Synthesis of Regulation Compliant Business Processes*

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Abstract—Organisations have to cope with large numbers of business rules and existing regulations governing the business in which they operate. Such rules are difficult to maintain due to their size and complexity, and it is increasingly challenging to ensure that each business process adheres to those rules. As such, automated extraction of business processes from rules has three clear advantages: (1) visualisation of all possible executions allowed by the rules, (2) automated execution and compliance by design, (3) identification of “inefficiencies” in the business rules. Existing approaches, however, only allow for the generation of partial traces based on input specifications and cannot handle many different input cases resulting in a full process. This paper presents a formal method to visualise and operationalise such sets of rules as a verifiable business process that is compliant by design, which allows us to analyse all possible execution paths. Additionally, we formally prove correctness of the business processes generated by our method. The approach is implemented in a tool and evaluated on both performance and correctness, showing that even for highly complex sets of rules the approach performs well and outperforms a well-known state-of-the-art approach. Evaluation on a real-life process shows the feasibility of the presented approach.

I. INTRODUCTION

It is a common scenario that organisations develop business rules manuals for their operations. Such business rules may specify constraints that apply to their business processes (e.g. a customer has to be older than 18 in order to be eligible for a loan). As organisations grow, so do their processes and business rules. As a consequence, the number of business rules is generally large [2].

In addition, organisations have to comply with regulations governing the business in which they operate. These regulations (as the etymology of the name itself, *regula* in latin means rule) consist of rules as well, and the compliance rules affect the procedures and processes described by the business rules. Although the combination of business rules and compliance rules allows organisations to determine whether such procedures and processes are compliant with the regulatory requirements, in general, for large rule sets the relationships among the rules (and the activities and conditions they describe) are difficult to understand and maintenance of those rule sets is complicated [2]. Furthermore, it is difficult to ensure that each business process adheres to those rules [3].

However, the combination of these rules implicitly contains the processes for organisations, as the constraints are specified by those rules. The extraction of business processes from rules has a number of clear advantages: (1) it helps to visualise all possible executions allowed by the rules, (2) the process can be implemented by a workflow engine (e.g. [4]) for automated execution and compliance by design, (3) it helps to identify “inefficiencies” in the business rules (e.g. rules with side effects that do not contribute to the goals of a process are visualised as separate paths in the process that only merge at the end, multiple rules for the same goal are visualised as redundant transitions, etc).

As a starting point to create our business processes we adopt a rule-based formalism, because rules are well suited to describe both the external constraints regulating the setting, and which actions the agent may undertake. In this way, rules are able to define relationships among the tasks to be executed to achieve a goal, are useful in understanding the behaviour of such tasks. By shifting the focus from the entire process to its basic building blocks, it can be identified which preconditions trigger the activation of a task, as well as what happens once a task completes its execution. This way of describing processes provides knowledge concerning the reasons why to execute a task instead of another and, implicitly, defines the control flow of a process as all the situations that do not violate any of the given relationships.

Existing approaches to transform rules into readable business process models, however, only produce partial traces based on input specifications [5][6], cannot handle many different input cases resulting in a full process [7] or duplicate tasks to represent every possible trace [8]. Therefore, to tackle these problems, this paper presents a new formal method to visualise and operationalise such sets of rules as a verifiable business process (i.e., it is possible to check whether the process is sound [9] and compliant [10]), by generating individual Petri nets for each case and subsequently merging them in such a way that duplication of tasks is avoided. The generated business process is compliant by design and allows us to analyse all possible execution paths within the rules (i.e. supporting all input cases). Furthermore, the generated business process is tailored to support and adhere to the business rules and regulations, as it is specifically designed to exactly represent the behaviour allowed by the rules.

