Non-local Similarity Based Nonnegative Tucker Decomposition for Hyperspectral Image Denoising

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Abstract—Compared with color or grayscale images, hyperspectral images deliver more informative representation of ground objects and enhance the performance of many recognition and classification applications. However, hyperspectral images are normally corrupted by various types noises, which has serious impact on the subsequent image processing tasks. In this paper, we propose a novel hyperspectral image denoising method based on tucker decomposition to model the non-local similarity across the spatial domain and global similarity along the spectral domain. In this method, 3D full band patches extracted from a hyperspectral image are grouped to form a 3rd-order tensor by utilizing the non-local similarity in a proper window size. In this way the task of image denoising is transformed into a high order tensor approximation problem, which can be solved by nonnegative tucker decomposition. Instead of traditional alternative least square based tucker decomposition, we propose a hierarchical least square based nonnegative tucker decomposition method to reduce the computational cost and improve the denoising effect. In addition, an iterative denoising strategy is adopted to achieve better denoising outcome in practice. Experimental results on three datasets show that the proposed method outperforms several state-of-the-art methods and significantly enhances the quality of the corrupted hyperspectral image.

Index Terms—Hyperspectral Image, Denoising, Nonnegative Tucker Decomposition, Non-local Similarity, Hierarchical Alternative Least Square

I. INTRODUCTION

Hyperspectral image can be considered as a set of spatially organized “pixels” where each pixel contains an entire spectral reflectance at a spatial location in the image over given light wavelengths. Compared with grayscale or color images which integrate the spectral responses at only one or several light wavelength intervals, hyperspectral images deliver more detailed spectral information of real scenes. Numerous research have been undertaken on hyperspectral images, for example, in feature extraction [1], [2], unmixing [3]–[5], classification [6]–[10], detection [11]–[13], band selection [14] and surveillance [15]. However, conventional hyperspectral imaging process suffers from issues such as limited illumination and short sensing time which introduce noises into the image acquisition step. Therefore, denoising has become a critical step to improve the quality of hyperspectral images.

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Over the past a few years, several hyperspectral image denoising methods have been proposed. The most straightforward way is to utilize traditional 2D denoising methods, such as NLM [16], K-SVD [17] and BM3D [18], to reduce noise band by band separately. Such band-wised methods cannot achieve superior effect in many cases since only spatial noise is removed. They may also destroy the correlation of image data in the spatial or the spectral domain. Therefore, spatial and spectral information should be considered jointly to effectively remove the noises.

Consequently, a hyperspectral image can be treated as a whole entity rather than band by band processing. Several 3D patch based denoising methods have been put forward, such as adaptive non-local means (ANLM3D) [19], 3D cube K-SVD [20] and block matching 4D filtering (BM4D) [21]. 3D patch based denoising methods can be implemented by processing small 3D patches in both spatial space and spectral space. In many cases, only local spectral correlation between a few adjacent bands is considered, and the high spectral redundancy in all continuous spectral bands has not been fully used. Novel denoising methods [22]–[24] have been proposed recently by combining non-local similarity across the spatial space and global redundancy along the spectral space. However, when the noises are strong, image denoising methods based on sparse coding and low rank constraint perform poorly because the dictionary learning step tends to remove some inherent information, *e.g.* textures and edges [22].

To further exploit the spatial-spectral information to generate better noise free estimation, a hyperspectral image can be treated as a stack of 2D gray images, which can be seen as a 3rd-order tensor. Accordingly, tensor based dictionary learning method was explored [25], and some algorithms were proposed to approximate tensor based on tensor low rank approximation [26]–[28] or tensor decomposition [29]. In particular, low-rank approximation (LRTA) [30] utilizes low rank tensor approximation [31] of the input image to achieve denoising and spectral dimensionality reduction. PARAFAC [29] method further extends the denoising approach by employing parallel factor analysis. Though both approaches take the correlation over different bands into consideration and try to eliminate the spectral redundancy of hyperspectral image, they ignore the non-local similarity of full band patches across the spatial domain. A denoising method based on a generalized multidimensional Wiener filter by using a 3rd-order tensor was proposed by Letexier and Bourennane [32]. Furthermore, some denoising method were developed based on basis of tensor decomposition [33]–[35]. Tensor based methods can be represented by tensor decomposition and PCA joint algorithm [36]

Fig. 1. Overview of the proposed hyperspectral image denoising method. Firstly, image patches extracted from noisy hyperspectral image are stacked as a 3rd-order tensor by non-local similarity matching. Then hierarchical alternative least square based nonnegative Tucker decomposition method is applied to get the denoised patches. Finally, the denoised patches are aggregated iteratively to get the clean image.

which take the hyperspectral properties into account. Although utilizing the correlation between different bands of the hyperspectral image, the spatial non-local similarity structures have been ignored, and so these approaches still have not reached the full potential for the denoising problem.

Aiming at the aforementioned issues, we propose a novel denoising method by considering both non-local similarity across the spatial domain and global correlation among the spectral domain. On one hand, a natural scene may contains a collection of patches grouped by non-local similarity over the spatial, composing of homologous aggregation of microstructures. The similarity among those patches is so called non-local similarity. Recovering these patches by several similar non-local patches can utilize the information redundancy across spatial space as well as maintain the texture characteristic of the hyperspectral image. On the other hand, hyperspectral image contains a large amount of spectral redundancy since some neighboring bands are highly correlated. These two kinds of prior knowledge are very helpful for various hyperspectral image recovery problems [37]–[40]. By this way, redundancy in both spatial space and spectral space can be made full use of.

Moreover, tensor decomposition based hyperspectral image denoising method is adopted in our model, which not only emphasizes the similarity among non-local full band patches but also utilizes the redundancy of spectral space. Specifically, we treat each full band patch as a 3rd-order tensor with two spatial modes and a spectral mode, and then build a 3rd-order tensor by stacking all these non-local similar full band patches as shown in Fig. 1. Then the denoising problem can be transformed into tensor recovery by utilizing the inherent spatial and spectral correlation of the full band patch groups.

