Numerical modeling of liquefaction in loose sand deposits subjected to ocean waves

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Abstract

The failure of marine structures is often attributed to liquefaction in loose sand deposits that are subjected to ocean waves. In this study, a two-dimensional integrated numerical model is developed to characterize the liquefaction behaviours of loosely deposited seabed foundations under various types of ocean waves. In the present model, Reynolds-Averaged Navier-Stokes (RANS) equations are used to simulate the surface wave motion, and Biot's consolidation equations are used to link the solid-pore fluid interactions in a porous medium. A poro-elasto-plastic solution is used to reproduce foundation behaviour under cyclic shearing. Unlike previous investigations, both oscillatory and residual soil responses were considered; they are coupled in an instantaneous approach. Verification of the model results to the previous centrifugal wave tests is carried out, obtaining fairly good agreement. Numerical examples show that foundation behaviour under various types of wave loading, particularly standing waves or a solitary wave, embodies a completely two-dimensional process in terms of residual pore pressure development. The parametric studies demonstrate that liquefaction caused by the build-up of pore pressures is more likely to occur in loosely deposited sand foundations with poor drainage and under large waves.

Keywords: Wave; Liquefaction; Loose sand deposit; Pore pressure accumulation.

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1. Introduction

Marine installations, such as breakwaters, pipelines, and offshore wind turbines have been widely constructed in coastal regions as a result of ever-increasing ocean engineering activities in the past few decades. One of the main concerns involved in the design of these structures is the instability of seabed foundations subjected to ocean waves. As typical marine sediments, loose sandy soils are widely distributed in the oceanic environment. In the practice of ocean engineering, it is sometimes unavoidable to select loose sand deposits as seabed foundations, i.e., submarine pipelines or cables embedded in loosely packed backfills. Understanding the mechanical behaviours of loose sand deposits to ocean waves could be beneficial for coastal engineers when evaluating the stability of marine structures throughout their design life.

Normally, the loose sandy soil has a low relative density of D_r , and S and P wave speeds, but a low standard penetration test (SPT) value. Under cyclic wave loading, soil particles in the loose sand deposits tend to rearrange their relative positions and become compacted in a manner, associated with the drainage of pore water. In this process, pore water pressure is likely to build up because of the effect of volumetric strain change on pores' storage capacity. Pore pressure can accumulate to a large value when the drainage condition is impeded, causing the soil to become unstable or even liquefied. The intensive use of both wave flume tests and geotechnical centrifugal tests has identified the occurrence of liquefaction in loose sand deposits [1, 2, 3, 4]. It has been recognized that the wave-induced cyclic stress ratio is the key parameter affecting the generation of residual pore pressure and the number of loading cycles required to cause liquefaction. The liquefaction resistance of a sandy seabed to cyclic loadings is found to be significantly affected by the soil state (i.e., void ratio, and effective confining pressure), grain characteristics (i.e., grain-size distribution and grain shape) and loading conditions (e.g., loading intensity, duration, frequency and even irregularity). In recent publications, it has been further proven to be affected by the shear stress conditions in the initial consolidation state [5, 6].

Accompanied by the experimental efforts, numerous theoretical frameworks-primarily involving analytical approximations and numerical modelling-have been developed to investigate the liquefaction susceptibility of loose sand deposits to ocean waves[7, 8, 9, 10, 3, 11]. For

example, Sekiguchi et al. [12] derived an analytical solution for residual pore pressure development using Laplace's transformation. The closed-form solution is capable of taking the cumulative contraction of soils under cyclic loading into account. However, in their analysis, the seabed thickness was much less than the assumed wavelength, which may be questionable. Sassa et al. [11] proposed a 1-D finite difference model to study the post-liquefaction process of loose seabed soils under progressive waves. In [11], the liquefied soil was treated as a heavy fluid based on potential flow theory and differentiated from the sub-liquefied soil using a moving boundary condition. Liu et al. [13] further extended [11]'s model by considering the viscous effect of liquefied layer and investigate the wave-induced residual liquefaction in the sandy seabed of finite thickness. Their numerical results indicate the need to consider the viscous effect in the post-liquefaction analysis. This framework has been further applied to a random wave case [14].

The aforementioned investigations are limited to 1-D models, neglecting the horizontal effective stress and pore pressure gradient along the wave propagation direction. In the practice of engineering, it is important to know the potential for liquefaction around the structure foundation or pipelines. Because of the presence of marine structures, the water wave field in the vicinity of structures will be modified, subsequently affecting wave-induced shear deformation in the soil profile. Additionally, for the case involving structures interacting with the water waves and seabed foundations, significant lateral forces and overturning momentum will be exerted on the structure, resulting in the foundation soil experiencing cyclic stresses in both the horizontal and vertical directions. With a one-dimensional numerical framework, it is impossible to capture the interactions between a wave, a structure (embedded or sitting on the seabed), and its deposited sandy foundation appropriately.

In this paper, we will introduce a simple but workable integrated numerical model with the capability of reproducing the liquefaction characteristics of loose sand deposits in two dimensions, using the theory of poro-elasto-plasticity, based on the existing 1-D framework [12, 3]. This model is first validated through comparison with centrifugal wave tests available in the literature to ensure its accuracy and effectiveness. After validation, it will be further applied to investigate the effects of key characteristics, including the nonlinearity and irregularity of cyclic loading conditions, the non-homogeneity of soil properties, the plasticity of soil behaviour, etc.,

on the interactions between progressive waves and loosely deposited sand foundations, through detailed parametric studies. Along with the progressive waves, the foundations' behavior under both standing waves and a solitary wave are presented and discussed at the end of the study.