The remainder of this paper is structured as follows. In Section II, we discuss the background with respect to rules for declarative processes. Subsequently, Section III describes

*This paper extends the work presented in [1] in the following ways: (i) it provides an updated and more efficient algorithm, (ii) it provides formal proofs of correctness of the algorithms, and (iii) it includes an extensive evaluation of the approach on both synthetic and real-life processes.

the preliminary definitions related to Petri nets. The method and the corresponding algorithms are described in Section IV, along with a running example, followed by a formal proof of correctness of the generated Petri net. The presented approach is evaluated in Section V. Lastly, Section VI presents the related work, while Section VII concludes the paper and discusses future work.

II. RULES FOR DECLARATIVE PROCESSES

A. Background

Governatori *et al.* [11][12] proposed an agent-oriented rule language for the declarative specifications of norm and goal compliant business processes. The main idea is that the set of rules can be partitioned into three subsets: a set of rules corresponding to the norms governing a process, a set of rules encoding the objectives/goals of an organisation to fulfil in their processes and, finally, a set of rules describing the “capabilities” of an organisation. The intuition behind the *capability rules* being that they model the set of activities/tasks an organisation is able to carry out, the preconditions required for each task, the effects of such tasks, and the relationships among tasks. The language upon which the rules are defined consists of a set of two types of literals: *condition literals* and *task literals*. The condition literals encode the preconditions and effects of tasks or, in general, state variables for a process, while each task literal corresponds to a task that could occur in a process. The three sets of rules have the form of *If...then* statements that can contain expressions like: *if c_1, t_1 then OBL t_2* meaning that there is a norm that specifies that the execution of task t_1 in presence of condition c_1 makes it obligatory to execute task t_2 ; and *if OBL c_i then t_j* whose reading is: task t_j requires task t_i to be executed when c is obligatory.

The logic proposed by [11][12] was devised to determine if, given a set of facts, it was possible to execute a process that achieve the objective of an organisation and at the same time complying with the norms governing the process. The underlying idea is that, given a set of facts, a derivation can be generated where all applicable rules fire and their conclusions have been added to the derivation. This derivation contains all tasks that are executable given the set of facts, which can be considered as the input for a process case (and instance of a business process). In addition, the derivation contains the literals corresponding to the conditions to trigger the execution of tasks or for activating obligations, the effects of the tasks, the obligations in force, and the expected goals. Note that while obligations and goals determine whether an execution of the process is compliant and meet the organisation objectives (thus influencing the activities or tasks included in the process), they are neither actions nor tasks. Therefore, rules for goals and norms do not directly contribute to the structure of the process: they only provide constraints on the tasks that can appear in a derivation. Goals and obligations can thus be considered as special kinds of conditions. Accordingly, if one ignores obligations, goals and the condition literals from a derivation, then a derivation is a sequence of tasks satisfying the constraints defined by the rules. This means that it is equivalent to a plan as defined in classical planning [13]. For these reasons, in this paper, we concentrate only on the

capability rules. Note, that, while the approach was originally intended for the logic proposed in [11][12], the method we are going to present in this paper can be used with any rule language satisfying derivation conditions described in the remainder of this section.

B. Technical definitions

Formally a (capability) rule is an expression $r: l_1, \dots, l_n \Rightarrow l_{n+1}$, where r is a label that uniquely identifies the rule, and each literal l_i is drawn from the set of literals $\text{Lit} = \text{Prop} \cup \{\neg p \mid p \in \text{Prop}\}$; Prop is a set of propositional atoms representing conditions c_i and tasks t_j . This form has the clear advantage that it immediately relates preconditions to the corresponding effect of performing the particular action. More specifically, we can identify the following three patterns. (1) $t \Rightarrow c$, where we can look at c as an effect of performing task t ; thus the effect represented by c holds after the execution of task t . (2) The pattern $c_1, \dots, c_n \Rightarrow t$ indicates that $c_1, \dots, c_n$ are preconditions for tasks t , and task t will be executed after the preconditions hold. (3) $t_1, \dots, t_n \Rightarrow t$ specifies that the combination of tasks $t_1, \dots, t_n$ triggers task t , and that task t appears in the process, if $t_1, \dots, t_n$ appear in the process (before t). In other words, this pattern describes relationships and dependencies among tasks in a process. In the rule given above, the meaning is that execution of tasks $t_1, \dots, t_n$ is required to trigger the execution of task t . It is possible to have negations of task literals, e.g. $\neg t$, meaning t does not appear in the process. Accordingly, a rule $t_1, c \Rightarrow \neg t_2$ means if task t_1 is executed, and c holds, then task t_2 does not appear in the process; similarly, $\neg t_1 \Rightarrow t_2$ means that if task t_1 does not appear in the process, then task t_2 is executed.