We solve the tensor recovery problem by adopting a hierarchical alternative least square optimization of nonnegative tensor decomposition and then aggregate the denoised 3D patches in the clean hyperspectral image iteratively. We demonstrate the effectiveness of the proposed method through experimental results on several hyperspectral image datasets. Our method outperforms many state-of-the-art approaches under several quantitative assessments.

The rest of the paper is organized as follows. We illustrate the basic theory of tensor in Section II. In Section III, by exploiting the non-local similarity across spatial space and spectral space, we introduce a denoising algorithm based on tensor decomposition for hyperspectral image. In Section IV, we present and analyze the experimental results. Finally, conclusions are drawn in Section V.

II. NOTATION AND PRELIMINARIES OF TENSOR

An N -way tensor $\mathcal{X} \in \mathbb{R}^{I_1 \times I_2 \times \dots \times I_N}$ has N indices and its elements are denoted by $x_{i_1 i_2 \dots i_N}$ where $1 \leq i_n \leq I_n$, $n = 1, 2, \dots, N$.

The scalar product of two tensors $\mathcal{X}$, $\mathcal{Y}$ is defined as $\langle \mathcal{X}, \mathcal{Y} \rangle = \sum_{i_1, i_2, \dots, i_N} x_{i_1 i_2 \dots i_N} y_{i_1 i_2 \dots i_N}$. The Frobenius norm of a tensor $\mathcal{X}$ is given by:

$$\|\mathcal{X}\|_F = \sqrt{\sum_{i_1=1}^{I_1} \sum_{i_2=1}^{I_2} \dots \sum_{i_N=1}^{I_N} x_{i_1 i_2 \dots i_N}^2} \quad (1)$$

Tensor mode- n unfolding: Tensor mode- n unfolding, which is also called tensor matricization, is analogous to vectorizing a matrix. Mode- n unfolding of $\mathcal{X} \in \mathbb{R}^{I_1 \times I_2 \times \dots \times I_N}$ re-arranges the elements of $\mathcal{X}$ to

form a matrix $\mathbf{X}_{(n)} \in \mathbb{R}^{I_n \times I_{n+1} I_{n+2} \cdots I_N I_1 I_2 \cdots I_{n-1}}$, where $I_{n+1} I_{n+2} \cdots I_N I_1 I_2 \cdots I_{n-1}$ is in cyclic order.

Tensor mode- n matrix product: Mode- n product of a tensor $\mathcal{X} \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_N}$ with a matrix $\mathbf{U} \in \mathbb{R}^{J \times I_n}$ is denoted by $\mathcal{X} \times_n \mathbf{U}$ and its size is $I_1 \times \cdots \times I_{n-1} \times J \times I_{n+1} \times \cdots \times I_N$. The produced tensor has the same size as $\mathcal{X}$ in each mode, except mode- n whose size is J . We have

$$(\mathcal{X} \times_n \mathbf{U})_{i_1 \cdots i_{n-1} j i_{n+1} \cdots i_N} = \sum x_{i_1 i_2 \cdots i_N} u_{j i_n} \quad (2)$$

Multiple matrix mode- n products can be performed in any order,

$$(\mathcal{X} \times_n \mathbf{A}) \times_m \mathbf{B} = (\mathcal{X} \times_m \mathbf{B}) \times_n \mathbf{A} \quad (3)$$

for $m \neq n$.

Tensor mode- n vector product: Mode- n vector product of a tensor $\mathcal{X} \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_N}$ with a vector $v \in \mathbb{R}^{I_n}$ is of size $I_1 \times \cdots \times I_{n-1} \times I_{n+1} \times \cdots \times I_N$ and is denoted by $\mathcal{X} \overline{\times}_n v$. Element-wise, we have

$$(\mathcal{X} \overline{\times}_n v)_{i_1 \cdots i_{n-1} i_{n+1} \cdots i_N} = \sum_{i_n=1}^{I_n} x_{i_1 i_2 \cdots i_N} v_{i_n}. \quad (4)$$

The basic tensor operations described above show that multiplying a 3-order tensor by a vector in one mode results in a two-way tensor, *i.e.* a matrix. It is possible to multiple a tensor by a vector in more than one mode as well. Multiplying a 3-order tensor by vectors in two modes results in a 1-order tensor, *i.e.* a vector. Multiplying in all modes results in a scalar.

Multiplication in all possible modes ($n = 1, 2, \cdots, N$) of a tensor $\mathcal{X}$ and a set of matrices $\mathbf{A}^{(n)}$ is defined as

$$\mathcal{X} \times \{\mathbf{A}\} = \mathcal{X} \times_1 \mathbf{A}^{(1)} \times_2 \mathbf{A}^{(2)} \cdots \times_N \mathbf{A}^{(N)} \quad (5)$$

Multiplication of a tensor $\mathcal{X}$ with all but one mode is denoted by

$$\mathcal{X} \times_{-n} \{\mathbf{A}\} = \mathcal{X} \times_1 \mathbf{A}^{(1)} \times_2 \mathbf{A}^{(2)} \cdots \times_{n-1} \mathbf{A}^{(n-1)} \times_{n+1} \mathbf{A}^{(n+1)} \cdots \times_N \mathbf{A}^{(N)} \quad (6)$$

III. HYPERSPECTRAL IMAGE DENOISING MODEL

In this section, we first introduce how non-local similar patches are grouped into a 3rd-order tensor. Then we describe the nonnegative Tucker decomposition method adopted for the 3rd-order tensor based on hierarchical alternative least square approach.

A. Non-local Similarity Patch Matching

Acito *et al.* [41] suggested that signal-independent noise in hyperspectral image can be modeled by Gaussian distribution. A noise free hyperspectral image $\mathcal{X}^*$ degraded by additive zero mean Gaussian noise can be modeled as

$$\mathcal{Y}^* = \mathcal{X}^* + \mathcal{N}^* \quad (7)$$

where $\mathcal{N}^*$ is the noise with a known variance σ^2 and zero mean. The ultimate goal of denoising is to estimate $\hat{\mathcal{X}}^*$ of the clean hyperspectral image $\mathcal{X}^*$ from noisy $\mathcal{Y}^*$.