2. Theoretical models

2.1. Wave model

In this study, RANS (Reynolds-Averaged Navier-Stokes) equations are used for governing the flow motion over a rigid seafloor. The mass and momentum conservation equations for a RANS solver can be expressed as follows:

$$\frac{\partial \langle u_{fi} \rangle}{\partial x_i} = 0, \quad (1)$$

$$\begin{aligned} \frac{\partial \rho_f \langle u_{fi} \rangle}{\partial t} + \frac{\partial \rho_f \langle u_{fi} \rangle \langle u_{fj} \rangle}{\partial x_j} = & -\frac{\partial \langle p \rangle^f}{\partial x_i} + \rho_f g_i \\ & + \frac{\partial}{\partial x_j} \left[\mu_f \left(\frac{\partial \langle u_{fi} \rangle}{\partial x_{fj}} + \frac{\partial \langle u_{fj} \rangle}{\partial x_{fi}} \right) \right] + \frac{\partial}{\partial x_j} (-\rho_f \langle u'_{fi} u'_{fj} \rangle), \end{aligned} \quad (2)$$

where u_{fi} is the flow velocity, x_i is a Cartesian coordinate, t is the time, ρ_f is the density of the fluid, p is the pressure, g_i is the acceleration caused by gravity, μ_f is the dynamic viscosity, and the relationship between the Reynolds stress term $-\rho_f \langle u'_{fi} u'_{fj} \rangle$ and the rates of strain of the flow field can be specified by the sophisticated $k - \epsilon$ turbulence model [15] as:

$$-\rho_f \langle u'_{fi} u'_{fj} \rangle = \mu_t \left[\frac{\partial \langle u_{fi} \rangle}{\partial x_j} + \frac{\partial \langle u_{fj} \rangle}{\partial x_i} \right] - \frac{2}{3} \rho_f \delta_{ij} k \quad (3)$$

where μ_t is the turbulent viscosity, which is considered a function of the turbulence kinetic energy (k) and turbulence dissipation rate (ϵ); and δ_{ij} is the Kronecker delta. Based on Eq. (3), Eq. (2) can be rewritten as follows:

$$\begin{aligned} \frac{\partial \rho_f \langle u_{fi} \rangle}{\partial t} + \frac{\partial \rho_f \langle u_{fi} \rangle \langle u_{fj} \rangle}{\partial x_j} = & -\frac{\partial}{\partial x_i} \left[\langle p \rangle + \frac{2}{3} \rho_f k \right] \\ & + \frac{\partial}{\partial x_j} \left[\mu_{eff} \left(\frac{\partial \langle u_{fi} \rangle}{\partial x_j} + \frac{\partial \langle u_{fj} \rangle}{\partial x_i} \right) \right] + \rho_f g_i \end{aligned} \quad (4)$$

in which $\mu_{eff} = \mu + \mu_t$ is the total effective viscosity. The symbol $\langle \rangle$ stands for Darcy's volume averaging operator, which can be defined as follows:

$$\langle a \rangle = \frac{1}{\mathcal{V}} \int_{\mathcal{V}_f} a d\mathcal{V} \quad (5)$$

in which $\mathcal{V}$ is the total averaging volume, and $\mathcal{V}_f$ is the portion of $\mathcal{V}$ that is occupied by the fluid. The relationship between the Darcy's volume averaging operator and intrinsic volume averaging is $\langle a \rangle = n \langle a \rangle^f$.

To generate the incident waves considered in this study, the internal wave maker method developed by Lin and Liu [16] is used; in that method, a mass source function ($S(x_i, t)$) is added in the continuity equation (Eq. (1)) in the source region, i.e. $\partial \langle u_i \rangle / \partial x_i = S(x_i, t)$. The value of $S(x_i, t)$ depends on both wave characteristics, and wave types. For a fifth-order Stokes wave, for example, the mass source function $S(x_i, t)$ can be expressed as follows:

$$S(x_i, t) = \sum_{i=1}^5 \frac{2C}{A} a_i \cos i \left(\frac{\pi}{2} - \omega t - \delta_p \right) \quad \text{in } \Omega \quad (6)$$

in which C is the wave phase velocity; ω is the wave frequency; δ_p is the phase shift constant; a_i is the wave amplitude associated with the i -th wave mode [17], which is a function of wave parameters including wave height (H), water depth (d) and wave period (T); Ω is the source region; and A is the area of source region. For a solitary wave, the corresponding source function is

$$S(x_i, t) = \frac{CH}{A} \text{sech}^2 \left[\sqrt{\frac{3H}{4d^3}} (4d/\sqrt{H/d} - Ct) \right] \quad (7)$$

In this study, the area (denoted by A) of the source region in which the source term wave-maker is acting, is predefined and remains constant during the simulation, as shown in Fig. 1. As recommended by [16], the source region has a height of 10% of the water depth ($0.65d \sim 0.75d$) and a width of 5% of the wavelength. More details can be found in [16].