The rules are then used to form (logical) derivations, where a *derivation* D , given a set of facts F represented as literals, is a sequence of literals $D(1), \dots, D(n)$, such that if $D(m+1) = l$ then either $l \in F$ or there is a rule $r: l_1, \dots, l_k \Rightarrow l$ such that for all $i \leq k$, $l_i \in D[1..m]$, where $D[1..m]$ is the initial sequence of length m of D .

The rules presented above can be linked to the sequential, parallel, and alternative patterns typical of business process modelling techniques to those that can be found in a logical derivation. Indeed, assume tasks A and B concur to obtain the resources needed for task C to start its execution. This means that C may bring about its effects only when both A and B have finished, and that A and B have no precedence order with respect to one another, that is they can be executed in parallel. From a logical perspective, all this information can simply be represented by a rule whose premises are literals A and B and which conclusion is C , i.e., something of the form ‘ $A, B \Rightarrow C$ ’. As such, a derivation can encode a possible order in which the tasks are executed to achieve a particular business goals according to the constraints specified by the rules themselves.

Note that, while the set of tasks triggered by a case (set of facts) is unique, multiple derivations are possible. For example, given the rule $t_1, t_2 \Rightarrow t_3$, the order in which t_1 and t_2 are executed does not matter. Accordingly, both t_1, t_2, t_3 and t_2, t_1, t_3 are valid derivations (and, consequently, plans conforming to the specification given by the rule). This means that, given a case, we can generate a set of plans corresponding to it, which can be understood as alternative ways in which the process

can be executed. Using the idea that a business process can be understood by a set of traces (where a trace is a sequence of tasks), we can establish a connection between a set of plans and a business process, where a process provides a concise (formal and graphical) representation of a set of plans, which are obtained from a single case and are combined using constructions modelling *and*-splits and *and*-joins. Moreover, given a set of rules, it is possible to give as input mutually exclusive sets of facts, where, for instance, each set of facts corresponds to a common set of instances for the process. For each case, a corresponding set of plans is created, where the mutually exclusive cases are subsequently merged, adopting *xor*-splits and *xor*-joins.

Let us explain how the derivation mechanism works by considering the following example. Consider the set of rules $R = \{r_0 : A \Rightarrow C, r_1 : B \Rightarrow D, r_2 : C, D \Rightarrow E\}$, and the facts (or case) A, B . It is easy to see that all the rules are applicable and actually fire producing the following six derivations:

(1) A, B, C, D, E	(4) B, A, C, D, E
(2) A, B, D, C, E	(5) B, A, D, C, E
(3) A, C, B, D, E	(6) B, D, A, C, E

Some considerations: A and B have no precedence order between each other. The same happens for the tuples: (C, D) , (A, D) , and (B, C) . However, A always precedes C , B always precedes D , and so do C and D for E . It is straightforward to see that such derivations can be visualised graphically in the Petri net below:

It is easy to see that for larger processes the amount of derivations is combinatorial and, therefore, intractable. In this paper, we present a method that directly uses the rules to derive the process, omitting the need for generating all derivations.

