Distributed Networked Set-Membership Filtering with Ellipsoidal State Estimations

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Abstract

This paper addresses the problem of distributed networked set-membership filtering with ellipsoidal state estimations for a class of discrete time-varying systems in the presence of unknown-but-bounded process and measurement noises. Both global and local ellipsoidal state estimations are provided to locate the true state (target) via a distributed filtering network. A new geometric method based on Minkowski sum is proposed to produce the global ellipsoidal estimation. A novel convex optimization approach is developed to derive some sufficient conditions on the existence of local networked set-membership filters and to obtain the local ellipsoidal estimations by exchanging information among neighboring filters via communication networks. An experiment is conducted based on a 2-kW single-phase grid-connected power generation system platform to demonstrate the feasibility and the effectiveness of the proposed method in the real application.

Keywords: Distributed networked set-membership filtering, ellipsoidal state estimation, geometric method, unknown-but-bounded noise, convex optimization approach.

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1. Introduction

Recently, the distributed filtering (or estimation) technology has become a promising research topic. This technology offers a number of filters in a distributed manner to independently estimate the state of a plant that resides in its connection (or communication) ranges and to share the estimation cooperatively by exchanging information with neighbors. This feature enables the distributed filtering to present great advantages, such as separating the computational load and facilitating fault detection, over the centralized filtering relying on a fusion to deal with all the measurements [4, 9]. Therefore, the distributed filtering has been widely applied in networked systems [15, 23, 42, 40, 29, 28, 13, 41], networked monitoring systems [5, 7, 24, 35, 18], multi-agent consensus [2, 20, 6, 25, 14, 12], and autonomous navigation [22, 30], and so on.

In the existing results, there are mainly two approaches for the distributed filtering, i.e., the distributed Kalman filtering (DKF) [2, 3, 9, 19, 21] and the distributed H_∞ filtering (DHF) [38, 5, 8, 10, 11, 25, 27, 36]. The DKF has collaborative information processing by Kalman filters in a distributed manner where each filter collects information around its neighbors connected by a communication network. However, the DKF is restrictive in application because it requires very detailed statistical information of process and measurement noises, and it may not be robust against plant uncertainty. Alternatively, the DHF usually safeguards against the worst case of disturbances and thus does not require knowledge of statistical properties for noises. It can minimize a bound on the variance of the estimation error over all possible disturbances while satisfying a certain prescribed H_∞ performance. Therefore, it is over-conservative and results in a better robust behavior to disturbance variation. However, both DKF and DHF focus on the optimization of the estimation error and only provide the *point-estimation* of system states. The region in which the true state resides is not exactly determined. How to locate the true state is crucial for some systems, especially for a tracking system. As a motivation, a *set-estimation* approach, i.e., the distributed networked set-membership filtering (DNSMF) approach, is proposed in this paper.

As is known, the set-membership filtering (SMF) can provide a set of state estimations in state space, which guarantees to contain the true state of the system in the presence of unknown-but-bounded noises [33, 34]. It is similar

with the interval algorithm [16, 26]. However, the interval algorithm mainly deals with system parameter uncertainty. Differently, the SMF deals with the uncertainty in system noise based on a determined system model. So far, there are many efficient approaches developed to obtain the solutions to SMF problems [31, 33, 34, 37]. Nevertheless, these approaches are all centralized ones. The distributed ones have not yet received adequate attention. Nowadays, owing to the development of networking technologies, an increasing number of practical systems are connected by networks. Thus the whole estimated system can be more complicated and more geographically dispersed such as the observation stations for locating satellites and rockets, which will naturally form a distributed estimation for locating the targets. Therefore, the problem of DNSMF for target estimation is of significance for practical applications. This is also the motivation for this paper to propose a DNSMF approach. However, there is a vital problem that DNSMF provides a number of local estimation sets of tiny difference due to the influence of different noises in the network. Thus, it is important to develop a new method to integrate all these local estimation sets in a global view, which is the so-called global state estimation method.

In order to obtain the global state estimation, there are two different methods can be used, i.e., the union method and the intersection method. The former is to build a set to contain the union of all the local estimation sets, which provides all possible regions for locating the true state. The latter is to find a set to contain the intersection of all the local estimation sets, which provides the most feasible region for locating the true state. Generally, the intersection method is more popular since it can offer a smaller set to improve the filtering performance, but it has the drawback that there may be even no intersection in case some incorrect measurements or impulse noises appearing. Compared with the intersection method, the union method is more fault-tolerant and reliable. Thus, in this paper, a union method will be adopted to produce the global estimation. And a new geometric method based on Minkowski sum is derived for calculating the global estimation.

In this paper, a DNSMF problem is investigated under unknown-but-bounded process and measurement noises, and some special features of the DNSMF are to be revealed in a distributed way. A global ellipsoidal state estimation is provided via the union method based on Minkowski sum to integrate the local ellipsoidal state estimation sets, which are produced by each local filter in a filtering network to locate the true state. In the filtering network, each local

filter provides an ellipsoidal state estimation by using local measurement and neighboring information. As a result, both global and local ellipsoidal estimations are provided to locate the true state. The main contributions of this paper can be summarized as follows: (i) A new geometric method based on Minkowski sum is proposed to produce the global ellipsoidal estimation, whose aim is to include the union of all the local ellipsoidal estimations; (ii) The techniques are presented for transforming and expanding the local ellipsoidal estimations to include the origin, which is crucial to determine the global ellipsoidal estimation; and (iii) A time-varying cooperative filtering approach is developed to derive some sufficient conditions for the existence of local set-membership filters by exchanging information among the neighboring filters. A recursive convex optimization algorithm is provided to determine the local ellipsoidal estimations.