2.2. Seabed model

In general, wave-induced pore water pressure in a loosely deposited sand foundation can be decomposed into two components, as shown in Fig. 2:

$$u_e = u_e^{(1)} + u_e^{(2)}, \text{ where } u_e^{(2)} = \frac{1}{T} \int u_e dt \quad (8)$$

where T is the wave period and u_e denotes the wave-induced pore pressure; $u_e^{(1)}$ represents the oscillatory component of pore water pressure whose temporal average $\bar{u}_e^{(1)}$ over the wave cycle $((t/T))$ is equal to zero and $u_e^{(2)}$ is termed as the residual component, which is caused by the volumetric contraction of the soil under cyclic shearing [10, 12, 11]. In this study, we will focus on the wave-induced residual soil response.

2.2.1. Biot's consolidation equations

In general, the geomaterials selected as foundations for coastal and offshore structures are inherently heterogeneous, and basically consist of two phases: a solid phase with a soil skeleton, and a fluid phase with water and air. Each of these phases has a different state of motion, leading to interactions between them. To better understand the mechanism of sea floor dynamics, an appropriate mathematical modelling of soil skeleton-pore fluid interaction is of paramount importance [18]. In this study, a mathematical model for the wave-seabed interaction is developed in two dimensions under plane strain idealization, in which Biot's consolidation equation [19] is used for linking the solid-pore fluid interaction in the porous seabed.

$$k_s \nabla^2 u_e^{(1)} - \gamma_w n_s \beta_s \frac{\partial u_e^{(1)}}{\partial t} = \gamma_w \frac{\partial \varepsilon_e}{\partial t}, \quad (9)$$

$$\frac{\partial \sigma'_x}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} = \frac{\partial u_e^{(1)}}{\partial x}, \quad (10)$$

$$\frac{\partial \tau_{xz}}{\partial z} + \frac{\partial \sigma'_z}{\partial z} + \rho g = \frac{\partial u_e^{(1)}}{\partial z}, \quad (11)$$

where ∇^2 is the Laplace operator; n_s is the porosity of soil; k_s is Darcy's permeability coefficient; σ'_x and σ'_z stand for the horizontal and vertical components of effective normal stresses, respectively; τ_{xz} is the shear stress; γ_w is the unit weight of pore fluid; and $\rho = \rho_f n_s + \rho_s (1 - n_s)$ is the average density of a porous seabed with ρ_f being the fluid density and ρ_s being the solid density. The elastic volume strain of the soil matrix ε_e and the compressibility of the pore fluid β_s can be respectively defined as follows:

$$\beta_s = \frac{1}{K_f} = \frac{1}{K_w} + \frac{1 - S_r}{P_{w0}}, \quad (12)$$

$$\varepsilon_s = \frac{\partial u_s}{\partial x} + \frac{\partial w_s}{\partial z} \quad (13)$$

where K_f is the apparent bulk modulus of the pore fluid, and generally, $K_f=2.29 \times 10^9$ N/m²; K_w is the true bulk modulus of elasticity of water; P_{w0} is the absolute water pressure; S_r is the degree of saturation; and u_s and w_s are soil displacements in the x - and z -directions, respectively.

2.2.2. Poro-elastoplastic solutions

Taking the oscillatory components associated with the surface wave motion as a source for the pore pressure generations, a two-dimensional poro-elastoplastic solution is developed to reproduce the foundation behaviour under cyclic shearing. The storage equation relevant to the poro-elastoplastic soil with contractancy can be expressed as a relationship between the plastic volumetric contraction of the soil (ε_p) and build-up of residual pore water pressure ($u_e^{(2)}$)

$$\frac{\partial u_e^{(2)}}{\partial t} = \frac{k_s}{K_v \gamma_w} \left(\frac{\partial^2 u_e^{(2)}}{\partial x^2} + \frac{\partial^2 u_e^{(2)}}{\partial z^2} \right) + \frac{1}{K_v} \frac{\partial \varepsilon_p}{\partial t}, \quad (14)$$

where K_v stands for the constrained modulus of soil and ε_p is the plastic volumetric contraction of the soil.

For loosely packed sand, the development of plastic volumetric strain ε_p under cyclic shearing is related to both the wave-induced shear stress and wave cycles [12, 11]:

$$\varepsilon_p(\xi, \chi) = \varepsilon_p^\infty(\chi) \cdot [1 - e^{-\beta \xi}], \quad \varepsilon_p^\infty(\chi) = R \cdot [e^{\alpha \chi} - 1], \quad (15)$$

where ξ is the wave repetitions related to the angular frequency of the waves (ω), which can be defined by $\xi = \omega t / 2\pi = t / T$; R , α and β are material parameters governing the rate of accumulation of plastic volumetric strain under cyclic shearing; ε_p^∞ stands for the amount of plastic volumetric strain with ξ ultimately approaching infinity; and χ stands for the cyclic shear stress ratio, which can be defined as follows:

$$\chi(x, z, t) = \frac{|\tau(x, z, t)|}{\sigma'_0}, \quad (16)$$

Unlike previous investigations, where the period-averaged shear stress amplitude is used as a source for pore pressure generations [12, 3, 11]. The quantity $\tau(x, z)$ in Eq. (16) represents the phase-resolved shear stress in the soil profile. With this new definition, both the oscillatory and residual mechanisms can be linked with instantaneous effects under the action of wave load. More details about this new definition can be referred to in [20, 21, 22]. The quantity σ'_0 in Eq. (16) is taken as the initial mean normal effective stress. In the case of wave-seabed interactions without marine structures, this stress can be simply expressed as follows [23, 9]:

$$\sigma'_0 = \frac{(1 + 2k_0)}{3} \gamma' z \quad (17)$$

in which γ' is the submerged specific weight of the soil and k_0 is the coefficient of lateral earth pressure.

