Simultaneous H_∞ stabilization for large-scale systems within distributed wireless networked control framework over fading channels

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Abstract

In this paper, the simultaneous H_∞ stabilization problem is investigated for a physically interconnected large-scale system which works in multiple operation modes. A distributed wireless networked control framework is introduced, in which the distributed dynamic output feedback controllers not only use the local measurements, but also receive the neighboring controllers' broadcasts via wireless networks. The channel fading in wireless communications is described as the Rice fading model. Our focus is on the design of the distributed controllers such that the large-scale system is mean-square stable in each operation mode and achieves a prescribed H_∞ disturbance attenuation level. By employing the Lyapunov functional method and related stochastic analysis techniques, a sufficient condition on the existence of desired controllers is presented, and the parameterization of the controller gains is derived. Finally, a numerical example is utilized to illustrate the feasibility of the proposed scheme.

Keywords: Simultaneous stabilization, multiple modes, channel fading, distributed networked controller, Rice fading model

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1. Introduction

With the rapid development of modern industry, economical and technological reasons in manufacturing and industrial processes have led to the arising of large-scale systems with interconnected subsystems. At present, there have been a plenty of results on complex control of large-scale systems [1, 2]. However, a realistic large-scale system such as chemical reaction system and photovoltaic power generation system, may work in multiple operation modes. Up to date, stability of large-scale systems under multiple operation modes has gained little attention, especially simultaneous stability of large-scale systems under multiple operation modes is still open yet challenging. In this paper, a novel simultaneous H_∞ stabilization problem is addressed within a distributed wireless networked control framework.

Control of large-scale systems has been studied in different manners, such as centralized control, decentralized control, and distributed control. In general, it is unrealistic to simultaneously stabilize a large-scale system under multiple operation modes based on a centralized control scheme since it requires intensive computational resources and powerful communication capacity. Although a decentralized scheme can be applied to the control problem [3, 4], it may lead to performance deterioration or instability in the presence of strong interactions of subsystems if the system works in a special operation mode. Compared with both centralized and decentralized control schemes, a distributed control scheme is a good choice due to the fact that it can provide a trade-off of computation, communication and system performance between them. Recently, distributed control has received increasing attention from researchers in the field and a great number of results have appeared [5–13]. For example, in [6], Wang and Lemmon has presented a distributed event-triggering scheme for both linear and nonlinear subsystems, where the subsystem broadcasts its state information to its neighbors only when the subsystem's local state error exceeds a specified threshold. Kazempour and Ghaisari [7] has designed the distributed sliding mode controller. In [8], a model-based networked distributed control system has been formulated as a time-dependent impulsive system by a 3-step interaction estimation algorithm. Razeghi-Jahromi and Seyedi [9, 10] have considered the stabilization problem of distributed control systems with model uncertainties and feedback delays. In [13], the distributed model predictive control scheme has

been proposed for large-scale systems over network information exchange. Moreover, more attention has been paid to the practical applications of distributed control strategies, see, e.g. [14–17], especially in the photovoltaic power generation and wind power generation fields. In this paper, a novel distributed wireless networked control framework is constructed, where the distributed dynamic output feedback (DOF) controllers not only use the local measurements, but also receive the neighboring controllers' broadcasts via wireless networks for simultaneous stability of the large-scale system under multiple operation modes. To the best of the authors' knowledge, there are few results solving the problem of simultaneous stabilization for the multimode physically interconnected large-scale system within a distributed wireless networked control framework.

On the other hand, due to the advances in communication technology, signals are often transmitted via wireless networks. The study on networked systems is gaining momentum, and a plenty of results have been derived by considering the network-induced imperfections, such as packet dropouts, signal quantization and communication delays [18–27]. However, the phenomenon of network-induced channel fading has been paid less attention despite its great impact on wireless communication. Wireless channels are known to be susceptible to fading effects [29]. In general, two main causes for channel fading effects are multipath transmission and shadowing from obstacles [28]. The fading is usually regarded as a stochastic process. That is, the fading can be described as a time-varying stochastic model which can represent the signal's change in both the phase and the amplitude. Recently, channel fading has gained some preliminary attention [29–35]. For example, Elia [29] is concerned with the remote mean-square stability of the system with different kinds of fading channels including the L th-order Rice fading channel and the analog erasure channel; In [30], Garone *et al.* consider the linear-quadratic-Gaussian control problem for MIMO systems over multiple erasure channels; In [31], by Lyapunov stability theory and linear matrix inequality (LMI) technique, the H_∞ fuzzy output feedback control problem for a class of T-S fuzzy systems with channel fading is investigated; Jiang and his colleagues [32] deal with the fault detection problem for a class of nonlinear networked systems with multiple fading measurements. Moreover, the state estimation and filtering problems for networked systems with fading measurements are studied in [33–35]. In this paper, the L th-order Rice fading model, which can simultaneously describe the network-induced delays, packet dropouts and channel fading, is adopted

to model the imperfect fading signals, when the distributed controllers exchange information via wireless networks.

In this paper, the simultaneous stabilization problem is addressed for the large-scale system under multiple operation modes. The objective is to design distributed controllers such that the overall closed-loop system is exponential mean-square stable in each operation mode with a prescribed H_∞ disturbance attenuation level. For this goal, a novel distributed wireless networked control framework is developed, where the distributed controllers can receive the neighboring controllers' broadcasts via wireless networks, to suppress the effects of strong coupling of the interconnected subsystems for better performance. A stochastic L th-order Rice fading model is introduced to describe the phenomenon of channel fading in wireless communications. Then, based on the Lyapunov functional method and the stochastic analysis approach, some sufficient conditions are established to guarantee the simultaneous stability and H_∞ performance of the resultant closed-loop system. The distributed controller gains are obtained via an iterative linear matrix inequality (ILMI) algorithm. To sum up, the main contributions of this paper can be highlighted as follows:

- 1) Within a novel distributed wireless networked control framework, the simultaneous stabilization method is introduced and applied to the multimode large-scale system;
- 2) A stochastic L th-order Rice fading model which can simultaneously describe the network-induced delays, packet dropouts and channel fadings, is considered, thereby better reflecting the reality of signal transmission in wireless channels between distributed controllers;
- 3) By defining a series of Lyapunov functions for the large-scale system in multiple operation modes, a sufficient condition on simultaneous stability is presented. Then, by introducing a parameter independent slack variable and employing an orthogonal complement technique, we succeed in separating the Lyapunov matrices and controller parameter matrix, thereby obtaining the parameterization of the controller gains via an ILMI algorithm.

Notations: The notation used is fairly standard. The expression $X > Y$ (respectively, $X \geq Y$), where X and Y are real symmetric matrices, means that $X - Y$ is positive definite (respectively, positive semi-definite). $\mathbb{E}\{x\}$ is the expectation of the stochastic variable x . $\text{diag}\{\cdot\}$ stands for

the block diagonal matrix. $\mathbb{D}\langle A_i \rangle$ for $i = 1, 2, \dots, N$, denotes the matrix $\text{diag}\{A_1, A_2, \dots, A_N\}$, and $\text{Col}\langle A_i \rangle$ for $i = 1, 2, \dots, N$, denotes the matrix $[A_1^T, A_2^T, \dots, A_N^T]^T$. The cardinality of a set is denoted by $|\cdot|$. $\text{Herm}\{G\}$ is equivalent to $G + G^T$. The symbol $\#$ is used as ellipsis for terms induced by symmetry. $G^\perp$ represents any orthogonal complement of G .

2. Problem Formulation and Preliminaries

This paper considers the simultaneous H_∞ stabilization problem of the large-scale system which works in multiple operating modes. Generally, the simultaneous H_∞ stabilization problem is to design a single fixed controller to simultaneously stabilize a finite number of independent plants and capture the H_∞ performance. While simultaneous stabilization as a branch of robust control has been found efficacious applications in a wide range of fields. We extend the simultaneous stabilization methodology to the multimode large-scale systems, because a realistic large-scale system such as chemical reaction system and photovoltaic power generation system, may work in multiple operation modes (each mode corresponds to a state space plant model). Therefore, the problem considered in this paper is simultaneous H_∞ stabilization problem of a large-scale system in multiple operating modes. For the stability in each operation mode, we intend to design the network based distributed DOF controllers (all the distributed controllers form a fixed controller network) to simultaneously stabilize the large-scale system in multiple operating modes, and achieve the prescribed H_∞ disturbance attenuation performance.

Consider the following large-scale system which works in multiple operation modes

$$\begin{cases} x_i^r(k+1) = A_i^r x_i^r(k) + B_i^r u_i^r(k) + E_i^r \omega_i^r(k) + \sum_{j=1, j \neq i}^N A_{ij}^r x_j^r(k), \\ z_i^r(k) = M_i^r x_i^r(k), \\ y_i^r(k) = C_i^r x_i^r(k) + D_i^r \omega_i^r(k), \end{cases} \quad \begin{matrix} i \in \mathcal{N} = \{1, 2, \dots, N\}, \\ r \in \mathcal{V} = \{1, 2, \dots, V\}, \end{matrix} \quad (1)$$

where N is the number of subsystems; V is the number of operation modes; $x_i^r(k) \in \mathbf{R}^{n_i}$ is the state of the i -th subsystem in the r -th mode; $u_i^r(k) \in \mathbf{R}^{m_i}$ is the control input; $\omega_i^r(k) \in \mathbf{R}^{v_i}$ is the disturbance input; $z_i^r(k) \in \mathbf{R}^{p_i}$ is the regulated output to be controlled; $y_i^r(k) \in \mathbf{R}^{q_i}$ is the output. Furthermore, A_i^r , B_i^r , E_i^r , A_{ij}^r , M_i^r , C_i^r , D_i^r are the constant matrices with compatible dimensions.