Traditional image denoising tasks has generalized the use of non-local similarity based denoising framework. Such framework also can be applied to hyperspectral images. Denote I_h , I_w and I_s as the height and the width in the spatial dimension and the number of spectral bands of an input hyperspectral image respectively, the hyperspectral image can be represented as a 3rd-order tensor $\mathcal{Y}^* \in \mathbb{R}^{I_h \times I_w \times I_s}$ with two spatial modes and one spectral mode. A group of 2D full band patches $\{P_{i,j}\}_{1 \leq i \leq I_h - I_p + 1, 1 \leq j \leq I_w - I_p + 1} \subset \mathbb{R}^{I_p \times I_s}$ is built to express the hyperspectral image by sweeping across the image spatially with overlaps, where each band of a full band patch is ordered lexicographically as a column vector. We can now reformulate all full band patches as a group of 2D patches $\Phi_{\mathcal{Y}} = \{\mathcal{Y}_i \in \mathbb{R}^{I_p \times I_s}\}_{i=1}^S$, where $S = (I_h - I_p + 1) \times (I_w - I_p + 1)$ is the number of patches over the whole hyperspectral image.

For a given local full band patch P_{ref} , we can find a collection of similar full band patches P_k from a predefined searching window by utilizing the non-local similarity matching, where $k \in [1, K]$ and K is the number of similar patches. The similarity between P_{ref} and P_k can be formulated as:

$$Sim(P_{ref}, P_k) = \sqrt{\|P_{ref} - P_k\|^2} \quad (8)$$

The smaller the $Sim(P_{ref}, P_k)$, the more similar these two patches turn out to be. Denote $\mathcal{Y} \in \mathbb{R}^{I_1 \times I_2 \times I_3}$ as the 3rd-order tensor stacked by P_{ref} and its non-local similar full band patches P_k in the search window, where we denote $I_1 = I_p^2$, $I_2 = I_s$, $I_3 = K$ for convenient notation. It not only utilizes the non-local similarity across spatial space, but also preserves the global correlation across spectral space.

Then the aforementioned denoising model for the corresponding true non-local similarity full band patches $\mathcal{X}$ from its corrupted $\mathcal{Y}$ is transformed into:

$$\mathcal{Y} = \mathcal{X} + \mathcal{N} \quad (9)$$

where $\mathcal{N}$ is the noise tensor of the stacked patches.

The corresponding noise free tensor $\mathcal{X}$ can be estimated from its corrupted $\mathcal{Y}$ by solving the following optimization problem:

$$\begin{aligned} \min_{\mathcal{G}, \mathbf{A}^{(1)}, \mathbf{A}^{(2)}, \mathbf{A}^{(3)}} & \|\mathcal{Y} - \mathcal{X}\|_F \\ \text{s.t.} & \quad \mathcal{X} = \mathcal{G} \times_1 \mathbf{A}^{(1)} \times_2 \mathbf{A}^{(2)} \times_3 \mathbf{A}^{(3)} \\ & \quad \mathbf{A}^{(n)T} \mathbf{A}^{(n)} = \mathbf{I}, n = 1, 2, 3 \end{aligned} \quad (10)$$

where $\mathbf{A}^{(1)} \in \mathbb{R}^{I_1 \times J_1}$, $\mathbf{A}^{(2)} \in \mathbb{R}^{I_2 \times J_2}$, $\mathbf{A}^{(3)} \in \mathbb{R}^{I_3 \times J_3}$ correspond to the basis vectors in the three modes of $\mathcal{Y}$ with $I_1 \geq J_1$, $I_2 \geq J_2$, and $I_3 \geq J_3$. Here $\mathcal{G} \in \mathbb{R}^{J_1 \times J_2 \times J_3}$ is the so-called core tensor and J_1, J_2, J_3 are the ranks responding to three modes. The AIC/MDL criterion [42], [43] can be used to determine the rank for our tensor model. In addition, $I_2 > J_2$ leads to dimensionality reduction in the spectral mode of corrupted tensor $\mathcal{Y}$.

Here we utilize the hierarchical alternative least square (ALS) based nonnegative Tucker decomposition to calculate the component $\mathcal{G}, \mathbf{A}^{(1)}, \mathbf{A}^{(2)}, \mathbf{A}^{(3)}$ of the noise free tensor $\mathcal{X}$.

B. Hierarchical ALS Algorithm for Nonnegative Tucker Decomposition

Due to matrix inversion, standard alternative least square (ALS) [44] suffers from high computational cost for large scale problem, and may not converge to a stable solution. Rather than solving the problem by ALS based tucker decomposition technique, we propose to use a hierarchical alternative least square approach for nonnegative tucker decomposition [45] (referred here as NTD-HALS algorithm), which estimates nonnegative components via a sequential iterative procedure. According to the nonnegative Tucker decomposition, the aforementioned denoising model can be converted into:

$$\begin{aligned}\mathcal{Y} &= \mathcal{X} + \mathcal{N} \\ &= \mathcal{G} \times_1 \mathbf{A}^{(1)} \times_2 \mathbf{A}^{(2)} \cdots \times_N \mathbf{A}^{(N)} + \mathcal{N} \\ &= \sum_{j_1=1}^{J_1} \sum_{j_2=1}^{J_2} \cdots \sum_{j_N=1}^{J_N} g_{j_1 j_2 \dots j_N} \mathbf{a}_{j_1}^{(1)} \circ \mathbf{a}_{j_2}^{(2)} \circ \cdots \circ \mathbf{a}_{j_N}^{(N)}\end{aligned}\quad (11)$$

where $\mathcal{Y} \in \mathbb{R}_+^{I_1 \times I_2 \times \dots \times I_N}$, $\circ$ is the vector outer product, and $\mathcal{X}$ denotes the noise free image to be estimated from the noisy $\mathcal{Y}$, and $\mathcal{G} \in \mathbb{R}_+^{J_1 \times J_2 \times \dots \times J_N}$ is the core tensor multiplied by a set of component matrices. $\mathbf{A}^{(n)} = [\mathbf{a}_1^{(n)}, \mathbf{a}_2^{(n)}, \dots, \mathbf{a}_{J_n}^{(n)}] \in \mathbb{R}_+^{I_n \times J_n}$ ($n = 1, 2, \dots, N$) represents nonnegative common factors. Instead of estimating the whole factor $\mathbf{A}^{(n)}$, we update each component $\mathbf{a}_{j_n}^{(n)}$ of the factor $\mathbf{A}^{(n)}$.