Following the procedure outlined in Sassa et al. [11], the rate $\partial \varepsilon_p / \partial t$ can be defined as follows:

$$\frac{\partial \varepsilon_p}{\partial \xi} = \beta \varepsilon_p^\infty(\chi) e^{-\beta \xi} = \beta \left[\varepsilon_p^\infty(\chi) - \varepsilon_p(\xi, \chi) \right]. \quad (18)$$

Substituting Eq. (18) into Eq. (14) and introducing ξ and χ , the equation relating to the residual pore pressure $u_e^{(2)}$ in a two-dimensional form becomes:

$$\frac{\partial u_e^{(2)}}{\partial \xi} = \frac{2\pi K_v k_s}{\omega \gamma_w} \left(\frac{\partial^2 u_e^{(2)}}{\partial x^2} + \frac{\partial^2 u_e^{(2)}}{\partial z^2} \right) + K_v R \beta e^{-\beta \xi} (e^{\alpha \chi} - 1). \quad (19)$$

2.3. Boundary conditions

To solve the wave field, appropriate boundary conditions are required. With respect to the mean flow field, the no-slip boundary condition is imposed on the seafloor surface ($u_{fi}=0$). The zero-stress condition is adopted on the mean free surface ($\tau_{fij} = 0$), whereas the effect of air

flow is neglected. For the turbulence field, the log-law distribution of the mean tangential velocity in the turbulent boundary layer is imposed near the rigid boundary. Both the turbulent kinetic energy (k) and its dissipation rate (ϵ) on the free surface are implemented with zero gradient boundary conditions, (i.e., $\frac{\partial k}{\partial n} = \frac{\partial \epsilon}{\partial n} = 0$, in which n is the unit normal on the free surface). The damping zone is located at two vertical boundaries, both of which are far from the concerned region. More detailed implementations about boundary conditions for the wave model are referred to in Lin and Liu [16].

At the seabed surface, both the vertical effective normal stress and the shear stress vanish, whereas the pore water pressure (u_e) is equal to the wave pressure obtained from the wave model. For the soil resting on an impermeable rigid bottom, zero displacements and no vertical flow occurs, i.e. $\partial u_e / \partial z = u_s = w_s = 0$. Both lateral boundaries of the seabed are horizontally fixed. In this study, the length of the computing domain for the seabed model was selected as three times of wave length (L). With these long distances, the effect of the lateral boundary condition can be eliminated.

2.4. Numerical scheme

The RANS equations are solved by a two-step projection method with finite volume discretization [24]. The computational domain is divided into a set of structured/unstructured meshes, in which all the fluid properties, such as pressure and velocities, are defined at the cell centroids. To convert cell centroid data to face centroids to evaluate the gradient of all the quantities at cell centroids, the least square linear reconstruction (LSLR) method developed by Barth [25] is applied. The forward time difference method is used for the discretization of the time derivative. To obtain computational stability, the time interval is automatically adjusted to satisfy the Courant-Friedrichs-Lewy condition and the diffusive limit condition [26], with a range between 0.005 s and 0.05 s.

The seabed model is constructed with the aid of the PDE module of Comsol Multiphysics [27], in which the finite element method is used to solve the governing equations, in combination with the second-order Lagrange elements to ensure second-order of accuracy in evaluating the dependent variables in the computational domain. The Generalized- α Method was used for the time integration when computing the dynamic soil response under cyclic loading. As a second order accurate numerical scheme [28], the Generalized- α Method is a one-step, three stage

implicit method, in which accelerations, velocities and displacements are uncoupled. The optimal Orthogonal FEM meshes are used in the computational domain, which are automatically generated by COMSOL software with a maximum element size scaling factor (MESSF) controlling the maximum allowed element size. The mesh was refined until no significant changes in the numerical solution were achieved.

2.5. Integration of wave and seabed models

There are three main steps in the model integration: (1) calculate the flow using the RANS solver; (2) use, in Biot's equations, the pressure at the seabed surface (obtained in the previous step), and calculate the phase-resolved geotechnical quantities such as the oscillating components of pore water pressure and shear stress in the soil, and (3) feed the obtained period-averaged shear stress in the soil, and solve the poro-elastoplastic solutions for the residual components of pore water pressure development.

The MATLAB interface in COMSOL Multiphysics enables researchers to more realistically simulate the physical process involved in wave-seabed interactions. In the one-way coupling process, a series of iterative calculation steps with transient studies are established using MATLAB script, as illustrated in Fig. 3. During a wave period (T), there are n computation steps. In the first step, study 1 is established and the wave load at $t = (1/n)T$ is applied on the seabed to calculate the seabed response during the time interval $\Delta t = (1/n)T$. Next, the solution of the first step is set as the initial condition for the second step (study 2) to simulate the seabed response at $t = (2/n)T$ with the corresponding wave load applied on the seabed. Therefore, the seabed response in the succeeding moments with time interval $\Delta t = (1/n)T$ is calculated in the same manner, i.e., the solution of study i is taken to initialize study $i+1$. In the numerical simulations, a fixed time interval $T/50$ is considered to provide a stable and precise solution over the entire computation.