The interconnected term $A_{ij}^r x_j^r$ represents dynamic couplings, which show how the j -th subsystem affects the i -th subsystem via state, when the large-scale system runs in the r -th mode. If $A_{ij}^r = 0$, it means there is no interconnection between the j -th subsystem and the i -th subsystem. Because of the influence of the dynamic couplings, the systems controlled with the decentralized control strategy have weaker performance and stability margins, especially when dynamic couplings between subsystems are strong.

Due to the complex structure and spatial distribution of the system (1), a novel distributed networked control framework is proposed, which can tune the trade-off between computation, communication and system performance. This framework in the r -th mode is shown in Fig. 1. It should be pointed out that Fig. 1 only shows the large-scale system works in the r modes. The other modes are not displayed in the diagram. In reality, the large-scale system considered in this paper works in multiple modes. Within the proposed framework, N DOF controllers form a fixed controller network to simultaneously stabilize the large-scale system in V operating modes, and achieve the prescribed H_∞ disturbance attenuation performance. Moreover, the controllers can communicate selectively via wireless networks. As a result, in this control structure, the states of the neighboring controllers are used to provide supplemental feedbacks for the local control to meet the stability and performance requirements. In addition, an unreliable wireless network is used for signal transmission between the controllers in this framework, where the information exchange between the j -th controller and the i -th controller over a multiple reflection path. The channel fading phenomenon and random time-delays have been considered in all the information exchange links by the stochastic Rice fading model. To the best of the authors knowledge, there is no relevant literature concerning the simultaneous H_∞ stabilization problem for the multimode large-scale system within such a distributed wireless networked control framework.

In order to describe the information exchange between the controllers in this framework, a graph theory is adopted. $\mathcal{G}_{cm} = (\mathcal{N}, \mathcal{E}_{cm})$ is a graph consisting of a set of controllers $\mathcal{N} = \{1, 2, \dots, N\}$, and a set of edges $\mathcal{E}_{cm} \in \mathcal{N} \times \mathcal{N}$. An edge from the j -th controller to the i -th controller denoted as an ordered pair $(i, j) \in \mathcal{E}_{cm}$ represents for a unidirectional information exchange link from the j -th controller to the i -th controller, that is, the i -th controller can receive information from the j -th controller, but not necessarily vice versa. In addition, it is assumed that

Figure 1: The distributed networked control framework in the r -th mode, $r \in \mathcal{V} = \{1, 2, \dots, V\}$, $i_l, \dots, i_s \in C_i^{in}$.

there is no edge between a controller and itself. The set of neighbors of the i -th controller is denoted by $C_i^{in} = \{j : (i, j) \in \mathcal{E}_{cm}\}$.

As the states of neighbors have direct effects on the subsystem according to (1), we need to provide certain available neighbors' information as supplemental feedbacks for local control to meet the stability and performance requirements. The state and measurement output information of neighbors, and the states of neighboring DOF controllers can be used as distributed compensation for the local controller. In consideration of the immeasurability and inflexibility of the neighbors' states and measurement outputs, we use the states of neighboring DOF controllers as supplemental feedbacks to reduce the effects of dynamic coupling. Therefore, the following network-based distributed DOF controllers are designed,

$$\begin{cases} \varepsilon_i(k+1) = K_i^a \varepsilon_i(k) + K_i^b y_i^r(k) + \sum_{j \in C_i^{in}} K_{ij} \hat{\varepsilon}_{ij}(k), \\ u_i(k) = K_i^c \varepsilon_i(k), \end{cases} \quad i \in \mathcal{N} = \{1, 2, \dots, N\}, \quad (2)$$

where $\varepsilon_i(k) \in \mathbf{R}^{d_i}$ is the state of the i -th local controller; $\hat{\varepsilon}_{ij}(k) \in \mathbf{R}^{d_j}$ is the received information from the j -th neighboring controller to the i -th local controller; $y_i^r(k) \in \mathbf{R}^{q_i}$ is the measurement output of the i -th plant; K_i^a , K_i^b , K_i^c and K_{ij} are the controller parameters with compatible dimensions to be determined. $K_{ij} \hat{\varepsilon}_{ij}(k)$ is a distributed law as supplemental feedback to improve both stability and performance according to the state of the j -th neighboring controller.

In this paper, an unreliable wireless network is used for signal transmission between the controllers, where the signal travels from the j -th controller to the i -th controller over a multiple-reflection path shown in Fig. 2. Due to multi-path propagation and the shadowing from obstacles, wireless channels are susceptible to fading effect. Therefore, in the presence of channel fading, the received signal of the i -th controller is no longer equivalent to the signal sent from the j -th controller. According to the Rice fading model in [29], the received signal $\hat{\varepsilon}_{ij}(k)$ from the j -th controller to the i -th controller can be modeled by

$$\hat{\varepsilon}_{ij}(k) = \varphi_{ij|\tau=0}(k) \varepsilon_j(k) + \sum_{\tau=1}^{L_{ij}} \varphi_{ij|\tau}(k) \varepsilon_j(k - \tau), \quad (3)$$

Figure 2: Multiple-reflection path.

where $\varphi_{ij|\tau}(k) \in \mathbf{R}$ ($\tau = 0, 1, 2, \dots, L_{ij}$) are the channel coefficients which are independent and identically distributed Gaussian random variables with mean $\bar{\varphi}_{ij|\tau}$ and variance $q_{ij|\tau}$, and L_{ij} is the order of the Rice fading model. In addition, to make the description meaningful and convenient, we set $L = \max\{L_{ij}|i \in \mathcal{N}, j \in C_i^{in}\}$, $L(k) = \min\{L, k\}$, and then the model (3) can be rewritten as

$$\hat{\varepsilon}_{ij}(k) = \varphi_{ij|\tau=0}(k)\varepsilon_j(k) + \sum_{\tau=1}^{L(k)} \varphi_{ij|\tau}(k)\varepsilon_j(k - \tau). \quad (4)$$

Remark 1. In practice, network environments are not ideal. Network-induced phenomena, such as channel fading, cyber-attacks, transmission delays, packet dropouts and disorder, time-varying transmission intervals, competition of multiple node accessing networks, and data quantization, may cause a series of problems. This paper only considers the channel fading which is regarded as the major cause of deterioration in system performance, through the wireless network communication. In our future research, it is expected to extend the results of this paper to other network-induced phenomena such as cyber-attacks [36, 37].

For the purpose of simplicity in the following analysis, we introduce the following notations:

$$x^r(k) = \begin{bmatrix} x_1^r(k) \\ x_2^r(k) \\ \vdots \\ x_N^r(k) \end{bmatrix}, \varepsilon(k) = \begin{bmatrix} \varepsilon_1(k) \\ \varepsilon_2(k) \\ \vdots \\ \varepsilon_N(k) \end{bmatrix}, \omega^r(k) = \begin{bmatrix} \omega_1^r(k) \\ \omega_2^r(k) \\ \vdots \\ \omega_N^r(k) \end{bmatrix}, z^r(k) = \begin{bmatrix} z_1^r(k) \\ z_2^r(k) \\ \vdots \\ z_N^r(k) \end{bmatrix}, \eta^r(k) = \begin{bmatrix} x^r(k) \\ \varepsilon(k) \end{bmatrix},$$

$$A^r = [A_{ij}^r]_{N \times N} \in \mathbf{R}^{N \times N} = \begin{bmatrix} A_1^r & A_{12}^r & \cdots & A_{1N}^r \\ A_{21}^r & A_2^r & \cdots & A_{2N}^r \\ \vdots & \vdots & \ddots & \vdots \\ A_{N1}^r & A_{N2}^r & \cdots & A_N^r \end{bmatrix}, \begin{aligned} B^r &= \text{diag}\{B_1^r, B_2^r, \dots, B_N^r\}, \\ C^r &= \text{diag}\{C_1^r, C_2^r, \dots, C_N^r\}, \\ D^r &= \text{diag}\{D_1^r, D_2^r, \dots, D_N^r\}, \\ E^r &= \text{diag}\{E_1^r, E_2^r, \dots, E_N^r\}, \\ M^r &= \text{diag}\{M_1^r, M_2^r, \dots, M_N^r\}; \end{aligned}$$

and

$K_{[\tau]} = [\mathcal{X}_{ij|\tau}]_{N \times N} \in \mathbb{R}^{N \times N}$, where $\mathcal{X}_{ij|\tau}$ are the elements of the matrix $K_{[\tau]}$ described as

$$\mathcal{X}_{ij|\tau} = \begin{cases} \varphi_{ij|\tau}(k)K_{ij}, & i = 1, 2, \dots, N; j \in C_i^{in}, \\ 0, & i = 1, 2, \dots, N; j \notin C_i^{in}, \end{cases} \quad \text{and } \varphi_{ij|\tau}(k) \text{ are the stochastic channel coefficients;}$$