The rest of this paper is organized as follows. An intuitive example of the DNSMF problem is formulated in Section 2. Both the global and the local ellipsoidal state estimations are presented in Section 3. Section 4 provides a real experiment to demonstrate the effectiveness of our algorithm. Conclusions and future directions are described in Section 5.

Notation. For symmetric matrices X and Y , the notation $X \geq Y$ (respectively, $X > Y$) means that $X - Y$ is positive semi-definite (respectively, positive definite). The superscript T stands for matrix transpose. $\text{trace}(P)$ denotes the trace of the matrix P . $\varepsilon(\hat{x}, P) = \{x : (x - \hat{x})^T P^{-1} (x - \hat{x}) \leq 1, P > 0\}$ denotes an ellipsoid. $\emptyset$ means an empty set. $\oplus$ denotes Minkowski sum. $\cup$ and $\cap$ represent the union of sets and the intersection of sets, respectively. $\mathbf{0} = \begin{bmatrix} 0 & \dots & 0 \end{bmatrix}^T \in \mathbb{R}^n$.

2. Problem Formulation

The objective of this paper is to design the distributed networked set-membership filters to estimate the state of an estimated system. Thus, an intuitive example of distributed filtering using N set-membership filters is provided in Fig. 1.

Remark 1. Fig. 1 shows a distributed form of the networked set-membership filters. It consists of N filters and each filter i has a communication domain $\mathbb{N}_i$ including its neighboring filters. For large-scale spatial and distributed systems, it is difficult to build a communication network to connect all the filters because a large-scale network may incur high cost and high communication overheads.

Figure 1: The structure of the distributed networked set-membership filtering

Thus, it is to be established a local communication network to exchange information between the i_{th} filter and its neighboring filters as the communication domain N_i , of which the interconnection topology is shown in Fig. 2. The state information will be collected only by the i_{th} filter from its neighboring filters in the N_i , and the i_{th} filter will not exchange information among the filters out of N_i . For practical interests, other architectures can also be applied for the distributed set-membership filters, such as a circle structure and a star structure.

Figure 2: The communication domain N_i of SMF i

Consider the following discrete time-varying system as the estimated system in Fig. 1, which is described by

$$x_{k+1} = F_k x_k + B_k w_k, \quad (1)$$

where $x_k \in \mathbb{R}^n$ is the system state; F_k and B_k are known time-varying matrices with appropriate dimensions; $w_k \in \mathbb{R}^r$ is the process noise which is assumed to be unknown but bounded, and to be confined to the following ellipsoid:

$$w_k \in \varepsilon(\mathbf{0}, Q_k) = \{w_k : w_k^T Q_k^{-1} w_k \leq 1\} \triangleq W_k, \quad (2)$$

where $Q_k = Q_k^T > 0$ is a known matrix with compatible dimensions. The initial state x_0 belongs to a given ellipsoid

$$x_0 \in \varepsilon(\hat{x}_0, P_0) = \{x_0 : (x_0 - \hat{x}_0)^T P_0^{-1} (x_0 - \hat{x}_0) \leq 1\} \triangleq X_0,$$

where $\hat{x}_0$ is an estimation of x_0 assumed to be given, and $P_0 = P_0^T > 0$ is a known matrix.

In Fig. 1, each set-membership filter has its own output measurement, which is described by

$$y_k^i = C_k^i x_k + D_k^i v_k^i, \quad i = 1, 2, \dots, N, \quad (3)$$

where $y_k^i \in \mathbb{R}^m$ is the measurement of the i_{th} set-membership filter; C_k^i and D_k^i are known matrices with appropriate dimensions; $v_k^i \in \mathbb{R}^p$ is the measurement noise which is assumed to be unknown but bounded, and to be confined to the following ellipsoid:

$$v_k^i \in \varepsilon(0, R_k^i) = \{v_k^i : (v_k^i)^T (R_k^i)^{-1} v_k^i \leq 1\} \triangleq V_k^i, \quad i = 1, \dots, N, \quad (4)$$

where $R_k^i = (R_k^i)^T > 0$ is a known matrix with compatible dimensions.

In DNSMF, each filter not only adopts its local measured information, but also takes the state information from its neighbors. The communication domain $\mathbb{N}_i$ contains the n_i filters that provide information for the set-membership filter i , see Fig. 1. The filter i estimates the state x_k , which is governed by

$$\hat{x}_{k+1}^i = G_k^i \hat{x}_k^i + L_k^i y_{k+1}^i + K_k^i \sum_{j \in \mathbb{N}_i} a_{ij} (\hat{x}_k^{ij} - \hat{x}_k^i), \quad (5)$$

where $\hat{x}_k^i \in \mathbb{R}^n$ is the estimation of the state x_k provided by the filter i ; $\hat{x}_k^{ij} \in \mathbb{R}^n$ is the estimation of the state x_k provided by the neighboring filter j in $\mathbb{N}_i$, where $j = 1, \dots, n_i$; G_k^i , L_k^i and K_k^i are the filter parameters to be determined; $a_{ij} \geq 0$ is the weighting coefficient that represents the influence of the filter j 's neighboring estimation on the filter i 's estimation. If $j = i$, then $a_{ij} = 0$. Therefore, each filter provides the state estimation which relies on its local measurement y_{k+1}^i and its neighboring estimations $\hat{x}_k^{ij}$ s received from its neighbors. Then, the obtained estimation is also transmitted to the neighbors for use.