III. PETRI NETS

Business processes can be modelled by using many different notations. Often these notations require further formalisation before formal verification can be applied. Petri nets (PN) are a popular modelling language used to formalise business processes [14]. Petri nets are mathematical models for the description of distributed systems. A Petri net [15] is a tuple (P, T, A, λ) , where P is a set of *places* and T is a set of *transitions*, such that $P \cap T = \emptyset$. *Arcs* form directed edges between place-transition pairs, such that $A \subseteq (P \times T) \cup (T \times P)$. $\lambda : P \cup T \rightarrow \mathcal{L}$ is a labelling function. The Petri net state, often referred to as the *net marking*, $M : P \rightarrow \mathbb{N}_0$ is a function that associates a place $p \in P$ with a natural number (viz., place tokens). A *marked net* $N = (P, T, A, \lambda, M_0)$ is a Petri net (P, T, A, λ) together with an *initial marking* M_0 .

Places and transitions are referred to as *nodes*. The *preset* of a node is denoted by $\bullet y = \{x \in P \cup T \mid (x, y) \in A\}$, and the *postset* of a node is denoted by $y \bullet = \{z \in P \cup T \mid (y, z) \in A\}$. Two places p_i and p_j are equal (i.e. $p_i = p_j$) iff $\bullet p_i = \bullet p_j \wedge p_i \bullet = p_j \bullet$. Two transitions t_i and t_j are equal (i.e. $t_i = t_j$) iff $\lambda(t_i) = \lambda(t_j)$.

If $\forall p \in \bullet t : M(p) > 0$, t is said to be *enabled*. The *firing* of t , denoted by $M \xrightarrow{t} M'$, leads to a new marking M' , with

$M'(p) = M(p) - 1$ if $p \in \bullet t$, $M'(p) = M(p) + 1$ if $p \in t \bullet$, and $M'(p) = M(p)$ otherwise. The marking M_n is said to be *reachable* from M if there exists a sequence of transition firings $\sigma = t_1 t_2 \dots t_n$ such that $M \xrightarrow{t_1} M_1 \xrightarrow{t_2} \dots \xrightarrow{t_n} M_n$.

A marking M of a net is *n-bounded* if $\forall p : M(p) \leq n$. A Petri net N is *n-bounded* if all its reachable markings are *n-bounded*. A Petri net N with initial marking M_0 is *live* if it is possible to eventually fire any transition of N by progressing through some further firing sequence [16]. Five categories of liveness are defined in [16]. For the needs of this paper, we can restrict ourselves to L_1 -live: t can be fired at least once in some firing sequence following M_0 . A Petri net N with initial marking M_0 is *deadlock-free* iff $\forall M : M_0 \rightarrow \dots \rightarrow M, \exists M' : M \rightarrow M'$. That is, any reachable marking M enables some transition t , such that there exists a subsequent marking M' , where $M' \neq M$. A Petri net N with initial marking M_0 is *sound* iff it is bounded, live and deadlock-free.

A *trace* is a sequence $\lambda(t_1), \lambda(t_2), \dots$ such that $\sigma = t_1, t_2, \dots$ is a sequence of firing transitions. However, certain control-flow behaviour (like certain exclusive (*xor*) and concurrent (*and*) constructs) requires additional transitions that do not correspond to a task literal. These transitions are commonly referred to as silent or τ transitions [17]. To improve clarity and for ease of reference, we will use a label for each τ transition as well throughout the paper. As such, the set of transition labels $\mathcal{L}$ comprises both labels corresponding to task literals and labels corresponding to τ transitions. A *visible trace* is a trace where all τ transitions have been removed (maintaining the order of the transitions representing task literals). For the remainder of this work, we refer to visible traces as traces.

IV. FROM RULES TO PETRI NETS

A. Method

In order to transform a set of rules R representing the regulations governing an organisation to a Petri net N , a number of steps are required, as shown graphically in Figure 1.

Fig. 1: Method overview.

First, some preprocessing of the rules is required to take into account only those rules that define the possible executions in the process. That is, we restrict the analysis to what we called capability rules. In case the language contains rules for obligations, goals and we transform obligations and goals in capability rules into conditions. From the resulting set of (capability) rules, we remove all rules with literals that are not in the set of all conclusions, as these have no effect on the resulting process.