The nonnegative Tucker decomposition is divided into two steps: first we update each nonnegative component $\mathbf{a}_{j_n}^{(n)}$ of the $\mathbf{A}^{(n)}$ sequentially, for $n = 1, 2, \dots, N$ and $j_n = 1, 2, \dots, J_n$, then we update the core tensor $\mathcal{G}$.

Update factors $\mathbf{A}^{(n)}$: The denoising model could be split into two parts to estimate $\mathbf{a}_{j_n}^{(n)}$, $n = 1, 2, \dots, N$, $j_n = 1, 2, \dots, J_n$. When the specific component $\mathbf{a}_{j_n}^{(n)}$ is not involved, the tensor consists of all rank-one tensors can be defined as:

$$\mathcal{Y}_{j_n}^- = \sum_{\substack{\bar{\mathbf{k}}=[k_1, k_2, \dots, k_N] \\ k_n \neq j_n}} g_{\bar{\mathbf{k}}} \mathbf{a}_{k_1}^{(1)} \circ \mathbf{a}_{k_2}^{(2)} \circ \cdots \circ \mathbf{a}_{k_N}^{(N)} \quad (12)$$

When the specific component $\mathbf{a}_{j_n}^{(n)}$ is involved, the tensor consists of all rank-one tensors can be denoted by:

$$\mathcal{Y}_{j_n}^+ = \sum_{\substack{\bar{\mathbf{k}}=[k_1, k_2, \dots, k_N] \\ k_n = j_n}} g_{\bar{\mathbf{k}}} \mathbf{a}_{k_1}^{(1)} \circ \cdots \circ \mathbf{a}_{k_{n-1}}^{(n-1)} \circ \mathbf{a}_{j_n}^{(n)} \circ \mathbf{a}_{k_{n+1}}^{(n+1)} \circ \cdots \circ \mathbf{a}_{k_N}^{(N)} \quad (13)$$

where $k_n = 1, 2, \dots, J_n$ and $\bar{\mathbf{k}} = [k_1, k_2, \dots, k_N]$ is the index vector.

The denoising model defined in Eq. (11) can be rewritten using the above notations as follows:

$$\mathcal{Y} = \mathcal{Y}_{j_n}^- + \mathcal{Y}_{j_n}^+ + \mathcal{N} \quad (14)$$

Then we define a residual tensor $\tilde{\mathcal{Y}}_{j_n}$ as:

$$\begin{aligned}\tilde{\mathcal{Y}}_{j_n} &= \mathcal{Y} - \mathcal{Y}_{j_n}^- \\ &= \mathcal{Y}_{j_n}^+ + \mathcal{N} \\ &= \mathcal{G}_{k_n=j_n \times -n} \{\mathbf{A}\} \times_n \mathbf{a}_{j_n}^{(n)} + \mathcal{N}\end{aligned}\quad (15)$$

Therefore, the component $\mathbf{a}_{j_n}^{(n)}$ can be updated from the residual tensor $\tilde{\mathcal{Y}}_{j_n}$ by

$$\mathbf{a}_{j_n}^{(n)} \leftarrow \frac{\max\{\xi, \langle \tilde{\mathcal{Y}}_{j_n}, \mathcal{G}_{k_n=j_n \times -n} \{\mathbf{A}\} \rangle_{-n}\}}{\langle \mathcal{G}_{k_n=j_n \times -n} \{\mathbf{A}\}, \mathcal{G}_{k_n=j_n \times -n} \{\mathbf{A}\} \rangle_{-n}} \quad (16)$$

for $j_n = 1, 2, \dots, J_n$ and $n = 1, 2, \dots, N$. However, the method is still of high computational complexity due to the computation cost of the residual tensor. We simplify Eq. (16) by replacing the residual tensor $\tilde{\mathcal{Y}}_{j_n}$ via Eq. (15) and algebraic operations. The solution can be transformed into:

$$\begin{aligned}\mathbf{a}_{j_n}^{(n)} &\leftarrow \frac{\max\{\xi, \langle \mathcal{G}_{k_n=j_n \times -n} \{\mathbf{A}\} \times_n \mathbf{a}_{j_n}^{(n)} + \mathcal{N}, \mathcal{G}_{k_n=j_n \times -n} \{\mathbf{A}\} \rangle_{-n}\}}{w_{j_n}} \\ &= \max\{\xi, \mathbf{a}_{j_n}^{(n)} + \frac{1}{w_{j_n}} \langle \mathcal{N}, \mathcal{G}_{k_n=j_n \times -n} \{\mathbf{A}\} \rangle_{-n}\}\end{aligned}\quad (17)$$

where $w_{j_n} = \langle \mathcal{G}_{k_n=j_n \times -n} \{\mathbf{A}\}, \mathcal{G}_{k_n=j_n \times -n} \{\mathbf{A}\} \rangle_{-n}$ is a scalar computed by current values. On the basis of above transformation, the vector $\mathbf{a}_{j_n}^{(n)}$ can be updated by the noise tensor $\mathcal{N}$ and known parameters.

$$\begin{aligned}\mathcal{N} &\leftarrow \mathcal{N} + \mathcal{Y}_{j_n}^+ - \hat{\mathcal{Y}}_{j_n}^+ \\ &= \mathcal{N} + \mathcal{G}_{k_n=j_n \times -n} \{\mathbf{A}\} \times_n (\mathbf{a}_{j_n}^{(n)} - \hat{\mathbf{a}}_{j_n}^{(n)})\end{aligned}\quad (18)$$

where $\hat{\mathbf{a}}_{j_n}^{(n)}$ is updated by the vector $\mathbf{a}_{j_n}^{(n)}$, and $\hat{\mathcal{Y}}_{j_n}^+$ is the updated residual tensor. Based on the above steps, the component $\mathbf{a}_{j_n}^{(n)}$ of the $\mathbf{A}^{(n)}$ can be updated sequentially by combining Eq. (17) and Eq. (18).