In DNSMF, each set-membership filter provides an ellipsoidal state estimation by using its own information and its neighboring information. Due to different measurements and unknown noises, each local filter provides a different ellipsoidal estimation, and the true state (the target) may locate anywhere

in each ellipsoid. For ensuring the reliability of the estimation result, a global ellipsoidal state estimation will be derived for including the union of all local estimations. As a result, the true states can be determined by the global estimation. For this purpose, the objective of this paper is to provide:

i) Local ellipsoidal state estimation: Compute an ellipsoid in which the estimated state resides based on the local set-membership filter, i.e., compute an ellipsoid X_k^i such that

$$X_k^i = \varepsilon(\hat{x}_k^i, P_k^i) = \{x_k \in \mathbb{R}^n | (x_k - \hat{x}_k^i)^T (P_k^i)^{-1} (x_k - \hat{x}_k^i) \leq 1\}. \quad (6)$$

ii) Global ellipsoidal state estimation: Compute a global ellipsoid containing the union of all local estimated ellipsoids, i.e., compute an ellipsoid X_k such that

$$X_k = \varepsilon(\hat{x}_k, P_k) \supseteq X_k^1 \cup \dots \cup X_k^i \cup \dots \cup X_k^N, \quad (7)$$

where X_k^i is the i_{th} local ellipsoidal state estimation.

3. Ellipsoidal State Estimation

In this section, an ellipsoid X_k will be derived in which the estimated state resides. Both global and local ellipsoidal state estimations are presented. A novel geometric method based on Minkowski sum will be proposed to compute the global ellipsoidal estimation to integrate the local ellipsoidal state estimations. The local estimations X_k^i s will be produced to satisfy the optimal criterion of an ellipsoid, and a convex optimization recursive algorithm will be provided for designing the local set-membership filters.

3.1. Global Ellipsoidal State Estimation

The global ellipsoidal state estimation is designed to cover the union of all the local estimation ellipsoids. In this sub-section, a geometric method is proposed to compute the global ellipsoidal state estimation based on Minkowski sum. In order to perform the global estimation, the definition of Minkowski sum and the minimum external ellipsoid of Minkowski sum are introduced as follows.

Definition 1. [17] Consider m ellipsoids $\varepsilon_i = \varepsilon(a_i, Q_i)$, $i = 1, \dots, m$, where $a_i \in \mathbb{R}^n$, $Q_i \in \ell(\mathbb{R}^n, \mathbb{R}^n)$ and $Q_i > 0$. Then, their Minkowski sum ε_s is defined as

$$\varepsilon_s = \varepsilon_1 \oplus \dots \oplus \varepsilon_i \oplus \dots \oplus \varepsilon_m = \{(e_1 + \dots + e_i + \dots + e_m) | e_1 \in \varepsilon_1, \dots, e_i \in \varepsilon_i, \dots, e_m \in \varepsilon_m\}.$$

Lemma 1. [17] The minimum external ellipsoid ε_{min} of the Minkowski sum ε_s of m ellipsoids $\varepsilon_i = \varepsilon(a_i, Q_i)$ is determined by

$$\varepsilon_{min} = \varepsilon(a, Q),$$

where

$$a = \sum_{i=1}^m a_i$$

and

$$Q = \sum_{i=1}^m p_i \cdot \left(\sum_{i=1}^m p_i^{-1} Q_i \right)$$

with $p_i = \sqrt{\text{trace}(Q_i)}$, $i = 1, \dots, m$. Therefore,

$$\varepsilon_{min} \supseteq \varepsilon_s = \varepsilon_1 \oplus \dots \oplus \varepsilon_i \oplus \dots \oplus \varepsilon_m.$$

Remark 2. Definition 1 gives a solution to form the geometric sum of local estimations, and the Lemma 1 is used to produce the final global ellipsoidal estimation. To some extent, Minkowski sum can be directly utilized to solve the problem of the geometric sum of several convex sets. But Minkowski sum is essentially the sum of vectors represented by those points in subsets, see Definition 1. As a result, the sum may not exactly the union of all the subsets. That is reason for generating a novel geometric algorithm based on Minkowski sum rather than directly applying Minkowski sum to find a set that includes the union of all the local estimation ellipsoids in our problem.

Before obtaining the set to cover the union of all the sets of the local estimation ellipsoids, the following theorem is firstly introduced to tell the feature of the union.

Theorem 1. Suppose that there are two ellipsoids $\varepsilon_1 = \varepsilon(a_1, Q_1)$ and $\varepsilon_2 = \varepsilon(a_2, Q_2)$. If there exists an external ellipsoid ε_e such that the Minkowski sum ε_s of ε_1 and ε_2 is included in ε_e , i.e., $\varepsilon_e \supseteq \varepsilon_s = \varepsilon_1 \oplus \varepsilon_2$, then we have the following statements:

- i) If $\mathbf{0} \in \varepsilon_1$, then $\varepsilon_2 \subseteq \varepsilon_e$.
- ii) If $\mathbf{0} \in \varepsilon_1 \cap \varepsilon_2$, then $\varepsilon_1 \cup \varepsilon_2 \subseteq \varepsilon_e$.