The resulting set of (preprocessed) rules will be used to produce the Petri net. However, only tasks that are potentially executed should be represented by a transition in N . As negative tasks represent the *absence* of a task in the execution,

these can be eliminated from the antecedents of the rules (e.g. $A, \neg B \Rightarrow C$ would be $A \Rightarrow C$). These negative tasks can be treated as additional “conditions”, which are satisfied automatically as they do not appear in the resulting net N and can, therefore, not be executed. Positive tasks that are in the set of facts represent the *initial state*. Positive tasks from the initial state that are in the antecedent of a rule can be removed as well, as they hold by default. In addition, rules containing a negative task in the consequence (e.g. $A \Rightarrow \neg B$) can be removed, as these rules result in the absence of a task, which is automatically satisfied by not including the respective transition in the resulting net N . The removal of negative tasks is shown in Algorithm 1 below.

Algorithm 1 Preprocessing rules

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1: function PREPROCESS( $R = \{r_1, \dots, r_n\}$ : a set of rules, initial state)
2:   for each  $r_i \in R$ , where  $r_i : l_1, \dots, l_n \Rightarrow l_{n+1}$  do
3:     if  $l_{n+1}$  is a negation of a task then
4:       remove  $r_i$ 
5:     else
6:       for each  $l_j \in r_i$  do
7:         if  $l_j$  is a negation of a task  $\vee$ 
8:            $l_j$  is a positive task in the initial state then
9:             delete  $l_j$  from  $\text{Antecedent}(r_i)$ 

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B. Obtaining a Petri net from rules

Once we have obtained the reduced rule set for each case cs_i , we can subsequently transform each set of reduced rules to a Petri net N_i corresponding to that case by using Algorithm 2. That is, the traces of each N_i contain exactly all possible derivations from the rules of its corresponding case cs_i .

The net N_i starts with an initial place p_s and subsequent transition t_s (Line 6–8). From there, the initial rules (i.e. rules with empty antecedent) are added (Line 9–11). For example, a rule $r_0 : \Rightarrow A$ would add an additional place and a transition A after t_s .

A rule $r_1 : A \Rightarrow B$ first adds a τ transition representing r_1 , followed by a place and a transition B after A , to realise the sequence from A to B induced by r_1 . Similarly, adding a rule $r_2 : A \Rightarrow C$ would add another outgoing place from A followed by a similar construct as created by r_1 leading to a transition C .

A rule with multiple tasks in its antecedent, e.g. $r_3 : B, C \Rightarrow D$, results in an *and-join* of B and C to D (Line 13–17).

Finally, all transitions t_i with $t_i \bullet = \emptyset$ are joined into a common end transition t_e (Line 31–32), which leads to the sink place p_e (Line 6–8).

When merged together, the resulting net can be visualised as follows:

These basic principles work well for simple scenarios that result in a structured Petri net (i.e. every split has its matching join). However, when two concurrent paths lead into the same transition without synchronisation, this may cause the transition to be executed more than once.

This can easily be illustrated by replacing r_3 in the previous example with $r_4 : B \Rightarrow D$ and $r_5 : C \Rightarrow D$. It is straightforward to see that the execution of A activates both branches, resulting in two tokens in p_x . Therefore, an additional construct is required to allow either r_4 or r_5 to produce a token for p_x and remove the remaining tokens.

To allow either r_4 or r_5 to produce a token for p_x , auxiliary transitions r'_4 and r'_5 are added in Line 23–25. A common input place $p_\oplus$ is introduced for r'_4 and r'_5 , to allow only one of them to fire (Line 19, 21 and 26). Auxiliary transitions r''_i are added to clean up remaining tokens (Line 23). The inputs of auxiliary transition r''_i are from all r'_j s except r'_i (Line 27–28). When r'_i fires, r''_i cleans up remaining tokens produced by all r'_j s. As a result, firing of r'_4 coincides with r_4 and firing of r'_5 coincides with r_5 .