Update the core tensor $\mathcal{G}$: Similarly, we attempt to update the core tensor $\mathcal{G}$ item by item so as to update the entry $g_{\bar{\mathbf{j}}}$ of the core tensor $\bar{\mathbf{j}} = [j_1, j_2, \dots, j_N]$. We define the tensor $\mathcal{Y}_{\bar{\mathbf{j}}}^-$ as the tensor excluding the rank one tensors $\mathbf{a}_{j_1}^{(1)}, \mathbf{a}_{j_2}^{(2)}, \dots, \mathbf{a}_{j_N}^{(N)}$,

$$\mathcal{Y}_{\bar{\mathbf{j}}}^- = \sum_{k_1 \neq j_1} \sum_{k_2 \neq j_2} \cdots \sum_{k_N \neq j_N} g_{\bar{\mathbf{k}}} \mathbf{a}_{k_1}^{(1)} \circ \mathbf{a}_{k_2}^{(2)} \circ \cdots \circ \mathbf{a}_{k_N}^{(N)} \quad (19)$$

The tensor composed by the components $\mathbf{a}_{j_1}^{(1)}, \mathbf{a}_{j_2}^{(2)}, \dots, \mathbf{a}_{j_N}^{(N)}$ can be denoted as

$$\mathcal{Y}_{\bar{\mathbf{j}}}^+ = g_{\bar{\mathbf{j}}} \mathbf{a}_{j_1}^{(1)} \circ \mathbf{a}_{j_2}^{(2)} \circ \cdots \circ \mathbf{a}_{j_N}^{(N)} \quad (20)$$

According the above notations, the denoising model can be rewritten as

$$\mathcal{Y} = \mathcal{Y}_{\bar{\mathbf{j}}}^- + \mathcal{Y}_{\bar{\mathbf{j}}}^+ + \mathcal{N} \quad (21)$$

Then a new residual tensor $\tilde{\mathcal{Y}}_{\bar{\mathbf{j}}}$ is defined as follows:

$$\begin{aligned}\tilde{\mathcal{Y}}_{\bar{\mathbf{j}}} &= \mathcal{Y} - \mathcal{Y}_{\bar{\mathbf{j}}}^- \\ &= \mathcal{Y}_{\bar{\mathbf{j}}}^+ + \mathcal{N} \\ &= g_{\bar{\mathbf{j}}} \mathbf{a}_{j_1}^{(1)} \circ \mathbf{a}_{j_2}^{(2)} \circ \cdots \circ \mathbf{a}_{j_N}^{(N)} + \mathcal{N}\end{aligned}\quad (22)$$

Suppose that the vectors $\mathbf{a}_{j_n}^{(n)}$ are the unit length vectors, i.e. $\mathbf{a}_{j_n}^{(n)T} \mathbf{a}_{j_n}^{(n)} = 1$. The entry $g_{\bar{\mathbf{j}}}$ can be derived by the following formula:

$$\begin{aligned}g_{\bar{\mathbf{j}}} &\leftarrow \tilde{\mathcal{Y}}_{\bar{\mathbf{j}}} \bar{\times}_1 \mathbf{a}_{j_1}^{(1)} \bar{\times}_2 \mathbf{a}_{j_2}^{(2)} \cdots \bar{\times}_N \mathbf{a}_{j_N}^{(N)} \\ &= (\mathcal{Y}_{\bar{\mathbf{j}}}^+ + \mathcal{N}) \bar{\times}_1 \mathbf{a}_{j_1}^{(1)} \bar{\times}_2 \mathbf{a}_{j_2}^{(2)} \cdots \bar{\times}_N \mathbf{a}_{j_N}^{(N)} \\ &= g_{\bar{\mathbf{j}}} + \mathcal{N} \bar{\times}_1 \mathbf{a}_{j_1}^{(1)} \bar{\times}_2 \mathbf{a}_{j_2}^{(2)} \cdots \bar{\times}_N \mathbf{a}_{j_N}^{(N)}\end{aligned}\quad (23)$$

Define $\hat{g}_{\bar{j}}$ as the new estimation of $g_{\bar{j}}$, $\hat{\mathcal{Y}}_{\bar{j}}^+$ to update $\mathcal{Y}_{\bar{j}}^+$. The update strategy of the noise tensor $\mathcal{N}$ can be deduced from Eq. (22).

$$\begin{aligned}\mathcal{N} &\leftarrow \mathcal{N} + \mathcal{Y}_{\bar{j}}^+ - \hat{\mathcal{Y}}_{\bar{j}}^+ \\ &= \mathcal{N} + (g_{\bar{j}} - \hat{g}_{\bar{j}}) \mathbf{a}_{j_1}^{(1)} \circ \mathbf{a}_{j_2}^{(2)} \circ \dots \circ \mathbf{a}_{j_N}^{(N)}\end{aligned}\quad (24)$$

The core tensor $\mathcal{G}$ can be derived by Eq. (23) and Eq. (24) sequentially.

Algorithm 1: Algorithm of the hyperspectral image denoising method.

Input: Noisy Hyperspectral Image $\mathcal{Y}^*$
Output: Denoised Hyperspectral Image $\mathcal{X}^{*(L)}$
Initialize $\mathcal{X}^{*(0)} = \mathcal{Y}^*$
for $l = 1 : L$ **do**
 Calculate $\mathcal{Y}^{*(l)}$ by Eq. (7)
 Construct the full band patches $\Phi_{\mathcal{Y}^{*(l)}}$
 Construct similar patches group set $\{\mathcal{Y}\}_{k=1}^K$ by Eq. (8)
 for each group $\mathcal{Y}$ **do**
 Calculate J_1, J_2, J_3 by AIC/MDL
 $\mathcal{N} = \mathcal{Y} - \mathcal{G} \times_1 \mathbf{A}^{(1)} \times_2 \mathbf{A}^{(2)} \times_3 \mathbf{A}^{(3)}$
 while not converge do
 /* Update $\mathbf{A}^{(n)}$ */
 for $n = 1$ to N **do**
 Update $\mathbf{A}^{(n)}$ by Eq. (17) and Eq. (18)
 end
 /* Update $\mathcal{G}$ */
 for each $\bar{j}$ in $\mathcal{G}$ **do**
 Update $g_{\bar{j}}$ by Eq. (23)
 Update $\mathcal{N}$ by Eq. (24)
 end
 Calculate $\mathcal{X} = \mathcal{G} \times_1 \mathbf{A}^{(1)} \times_2 \mathbf{A}^{(2)} \times_3 \mathbf{A}^{(3)}$
 Aggregate $\mathcal{X}$ to form the clean $\mathcal{X}^{*(l)}$
 end
 end
end