3. Validations of the present model

Sassa et al. [3] used a balanced-beam centrifuge to investigate the cyclic behaviour of sandy soil deposits under the actions of a wave load. The centrifugal wave tests were carried out in a wave channel under a steady-state centrifugal acceleration of 50 g because the viscosity of the

silicone oil used in the test was 50 times that of water. The test programme consisted of either progressive- or standing-wave tests on loosely packed, fresh deposits of soil (British Standard sieve 100/170). The incoming waves were generated using a quasi-flap-type wave generator at a frequency of $f=11$ Hz with a fluid depth of $d=90$ mm. The measured pore pressure-time histories in test P4-1 for progressive waves (sub-liquefaction case) and test S5-1 for standing waves (liquefaction case) are used in the verification performed in this study.

The computational setup is arranged similar to the experimental setup of [3], with a scaling factor of 50. The input data are given here: wave period (T)= 4.5 s; water depth (d)=4.5 m; soil porosity (n_s)=0.5; $\alpha=55$; $R=1.8\times10^{-5}$; and $\beta=0.04$. The simulated times series of wave-induced residual pore water pressures are compared with the measured ones of [3], as illustrated in Fig. 4. Overall, the computed results of residual pore pressure development with time including the build-up of pore pressure to a maximum point and subsequent dissipation attributable to soil compaction, are in good agreement with those observed in the geotechnical centrifugal wave tests [3], especially for the no-liquefaction case (P4-1) under progressive waves. The discrepancy between simulated results and experimental data in case S5-1 under standing waves, however, may be attributable to the lateral soil movement in a sediment trench of limited length in the centrifugal test, giving rise to the accumulation of pore pressure at both lateral sides close to the standing-wave nodes and subsequently affecting the performance of the antinode in the middle tested section. Furthermore, the post-liquefaction process, such as that in [11], is not incorporated into the present numerical framework. A complete model covering the entire liquefaction/dissipation process in the liquefaction regime, in which the actual liquefaction process also takes place with the soil-water mixture acting as a fluid, is required in future investigations.

4. Results and discussions

In this section, the developed numerical model has been scaled to field conditions to investigate the wave-induced residual soil response in a loosely deposited sand foundation, as shown in Fig. 5. In the numerical calculations, a flat seabed foundation 400 m long and 20 m thick is considered the computational domain, which is discretized by a 4000×20 mesh grid system with $\Delta x=0.1$ m and $\Delta z=1$ m. The incident waves with varying wave steepness (H/L_0) are generated in the wave maker and propagate in the positive x -direction, whereas the z -direction is upward

from the interface between the water and porous seabed. The wave characteristics and properties of the sandy bed used in the numerical simulations are listed in Table 1, unless specified in the parametric studies.

4.1. Progressive waves over a homogeneous sandy bed

4.1.1. Progressive nature of liquefaction

As described in the previous section, when subjected to ocean waves, the soil element of the seabed will undergo a cyclic shear stress variation, similar to those induced by earthquakes. For a loosely deposited sand foundations whose soil skeleton is sensitive to contraction under cyclic shearing, this may result in a significant pore pressure build up when the drainage condition is impeded. It is well known that liquefaction occurs when the accumulated residual pore pressure reaches the overburden soil weight [23, 9], i.e. $u_e^{(2)} = \sigma'_0$. Figure 6(a) shows the development of the liquefaction zone in 2-D under progressive waves. As seen, not surprisingly, the potential for liquefaction initially occurs in the shallow soil layers and progressively advances downward as the wave propagates, until reaching a balanced state, where the maximum liquefaction depth occurs. The pattern of the liquefied region, although not significant, shows some differences in the horizontal direction, which is likely attributable to the new definition of the source for the pore pressure generations considered in the present elastoplastic framework. Compared to the classical period-averaged pore pressure build-up models, the phase-resolved shear stress in the source term causes the horizontal distribution of the liquefaction zone to be related to the incident of the wave phases, as demonstrated by Jeng and Zhao [20].

Further interpretations of the progressive nature of the liquefaction zone can be made by plotting the depth variations of pore pressure accumulation with time. Since there are no significant changes in the horizontal distributions in the case of progressive waves, only the simulated results along the centre line of the computational domain are presented, as shown in Figure 6(b). It is found that pore pressures in soil elements at different depths tend to develop at varying rates. This phenomenon is caused by diffusion within and dissipation out of the free draining boundary at the top of the deposit. In the near-surface region, the accumulated residual pore pressure could easily overcome the overburden pressure, causing the soil to become liquefied. The liquefaction front progressively travels downwards as the wave continues and stops at a

certain depth after approximately 20 wave repetitions, where the maximum liquefaction depth (L_m) is reached. Numerical results also indicate that the present model is capable of capturing the features of the post-liquefaction behaviour of loose seabed soil, at least to some extent. As seen, the process of the building up of pore pressure and liquefaction in the upper soil layers is followed by pore pressure dissipation after 10 wave repetitions, which is likely attributable to the progressive compaction of the soil [29].