Let us assume a scenario where both r_4 and r_5 have fired, resulting in a token in x_1 and a token in x_2 respectively. If r'_4 fires, it consumes tokens from $p_\oplus$ and x_1 and produces a token to p_x to allow D to fire and produce a token to p_8 and auxiliary place $p_\sim$ (highlighted in purple). The remaining token in x_2 is consumed along with the token in $p_\sim$ by r''_4 , which produces a token for $p_\blacksquare$ (highlighted in orange). Place $p_\sim$ is the common place for auxiliary transitions r''_i allowing only one of them to execute (Line 29–30). Place p_8 is then connected to any subsequent rules having D in their antecedent (e.g. $r_6 : D \Rightarrow E$), whereas $p_\blacksquare$ is connected to the final transition to ensure that all remaining tokens are consumed (Line 33–34).

Fig. 2: Petri net showing the merge of two branches without synchronisation.

This construct ensures that, while having two rules producing D , only one execution of D is possible, without having

any orphaned tokens after finishing execution of the process. The entire net can be visualised as shown in Figure 2.

Algorithm 2 Converting rules to a Petri Net

$|r_i|^l$ = number of literals in the body of rule r_i
 $x \mapsto y$: arc from x to y
 $x \mapsto y$: $x \mapsto p$ and $p \mapsto y$, p is a place, x and y are transitions

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1: function CONVERTToPN( $R_C = \{r_1, \dots, r_n\}$ : a set of rules from a single case, initial state)
2:   PREPROCESS( $R_C$ , initial state) ▷ Algorithm 1
3:   Petri Net  $N$ : initially empty
4:   for each  $l_i$  in the body or head of each rule do
5:     add transition  $t_{l_i}$  to  $N$ 
6:   add transitions  $t_{-s}$  and  $t_{-e}$  to  $N$ 
7:   add places  $p_{-s}$  and  $p_{-e}$  to  $N$ 
8:   ( $p_{-s} \mapsto t_{-s}$ ) and ( $t_{-e} \mapsto p_{-e}$ )
9:   for each  $r_i \in R_C$  with  $r_i : l_1, \dots, l_n \Rightarrow l_{n+1}$  do
10:    if  $|r_i|^l = 0$  then
11:      ( $t_{-s} \mapsto t_{l_{n+1}}$ )
12:    else
13:      add transition  $t_{r_i}$ 
14:      for each  $l_j \in \{l_1, \dots, l_n\}$  do
15:         $t_{l_j} \mapsto t_{r_i}$ 
16:    if |rules with head  $l_{n+1}$ | = 1 then
17:       $t_{r_i} \mapsto t_{l_{n+1}}$ 
18:    else if |rules with head  $l_{n+1}$ | > 1,  $R'_C = \{r_1, \dots, r_m\}$  then
19:      add places  $p_{\oplus}$ ,  $p_x$ ,  $p_{\sim}$  and  $p_{\blacksquare}$  to  $N$  ▷  $p_{\blacksquare}$  is end point
20:       $p_x \mapsto t_{l_{n+1}} \mapsto p_{\sim}$ 
21:       $t_{-s} \mapsto p_{\oplus}$ 
22:      for each rule  $r_i \in R'_C$  do
23:        add transitions  $t_{r_i}$ ,  $t_{r'_i}$  and  $t_{r''_i}$  to  $N$ 
24:        add place  $x_i$  to  $N$ 
25:         $t_{r_i} \mapsto x_i \mapsto t_{r'_i} \mapsto p_x$ 
26:         $p_{\oplus} \mapsto t_{r'_i}$ 
27:      for each  $r_j \in R'_C, j \neq i$  do
28:         $x_j \mapsto t_{r''_i}$ 
29:      for each rule  $r_i \in R'_C$  do
30:         $p_{\sim} \mapsto t_{r''_i} \mapsto p_{\blacksquare}$ 
31:    for each transition  $t_i$  in  $N$ :  $t_i \bullet = \emptyset$  do
32:       $t_i \mapsto t_{-e}$ 
33:    for each place  $p_{\blacksquare}$  in  $N$  for end points do
34:       $p_{\blacksquare} \mapsto t_{-e}$ 
35:    return  $N$ 

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C. Merging cases

The algorithm described in the previous subsection is used for transforming individual cases into Petri nets. The final step is to merge the Petri nets of these individual cases into a combined Petri net that represents the entire process without duplicating transitions, which is achieved through Algorithm 3.