C. Iterative Denoising Optimization

Denoising once may not guarantee to get a desired result in practice. Therefore, we propose an iterative denoising strategy for the proposed method, which is a process to take the trade-off between bias and variance. A single iteration can be done by:

$$\mathcal{Y}^{*(l)} = \mathcal{X}^{*(l-1)} + \delta(\mathcal{Y}^* - \mathcal{X}^{*(l-1)}) \quad (25)$$

where l is the iteration number, $\delta \in (0, 1)$ is the residual proportionality coefficient. By utilizing Eq. (25) iteratively, the denoised effect could be optimized. Nevertheless, with the increase of iterations, the time consumption tends to become excessively expensive. Hence, the maximum iteration number L should be properly set to balance the time cost and the experiment results.

The algorithm for the proposed hyperspectral image denoising method is summarized in Algorithm 1.

D. Analysis of Computational Complexity

Here, we analyze the computational complexity of the proposed method. The computation of non-local similarity patch matching involves full band patch construction and similar patch group set construction. In fact, full band patches can be generated by a sliding window which is related to the patch size and the size of the hyperspectral image in spatial space. Thus this part only considers the time complexity of similar patch group set construction. When the full band patches $\Phi_{\mathcal{Y}^*}$ are given, cost of similar patch construction is $\mathcal{O}(KSI_p^2I_s)$, where K is the number of similar patches, and S is the number of full band patches. Since hierarchical ALS algorithm for nonnegative Tucker decomposition is only related to the size of full band patches, the computational complexity of this part is $\mathcal{O}(KI_p^2I_s)$. Thus, the overall per iteration computational complexity of our method is $\mathcal{O}((S+1)KI_p^2I_s)$.

IV. EXPERIMENTAL RESULTS

To demonstrate the effectiveness of the proposed method, we compare it with seven recently developed denoising methods on a simulated hyperspectral image dataset and three hyperspectral image datasets: Columbia dataset [46], Indian Pines dataset [47], Pavia University dataset [48] and Washington DC Mall dataset [47].

The alternative denoising methods include band-wise K-SVD [17] and band-wise BM3D [18]; 3D-cube-based approaches K-SVD [20] and non-local transform domain filter for volumetric data (BM4D) [21] which are extended from 2D methods, state-of-the-art tensor-based approaches low rank tensor approximation (LRTA) [30] and PARAFAC [29], and denoising algorithm via sparse representation and low rank constraint (SPA+LR) [22]. All the parameters of the competing algorithm are optimally assigned or automatically chosen according to the reference papers.

To evaluate the performance of the denoising methods, picture quality indices (PQI), *e.g.* peak signal-to-noise ratio (PSNR), structure similarity (SSIM) and feature similarity (FSIM), are used to measure the similarity between the denoised image and the original hyperspectral image which is assumed to be the ground truth. PSNR and SSIM evaluate the similarity between the target image and the reference image based on mean square error (MSE) and structural consistency, respectively. SSIM emphasizes the perceptual consistency with the reference image. The higher three measures are, the closer the denoised hyperspectral image is to the ground truth. The denoising performance evaluation metrics for each hyperspectral image are calculated as the average of the band-wise results.

We corrupted a clean hyperspectral image with an additive zero mean Gaussian noise whose σ ranges from 10 to 60. The patch size is set to 6 pixels and the number of similar patches is set to 50 empirically. The size of full band patch searching window depends on the size of input hyperspectral image, ranging from 30×30 to 80×80 pixels for all datasets. To demonstrate the influence of window size, we show the average PSNR on the Columbia dataset in Table. II. It shows that the performance will be improved when searching window

Fig. 2. (a) The images at band 620nm of *stuffed toy*, *cloth* and *feather*; (b) The corresponding images corrupted by the mixture of $\sigma = 40$ Gaussian noise; (c)-(f) The restored images obtained by 4 of the comparing 8 denoising method. The demarcated areas in each image is amplified at a 4 times larger scale for easy observation of details.

TABLE I
PERFORMANCE COMPARISON OF THE COMPETING METHODS ON THE COLUMBIA DATASET.

Methods	$\sigma = 20$			$\sigma = 40$			$\sigma = 60$		
	PSNR	SSIM	FSIM	PSNR	SSIM	FSIM	PSNR	SSIM	FSIM
Noisy	17.59	0.2433	0.6253	14.27	0.1870	0.5254	11.38	0.0962	0.4174
PARAFAC	35.36	0.8932	0.9599	31.52	0.7609	0.9246	29.30	0.7857	0.8775
LRTA	37.67	0.9437	0.9722	34.28	0.8212	0.9223	31.53	0.8036	0.9028
BwK-SVD	33.95	0.8002	0.9603	29.65	0.7157	0.9200	27.89	0.7079	0.8564
K-SVD	36.02	0.9397	0.9721	33.41	0.7285	0.9203	31.90	0.8110	0.9123
BwBM3D	38.21	0.9497	0.9747	34.94	0.8802	0.9358	33.84	0.8761	0.9251
BM4D	40.57	0.9665	0.9878	37.79	0.9105	0.9583	34.62	0.8984	0.9399
SPA+LR	40.96	0.9707	0.9813	38.16	0.9334	0.9716	35.41	0.9012	0.9421
Ours	42.87	0.9894	0.9946	39.85	0.9472	0.9760	36.26	0.9048	0.9564

(a)

(b)

(c)

Fig. 3. (a) Simulated RGB image of *sponges*; (b) The spectral reflectance difference curves of 8 competing methods at pixel (20, 200); (c) The spectral reflectance difference curves of 8 competing methods at pixel (80, 380). The noise free images are corrupted by Gaussian noise of $\sigma = 30$.

TABLE II
PSNR PERFORMANCE FOR DIFFERENT SEARCHING WINDOW SIZES ON
THE COLUMBIA DATASET.