Proof: i) Obviously, when $\mathbf{0} \in \varepsilon_1$, $\varepsilon_s = \varepsilon_1 \oplus \varepsilon_2$ can be rewritten as

$$\begin{aligned} \varepsilon_s &= \{(\mathbf{0} + e_2) | e_2 \in \varepsilon_2\} \cup \{(e_1 + e_2) | e_1 \in \varepsilon_1, e_1 \neq \mathbf{0}, e_2 \in \varepsilon_2\} \\ &= \varepsilon_2 \cup \{(e_1 + e_2) | e_1 \in \varepsilon_1, e_1 \neq \mathbf{0}, e_2 \in \varepsilon_2\}. \end{aligned}$$

Since $\varepsilon_e \supseteq \varepsilon_s = \varepsilon_1 \oplus \varepsilon_2$, there is $\varepsilon_2 \subseteq \varepsilon_e$.

ii) Based on the result of i), we have

$$\mathbf{0} \in \varepsilon_1 \cap \varepsilon_2 \Rightarrow \left\{ \begin{array}{l} \mathbf{0} \in \varepsilon_1 \Rightarrow \varepsilon_2 \subseteq \varepsilon_e \\ \mathbf{0} \in \varepsilon_2 \Rightarrow \varepsilon_1 \subseteq \varepsilon_e \end{array} \right\} \Rightarrow \varepsilon_1 \cup \varepsilon_2 \subseteq \varepsilon_e,$$

which ends the proof.

Remark 3. Theorem 1 reveals the important feature of the union of two ellipsoids. If two ellipsoids ε_1 and ε_2 both contain the origin, their union will be included in an external ellipsoid ε_e of their Minkowski sum ε_s . Therefore, an external ellipsoid can be found through this feature to cover the union of the local ellipsoids ε_i . And without loss of generality, the minimal external ellipsoid ε_{min} is chosen as the external ellipsoid in the following steps, which can be easily calculated by Lemma 1.

In order to clearly explain the feature described in Theorem 1, an example is provided in Fig. 3. In Fig. 3(a), $\varepsilon_1 = \varepsilon(a_1, Q_1)$, where $a_1 = [1, 2]^T$ and $Q_1 = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$, and $\varepsilon_2 = \varepsilon(a_2, Q_2)$, where $a_2 = [7, 8]^T$ and $Q_2 = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$. It is clear that $\mathbf{0} \in \varepsilon_1$ and thus ε_2 is included in the external ellipsoid ε_{min} of the Minkowski sum of ε_1 and ε_2 , i.e., $\varepsilon_2 \subseteq \varepsilon_{min}$. In Fig. 3(b), $\varepsilon_1 = \varepsilon(a_1, Q_1)$, where $a_1 = [1, 2]^T$ and $Q_1 = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$, and $\varepsilon_2 = \varepsilon(a_2, Q_2)$, where $a_2 = [0.5, 0.5]^T$ and $Q_2 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$. It is clear that $\mathbf{0} \in \varepsilon_1 \cap \varepsilon_2$ and therefore $\varepsilon_1 \cup \varepsilon_2 \subseteq \varepsilon_{min}$.

Remark 4. Fig. 3 gives an illustrative example to reveal the feature described in Theorem 1, which also indicates that Minkowski sum cannot make sure to cover the union of local ellipsoids. Moreover, Fig. 3 also reveals that the including of the origin is the key point of the proposed algorithm for the reliability of the global estimation.

Inspired by Theorem 1, the following theorem can be obtained as an expansion from two ellipsoids to m ellipsoids.

Theorem 2. Consider m ellipsoids $\varepsilon_i = \varepsilon(a_i, Q_i), i = 1, \dots, m$. If $\mathbf{0} \in \varepsilon_1 \cap \dots \cap \varepsilon_m$, then $\varepsilon_1 \cup \dots \cup \varepsilon_m \subseteq \varepsilon_{min}$, where ε_{min} is the minimum external ellipsoid of the Minkowski sum ε_s defined by $\varepsilon_s = \varepsilon_1 \oplus \varepsilon_2 \oplus \dots \oplus \varepsilon_m$.

Figure 3: Examples of two ellipsoids and the minimum external ellipsoid of their Minkowski sum

Proof: From Definition 1, since $\mathbf{0} \in \varepsilon_1 \cap \dots \cap \varepsilon_m$, the Minkowski sum $\varepsilon_s = \varepsilon_1 \oplus \varepsilon_2 \oplus \dots \oplus \varepsilon_m$ can be rewritten as

$$\begin{aligned} \varepsilon_s = & \{(e_1 + \mathbf{0} + \dots + \mathbf{0}) | e_1 \in \varepsilon_1\} \cup \{(e_1 + \dots + e_m) | e_i \in \varepsilon_i, i = 1, 2, \dots, m, e_j \neq \mathbf{0}, \\ & j = 2, 3, \dots, m\} \cup \{(\mathbf{0} + e_2 + \mathbf{0} \dots + \mathbf{0}) | e_2 \in \varepsilon_2\} \cup \{(e_1 + \dots + e_m) | e_i \in \varepsilon_i, \\ & i = 1, 2, \dots, m, e_j \neq \mathbf{0}, j = 1, 3, \dots, m\} \cup \dots \cup \{(\mathbf{0} + \dots + \mathbf{0} + e_m) | e_m \in \varepsilon_m\} \\ & \cup \{(e_1 + \dots + e_m) | e_i \in \varepsilon_i, i = 1, 2, \dots, m, e_j \neq \mathbf{0}, j = 1, 2, \dots, m-1\} \\ = & \varepsilon_1 \cup \varepsilon_2 \cup \dots \cup \varepsilon_m \cup \{(e_1 + \dots + e_m) | e_i \in \varepsilon_i, i = 1, 2, \dots, m, e_j \neq \mathbf{0}, j = 2, 3, \dots, m\} \\ & \cup \{(e_1 + \dots + e_m) | e_i \in \varepsilon_i, i = 1, 2, \dots, m, e_j \neq \mathbf{0}, j = 1, 3, \dots, m\} \cup \dots \\ & \cup \{(e_1 + \dots + e_m) | e_i \in \varepsilon_i, i = 1, 2, \dots, m, e_j \neq \mathbf{0}, j = 1, 2, \dots, m-1\}, \end{aligned}$$

which implies that $\varepsilon_1 \cup \varepsilon_2 \cup \dots \cup \varepsilon_m \subseteq \varepsilon_s$. Through Lemma 1, there is

$$\varepsilon_1 \cup \varepsilon_2 \cup \dots \cup \varepsilon_m \subseteq \varepsilon_{min},$$

which ends the proof.