Let us illustrate the merging with a small example, where we have two different cases: *Case* $cs_1 = \{r_1 : \Rightarrow A, r_2 : A \Rightarrow C, r_3 : C \Rightarrow D\}$ and *Case* $cs_2 = \{r_4 : \Rightarrow B, r_5 : B \Rightarrow C, r_6 : C \Rightarrow E\}$. The Petri nets N_1 and N_2 , representing cs_1 and cs_2 respectively, are shown graphically in Figure 3. Note that the two cases differ on the first and final transition to be executed, but contain the mutual transition C in between.

Fig. 3: Petri nets N_1 and N_2 for cases cs_1 and cs_2 .

Furthermore, A co-occurs with D and B co-occurs with E , but A and E or B and D cannot occur in the same trace. As such, p_s and t_s are common, where the outgoing place (labeled xor_1) of t_s marks the split to either A (case cs_1) or B (case

cs_2). Next, A leads to C via r_2 , whereas B leads to C via r_5 , having xor_2 as xor -join place.

Similarly, from C there exist two rules (r_3 and r_6), leading to D and E respectively. Together with the common *end* transition t_e and place p_e , the resulting Petri net would look as follows:

Fig. 4: Merged Petri net N_M without preserving dependencies.

However, we want to obtain a condition-free representation of subsequent exclusive branches that maintain their dependencies without duplicating transitions. Although transition C should not be duplicated in the merged Petri net, dependencies between A and D should remain intact (similar for B and E). In the example shown in Figure 4, A could be followed by E , which is incorrect as both correspond to different cases.¹

When the presets and/or postsets of common transitions (i.e. t_s , C and t_e) are different, additional τ transitions are required to enforce these case-specific dependencies. For instance, transition t_s has different postsets for cs_1 and cs_2 (Line 28–34). As such, we add C_1^s and C_2^s after xor_1 (Line 31–32). C_1^s will be connected to A and C_2^s will be connected to B (Line 33–34). The common subset of postsets will repeat in the resulting Petri net (Line 24–26). Subsequently, we can add two τ transitions representing cs_1 and cs_2 (Line 2). These two transitions are mutually exclusive (as shown below in Figure 5), such that the chosen path after xor_1 can be determined by the case currently being executed. That is, C_1^s may only fire with a token in xor_1 and after cs_1 has fired. This way, the execution of A is specific to case cs_1 , whereas the execution of B is specific to case cs_2 .

Fig. 5: Enforcing cases in the merged Petri net (splits).

In case of common transitions with different presets, a similar procedure is required to ensure that all incoming conditions are satisfied (Line 8–19). For example, we may have a similar rule in two cases, where in one case a certain

¹Alternatively, a common prefix can be computed, from where every subsequent difference between the input Petri nets is resolved by a different branch. The resulting merged Petri net will effectively be a process tree, where the ends of all branches are joined together by a common end transition. This leads to an abundance of traces (i.e. the amount of traces equals the amount of cases), which would i) reduce understandability and maintainability and ii) increases the risk of inconsistencies when a transition needs to be changed (as a change of that transition is required for every occurrence of that transition in the process tree, which is potentially as much as the amount of cases).

input condition is true at the initial state and, in the other case, that condition may be derived by another rule. Consequently, the inputs of that common transition may differ, requiring a synchronisation step to only proceed when all incoming transitions have fired. As such, we add a τ transition C_i^j for each incoming path.

Fig. 6: Enforcing cases in the merged Petri net (joins).

The full resulting Petri net N_M is represented in Figure 7. Note that the purpose of the C^s transitions here is immediately clear, as the co-occurrence of A and D for case cs_1 (and B and E for case cs_2) is warranted. In Figure 7, the execution of the transitions in case cs_1 is highlighted in purple, while the auxiliary routing to preserve dependencies is highlighted in orange.