Window Size	$\sigma = 20$	$\sigma = 40$	$\sigma = 60$
30×30	39.63	37.14	33.93
40×40	39.45	37.59	34.12
50×50	39.72	38.07	34.51
60×60	41.54	38.46	35.39
70×70	41.65	38.68	35.41
80×80	41.59	38.66	35.27

size increases in a range, and might get worse when searching window becomes too large. The residual proportionality coefficient δ is set to 0.1 and the maximum iteration number L is determined automatically according to σ , usually less than three times of σ for balancing the time cost and denoising effect.

A. Results on Columbia Dataset

The Columbia Dataset [46] contains 32 real-world scenes formed by a wide variety of real-world materials and objects. Each hyperspectral image has 31 bands, containing reflectance data collected from 400nm to 700nm with 10nm step. Each band is of 512×512 in size.

In Fig. 2, we show band at 620nm of *stuffed toy*, *cloth* and *feather* images to demonstrate the performance of various methods. Our method is compared with K-SVD, BM4D, and SPA+LR. Demarcated areas in the scene have been enlarged for 4 times in the figure for easy observation of details. It is obvious that the proposed method outperforms the alternative methods in the recovery of both finer-grained textures and coarser-grained structures.

Table I lists the performance of all methods, which shows clear advantage of the proposed method over alternative. To further illustrate such superiority, we demonstrate in Fig. 3 the spectral reflectance difference curves of all competing methods using image *sponges*. Spectral reflectance difference curves of a hyperspectral image denoising method at two spatial locations are obtained by sequentially interpolating 31 elements of the deviation between the restored and the clean hyperspectral image along their spectral domain. The curves of our proposed method are smoother than other methods and the mean square error is smaller. This further proves that our method achieves better approximation than alternative methods.

B. Results on Indian Pines Dataset

Indian Pines scene was collected by the AVIRIS sensor over the Indian Pines test site in North-western Indiana. The image consists of 145×145 pixels and 224 spectral bands.

In Fig. 5, we show the denoising performance at the 45th band of the Indian Pines dataset under Gaussian noise with σ ranging from 10 to 50. Our method is capable of properly removing the different degrees of Gaussian noises while finely preserving the underlying structure in the hyperspectral image. More noise is added, the worse is the denoising effect. From

Table III, we can infer that the proposed method evidently outperforms all other competing methods with $\sigma = 20, 30, 40, 50$ respectively.

For ease of observation, we illustrate an example image located at the 45th band of Indian Pines dataset in Fig. 4. Among alternative methods, BM4D and SPA+LR perform comparatively better in structure preserving. However, the images restored by them have more blurred edges than those obtained by our method. Compared with traditional non-local similarity based and dictionary learning based methods, our method restores the corrupted image preferably because it utilizes properties in both spectral and spatial domains. Fig. 6 shows the denoising performance of Indian Pines dataset for band 1 to 60, in the case of $\sigma = 20$.

C. Results on Pavia University Dataset

Hyperspectral image acquired using ROSIS over the Pavia University is chosen for this experiment. Pavia University dataset contains 103 spectral bands with a spectral coverage ranging from 0.43 to 0.86 μm and a spatial resolution of 1.3 meter per pixel. This image is of the size $610 \times 340 \times 103$.

Table IV lists the performance of all comparing methods on the Pavia University dataset corresponding to various degrees of noises. For each noise parameter, the PQI value for each competing HSI denoising method has been calculated and recorded. From these results, the advantage of the proposed method can be clearly observed. We show in Fig. 7 the 15th band with 340×340 window size extracted from the Pavia University dataset for further assessment of our method. It is obvious that the proposed method outperforms the comparing methods. The PSNR analysis of each spectrum presented in Fig. 8 illustrates that the denoising effect of the proposed method at each band of the Pavia University Dataset with the noise of $\sigma = 30$ is superior than any other comparing methods. This proves again that combining non-local similarity and tensor decomposition can preserve both spatial information and spectral information, which leads to prominent advantages in denoising effect.

D. Results on Washington DC Mall Dataset

Washington DC Mall Dataset was collected using an airborne system containing 210 spectral bands from 0.4 to 2.4 μm in the visible to infrared spectrum. It contains 1028 scan lines with 307 pixels in each scan line.

We used a patch of size $300 \times 300 \times 190$ in the Washington DC image for experiments. The PSNR, SSIM and FSIM are presented in Table V. This table shows that, of the nine methods, the proposed non-local similarity based denoising algorithm still works very effectively. The results of all methods at the 15th band of the Washington DC Mall image are shown in Fig. 9. It can be concluded that the image restored by our method is capable of properly removing the Gaussian noise while finely preserving the structure underlying in the hyperspectral image. Not only the noise is well suppressed, but also the detailed information, both in edges and textures, is well preserved. Comparison of PSNR values among various methods on the Washington DC Dataset in Fig. 10 also shows

Fig. 4. (a) The Original image located at the 45th band in the Indian Pines dataset; (b) The corrupted image by Gaussian noise of $\sigma = 20$; (c)-(f) The denoised hyperspectral images generated by the alternative methods.

TABLE III
PERFORMANCE COMPARISON OF THE COMPETING METHODS ON THE INDIAN PINES DATASET.

Methods	$\sigma = 20$			$\sigma = 30$			$\sigma = 40$			$\sigma = 50$		
	PSNR	SSIM	FSIM	PSNR	SSIM	FSIM	PSNR	SSIM	FSIM	PSNR	SSIM	FSIM
Noisy	22.134	0.394	0.730	18.617	0.117	0.407	16.113	0.081	0.352	14.175	0.054	0.302
LRTA [30]	38.640	0.902	0.936	36.543	0.854	0.905	34.832	0.826	0.896	34.328	0.826	0.889
PARAFAC [29]	38.877	0.904	0.933	35.225	0.830	0.891	29.536	0.582	0.777	27.479	0.463	0.716
BwK-SVD [17]	30.584	0.678	0.863	28.947	0.582	0.775	28.129	0.529	0.760	27.349	0.486	0.745
K-SVD [20]	35.329	0.836	0.893	33.533	0.769	0.860	33.459	0.781	0.867	22.116	0.203	0.535
BwBM3D [18]	36.736	0.867	0.884	35.174	0.833	0.863	33.358	0.794	0.854	33.002	0.774	0.838
BM4D [21]	38.642	0.906	0.917	36.826	0.874	0.898	35.105	0.845	0.892	34.069	0.819	0.881
SPA+LR [22]	41.143	0.931	0.951	38.301	0.903	0.930	36.328	0.838	0.899	35.523	0.869	0.914
Ours	41.937	0.946	0.975	39.026	0.912	0.939	37.012	0.862	0.917	35.972	0.893	0.926

TABLE IV
PERFORMANCE COMPARISON OF THE COMPETING METHODS ON THE PAVIA UNIVERSITY DATASET.