Remark 5. Theorem 2 provides an expanded solution of Theorem 1 that the union of m ellipsoids can be included in the minimal external ellipsoid of their Minkowski sum. To sum up, the union of all the ellipsoids $\{\varepsilon_1, \dots, \varepsilon_m\}$ can be included in the minimal external ellipsoid ε_{min} of their Minkowski sum ε_s if they all contain the origin.

We now in a position to derive the global ellipsoidal estimation X_k . Suppose that the local ellipsoidal estimations are described as $X_k^i = \varepsilon(\hat{x}_k^i, P_k^i)$, where $\hat{x}_k^i$

is the estimated center and P_k^i is its corresponding shape matrix at instant k , $i = 1, 2, \dots, N$. Firstly, one needs to obtain a group of new ellipsoids $\bar{X}_k^i$ s via the translation of the initial estimations X_k^i s in position to satisfy the requirements that a) the mean value of the centers of $\bar{X}_k^i$ s is zero; b) the relative position of $\bar{X}_k^i$ s among each other is the same as that of the initial estimations X_k^i s. That is to say, the $\bar{X}_k^i$ is obtained by transforming the X_k^i as

$$\bar{X}_k^i = \varepsilon(\bar{x}_k^i, P_k^i) = \varepsilon(\hat{x}_k^i - \bar{x}_k, P_k^i), \quad i = 1, 2, \dots, N, \quad (8)$$

where

$$\bar{x}_k = \frac{1}{N} \sum_{i=1}^N \hat{x}_k^i.$$

Secondly, one needs to judge whether each $\bar{X}_k^i$ includes the origin, i.e., $\mathbf{0} \in \bar{X}_k^i$. If $\mathbf{0} \in \bar{X}_k^i$, the $\bar{X}_k^i$ can be used to build the global estimation directly in the next steps. Once $\mathbf{0} \notin \bar{X}_k^i$, another new ellipsoid S_k^i will be constructed to include both $\bar{X}_k^i$ and the origin, of which the construction is as follows.

In order to construct a S_k^i , an ellipsoid $\Theta_k^i = \varepsilon(\mathbf{0}, Q_k^i)$ centered at the origin needs to be produced, which should be intersected with $\bar{X}_k^i$. Thus, Θ_k^i has the following properties:

- i) There is a point $a \in \bar{X}_k^i \cap \Theta_k^i$, i.e., $\bar{X}_k^i \cap \Theta_k^i \neq \emptyset$.
- ii) There is a point $b \in \Theta_k^i$ such that $a + b = \mathbf{0}$, i.e., b is the symmetric point of a .

Based on Definition 1, we have

$$\begin{aligned} \bar{X}_k^i \oplus \Theta_k^i &= \{a + b\} \cup \{(\bar{x}^i + \theta^i) | \bar{x}^i \in \bar{X}_k^i, \bar{x}^i \neq a, \theta^i \in \Theta_k^i\} \\ &\quad \cup \{(\bar{x}^i + \theta^i) | \bar{x}^i \in \bar{X}_k^i, \theta^i \in \Theta_k^i, \theta^i \neq b\} \\ &= \{\mathbf{0}\} \cup \{(\bar{x}^i + \theta^i) | \bar{x}^i \in \bar{X}_k^i, \bar{x}^i \neq a, \theta^i \in \Theta_k^i\} \\ &\quad \cup \{(\bar{x}^i + \theta^i) | \bar{x}^i \in \bar{X}_k^i, \theta^i \in \Theta_k^i, \theta^i \neq b\}. \end{aligned}$$

Thus, it is clear that $\mathbf{0} \in \bar{X}_k^i \oplus \Theta_k^i$.

After producing an ellipsoid $\Theta_k^i = \varepsilon(\mathbf{0}, Q_k^i)$, one still needs to minimize the Θ_k^i to make it tangential to the $\bar{X}_k^i$. Therefore, the following optimization method is proposed to generate the minimal ellipsoid Θ_k^i .

$$\begin{aligned} \min \quad & \text{trace}(Q_k^i). \\ \text{s.t.} \quad & Q_k^i > 0 \\ & \bar{X}_k^i \cap \Theta_k^i \neq \emptyset \end{aligned}$$

When Θ_k^i is generated, the sum $\bar{X}_k^i \oplus \Theta_k^i$ is obtained. According to Lemma 1, the minimum external ellipsoid of the Minkowski sum of $\bar{X}_k^i$ and Θ_k^i is obtained by $\varepsilon(s_k^i, M_k^i)$, where

$$s_k^i = \bar{x}_k^i + \mathbf{0} = \bar{x}_k^i$$

and

$$M_k^i = (p_1 + p_2) (p_1^{-1} P_k^i + p_2^{-1} Q_k^i)$$

with $p_1 = \sqrt{\text{trace}(P_k^i)}$ and $p_2 = \sqrt{\text{trace}(Q_k^i)}$. Then, $\mathbf{0} \in \bar{X}_k^i \oplus \Theta_k^i \subseteq \varepsilon(s_k^i, M_k^i)$. Thus, for the convenience of description, the form of S_k^i can be summarized as

$$S_k^i = \varepsilon(\bar{s}_k^i, \bar{P}_k^i) = \begin{cases} \bar{X}_k^i & , \quad \mathbf{0} \in \bar{X}_k^i, \\ \varepsilon(s_k^i, M_k^i) & , \quad \mathbf{0} \notin \bar{X}_k^i, \end{cases} \quad (9)$$

where the $\bar{s}_k^i$ and the $\bar{P}_k^i$ are the center and shape matrix of the S_k^i respectively. According to Theorem 1 and (9), it is obvious that $\bar{X}_k^i \cup \{\mathbf{0}\} \subseteq S_k^i$.