4.1.2. Effects of wave characteristics

Wave characteristics, such as wave steepness (H/L_0 , where L_0 is the wave length calculated from linear wave theory [30]), directly affect the loading intensity and subsequent liquefaction susceptibility of foundation soils in loose sand deposits. Adopting the present numerical model, the effects of wave steepness on the wave-induced residual soil response is first investigated in this section. Fig. 7(a) shows the time history of residual pore pressure ($u_e^{(2)}/\gamma_w d$) development at a given point ($x/L_0=1.5$ $z/h=-0.1$) under different wave loadings, with initial incident wave steepness (H/L_0) varying from 0.036 to 0.063. It is found that all curves with different values of H/L_0 display a similar trend, where the rate of pore pressure generation is much greater than those of pore pressure dissipations in the early stage, and the pore pressures gradually increase to a maximum point (where the generation and dissipation of residual pore pressure are in balance) after approximately 10 wave cycles. Next, the dissipation of residual pore pressure becomes more dominant because of the increasing compaction of the foundation soil, and a gradual decrease in pore pressure with time can be observed.

Fig. 7(b) shows the effects of wave steepness on the vertical distribution of the accumulated residual pore pressure ($u_e^{(2)}/\gamma_w d$) after 20 wave repetitions when the seabed has reached its maximum liquefaction depth. As we have seen, the accumulated pore pressure increases very rapidly within the top 20% of its thickness, where a maximum value is reached. Then significant pore pressure attenuation with depth is observed.

It is found that the water waves with large wave steepness cause wave-induced residual pore pressure to build up in loosely deposited sand more significantly. Correspondingly, the maximum liquefaction depth increases as H/L_0 increases.

The wave generally exhibits some behaviours of nonlinearity because of the wave-wave interaction, particularly in shallow water zones. The nonlinearity of ocean waves is normally

associated with a steeper wave crest and a flatter wave trough in surface water elevations. It may also play an important role in the evaluation of wave-induced residual soil responses in marine sediments [31, 20]. The effects of wave nonlinearity on the development of residual pore pressure and potential of liquefaction can be seen in Fig. 8, which shows that the residual pore pressure builds up faster and can accumulate to a large value in the case of nonlinear waves. This result means that the use of linear wave theory in the case of large wave steepness may result in underestimated potential for liquefaction because of the buildup of pore pressure. Consequently, to ensure the accuracy of loading application and the resulting dynamics of soil foundations, the fifth-order stokes wave theory is adopted in the numerical wave maker to generate the incident waves considered in this study.

In a realistic oceanic environment, the wave motions generated by winds are random in nature, which may impose irregular cycles in the magnitudes of wave loading, yielding more complex consequences on the liquefaction susceptibility of sandy soils. Taking the developed model, the effects of wave randomness on the cyclic behaviour of loose sand deposits is investigated.

In the computation, an irregular wave train composed of 50 linear wave modes with different frequencies and wave heights is generated in the numerical wave flume. Fig. 9 shows the time histories of water surface elevation and the resulting wave pressure fluctuations on the seabed surface; random waves display some loading irregularity, both in frequency and magnitude, compared with regular waves. The effects of loading irregularity on the liquefaction susceptibility of sandy soils are also investigated in this study. The predicted results are presented in Fig. 10. In comparison, the predicted results under regular waves that are identical to the peak wave component in the irregular wave train are also incorporated. It is found that the irregularity of cyclic loading causes the development of residual pore pressure to become implicit compared to that of regular waves. In addition, when subjected to irregular waves, the loose sandy seabed exhibits higher liquefaction resistance compared to those subjected to regular waves, which is consistent to the experimental findings of [5].

4.1.3. Effects of soil characteristics

In addition to wave characteristics, soil properties have also been found to significantly affect the residual pore pressure development involved in wave-seabed interactions [20]. Among these, α and R are related to the plastic volumetric strain ϵ_p when the loading cycle ultimately approaches infinity in the form of $\epsilon_p^\infty = R(e^{\alpha\chi})$, with χ being the cyclic shear stress ratio. β is another important material parameter dominating the rates of pore pressure accumulation and dissipation associated with the plastic volumetric strain ϵ_p . Based on the standard wave parameters listed in Table 1, we will further examine the effects of the model parameters on the residual soil response in the loosely deposited seabed foundations with different drainage conditions (k_s) under cyclic loadings.

Fig. 11 illustrates the effects of model parameters (α , β and R) and soil permeability (k_s) on residual pore pressure development at a given point ($x/L_0=1.5$, $z/h=-0.1$) in foundation soil under cyclic shearing. As seen, residual pore pressure can accumulate to a large value as the model parameters (α , β and R) increase and soil permeability decreases. The maximum value is often reached within the first several loading impacts, followed by a rapid dissipation caused by soil compaction. However, the duration period for the pore pressure accumulation to its maxima increases with the decrease of soil permeability, which is evident because the energy and pore pressure are more difficult to dissipate in soil with poor drainage conditions. Furthermore, it is found that the rates of pore pressure generation and dissipation in the soil are magnified with a large value of the plastic parameter β .

The effects of the plastic parameters (α , β and R) and soil permeability (k_s) on the depth variation of accumulated residual pore pressure ($u_e^{(2)}/\gamma_w d$) along the centre line of the computation domain are also examined, as shown in Fig. 12. Despite variations in the plastic parameters or soil permeability, each profile of $u_e^{(2)}/\gamma_w d$ varying with z/h takes the shape of a parabola-like curve after 20 wave repetitions. In addition, the soil tends to be liquefied deeper with increasing α , β , R and decreasing soil permeability (k_s).