Methods	$\sigma = 20$			$\sigma = 30$			$\sigma = 40$			$\sigma = 50$		
	PSNR	SSIM	FSIM	PSNR	SSIM	FSIM	PSNR	SSIM	FSIM	PSNR	SSIM	FSIM
Noisy	22.143	0.386	0.710	18.616	0.190	0.538	16.122	0.164	0.519	14.179	0.087	0.399
LRTA [30]	33.158	0.821	0.917	31.432	0.766	0.886	30.069	0.714	0.856	29.157	0.674	0.837
PARAFAC [29]	29.759	0.734	0.851	29.556	0.718	0.847	29.286	0.697	0.842	28.951	0.672	0.834
BwK-SVD [17]	28.335	0.687	0.856	26.626	0.596	0.812	25.872	0.504	0.623	24.715	0.436	0.517
K-SVD [20]	34.372	0.917	0.942	32.319	0.847	0.893	30.410	0.828	0.903	31.328	0.804	0.862
BwBM3D [18]	31.994	0.734	0.851	30.338	0.770	0.853	29.196	0.729	0.829	28.461	0.702	0.805
BM4D [21]	36.342	0.931	0.958	34.491	0.873	0.920	32.450	0.861	0.911	31.981	0.814	0.880
SPA+LR [22]	37.542	0.948	0.975	34.503	0.875	0.933	33.519	0.893	0.942	32.187	0.819	0.901
Ours	39.937	0.969	0.984	37.728	0.956	0.961	36.012	0.927	0.956	34.472	0.903	0.928

TABLE V
PERFORMANCE COMPARISON OF THE COMPETING METHODS ON THE WASHINGTON DC MALL DATASET.

Methods	$\sigma = 20$			$\sigma = 30$			$\sigma = 40$			$\sigma = 50$		
	PSNR	SSIM	FSIM	PSNR	SSIM	FSIM	PSNR	SSIM	FSIM	PSNR	SSIM	FSIM
Noisy	22.139	0.392	0.688	18.618	0.249	0.528	16.119	0.168	0.506	14.184	0.120	0.449
LRTA [30]	35.672	0.909	0.947	33.368	0.862	0.919	31.735	0.816	0.893	30.448	0.770	0.870
PARAFAC [29]	31.674	0.838	0.900	31.307	0.824	0.893	30.836	0.805	0.885	30.159	0.776	0.874
BwK-SVD [17]	28.687	0.672	0.845	27.253	0.580	0.804	26.408	0.524	0.780	25.809	0.488	0.766
K-SVD [20]	36.132	0.939	0.902	32.426	0.865	0.902	34.41	0.728	0.878	31.616	0.846	0.900
BwBM3D [18]	31.072	0.826	0.884	29.475	0.765	0.843	28.384	0.716	0.812	27.668	0.678	0.779
BM4D [21]	41.056	0.951	0.928	38.770	0.927	0.908	37.156	0.907	0.891	35.909	0.887	0.878
SPA+LR [22]	40.967	0.960	0.945	39.564	0.941	0.932	38.072	0.917	0.911	36.742	0.894	0.894
Ours	42.870	0.974	0.956	40.371	0.954	0.939	39.56	0.938	0.926	37.263	0.917	0.922

Fig. 5. (a) The original image located at the 20th band in the Indian Pines dataset; (b)–(f) The denoised images obtained by our method with each images corrupted by different Gaussian noises ranging from 10 to 50.

Fig. 6. Comparison of PSNR values among various methods on the Indian Pines Dataset with $\sigma = 20$.

that the proposed algorithm achieves higher performance than others in the case of $\sigma = 30$. These results demonstrate the effectiveness of our denoising method.

V. CONCLUSION

In this paper, we have introduced a novel hyperspectral image denoising method based on tensor decomposition which utilizes both non-local similarity across the spatial space and global redundancy along the spectral space. Unlike previous vector or matrix based approaches, the proposed method considers the hyperspectral image as a 3D tensor and simultaneously exploits the spatial-spectral information of the hyperspectral image, thus boosts the denoising performance. We solved this model by a hierarchical ALS based nonnegative Tucker decomposition rather than traditional approach so as to achieve better denoising effect. Experiments on three real hyperspectral image datasets demonstrated that the proposed method outperforms several well-known denoising approaches in terms of both visual quality and PSNR metrics. Last but not

least, the proposed tensor-based framework can also be applied to other hyperspectral image processing problems (e.g., reconstruction and unmixing) and enhance the performance in those applications. In the optimization step, though each component is updated sequentially, the time complexity of our solution still can be improved, which will be our future research task.

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Fig. 7. (a) The Original image located at the 15th band in the Pavia University dataset; (b) The corrupted image with Gaussian noise of $\sigma = 30$; (c)-(f) The denoised hyperspectral images obtained by the comparing denoising methods. Image patches at the bottom-left are zoomed-in views from the top-right small patches.

Fig. 8. Comparison of PSNR values among various methods on the Pavia University Dataset with $\sigma = 30$.

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Fig. 9. (a) The Original image located at the 15th band in the Washington DC Mall dataset; (b) The corrupted image by Gaussian noise of $\sigma = 30$; (c)-(f) The denoised hyperspectral images obtained by the comparing denoising methods. Image patches at the bottom-left are zoomed-in views from the top-right small patches.

Fig. 10. Comparison of PSNR values among various methods on the Washington DC Dataset with $\sigma = 30$.

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