Remark 6. The second step is to form the ellipsoid S_k^i to contain both the origin and $\bar{X}_k^i$ in order to apply Theorem 2 for the reliability of producing the global estimation. The inclusion of the $\bar{X}_k^i$ is equal to the inclusion of the true state, and the inclusion of the origin can help to apply Theorem 2. Thus, both the $\bar{X}_k^i$ and the origin are significant to be covered in this step.

Thirdly, an ellipsoid $\bar{S}_k$ is to be found to enclose all the $S_k^i, i = 1, 2, \dots, N$. According to Lemma 1, $\bar{S}_k$ can be calculated as

$$\bar{S}_k = \varepsilon(\bar{s}_k, \bar{P}_k), \quad (10)$$

where

$$\bar{s}_k = \sum_{i=1}^N \bar{s}_k^i$$

and

$$\bar{P}_k = \sum_{i=1}^N \bar{p}_k^i \cdot \left(\sum_{i=1}^N (\bar{p}_k^i)^{-1} \bar{P}_k^i \right)$$

with $\bar{p}_k^i = \sqrt{\text{trace}(\bar{P}_k^i)}, i = 1, \dots, N$. Therefore,

$$\bar{S}_k = \varepsilon(\bar{s}_k, \bar{P}_k) \supseteq S_k^1 \oplus S_k^2 \oplus \dots \oplus S_k^N.$$

Owing to $\mathbf{0} \in S_k^1 \cap \dots \cap S_k^N$ and Theorem 2, it is obvious that

$$\bar{S}_k \supseteq S_k^1 \cup \dots \cup S_k^N.$$

Finally, through the translation in position again, the $\bar{S}_k$ should be transformed back to the original mean point of the centers of the original local estimations, i.e.,

$$X_k = \varepsilon(\hat{x}_k, P_k) = \varepsilon(\bar{s}_k + \bar{x}_k, \bar{P}_k). \quad (11)$$

Thus, the X_k in (7) is obtained.

In summary, the following procedures are formed to compute the global ellipsoidal state estimation (Global Estimation Algorithm):

- i) Transform the local ellipsoidal state estimations X_k^i s into $\bar{X}_k^i$ s by (8).
- ii) Generate the expanded ellipsoids S_k^i s by (9).
- iii) Produce the ellipsoid $\bar{S}_k$ for covering the union of all the expanded ellipsoids S_k^i s by (10).
- iv) Transform $\bar{S}_k$ to obtain the global ellipsoidal estimation X_k by (11).

Remark 7. *The whole procedure of the global estimation algorithm is operating on the transformation of the local estimation sets. Note that if the generated global ellipsoid can cover these transformed local estimations, it can definitely contain the union of initial local estimations after translation. Thus, the first step is to transform the initial local estimations by translation towards the origin for the convenience of the calculation. The second and the third steps are to generate the global ellipsoidal set over these transformed ellipsoids based on Theorem 1 and 2. The final step is to return the produced global ellipsoid over the initial local estimations by translation, which can obviously provide the required global estimation set.*

Therefore, the global ellipsoidal state estimation can be obtained as long as all the local estimations X_k^i s are known. In the following sub-section, the local ellipsoidal state estimation will be performed.

3.2. Local Ellipsoidal State Estimation

In this subsection, we provide a convex optimization recursive algorithm for computing the local state estimation ellipsoids X_k^i s for DNSMF. The conditions on the existence of the local ellipsoidal estimation X_k^i are derived in the following theorem satisfying the optimal criterion of an ellipsoid in which the estimated state resides.

Theorem 3. *For the system (1)-(3), suppose that the state x_k belongs to its state estimation ellipsoid $\varepsilon(\hat{x}_k^i, P_k^i)$. Then, the one-step ahead state x_{k+1} resides*

in its ellipsoidal state estimation $\varepsilon(\hat{x}_{k+1}^i, P_{k+1}^i)$ if there exist matrices $P_{k+1}^i > 0$, G_k^i , L_k^i , K_k^i , and positive scalars τ_1^i, τ_2^i and τ_3^i such that

$$\begin{bmatrix} -P_{k+1}^i & \Phi_k^i \\ \Phi_k^{iT} & -\Psi(\tau_1^i, \tau_2^i, \tau_3^i) \end{bmatrix} \leq 0, \quad (12)$$

where

$$\begin{aligned} \Phi_k^i &= [(I - L_k^i C_{k+1}^i) F_k \hat{x}_k^i - G_k^i \hat{x}_k^i - K_k^i \sum_{j \in \mathbb{N}_i} a_{ij} (\hat{x}_k^{ij} - \hat{x}_k^i), (I - L_k^i C_{k+1}^i) F_k E_k^i, \\ &\quad (I - L_k^i C_{k+1}^i) B_k, -L_k^i D_{k+1}^i], \\ \Psi(\tau_1^i, \tau_2^i, \tau_3^i) &= \text{diag} \left\{ \Pi_k^i, \tau_1^i I, \tau_2^i Q_k^{-1}, \tau_3^i (R_{k+1}^i)^{-1} \right\}, \\ \Pi_k^i &= 1 - \tau_1^i - \tau_2^i - \tau_3^i. \end{aligned}$$