$\bar{K}_{[\tau]} = [\bar{\mathcal{K}}_{ij|\tau}]_{N \times N} \in \mathbb{R}^{N \times N}$, where $\bar{\mathcal{K}}_{ij|\tau}$ are the elements of the matrix $\bar{K}_{[\tau]}$ described as

$$\bar{\mathcal{K}}_{ij|\tau} = \begin{cases} \bar{\varphi}_{ij|\tau}K_{ij}, & i = 1, 2, \dots, N; j \in C_i^{in}, \\ 0, & i = 1, 2, \dots, N; j \notin C_i^{in}, \end{cases} \quad \text{and } \bar{\varphi}_{ij|\tau} \text{ are the means of the channel coefficients;}$$

$$K_A = \text{diag}\{K_1^a, K_2^a, \dots, K_N^a\}, K_B = \text{diag}\{K_1^b, K_2^b, \dots, K_N^b\}, K_C = \text{diag}\{K_1^c, K_2^c, \dots, K_N^c\}.$$

(5)

Then, by substituting the controller (2) and the Rice fading model (4) into the system (1), we obtain the following overall closed-loop system in V operation modes

$$\begin{cases} \eta^r(k+1) = \begin{bmatrix} A^r & B^r K_C \\ K_B C^r & K_A + K_{[0]} \end{bmatrix} \eta^r(k) + \sum_{\tau=1}^{L(k)} \begin{bmatrix} 0 & 0 \\ 0 & K_{[\tau]} \end{bmatrix} \eta^r(k-\tau) + \begin{bmatrix} E^r \\ K_B D^r \end{bmatrix} \omega^r(k) \\ z^r(k) = \begin{bmatrix} M^r & 0 \end{bmatrix} \eta^r(k), \quad r \in \mathcal{V} = \{1, 2, \dots, V\}. \end{cases} \quad (6)$$

Since there exist stochastic parameters $\varphi_{ij|\tau}$ in the closed-loop system (6), we need separate the stochastic parameters and deterministic parameters for convenience in the following analysis. Then, the closed-loop system (6) can be rewritten as

$$\begin{cases} \eta^r(k+1) = \bar{A}^r \eta^r(k) + \sum_{i=1}^N \sum_{j \in C_i^{in}} (\varphi_{ij|\tau=0}(k) - \bar{\varphi}_{ij|\tau=0}) \bar{H}_{ij|0}^r \eta^r(k) \\ \quad + \sum_{\tau=1}^{L(k)} \bar{F}_{[\tau]} \eta^r(k-\tau) + \sum_{\tau=1}^{L(k)} \sum_{i=1}^N \sum_{j \in C_i^{in}} (\varphi_{ij|\tau}(k) - \bar{\varphi}_{ij|\tau}) \bar{H}_{ij|\tau}^r \eta^r(k-\tau) + \bar{B}^r \omega^r(k), \\ z^r(k) = \bar{M}^r \eta^r(k), \quad r \in \mathcal{V} = \{1, 2, \dots, V\}, \end{cases} \quad (7)$$

where

$$\begin{aligned} \bar{A}^r &= \vec{A}^r + \vec{B}^r (K + \vec{K}_{[0]}) \vec{C}^r, \quad \bar{H}_{ij|\tau}^r = \frac{\Pi_i \vec{K}_{[\tau]} \Psi_j}{\varphi_{ij|\tau}}, \quad \bar{B}^r = \vec{E}^r + \vec{B}^r K \vec{D}^r, \\ \bar{M}^r &= \vec{M}^r = \begin{bmatrix} M^r & 0 \end{bmatrix}, \quad \bar{F}_{[\tau]} = \nabla^T \vec{K}_{[\tau]} \nabla, \end{aligned} \quad (8)$$

and

$$\begin{aligned}
 \vec{A}^r &= \begin{bmatrix} A^r & 0 \\ 0 & 0 \end{bmatrix}, \vec{B}^r = \begin{bmatrix} 0 & B^r \\ I & 0 \end{bmatrix}, \vec{C}^r = \begin{bmatrix} 0 & I \\ C^r & 0 \end{bmatrix}, \nabla = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}, \\
 \vec{D}^r &= \begin{bmatrix} 0 \\ D^r \end{bmatrix}, \vec{E}^r = \begin{bmatrix} E^r \\ 0 \end{bmatrix}, \Pi_i = \begin{bmatrix} 0 & 0 \\ \varphi_{(i)} & 0 \end{bmatrix}, \Psi_j = \begin{bmatrix} 0 & \varphi_{(j)} \\ 0 & 0 \end{bmatrix}, \\
 K &= \begin{bmatrix} K_A & K_B \\ K_C & 0 \end{bmatrix}, \vec{K}_{[\tau]} = \begin{bmatrix} \bar{K}_{[\tau]} & 0 \\ 0 & 0 \end{bmatrix}, \varphi_{(i)} = \text{diag}\{\underbrace{0, \dots, 0}_{i-1}, I_{d_i}, \underbrace{0, \dots, 0}_{N-i}\}.
 \end{aligned} \tag{9}$$

Before formulating the problem to be discussed, we first introduce the following definition.

Definition 1 ([24]). *The closed-loop system is said to be exponentially mean-square stable, if with $\omega(k) = 0$, there exist constants $\rho > 0$ and $\tau \in (0, 1)$ satisfying*

$$\mathbb{E} \{ \|\eta(k)\|^2 \} \leq \rho \tau^k \mathbb{E} \{ \|\eta(0)\|^2 \}, \quad \text{for all } \eta(0) \in \mathbf{R}^n, \quad k \in \mathbb{I}^+. \tag{10}$$

With *Definition 1*, the simultaneous H_∞ stabilization (SHS) problem to be solved in this paper is addressed as follows.

Problem 1 (SHS). *Design N distributed DOF controllers (2) such that*

- (RM1) *The overall closed-loop system (7) in each operation mode is exponentially mean-square stable; and*
- (RM2) *For the given constants $\gamma^r > 0$ and all nonzero $\omega^r(k)$, under the zero-initial conditions, the controlled output $z^r(k)$ satisfies*

$$\sum_{k=0}^{\infty} \mathbb{E} \{ \|z^r(k)\|^2 \} < (\gamma^r)^2 \sum_{k=0}^{\infty} \mathbb{E} \{ \|\omega^r(k)\|^2 \}, \quad \text{with } r \in \mathcal{V} = \{1, 2, \dots, V\}. \tag{11}$$

Remark 2. *In the different working modes, the system may face the different types of bounded disturbances $\omega^r(k)$, and even has the different performance requirements. Compared with the global H_∞ index γ , in order to reduce the conservatism, this paper uses multiple H_∞ indexes γ^r ($r = \{1, 2, \dots, V\}$, and V is the number of working modes) to satisfy the different H_∞ performance*

in each mode, such that, the considered multimode system achieve the optimal (or suboptimal) H_∞ disturbance attenuation performance in each mode.

Remark 3. The feature of multimode system considered in this paper is simultaneous stability in each operation mode. That is, a novel distributed simultaneous stabilization methodology is introduced, instead of the conventional control strategies such as sliding-mode control and switching control. The main difficulty and challenge is how to parameterize the simultaneous stabilization controller gains, because its parameters are fixed in any mode. If the parameterization of the controller gains is determined by the Lyapunov matrices or system parameters such as $K = YP^{-1}$ in [42], it is quite difficult to find the solution of simultaneous stabilization controller gains. The main reason is that the Lyapunov matrices and system parameters are different in any mode, while the controller gains are fixed and uniform in any mode by simultaneous stabilization approach. In this paper, based on an orthogonal complement technique, the gains of the simultaneous stabilization controllers can be parameterized independently of the Lyapunov matrices and system parameters.

3. Analysis of Simultaneous H_∞ Stabilization Problem

In this section, by employing the Lyapunov functional method and the stochastic analysis technique, sufficient conditions are provided to guarantee the stability and the prescribed H_∞ performance for the overall closed-loop systems (7) in each operation mode.

First of all, we provide the following analysis result and theorem which serves as a theoretical basis for the subsequent design issue.

