41.5	4.2/45.4/36.3	4.7/36.1/39.8	4.7/52.7/35.7	4.0/36.7/53.3
	A	4.3/48.3/29.9	3.9/25.1/35.8	4.6/47.1/28.4	3.9/25.4/38.2	4.9/52.6/29.0	4.9/35.9/36.0

⁽¹⁾: L = Low MOE veneers, M = Medium MOE veneers, H = High MOE veneers and A = All veneers

Figure 1: Stress-strain curve relationship for wood loaded parallel to the grain

Figure 2: (a) LVL beam in bending, (b) linear strain profile, (c) non-linear stress profile on flat, and (d) non-linear stress profile on edge

Figure 3: LVL beam in four-point bending

Figure 4: Limiting factor for MOR

Figure 6: Experimental procedure used to validate the model

Figure 7: Four-point bending test set-up for model calibration

Figure 8: Comparison between experimental and numerical results for Panels 1 and 3, (a) flat and (b) edge bending

(a)

(b)

Figure 9: Bending MOR simulated distributions on (a) flat and (b) edge bending