4.2. Progressive waves over a heterogeneous sandy bed

Accompanied by the deposition processes in the geological history, marine sediments normally displays certain degrees of inhomogeneity in the real ocean environment. To date, numerous studies have been carried out to investigate the wave-induced oscillatory soil response

in a poro-elastic seabed with inhomogeneous soil behaviour [32, 33, 34]. In these studies, the sediment heterogeneity is essentially reproduced by using two different approaches: layered seabed with constant soil property within each layer; and a uniformed seabed with varying soil properties along depth, i.e., a Gibson soil [35]. Based on these approaches, we will further investigate the wave-induced residual soil response in heterogeneous marine sediments in this section using the developed integrated numerical model.

4.2.1. Soil response in a layered seabed

The inhomogeneous soil behaviour is first reproduced in the form of a layered seabed, in which a cover layer 5 metres thick is considered and backfilled with coarse sand with large soil permeability, whereas the thickness of the remainder of the soil layers is 15 metres, consisting of finer material with poor drainage. In the computation, the soil parameters are considered constant in each layer, whereas at the interface between two layers, the displacements of soil particles and the pore water pressures are set to be consistent. Other boundary settings are the same as the homogeneous problem considered in the previous section.

Fig. 13 shows the development of the liquefaction potential in a multi-layered seabed with wave repetitions. As a comparison, the computed results in the case of a uniformed seabed is also incorporated into the analysis. As seen, the content of coarse backfilled material in a multi-layered seabed significantly affects its drainage conditions, in which the pore pressure dissipates more quickly and therefore is less likely to build up than the content in a uniformed seabed. However, this has a less significant effect on the development of the liquefaction front, especially in the early stage of cyclic loadings. One possible reason for this result is the loading intensity considered in this study, which is strong enough to render the soil liquefied, especially in the near-surface soil layers with less significant overburden pressure. Therefore, in the early loading stage, no significant difference can be observed in the development of the liquefaction front between a uniform seabed or foundation that consists of multiple soil layers. The discrepancy increases as the liquefaction front progressively advances downward with increasing wave repetitions.

4.2.2. Soil response in a Gibson soil

In addition to a layered seabed, a typical type of coastal sediment heterogeneity would involve variations in the soil properties along depth [35]. This variation is accompanied by the deposition process of marine sediments in the geological history. To further investigate the effects of soil inhomogeneity on the wave-induced residual soil response in a sandy bed that is loosely deposited, a more general case of heterogeneous sediment is considered in this study, in which the inhomogeneity of soil behaviour is reproduced by varying its permeability and shear modulus linearly with depth, i.e. $k_s/k_0 = 1 + 0.9z/h$, $G/G_0 = 1 - 9z/h$ [36].

As shown in Fig. 14, throughout the period, the wave-induced pore pressure is more likely to build up, and hence the soil tends to be liquefied deeper in a seabed with heterogeneous soil behaviour. However, the dissipation of pore water pressure in the upper soil layer tends to be enhanced by deposit heterogeneity. Correspondingly, the discrepancy of liquefaction depth between a heterogeneous seabed and a homogeneous one decreases as time increases. The largest difference in the liquefaction depth is approximately 5 metres, which occurs after approximately 10 wave repetitions. This result demonstrates that using the ideal homogeneous soil assumption may significantly underestimate the wave-induced residual pore pressure development and the resulting liquefaction depth in a loosely deposited sand foundation and should be taken into consideration when the model is applied in engineering practice.

4.3. Various types of wave loadings

In addition to the progressive wave loading, the present model's ability to reproduce the foundation behaviour under other types of wave loadings is investigated in this study. Herein, we consider two typical waves that commonly exist around the marine structures in engineering practice. The first is standing waves, which normally occur in front of caisson-type breakwaters; and the second is the solitary wave, which can simulate a tsunami to some extent.

4.3.1. Foundation behaviour under standing waves

Standing-wave-induced liquefaction is first discussed in this section. In the numerical simulation, an impervious wall is set to stand in the way of where a wave propagates. Standing waves are formed in front of the wall as a result of wave interference between incident and reflected waves.

Snapshots of standing-wave-induced residual pore pressure development are illustrated in Fig. 15, which shows that the horizontal distribution of accumulated pore pressure is quite different from that in the case of progressive waves. Under standing-wave loading, the pore pressure distribution is strongly x -dependent, which is likely attributable to the difference in shear strains caused by the surface water waves. As demonstrated by Hsu and Jeng [37], the shear strains caused by standing waves, are most significant under the node, gradually decrease towards the antinodes, and eventually cease to exist at the antinode section. Correspondingly, the pore pressure is most likely to develop in the node section, which is consistent with observations in laboratory experiments [38] and geotechnical centrifuge tests [3]. The vertical distribution of residual pore pressure development is similar to that under progressive waves. It emerges at shallow soil layers and close to the mudline and then spreads sideways and advances downward as the wave continues. The pore pressure in shallow soil layers begins to release after certain wave periods as the result of soil compaction during the post-liquefaction process.

4.3.2. *Foundation behavior under a solitary wave*

Tsunamis are typically caused by large, undersea earthquakes. With their extremely high wave height and high water travelling speed, they can cause widespread destruction and erosion to both coastlines and coast infrastructure. Therefore, for coastal engineers involved in the foundation design of coastal infrastructures, it is important and necessary to understand the foundation behaviours underlying a tsunami.