Theorem 1. For given constants $\gamma^r > 0$, the overall closed-loop system (7) in each operation mode is exponentially mean-square stable and the H_∞ performance constraint (11) is satisfied for

all nonzero $\omega_r(k)$, if there exist $X^r > 0, R_1^r, R_2^r, \dots, R_L^r > 0$ satisfying

$$\begin{bmatrix} \mathbb{Z}_0 & (\bar{A}^r)^T X^r \bar{F}_{[1]} & (\bar{A}^r)^T X^r \bar{F}_{[2]} & \cdots & (\bar{A}^r)^T X^r \bar{F}_{[L]} & (\bar{A}^r)^T X^r \bar{B}^r \\ (\bar{F}_{[1]})^T X^r \bar{A}^r & \mathbb{Z}_1 & (\bar{F}_{[1]})^T X^r \bar{F}_{[2]} & \cdots & (\bar{F}_{[1]})^T X^r \bar{F}_{[L]} & (\bar{F}_{[1]})^T X^r \bar{B}^r \\ (\bar{F}_{[2]})^T X^r \bar{A}^r & (\bar{F}_{[2]})^T X^r \bar{F}_{[1]} & \mathbb{Z}_2 & \cdots & (\bar{F}_{[2]})^T X^r \bar{F}_{[L]} & (\bar{F}_{[2]})^T X^r \bar{B}^r \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ (\bar{F}_{[L]})^T X^r \bar{A}^r & (\bar{F}_{[L]})^T X^r \bar{F}_{[1]} & (\bar{F}_{[L]})^T X^r \bar{F}_{[2]} & \cdots & \mathbb{Z}_L & (\bar{F}_{[L]})^T X^r \bar{B}^r \\ (\bar{B}^r)^T X^r \bar{A}^r & (\bar{B}^r)^T X^r \bar{F}_{[1]} & (\bar{B}^r)^T X^r \bar{F}_{[2]} & \cdots & (\bar{B}^r)^T X^r \bar{F}_{[L]} & (\bar{B}^r)^T X^r \bar{B}^r - (\gamma^r)^2 I \end{bmatrix} < 0, \quad (12)$$

where $r \in \mathcal{V} = \{1, 2, \dots, V\}$,

$$\begin{aligned} \mathbb{Z}_0 &= (\bar{A}^r)^T X^r \bar{A}^r + (\bar{M}^r)^T \bar{M}^r + \sum_{i=1}^N \sum_{j \in C_i^{\text{in}}} q_{ij|0} (\bar{H}_{ij|0}^r)^T X^r \bar{H}_{ij|0}^r - X^r + \sum_{n=1}^L R_n^r, \\ \mathbb{Z}_1 &= (\bar{F}_{[1]})^T X^r \bar{F}_{[1]} + \sum_{i=1}^N \sum_{j \in C_i^{\text{in}}} q_{ij|1} (\bar{H}_{ij|1}^r)^T X^r \bar{H}_{ij|1}^r - R_1^r, \\ \mathbb{Z}_2 &= (\bar{F}_{[2]})^T X^r \bar{F}_{[2]} + \sum_{i=1}^N \sum_{j \in C_i^{\text{in}}} q_{ij|2} (\bar{H}_{ij|2}^r)^T X^r \bar{H}_{ij|2}^r - R_2^r, \\ \mathbb{Z}_L &= (\bar{F}_{[L]})^T X^r \bar{F}_{[L]} + \sum_{i=1}^N \sum_{j \in C_i^{\text{in}}} q_{ij|L} (\bar{H}_{ij|L}^r)^T X^r \bar{H}_{ij|L}^r - R_L^r. \end{aligned} \quad (13)$$

Proof. Define the following Lyapunov functions for the system (7) in multiple operation modes

$$\mathbb{V}^r(k) = \mathbb{V}_1^r(k) + \mathbb{V}_2^r(k), \quad (14)$$

where

$$\mathbb{V}_1^r(k) = (\eta^r(k))^T X^r \eta^r(k), \quad \mathbb{V}_2^r(k) = \sum_{n=1}^L \sum_{m=k-n}^{k-1} (\eta^r(m))^T R_n^r \eta^r(m), \quad r \in \mathcal{V} = \{1, 2, \dots, V\}.$$

$X^r, R_1^r, R_2^r, \dots, R_L^r$, are positive definite matrices. When $\omega^r(k) = 0$, it follows from (7) that

$$\mathbb{E}\{\Delta \mathbb{V}_1^r(k) | \eta^r(k)\} = \mathbb{E}\{(\eta^r(k+1))^T X^r \eta^r(k+1) - (\eta^r(k))^T X^r \eta^r(k)\} = (\tilde{\eta}^r(k))^T \mathcal{U}_1^r \tilde{\eta}^r(k), \quad (15)$$

where $(\tilde{\eta}^r(k))^T = \begin{bmatrix} (\eta^r(k))^T & (\eta^r(k-1))^T & \cdots & (\eta^r(k-2))^T & (\eta^r(k-L))^T \end{bmatrix}^T$,

$$\mathcal{U}_1^r = \begin{bmatrix} \mathbb{Z}_0 - \sum_{n=1}^L R_n^r & (\bar{A}^r)^T X^r \bar{F}_{[1]} & (\bar{A}^r)^T X^r \bar{F}_{[2]} & \cdots & (\bar{A}^r)^T X^r \bar{F}_{[L]} \\ (\bar{F}_{[1]})^T X^r \bar{A}^r & \mathbb{Z}_1 + R_1^r & (\bar{F}_{[1]})^T X^r \bar{F}_{[2]} & \cdots & (\bar{F}_{[1]})^T X^r \bar{F}_{[L]} \\ (\bar{F}_{[2]})^T X^r \bar{A}^r & (\bar{F}_{[2]})^T X^r \bar{F}_{[1]} & \mathbb{Z}_2 + R_2^r & \cdots & (\bar{F}_{[2]})^T X^r \bar{F}_{[L]} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ (\bar{F}_{[L]})^T X^r \bar{A}^r & (\bar{F}_{[L]})^T X^r \bar{F}_{[1]} & (\bar{F}_{[L]})^T X^r \bar{F}_{[2]} & \cdots & \mathbb{Z}_L + R_L^r \end{bmatrix},$$

and

$$\begin{aligned} \mathbb{E}\{\Delta \mathbb{V}_2^r(k) | \eta^r(k)\} &= \sum_{n=1}^L \sum_{m=k+1-n}^k (\eta^r(m))^T R_n^r \eta^r(m) - \sum_{n=1}^L \sum_{m=k-n}^{k-1} (\eta^r(m))^T R_n^r \eta^r(m) \\ &= \sum_{n=1}^L \{(\eta^r(k))^T R_n^r \eta^r(k) - (\eta^r(k-n))^T R_n^r \eta^r(k-n)\} \\ &= (\tilde{\eta}^r(k))^T \mathcal{U}_2^r \tilde{\eta}^r(k), \end{aligned} \quad (16)$$

where $\mathcal{U}_2^r = \text{diag}\{\sum_{n=1}^L R_n^r, -R_1^r, -R_2^r, \dots, -R_L^r\}$. Next, summing (15) and (16), we obtain that

$$\mathbb{E}\{\Delta \mathbb{V}^r(k) | \eta^r(k)\} = \mathbb{E}\{\Delta \mathbb{V}_1^r(k) | \eta^r(k)\} + \mathbb{E}\{\Delta \mathbb{V}_2^r(k) | \eta^r(k)\} = (\tilde{\eta}^r(k))^T (\mathcal{U}_1^r + \mathcal{U}_2^r) \tilde{\eta}^r(k). \quad (17)$$

It follows from (12) that $\mathcal{U}_1^r + \mathcal{U}_2^r < 0$ and $\mathbb{E}\{\Delta \mathbb{V}^r(k) | \eta^r(k)\} < 0$. Now, along the same line of the proof for Theorem 1 in [40], it can verify that the closed-loop system (7) in each operation mode with $\omega^r(k) = 0$ is exponentially mean-square stable.

Next, let us describe the H_∞ performance constraint of the system (7). For all nonzero $\omega^r(k)$, it follows from (7) and (14) that

$$\begin{aligned} &\mathbb{E}\{\mathbb{V}^r(k+1)\} - \mathbb{E}\{\mathbb{V}^r(k)\} + \mathbb{E}\{(z^r(k))^T z^r(k)\} - (\gamma^r)^2 \mathbb{E}\{(\omega^r(k))^T \omega^r(k)\} = \\ &(\tilde{\eta}^r(k))^T \begin{bmatrix} \mathbb{Z}_0 & (\bar{A}^r)^T X^r \bar{F}_{[1]} & (\bar{A}^r)^T X^r \bar{F}_{[2]} & \cdots & (\bar{A}^r)^T X^r \bar{F}_{[L]} & (\bar{A}^r)^T X^r \bar{B}^r \\ (\bar{F}_{[1]})^T X^r \bar{A}^r & \mathbb{Z}_1 & (\bar{F}_{[1]})^T X^r \bar{F}_{[2]} & \cdots & (\bar{F}_{[1]})^T X^r \bar{F}_{[L]} & (\bar{F}_{[1]})^T X^r \bar{B}^r \\ (\bar{F}_{[2]})^T X^r \bar{A}^r & (\bar{F}_{[2]})^T X^r \bar{F}_{[1]} & \mathbb{Z}_2 & \cdots & (\bar{F}_{[2]})^T X^r \bar{F}_{[L]} & (\bar{F}_{[2]})^T X^r \bar{B}^r \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ (\bar{F}_{[L]})^T X^r \bar{A}^r & (\bar{F}_{[L]})^T X^r \bar{F}_{[1]} & (\bar{F}_{[L]})^T X^r \bar{F}_{[2]} & \cdots & \mathbb{Z}_L & (\bar{F}_{[L]})^T X^r \bar{B}^r \\ (\bar{B}^r)^T X^r \bar{A}^r & (\bar{B}^r)^T X^r \bar{F}_{[1]} & (\bar{B}^r)^T X^r \bar{F}_{[2]} & \cdots & (\bar{B}^r)^T X^r \bar{F}_{[L]} & (\bar{B}^r)^T X^r \bar{B}^r - (\gamma^r)^2 I \end{bmatrix} \tilde{\eta}^r(k). \end{aligned} \quad (18)$$