Proof: From (1), (3) and (5), we have

$$\begin{aligned} x_{k+1} - \hat{x}_{k+1}^i &= (I - L_k^i C_{k+1}^i) F_k x_k - G_k^i \hat{x}_k^i + (I - L_k^i C_{k+1}^i) B_k w_k \\ &\quad - K_k^i \sum_{j \in \mathbb{N}_i} a_{ij} (\hat{x}_k^{ij} - \hat{x}_k^i) - L_k^i D_{k+1}^i v_{k+1}^i. \end{aligned}$$

If x_k resides in $\varepsilon(\hat{x}_k^i, P_k^i)$, there exists z_k^i with $\|z_k^i\| \leq 1$ such that

$$x_k = \hat{x}_k^i + E_k^i z_k^i,$$

where E_k^i is the factorization of $P_k^i = E_k^i E_k^{iT}$. Then, for all $j \in \mathbb{N}_i$, we have

$$\begin{aligned} x_{k+1} - \hat{x}_{k+1}^i &= [(I - L_k^i C_{k+1}^i) F_k - G_k^i] \hat{x}_k^i - K_k^i \sum_{j \in \mathbb{N}_i} a_{ij} (\hat{x}_k^{ij} - \hat{x}_k^i) \\ &\quad + (I - L_k^i C_{k+1}^i) F_k E_k^i z_k^i + (I - L_k^i C_{k+1}^i) B_k w_k - L_k^i D_{k+1}^i v_{k+1}^i. \end{aligned}$$

Denoting $\eta_k^i = [1, z_k^{iT}, w_k^T, v_{k+1}^{iT}]^T$, we obtain a compact form as

$$x_{k+1} - \hat{x}_{k+1}^i = \Phi_k^i \eta_k^i.$$

Hence, $(x_{k+1} - \hat{x}_{k+1}^i)^T (P_{k+1}^i)^{-1} (x_{k+1} - \hat{x}_{k+1}^i) \leq 1$ can be written as

$$\eta_k^{iT} [\Phi_k^{iT} (P_{k+1}^i)^{-1} \Phi_k^i - \text{diag}\{1, 0_{d_x+d_w+d_v}\}] \eta_k^i \leq 0, \quad (13)$$

where 0_a is a zero matrix with the dimension $a \times a$.

In addition, $\|z_k^i\| \leq 1$ can be expressed as

$$\eta_k^{iT} \text{diag}\{-1, I, 0_{d_w+d_v}\} \eta_k^i \leq 0, \quad (14)$$

and by using (2) and (4), we have

$$\eta_k^{iT} \text{diag}\{-1, 0_{d_x}, Q_k^{-1}, 0_{d_v}\} \eta_k^i \leq 0, \quad (15)$$

$$\eta_k^{iT} \text{diag}\{-1, 0_{d_x+d_w}, (R_{k+1}^i)^{-1}\} \eta_k^i \leq 0. \quad (16)$$

By using an $\mathcal{S}$ -procedure, one can see that (13)-(16) hold if there exist positive scalars τ_1^i, τ_2^i and τ_3^i such that

$$\begin{aligned} & \Phi_k^{iT} (P_{k+1}^i)^{-1} \Phi_k^i - \text{diag}\{1, 0_{d_x+d_w+d_v}\} - \tau_1^i \text{diag}\{-1, I, 0_{d_w+d_v}\} \\ & - \tau_2^i \text{diag}\{-1, 0_{d_x}, Q_k^{-1}, 0_{d_v}\} - \tau_3^i \text{diag}\{-1, 0_{d_x+d_w}, (R_{k+1}^i)^{-1}\} \leq 0, \end{aligned}$$

which can be rewritten in a compact form as

$$\Phi_k^{iT} (P_{k+1}^i)^{-1} \Phi_k^i - \Psi(\tau_1^i, \tau_2^i, \tau_3^i) \leq 0. \quad (17)$$

By using *Schur Complements*, (17) is equivalent to (12), which ends the proof.

Theorem 3 provides a method to obtain the local state estimation ellipsoids for DNSMF. In order to find the minimum ellipsoids, a convex optimization approach is derived to determine the optimal ellipsoids. P_{k+1}^i are obtained by solving the following optimization problem:

$$\begin{aligned} & \min_{G_k^i, L_k^i, K_k^i, \tau_1^i, \tau_2^i, \tau_3^i} \text{trace}(P_{k+1}^i), \\ & \text{s.t.} \quad (12) \end{aligned} \quad (18)$$

which is easily solved by the existing semi-definite programming via an interior-point algorithm [32]. So far the optimal local ellipsoidal state estimations are obtained, and the global ellipsoidal state estimation can be calculated by (8)-(11).

Note that the proposed DNSMF is a recursive approach with solving a feasible LMI problem via an interior-point algorithm, which has a polynomial-time complexity. The row size of the LMI constraint (12) is $\gamma = 2n + r + p + 1$ and the number of scalar decision variables is $\alpha = \frac{3}{2}n^2 + \frac{5}{2}n + 3$. Thus, the computational complexity of the proposed approach is proportional to $\gamma\alpha^3$. Compared with the DKF approach, which is only a recursive algorithm without LMIs, the proposed approach has more computational complexity. However, the advantage of applying the proposed approach is that it can operate under the unknown-but-bounded noise, which is not suitable for performing the DKF approach. Furthermore, the proposed DNSMF has a good scalability because it is easy to add local filters with expanding the communication domains for the

information exchange, which is similar to the DKF approach. The performance of each local filter is independent of the total number of local filters. But more neighboring filters in the communication domain $\mathbb{N}_i$ indicates better accuracy for the i_{th} local estimation.