In real ocean environments, a tsunami propagates with several solitary waves. To simplify the situation, only one solitary wave propagating over the porous seabed is investigated in this study. As shown in Fig. 16(a), the residual pore pressure starts to accumulate once a solitary wave propagates over the porous seabed. As the wave continues, the accumulated pore pressure in the initial position will not immediately dissipate (Fig. 16(b)). Instead, the area of residual pore pressure development spreads sideways and continuously advances downward (Fig. 16(c) and (d)). The pore pressure accumulation under solitary wave loading embodies a completely two-dimensional development process, similar to that under standing waves. As one assumption can be expected, if more solitary waves continue to propagate over the seabed, the pore pressure will be linked together as a whole in the upper soil layers, which significantly compromises the stability of the foundation soil and its supported marine structures.

It is worth noticing that unlike ocean waves caused by wind, a solitary wave does not yield cyclic shear stress and induces a type of monotonic shearing with a stress reversal. However, the theoretical part of this study for determining the build-up of residual pore pressure is based on the assumption that the soil is undergoing cyclic shearing. A more sophisticated model, such as a constitutive elastoplastic model with physical validation regarding the residual pore pressure under a solitary wave, is required in future investigations.

5. Conclusion

In this study, a two-dimensional poro-elastoplastic model for a wave-induced residual soil response in loosely deposited sand foundations is proposed. The proposed model was validated by the centrifuge wave tests [3] available in the literature, obtaining fairly good agreement. Based on the numerical results presented, the following conclusions can be drawn:

1. The foundation behaviour under various types of wave loading, particularly standing waves or a solitary wave, embodies a completely two-dimensional process in terms of residual pore pressure development. This result demonstrates that the present model is capable of capturing the features of wave-induced residual soil response in 2-D, which allows it to be further applied to more general cases with marine structures.
2. Liquefaction occurring in loosely deposited sand foundations because of the buildup of pore pressure is a progressive process. It emerges at the shallow soil layers that are close to the mudline and spreads sideways; meanwhile, it advances downwards as the surface water wave continues until reaching a balanced state after certain wave cycles, where the maximum liquefaction occurs.
3. The wave nonlinearity, along with the irregularity and non-homogeneity of soil behaviour, significantly affects the wave-induced residual soil response in loosely deposited sand foundations. Assuming a linear monochromatic wave that propagates over a homogeneous soil deposits, the residual pore pressure development and the liquefaction susceptibility of a loose sandy seabed may be significantly underestimated.

4. Parametric studies indicate that the plastic volumetric contraction leading to the build-up of residual pore pressure is more likely to occur in loosely deposited sand foundations with poor drainage conditions and under large waves. The maximum liquefaction depth increases as wave steepness (H/L_0) and plastic parameters (α, R) increase, and as soil permeability (k_s) and the plastic parameter β decrease.

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Table 1: Input data in the numerical examples.

Characteristics	Value	Unit
<i>Wave characteristics</i>		
Incident wave steepness (H/L_0)	0.063 or various	—
Relative water depth (d/L_0)	0.143	—
Wave period (T)	10	[s]
<i>Soil characteristics</i>		
Permeability	1.0×10^{-5} or various	[m/s]
Poisson's ratio (μ_s)	0.333	—
Young's modulus (E_s)	2×10^7	[Pa]
Porosity	0.51	—
Degree of saturation	100	%
Plastic parameter (R)	5×10^{-5} or various	—
Plastic parameter (α)	55 or various	—
Plastic parameter (β)	0.04 or various	—

Figure 1: Schematic of the numerical wave flume considered in this study.

Figure 2: Mechanism of wave-induced pore pressures in a porous seabed.

Figure 3: Numerical scheme (a wave period is divided into $n=50$ components, $i=2, 3, \dots, N-1$; N is the total number of calculation steps).

(a) Progressive wave (P4-1)

(b) Standing wave (S5-1)

Figure 4: Validation of the wave-induced development of residual pore pressure against the geotechnical centrifugal wave test (Sassa and Sekiguchi, 1999).

Figure 5: Schematic of the computation considered in this study.

Figure 6: Development of residual pore pressure and the resulting potential for liquefaction in a loosely deposited sandy bed subjected to progressive waves.

Figure 7: Effects of wave steepness (H/L_0) on the (a) time histories of pore pressure development; and (b) vertical distributions of accumulated pore pressure after 20 wave repetitions.

Figure 8: Effects of wave nonlinearity on the (a) time histories of pore pressure development; (b) vertical distributions of accumulated pore pressure after 20 wave repetitions.

Figure 9: Time histories of irregular wave trains and the resulting loading intensities on the seabed surface.

Figure 10: Effects of loading irregularity on the (a) time histories of pore pressure development; and (b) vertical distributions of accumulated pore pressure after 20 wave repetitions.

(a) various α

(b) various β

(c) various R

(d) various permeability k_s

Figure 11: Time histories of wave-induced residual pore pressure development for (a1) various empirical coefficient α , (a2) various empirical coefficients β , (a3) various empirical coefficients R , and (a4) various soil permeability k_s .

(a) various α

(b) various β

(c) various R

(d) various permeability k_s

Figure 12: Vertical distribution of accumulated residual pore pressure for (a1) various empirical coefficients α , (a2) various empirical coefficients β , (a3) various empirical coefficients R , and (a4) various soil permeability k_s .

Figure 13: Development of wave-induced liquefaction potential with time in a multi-layered seabed.

Figure 14: Development of the wave-induced liquefaction potential over time in a heterogeneous seabed with varying soil properties along depth.

Figure 15: Distribution of the wave-induced residual pore pressure under solitary wave loading.

Figure 16: Distribution of the wave-induced residual pore pressure under solitary wave loading.