From (12) and (18), we know that

$$\mathbb{E}\{\mathbb{V}^r(k+1)\} - \mathbb{E}\{\mathbb{V}^r(k)\} + \mathbb{E}\{(z^r(k))^T z^r(k)\} - (\gamma^r)^2 \mathbb{E}\{(\omega^r(k))^T \omega^r(k)\} < 0. \quad (19)$$

Later, summing up (19) from zero to ∞ with respect to k yields that

$$\sum_{k=0}^{\infty} \mathbb{E}\{\|z^r(k)\|^2\} < (\gamma^r)^2 \sum_{k=0}^{\infty} \mathbb{E}\{\|\omega^r(k)\|^2\}, \quad \text{with } r = 1, 2, \dots, V. \quad (20)$$

The above inequality (20) means that the H_∞ performance constraint (11) is satisfied for all nonzero $\omega_r(k)$. $\square$

Remark 4. The system considered in this paper is the multimode system (each mode corresponds to a mathematical model). We have to construct a series of Lyapunov functions in (14) for the system in multiple operation modes. As a result, the controller parameter K is coupled with the multiple Lyapunov matrices $X^r, R_1^r, R_2^r, \dots, R_L^r$, $r \in \mathcal{V} = \{1, 2, \dots, V\}$ used for building the sufficient simultaneous stability condition and the other system parameters such as $A^r, B^r, C^r, D^r, E^r, M^r$ ($r \in \mathcal{V} = \{1, 2, \dots, V\}$) in Theorem 1. These couplings will lead to difficulty and infeasibility in the parameterization of the gains, because both the Lyapunov matrices $X^r, R_1^r, R_2^r, \dots, R_L^r$ and system parameters are different under the different modes ($r \in \mathcal{V} = \{1, 2, \dots, V\}$), but the controller parameters $K_i^a, K_i^b, K_i^c, K_{ij}$ are fixed and uniform under the different modes by simultaneous stabilization approach. Therefore, that the controller gains independent of the Lyapunov matrices and the system parameters is the prerequisite for the parameterization of the gains of the simultaneous controllers. To overcome the difficulties encountered, by introducing a parameter independent slack variable and employing an orthogonal complement technique, a method to separate the controller parameter matrices, the Lyapunov matrices and the system matrices in the LMI-based condition for the existence of the controller will be developed later.

On the basis of Theorem 1, by Schur complement lemma and linear matrix transformation, we provide the following corollary.

Corollary 1. For given some constants $\gamma^r > 0$, the overall closed-loop system (7) in each operation mode is exponentially mean-square stable and the H_∞ performance constraint (11) is satisfied for

all nonzero $\omega_r(k)$, if there exist $P^r > 0$, $Q_1^r, Q_2^r, \dots, Q_L^r > 0$ satisfying

$$\begin{bmatrix} -P^r + \sum_{n=1}^L Q_n^r & \# & \# & \cdots & \# & \# & \# & \# & \# & \cdots & \# & \# & \# & \# \\ 0 & -Q_1^r & \# & \cdots & \# & \# & \# & \# & \# & \cdots & \# & \# & \# & \# \\ 0 & 0 & -Q_2^r & \cdots & \# & \# & \# & \# & \# & \cdots & \# & \# & \# & \# \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -Q_L^r & \# & \# & \# & \# & \cdots & \# & \# & \# & \# \\ P^r \bar{A}^r & P^r \bar{F}_{[1]}^r & P^r \bar{F}_{[2]}^r & \cdots & P^r \bar{F}_{[L]}^r & -P^r & \# & \# & \# & \cdots & \# & \# & \# & \# \\ P_{\mathfrak{M}}^r \bar{H}_{[0]}^r & 0 & 0 & \cdots & 0 & 0 & -P_{\mathfrak{M}}^r & \# & \# & \cdots & \# & \# & \# & \# \\ 0 & P_{\mathfrak{M}}^r \bar{H}_{[1]}^r & 0 & \cdots & 0 & 0 & 0 & -P_{\mathfrak{M}}^r & \# & \cdots & \# & \# & \# & \# \\ 0 & 0 & P_{\mathfrak{M}}^r \bar{H}_{[2]}^r & \cdots & 0 & 0 & 0 & 0 & -P_{\mathfrak{M}}^r & \cdots & \# & \# & \# & \# \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & P_{\mathfrak{M}}^r \bar{H}_{[L]}^r & 0 & 0 & 0 & 0 & \cdots & -P_{\mathfrak{M}}^r & \# & \# & \# \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & -P^r & \# & \# \\ 0 & 0 & 0 & \cdots & 0 & (\bar{B}^r)^T P^r & 0 & 0 & 0 & \cdots & 0 & 0 & -\gamma^r I & \# \\ \bar{L}^r & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & -\gamma^r I \end{bmatrix} < 0, \quad (21)$$

where

$$P_{\mathfrak{M}}^r = \text{diag}\{\underbrace{P^r, P^r, \dots, P^r}_{|\mathcal{E}_{cm}|}, \bar{H}_{[\tau]}^r\} = \begin{bmatrix} \text{Col}\langle \sqrt{q_{1j|\tau}} \bar{H}_{1j|\tau}^r \rangle \text{ for } j \in C_1^{in} \\ \text{Col}\langle \sqrt{q_{2j|\tau}} \bar{H}_{2j|\tau}^r \rangle \text{ for } j \in C_2^{in} \\ \vdots \\ \text{Col}\langle \sqrt{q_{Nj|\tau}} \bar{H}_{Nj|\tau}^r \rangle \text{ for } j \in C_N^{in} \end{bmatrix}, \tau = 0, 1, 2, \dots, L,$$

$r \in \mathcal{V} = \{1, 2, \dots, V\}$, $\bar{A}^r, \bar{B}^r, \bar{L}^r, \bar{F}_{[\tau]}^r$ and $\bar{H}_{ij|\tau}^r$ have been defined in (8).

Proof. In view of Theorem 1, we just need to prove that (21) is derived from (12). Firstly, defining

$$\begin{aligned} \mathbb{X}^r &= \text{diag}\{\underbrace{X_{\mathfrak{M}}^r, X_{\mathfrak{M}}^r, \dots, X_{\mathfrak{M}}^r}_{L+1}, X^r\} \text{ with } X_{\mathfrak{M}}^r = \text{diag}\{\underbrace{X^r, X^r, \dots, X^r}_{|\mathcal{E}_{cm}|}\}, \\ \mathbb{H}^r &= \text{diag}\{\bar{H}_{[0]}^r, \bar{H}_{[1]}^r, \bar{H}_{[2]}^r, \dots, \bar{H}_{[L]}^r, 0\}, \quad \mathbb{A}^r = \begin{bmatrix} \bar{A}^r & \bar{F}_{[1]}^r & \bar{F}_{[2]}^r & \cdots & \bar{F}_{[L]}^r & \bar{B}^r \end{bmatrix}, \\ \mathbb{M}^r &= \begin{bmatrix} \bar{M}^r & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}, \quad \mathbb{Z}^r = \text{diag}\{-X^r + \sum_{n=1}^L R_n^r, -R_1^r, -R_2^r, \dots, -R_L^r, -(\gamma^r)^2 I\}, \end{aligned}$$

the inequality (12) in Theorem 1 can be rewritten as

$$\mathbb{Z}^r + (\mathbb{H}^r)^T \mathbb{X}^r \mathbb{H}^r + (\mathbb{A}^r)^T X^r \mathbb{A}^r + (\mathbb{M}^r)^T \mathbb{M}^r < 0.$$

Then, we have

$$\mathbb{Z}^r + \begin{bmatrix} \mathbb{H}^r \\ \mathbb{A}^r \\ \mathbb{M}^r \end{bmatrix}^T \begin{bmatrix} \mathbb{X}^r & 0 & 0 \\ 0 & X^r & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} \mathbb{H}^r \\ \mathbb{A}^r \\ \mathbb{M}^r \end{bmatrix} < 0.$$

Next, via Schur complement, yield that

$$\left[\begin{array}{c|c} \mathbb{Z}^r & \begin{bmatrix} (\mathbb{H}^r)^T & (\mathbb{A}^r)^T & (\mathbb{M}^r)^T \end{bmatrix} \begin{bmatrix} \mathbb{X}^r & 0 & 0 \\ 0 & X^r & 0 \\ 0 & 0 & I \end{bmatrix} \\ \hline \begin{bmatrix} \mathbb{X}^r & 0 & 0 \\ 0 & X^r & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} \mathbb{H}^r \\ \mathbb{A}^r \\ \mathbb{M}^r \end{bmatrix} & - \begin{bmatrix} \mathbb{X}^r & 0 & 0 \\ 0 & X^r & 0 \\ 0 & 0 & I \end{bmatrix} \end{array} \right] < 0.$$

Later, letting $P^r = \frac{X^r}{\gamma^r}$, $Q_1^r = \frac{R_1^r}{\gamma^r}$, $Q_2^r = \frac{R_2^r}{\gamma^r}, \dots, Q_L^r = \frac{R_L^r}{\gamma^r}$, and performing the congruence transformation, we get (21). This completes the proof. $\square$

In the Corollary 1, the multiple Lyapunov matrices P^r used for constructing the sufficient stability condition are coupled with the controller parameter matrix K and $\vec{K}_{[\tau]}$. This coupling will bring a lot of difficulties in the further matrix linearization and the parametrization of the controller gains. Therefore, we introduce the following lemma that will be used in the separation of the Lyapunov matrices and controller parameter matrix.