4. Simulation Results

In order to verify the effectiveness and to show the implementation of the proposed algorithm on real world datasets, in this section, an experiment is conducted based on a 2-kW single-phase grid-connected power generation system, which parameters are exhibited in Table 1.

Table 1: The parameters of the model system

DC power source	360V
Power	2kW
LC filter	$L : 8mH; C : 50\mu F$
RLC load	$R : 24.2\Omega; L : 77mH; C : 132\mu F$
Grid voltage	$220\sqrt{2}V$
Grid impedance	$L_g : 1mH; R_g : 0.1\Omega$
Fundamental frequency	50Hz
Sampling frequency	400Hz

Based on the employed set-up, a discrete-time model system in state space is built to track the state of voltage at the point of common coupling (PCC) of the grid-connected generation system, which is formed as

$$x_{k+1} = \begin{bmatrix} \cos(\omega T) & -\sin(\omega T) \\ \sin(\omega T) & \cos(\omega T) \end{bmatrix} x_k + \begin{bmatrix} \Delta u \cos(\omega k T + \omega T) \\ \Delta u \sin(\omega k T + \omega T) \end{bmatrix},$$

where the state $x_k = [x_{1k}, x_{2k}]^T$, $x_{1k} \in \mathbb{R}$ and $x_{2k} \in \mathbb{R}$; $\omega = 100\pi rad/s$ represents the angular fundamental frequency; k is the k_{th} sampling; $T = 2.5ms$ is the sampling period; $\Delta u = 10^{-3}U_0 \sin(\omega k T)$ is the voltage deviation between two sampling intervals, where $U_0 = 220\sqrt{2}V$ is the voltage amplitude.

In the proposed module of filtering, the designed methods are provided by programming in Matlab, where there are four filters in the designed filtering network. Each filter i will communicate with its two neighboring filters in the domain $\mathbb{N}_i$, see Fig. 4. According to the Metropolis weights method¹ [1], the

Figure 4: The communication topology graph of the filtering network

weighted matrix of the communication topology is

$$[a_{ij}] = \begin{bmatrix} 0 & 1/3 & 0 & 1/3 \\ 1/3 & 0 & 1/3 & 0 \\ 0 & 1/3 & 0 & 1/3 \\ 1/3 & 0 & 1/3 & 0 \end{bmatrix}.$$

Then the measurement equation is

$$y_k^i = \begin{bmatrix} 1 + 0.1i & 0 \end{bmatrix} x_k + \begin{bmatrix} 1 + 0.1i \end{bmatrix} v_k^i,$$

where $v_k^i = 0.02U_0 \sin(\omega kT)(1 + 0.1i)$, $i = 1, 2, 3, 4$.

Additionally, $Q_k = \begin{bmatrix} (0.002U_0)^2 & 0 \\ 0 & (0.002U_0)^2 \end{bmatrix}$; $R_k^i = (0.03U_0)^2(1 + 0.1i)$, $i = 1, 2, 3, 4$.

The initial conditions of the states are $x_0 = [220, 220]^T$, $\hat{x}_0^1 = [300.53, -80.53]^T$, $\hat{x}_0^2 = [295.9, 96.14]^T$, $\hat{x}_0^3 = [-182.88, 251.71]^T$, $\hat{x}_0^4 = [-287.44, -119.07]^T$, respectively and $P_0^1 = P_0^2 = P_0^3 = P_0^4 = \begin{bmatrix} 4U_0^2 & 0 \\ 0 & 4U_0^2 \end{bmatrix}$.

The simulation results of the proposed method are obtained under Matlab R2012a from $k = 1$ to $k = 50$. By solving the optimization problem (18), the results for the phase planes of the position of the true states together with their estimations are depicted in Fig. 5. It is clear that all the true states are included in the estimated ellipsoids, which indicates the good performance of the local state estimations.

¹Metropolis weights: $a_{ij} = 1/(1 + \max[|N_i|, |N_j|])$

In order to integrate these local estimations, the global ellipsoidal estimation method proposed in Section 3.1 is applied. Fig. 6 illustrates the global state estimation results. Particularly, each global estimation contains the union of all local estimations at each time instant, which also reveals the good performance of the global state estimations.

Figure 5: The ellipsoidal state estimation results of each filter

Figure 6: The global ellipsoidal state estimations

Remark 8. *Note that the experiment is conducted under the unknown-but-bounded noises, which is not suitable for performing the DKF approach. Thus, the comparison between the estimation of the DNSMF and that of the DKF cannot be made in this experiment. But one can evidently see that the proposed algorithm is effective and successful under the unknown-but-bounded noises.*

5. Conclusion

This paper has provided a new approach to deal with the DNSMF problem with ellipsoidal state estimation in the presence of the unknown-but-bounded process and measurement noises. The true state has been located by both local and global ellipsoidal estimations. Moreover, the global estimation can successfully integrate the local estimations by including the union of all the local estimations via the proposed geometric algorithm. The feasibility and effectiveness of the proposed filtering methods have been demonstrated by an illustrative experiment. The presented approach is very useful for target tracking and fault location. The perfect information exchanged among the distributed set-membership filters is considered in this paper. In the future, a challenging research topic will focus on imperfect information among the distributed filter exchange, such as data dropouts, time-delay, and transmission errors.

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Distributed Networked Set-Membership Filtering with Ellipsoidal State Estimations

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