Lemma 1 ([39]). Let $J = J^T \in \mathbf{R}^{n \times n}$, G_1 and G_2 be real matrices with appropriate dimensions. Then,

$$J + G_1^T \Delta^T G_2 + G_2^T \Delta G_1 < 0,$$

holds for some Δ if and only if

$$(G_1^\perp)^T J G_1^\perp < 0, (G_2^\perp)^T J G_2^\perp < 0. \quad (22)$$

If when G_1 or G_2 has full rank n , the first or second inequality in (22) shall disappear.

Next, on the basis of Lemma 1, we will provide an efficient approach to separate the Lyapunov matrices and controller parameter matrix. Before proceeding, we denote

$$q_{[\tau]} = \begin{bmatrix} \mathbb{D}\langle \frac{\sqrt{q_{1j\tau}}}{\varphi_{1j\tau}} \rangle \text{for } j \in C_1^{in} & 0 & \cdots & 0 \\ 0 & \mathbb{D}\langle \frac{\sqrt{q_{2j\tau}}}{\varphi_{2j\tau}} \rangle \text{for } j \in C_2^{in} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbb{D}\langle \frac{\sqrt{q_{Nj\tau}}}{\varphi_{Nj\tau}} \rangle \text{for } j \in C_N^{in} \end{bmatrix}, \tau = 0, 1, 2, \dots, L,$$

$$\check{q} = \text{diag}\{\underbrace{q_{[0]}I, q_{[1]}I, q_{[2]}I, \dots, q_{[L]}I}_{L+1}\}, \check{A}^r = \begin{bmatrix} \check{A}^r & \underbrace{0, 0, \dots, 0}_L \end{bmatrix}, \check{B}^r = \begin{bmatrix} \check{B}^r & \underbrace{\nabla^T, \nabla^T, \dots, \nabla^T}_L \end{bmatrix},$$

$$\check{C}^r = \text{diag}\{\check{C}^r, \underbrace{\nabla, \nabla, \dots, \nabla}_L\}, \check{E}^r = \check{E}, \check{D}^r = \begin{bmatrix} (\check{D}^r)^T & \underbrace{0, 0, \dots, 0}_L \end{bmatrix}^T, \check{M}^r = \begin{bmatrix} \check{M}^r & \underbrace{0, 0, \dots, 0}_L \end{bmatrix},$$

$$\vec{\Pi}_i = \text{diag}\{\underbrace{\Pi_i, \Pi_i, \dots, \Pi_i}_{|C_i^{in}|}\}, \Psi = \begin{bmatrix} \text{Col}\langle \Psi_j \rangle \text{for } j \in C_1^{in} \\ \text{Col}\langle \Psi_j \rangle \text{for } j \in C_2^{in} \\ \vdots \\ \text{Col}\langle \Psi_j \rangle \text{for } j \in C_N^{in} \end{bmatrix}, \check{\Pi} = \text{diag}\{\underbrace{\Pi, \Pi, \Pi, \dots, \Pi}_{L+1}\},$$

$$\Pi = \text{diag}\{\underbrace{\vec{\Pi}_1, \vec{\Pi}_2, \dots, \vec{\Pi}_N}_N\}, \check{\Psi} = \text{diag}\{\underbrace{\Psi, \Psi, \Psi, \dots, \Psi}_{L+1}\},$$

$$\hat{K}_{[\tau]} = \text{diag}\{\underbrace{\hat{K}_{[\tau]}, \hat{K}_{[\tau]}, \dots, \hat{K}_{[\tau]}}_{|\mathcal{E}_{cm}|}\}, \text{with } \tau = 0, 1, 2, \dots, L,$$

$$\check{K}_2 = \text{diag}\{\underbrace{\hat{K}_{[0]}, \hat{K}_{[1]}, \hat{K}_{[2]}, \dots, \hat{K}_{[L]}}_{L+1}\}, \check{K}_1 = \text{diag}\{K + \check{K}_{[0]}, \underbrace{\check{K}_{[1]}, \check{K}_{[2]}, \dots, \check{K}_{[L]}}_L\}, \quad (23)$$

and

$$\begin{aligned}
 \tilde{A}^r &= \begin{bmatrix} -\frac{1}{2}I & 0 & 0 & 0 \\ \check{A}^r & -\frac{1}{2}I & 0 & 0 \\ 0 & 0 & -\frac{1}{2}I & 0 \\ 0 & 0 & 0 & -\frac{1}{2}I \end{bmatrix}, \tilde{B}^r = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \check{B}^r & 0 & 0 \\ 0 & 0 & \check{q}\check{\Pi} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tilde{C}^r = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \check{C}^r & 0 & 0 & 0 \\ \check{\Psi} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \\
 \tilde{E}^r &= \begin{bmatrix} 0 \\ \check{E}^r \\ 0 \\ 0 \end{bmatrix}, \tilde{D}^r = \begin{bmatrix} 0 \\ \check{D}^r \\ 0 \\ 0 \end{bmatrix}, \tilde{M}^r = \begin{bmatrix} \check{M}^r & 0 & 0 & 0 \end{bmatrix}, \tilde{K} = \text{diag}\{0, \check{K}_1, \check{K}_2, 0\}, \\
 \tilde{P}^r &= \text{diag}\{P^r - \sum_n^L Q_n^r, Q_1^r, Q_2^r, \dots, Q_L^r, P^r, \underbrace{P^r, P^r, \dots, P^r}_{(L+1)*|\mathcal{E}_{cm}|}, P^r\}.
 \end{aligned} \tag{24}$$

Then, from *Corollary 1*, the following theorem lying in the separation of the Lyapunov matrices P^r and controller parameter matrix can be derived based on *Lemma 1*.

Theorem 2. For given constants $\gamma^r > 0$, the overall closed-loop system (7) in each operation mode is exponentially mean-square stable and the H_∞ performance constraint (11) is satisfied for all nonzero $\omega_r(k)$, if there exist $P^r > 0$, $Q_1^r, Q_2^r, \dots, Q_L^r > 0$, an arbitrary $S > 0$, a large scalar $\theta > 0$, and some real matrices K_A, K_B, K_C, K_{ij} satisfying the following inequalities,

$$((\Theta^r)^\perp)^T \begin{bmatrix} \Phi^r(\theta) & \# & \# \\ \Xi^r & -2\mathfrak{Y}_+(\theta) & \# \\ \Omega^r & 0 & -\gamma^r I \end{bmatrix} (\Theta^r)^\perp < 0, r \in \mathcal{V} = \{1, 2, \dots, V\}, \tag{25}$$

where

$$\begin{aligned} \Phi^r(\theta) &= \begin{bmatrix} -2\theta(\tilde{C}^r)^T \tilde{K}^T \tilde{S} \tilde{K} \tilde{C}^r & \# & \# \\ 2\theta \tilde{S} \tilde{K} \tilde{C}^r & -2\theta \tilde{S} & \# \\ -2\theta(\tilde{D}^r)^T \tilde{K}^T \tilde{S} \tilde{K} \tilde{C}^r & 2\theta(\tilde{D}^r)^T \tilde{K}^T \tilde{S} & -2\theta(\tilde{D}^r)^T \tilde{K}^T \tilde{S} \tilde{K} \tilde{D}^r - \gamma^r I \end{bmatrix}, \\ \Xi^r &= \begin{bmatrix} \tilde{P}^r & 0 & 0 \\ \tilde{S} \tilde{K} \tilde{C}^r & -\tilde{S} & \tilde{S} \tilde{K} \tilde{D}^r \\ \tilde{P}^r & 0 & 0 \\ 0 & 0 & \tilde{S} \tilde{K} \tilde{D}^r \end{bmatrix} \quad \Theta^r = \begin{bmatrix} \tilde{A}^r & \tilde{B}^r & \tilde{E}^r & -I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -I & 0 & 0 \end{bmatrix}, \\ \Omega^r &= \begin{bmatrix} \tilde{M}^r & 0 & 0 \end{bmatrix}, \quad \mathfrak{I}_+(\theta) = \text{diag}\{0, \tilde{S}, 0, \theta \tilde{S}\}, \end{aligned} \quad (26)$$

$\tilde{S} = \text{diag}\{ \underbrace{S, S, \dots, S}_{2L+3+(L+1)*|\mathcal{E}_{cm}|} \}$, $\tilde{A}^r, \tilde{B}^r, \tilde{E}^r, \tilde{C}^r, \tilde{D}^r, \tilde{M}^r, \tilde{K}, \tilde{P}^r$ have been denoted in (24).

Along the same line of the proof of [38, Theorem 2], *Theorem 2* in this paper is easy to be proved via Schur complement and *Lemma 1* (see in detail, [38]).

Note that the matrix inequalities (25) still contain some nonlinear terms. Then the LMI technique can not be used directly. In the next section, we aim to deal with these nonlinear terms and develop an ILMI method to design the controllers.

4. Controller Design and Algorithm

In this section, a sufficient condition on the existence of the desired controller is developed. The distributed DOF controllers which satisfy the requirements (RM1) and (RM2) are found in the following theorem.

Theorem 3. For given constants $\gamma^r > 0$, the closed-loop stochastic system (7) in each operation mode is exponentially mean-square stable and the H_∞ constraint (11) is satisfied, if there exist $P^r > 0$, $Q_1^r, Q_2^r, \dots, Q_L^r > 0$, W^r , U^r , a diagonal $S > 0$, Y and $\vec{Y}_{[\tau]}$ with the prescribed pattern, and a large scalar $\theta > 0$ satisfying

$$\mathfrak{G}^r \triangleq ((\Theta^r)^\perp)^T \begin{bmatrix} \mathfrak{W}^r(\theta) & \# & \# \\ \mathfrak{X}^r & -2\mathfrak{I}_+(\theta) & \# \\ \Omega^r & 0 & -\gamma^r I \end{bmatrix} (\Theta^r)^\perp < 0, r \in \mathcal{V} = \{1, 2, \dots, V\}, \quad (27)$$

where $S = \text{diag}\{S_1^a, S_2^a, \dots, S_N^a, S_1^c, S_2^c, \dots, S_N^c\}$,

$$\begin{aligned} \mathfrak{W}^r(\theta) &= 2\theta \begin{bmatrix} -\text{Herm}\{(\tilde{C}^r)^T \tilde{Y}^T W^r\} + (W^r)^T \tilde{S} W^r & \# & \# \\ \tilde{Y} \tilde{C}^r & -\tilde{S} & \# \\ -(\tilde{D}^r)^T \tilde{Y}^T W^r - (U^r)^T \tilde{Y} \tilde{C}^r + (U^r)^T \tilde{S} W^r & (\tilde{D}^r)^T \tilde{Y}^T & -\text{Herm}\{(\tilde{D}^r)^T \tilde{Y}^T U^r\} + (U^r)^T \tilde{S} U^r - \frac{\gamma^r}{2\theta} I \end{bmatrix}, \\ \mathfrak{Y}^r &= \begin{bmatrix} \tilde{P}^r & 0 & 0 \\ \tilde{Y} \tilde{C}^r & -\tilde{S} & \tilde{Y} \tilde{D}^r \\ \tilde{P}^r & 0 & 0 \\ 0 & 0 & \tilde{Y} \tilde{D}^r \end{bmatrix} \begin{bmatrix} \tilde{Y} = \text{diag}\{0, Y + \underbrace{\tilde{Y}_{[0]}, \tilde{Y}_{[1]}, \tilde{Y}_{[2]}, \dots, \tilde{Y}_{[L]}}_L, \underbrace{\tilde{Y}_{[0]}, \tilde{Y}_{[0]}, \dots, \tilde{Y}_{[0]}}_{|\mathcal{E}_{cm}|}, \\ \underbrace{\tilde{Y}_{[1]}, \tilde{Y}_{[1]}, \dots, \tilde{Y}_{[1]}}_{|\mathcal{E}_{cm}|}, \underbrace{\tilde{Y}_{[2]}, \tilde{Y}_{[2]}, \dots, \tilde{Y}_{[2]}}_{|\mathcal{E}_{cm}|}, \dots, \underbrace{\tilde{Y}_{[L]}, \tilde{Y}_{[L]}, \dots, \tilde{Y}_{[L]}}_{|\mathcal{E}_{cm}|}, 0 \end{bmatrix}, \quad (28) \\ Y &= \begin{bmatrix} Y_A & Y_B \\ Y_C & 0 \end{bmatrix}, \text{ with } \begin{aligned} Y_A &= \text{diag}\{Y_1^a, Y_2^a, \dots, Y_N^a\}, & Y_C &= \text{diag}\{Y_1^c, Y_2^c, \dots, Y_N^c\}, \\ Y_B &= \text{diag}\{Y_1^b, Y_2^b, \dots, Y_N^b\}, \end{aligned} \\ \tilde{Y}_{[\tau]} &= \begin{bmatrix} \tilde{Y}_{[\tau]} & 0 \\ 0 & 0 \end{bmatrix}, \tilde{Y}_{[\tau]} = [\tilde{Y}_{ij[\tau]}]_{N \times N} \in R^{N \times N}, \text{ with } \tilde{Y}_{ij[\tau]} = \begin{cases} \tilde{\varphi}_{ij[\tau]} Y_{ij}, & i = 1, 2, \dots, N; j \in C_i^{\text{in}}, \\ 0, & i = 1, 2, \dots, N; j \notin C_i^{\text{in}}, \end{cases} \end{aligned}$$

and $\tilde{P}^r, \tilde{C}^r, \tilde{D}^r$ are defined in (24), $\Theta^r, \Omega^r, \tilde{S}, \mathfrak{Y}_+(\theta)$ are defined in (26). Then, there exist the controllers which satisfy the requirements (RM1) and (RM2). The distributed controllers parameters can be determined by

$$K_i^a = (S_i^a)^{-1} Y_i^a, \quad K_{ij} = (S_i^a)^{-1} Y_{ij}, \quad K_i^b = (S_i^a)^{-1} Y_i^b, \quad K_i^c = (S_i^c)^{-1} Y_i^c, \quad i \in \mathcal{N}, j \in C_i^{\text{in}}. \quad (29)$$

By an equivalent transformation and setting $\tilde{Y} = \tilde{S} \tilde{K}$, $W^r = \tilde{K} \tilde{C}^r = \tilde{S}^{-1} \tilde{Y} \tilde{C}^r$, $U^r = \tilde{K} \tilde{D}^r = \tilde{S}^{-1} \tilde{Y} \tilde{D}^r$, the matrix inequalities (25) can be converted to (27) and vice versa. Thus, it is straightforward that Theorem 3 can be derived from Theorem 2. The process of proof is omitted here.

The matrix inequalities (27) still are dependent on the parameter θ , and contain nonlinear terms. However, when we fix θ , W^r and U^r , the conditions in (27) become LMIs. Thence, the distributed DOF controllers can be obtained via seeking solutions of LMIs (27) in Theorem 3. Moreover, the scalar σ satisfying (27) $< \sigma I$ achieves its minimum, when $\theta \rightarrow \infty$, $W^r = \tilde{K} \tilde{C}^r = \tilde{S}^{-1} \tilde{Y} \tilde{C}^r$ and $U^r = \tilde{K} \tilde{D}^r = \tilde{S}^{-1} \tilde{Y} \tilde{D}^r$, which help us set up an iteration criteria to design iterative algorithm. From the above analysis, on the basis of Theorem 3, an iterative LMI algorithm is established to seek a feasible solution of distributed controllers.

Algorithm 1:

Step 1. Set $\nu = 1$, $^{(\nu)}\sigma > 0$ to be a large number, and select initial values θ which is a sufficiently large scalar, $^{(\nu)}W^r$ and $^{(\nu)}U^r$ which are the state-feedback matrices.

Step 2. For the fixed θ , $^{(\nu)}W^r$, $^{(\nu)}U^r$, find a set of optimal solution $^{(\nu+1)}\sigma$, $^{(\nu)}S$, $^{(\nu)}Y$, such that, minimize σ subject to the following LMI constraints,

$$Q_1^r > 0, \quad Q_2^r > 0, \quad \dots, \quad Q_L^r > 0, \quad P^r > 0, \quad S > 0, \quad \mathfrak{G}^r < \sigma I. \quad (30)$$

Step 3. If $^{(\nu+1)}\sigma < 0$, then there exist the desired controllers which satisfy the requirements (RM1) and (RM2), and the controller parameters can be determined by

$$K_i^a = (S_i^a)^{-1} Y_i^a, \quad K_{ij} = (S_i^a)^{-1} Y_{ij}, \quad K_i^b = (S_i^a)^{-1} Y_i^b, \quad K_i^c = (S_i^c)^{-1} Y_i^c, \quad i \in \mathcal{N}, j \in C_i^{in}. \quad (31)$$

End.

Else if $|^{(\nu+1)}\sigma - ^{(\nu)}\sigma| \leq \varrho$, where ϱ is a pre-determined tolerance, **then** go to **Step 4**.

Else set

$$^{(\nu+1)}W^r = (^{(\nu)}\tilde{S})^{-1} (^{(\nu)}\tilde{Y})\tilde{C}^r, \quad ^{(\nu+1)}U^r = (^{(\nu)}\tilde{S})^{-1} (^{(\nu)}\tilde{Y})\tilde{D}^r, \quad (32)$$

and $\nu = \nu + 1$, **then** go to **Step 2**.

Step 4. There is no solution. Stop or choose other initial values.

Remark 5. With the computation complexity of Algorithm 1, it is not easy to solve the optimal H_∞ control problem. This paper only considers the suboptimal H_∞ control problem to pursue a suboptimal system performance. The index gamma $\gamma^r (r = \{1, 2, \dots, V\})$ are the given positive constants in Algorithm 1. The optimal H_∞ control problem of simultaneous stabilization is still a challenging issue, leaving a research topic in the near future.

Remark 6. For the solution of the controllers' gains, we need to calculate LMIs (30) with large dimensions, which give rise to much difficulty in off-line numerical computation. By the proposed algorithm, the computational complexity depends on the size of LMI (27) in Theorem 3. We recall

the variable dimensions from the subsystem state $x_i^r(k) \in \mathbf{R}^{n_i}$, the controller state $\varepsilon_i(k) \in \mathbf{R}^{d_i}$, the control input $u_i^r(k) \in \mathbf{R}^{m_i}$, the disturbance input $\omega_i^r(k) \in \mathbf{R}^{v_i}$, $i \in \{1, 2, \dots, N\}$. Then, the size of LMI (27) in Theorem 3 can be summarized as $(3 + L + L|\mathcal{E}_{cm}| + |\mathcal{E}_{cm}|)\mathbf{n} + (12 + 7L + 4L|\mathcal{E}_{cm}| + 4|\mathcal{E}_{cm}|)\mathbf{d} + (9 + 6L + 3L|\mathcal{E}_{cm}| + 3|\mathcal{E}_{cm}|)\mathbf{m} + 2\mathbf{v}$, where $|\mathcal{E}_{cm}|$ is the number of the information exchange links between the controllers, L is the order of the Rice fading model, and $\mathbf{n} = n_1 + n_2 + \dots + n_N$, $\mathbf{d} = d_1 + d_2 + \dots + d_N$, $\mathbf{m} = m_1 + m_2 + \dots + m_N$, $\mathbf{v} = v_1 + v_2 + \dots + v_N$. With the increase of the number of subsystems, the size of LMI (27) would be much increased. To reduce the computational burden, we can decrease the number of the information exchange links and the network size while keeping the essential information. Moreover, noting that the computational burden is mainly caused by the basic mathematical operations, how to improve the computational efficiency is a very active issue in the areas of computational mathematics, optimization, and operational research community. Therefore, the further progress can be expected in the future.

Remark 7. The industrial control systems require much higher real-time capability, especially, in the large-scale system. However, the complicated off-line calculation has no direct influence on real-time control by our proposed approach. In the distributed controllers (2), we use the low dimension control gains K_i^a , K_i^b , K_i^c , K_{ij} . Once the control gains were determined via solving LMIs in Algorithm 1, it is effective and fast to calculate the local control input u_i by (2). The real-time computational burden is low. As a result, when the proposed distributed networked control framework is used to control the large systems, our proposed method costs low real-time computation and communication.

5. Simulation Example

To illustrate feasibility and operability of the proposed scheme, consider the following system

$$\begin{cases} x_i^r(k+1) = A_i^r x_i^r(k) + B_i^r u_i^r(k) + E_i^r \omega_i^r(k) + \sum_{j=1, j \neq i}^N A_{ij}^r x_j^r(k), \\ z_i^r(k) = M_i^r x_i^r(k), \\ y_i^r(k) = C_i^r x_i^r(k) + D_i^r \omega_i^r(k), \end{cases} \quad \begin{aligned} i &\in \mathcal{N} = \{1, 2, 3\}, \\ r &\in \mathcal{V} = \{1, 2\}, \end{aligned} \quad (33)$$

where the parameters of three subsystems in two modes are shown in Table 1:

Table 1: Parameters of three subsystems in two operation modes

	$r = 1$ (the first operation mode)	$r = 2$ (the second operation mode)
$i = 1$	$A_1^1 = -0.3, A_{12}^1 = 0.25, A_{13}^1 = 0.01,$ $B_1^1 = 0.025, E_1^1 = 0.02, L_1^1 = 0.15,$ $C_1^1 = 0.1, D_1^1 = 0.1.$	$A_1^2 = -0.4, A_{12}^2 = 0.12, A_{13}^2 = 0.02,$ $B_1^2 = 0.01, E_1^2 = 0.015, L_1^2 = 0.2,$ $C_1^2 = 0.1, D_1^2 = 0.12.$
$i = 2$	$A_2^1 = -0.2, A_{21}^1 = -0.01, A_{23}^1 = 0,$ $B_2^1 = 0.012, E_2^1 = 0.015, L_2^1 = 0.2,$ $C_2^1 = 0.1, D_2^1 = 0.15.$	$A_2^2 = -0.1, A_{21}^2 = -0.05, A_{23}^2 = 0.03,$ $B_2^2 = 0.02, E_2^2 = 0.025, L_2^2 = 0.1,$ $C_2^2 = 0.1, D_2^2 = 0.08.$
$i = 3$	$A_3^1 = 0.2, A_{31}^1 = -0.05, A_{32}^1 = 0.25,$ $B_3^1 = 0.03, E_3^1 = 0.012, L_3^1 = 0.12,$ $C_3^1 = 0.1, D_3^1 = 0.12.$	$A_3^2 = 0.4, A_{31}^2 = 0.04, A_{32}^2 = 0.06,$ $B_3^2 = 0.015, E_3^2 = 0.01, L_3^2 = 0.25,$ $C_3^2 = 0.1, D_3^2 = 0.15.$

The distributed topology describing the information exchange between controllers is represented by a directed graph $\mathcal{G}_{cm} = (\mathcal{N}, \mathcal{E}_{cm})$ with the set of the controller nodes $\mathcal{N} = \{1, 2, 3\}$, and the set of edges $\mathcal{E}_{cm} = \{(2, 1), (3, 2)\}$. That is to say, the second controller can receive the first controller's information and the third controller can receive the second controller's information. Moreover, as we use an unreliable wireless network for information transmission between the controllers, the channel fading is considered and described by the following Rice fading model

$$\hat{\varepsilon}_{ij}(k) = \varphi_{ij|\tau=0}(k)\varepsilon_j(k) + \sum_{\tau=1}^{L(k)} \varphi_{ij|\tau}(k)\varepsilon_j(k-\tau), \quad L(k) = \min\{L, k\}. \quad (34)$$

In this example, the order of the Rice fading model is $L = 1$, the mathematical expectations and the variances of channel coefficients are $\bar{\varphi}_{21|0} = 0.4, \bar{\varphi}_{21|1} = 0.2, \bar{\varphi}_{32|0} = 0.2, \bar{\varphi}_{32|1} = 0.4, q_{21|0} = 0.38, q_{21|1} = 1, q_{32|0} = 1, q_{32|1} = 0.38$. Then, via *Algorithm 1* and *Yalmip* toolbox, we can have the following distributed controllers parameters:

$$\begin{aligned} K_1^a &= -0.0263, \quad K_1^b = -0.1926, \quad K_1^c = -2.7256, \\ K_2^a &= 0.0008, \quad K_{21} = 0.0034, \quad K_2^b = 0.0676, \quad K_2^c = 0.9710, \\ K_3^a &= -0.0214, \quad K_{32} = 0.0128, \quad K_3^b = -0.1863, \quad K_3^c = 5.5745. \end{aligned}$$

In the simulation, $\omega_1^1(k), \omega_1^2(k)$ are the Gaussian white noise disturbances with variance $R = 0.1$, and $\omega_2^1(k) = \omega_2^2(k) = 1/k, \omega_3^1(k) = \omega_3^2(k) = 1/(k * k)$ are the bounded disturbances. The initial values are $x_1^1(0) = x_1^2(0) = x_2^1(0) = x_2^2(0) = x_3^1(0) = x_3^2(0) = 0, x_1^1(1) = x_1^2(1) = 0.5, x_2^1(1) = x_2^2(1) = -0.5, x_3^1(1) = x_3^2(1) = 0.2$. Simulation results under two different operation modes

are given in Fig.3 and Fig.4, respectively. We can see from Fig.3-4 that, no matter which mode the large-scale system works in, the designed controllers always guarantee the stability. The results have confirmed the feasibility and operability of the proposed method.

Figure 3: Curves of the state response in the 1st mode.

Figure 4: Curves of the state response in the 2nd mode.

6. Conclusion

This paper has addressed a novel simultaneous H_∞ stabilization problem for the multimode large-scale system, which is composed of physically interconnected subsystems. A distributed networked control framework has been developed, where the controllers can exchange information via an unreliable wireless network with channel fading. A sufficient condition on the existence of the desired controllers has been provided by the Lyapunov functional method and the stochastic analysis technique. The designed controllers can guarantee the simultaneous stability and the H_∞ performance of a large-scale system in multiple operation modes. The solution of the distributed

controller parameters is characterized by solving a convex optimization problem. Finally, a numerical simulation example has been provided to illustrate the feasibility of the proposed scheme.

Simultaneous stabilization methodology has found efficacious applications in a wide range of fields, because complex practical systems may also work in multiple operation modes. In the future, we will extend our results to the multimode sampled-data systems. The simultaneous stabilization problem of aperiodic sampled-data control systems such as multi-rate sampled-data systems [41] and sampled-data systems under noisy sampling intervals [42] will be considered as a future possible research topic. The analysis/synthesis problems will be investigated for the sampled-data simultaneous stabilization controller.

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