Fig. 7: Merged Petri net N_M from N_1 and N_2 .

The examples shown so far comprise different cases with distinctive paths that merge somewhere in the process. However, a situation where a transition occurring in one case is *skipped* in another case requires additional measures, as there is no immediate transition following the *xor* (as it is skipped). As such, certain C^s transitions may end up without outgoing arcs. This problem can be illustrated with two cases: $Case\ cs_1 = \{r_1 : \Rightarrow A, r_2 : A \Rightarrow B, r_3 : B \Rightarrow D\}$ and $Case\ cs_2 = \{r_4 : \Rightarrow A, r_5 : A \Rightarrow B, r_6 : A \Rightarrow C, r_7 : B \Rightarrow D, r_8 : B \Rightarrow E\}$, resulting in traces $\langle A, B, D \rangle$ and $\langle A, B, C, D, E \rangle$ or $\langle A, C, B, D, E \rangle$ respectively, as shown graphically in Figure 8. A merge of both Petri nets would need to comprise an optional concurrent execution of C with B and an optional concurrent execution of E with D , as C and E only occur in cs_2 and not in cs_1 .

Fig. 8: Example resulting in a skipped transition (C and E).

Using the basic principles outlined above, however, the corresponding Petri net N_M would be generated as shown in Figure 9. Note that there are three transitions (indicated with a light gray background with dashed borders) that are not fully connected: C_2^s and C_4^s have an empty postset, whereas C_2^j has

Algorithm 3 Merging Petri Nets

```

Proc: boolean flag for each arc and transition, initially false
1: function MERGEPNs( $N_1, N_2, \dots, N_n$ )
2:   add place  $x$ , transitions  $cs_1, cs_2, \dots, cs_n$  to  $N_M$ 
3:    $t_{-s} \mapsto x, x \mapsto cs_1, x \mapsto cs_2, \dots, x \mapsto cs_n$ 
4:   for  $i = 1$  to  $n$  do
5:     for each  $t_j \in N_i$ , Proc[ $t_j$ ] = false do
6:       Proc[ $t_j$ ] = true
7:        $T_c = \{t_k \mid t_k \in N_{k'}, k' = i, \dots, n, t_k = t_j\}$   $\triangleright$  common transitions
8:        $D_c = \{p \mid \forall t_k \in T_c, p \in \bullet t_k\}$   $\triangleright$  common inputs
9:       for all  $p \in D_c$  do
10:        add place node  $p$  to  $N_M$ 
11:         $p \mapsto t_j$ , Proc[ $p \mapsto t_j$ ] = true
12:       for all  $t_k \in T_c$  do
13:         $D_k = \bullet t_k - D_c$   $\triangleright$  uncommon inputs
14:        if place node  $xor \notin \bullet t_k$  then
15:          add place node  $xor$  to  $N_M$ ,  $xor \mapsto t_k$ 
16:        if  $t'_k = C^j(p \in D_k) \notin N_M$  then
17:          add  $t'_k$  to  $N_M$ ,  $t'_k \mapsto xor$ 
18:        for each  $p \in D_k$  do
19:          add place  $p$  to  $N_M$ , add  $p \mapsto t'_k$ , Proc[ $p \mapsto t_k$ ] = true
20:        if  $D_k = \emptyset$  then
21:          if place node  $p_{Cj0} \notin N_M$  then
22:            add place node  $p_{Cj0}$  to  $N_M$ ,  $p_{Cj0} \mapsto t'_k$ 
23:             $cs_x \mapsto p_{Cj0}$   $\triangleright$   $cs_x$  is the corresponding case transition
24:         $D_c = \{p \mid \forall t_k \in T_c, p \in t_k \bullet\}$   $\triangleright$  common outputs
25:        for all  $p \in D_c$  do
26:          add place node  $p$  to  $N_M$ ,  $t_j \mapsto p$ , Proc[ $t_j \mapsto p$ ] = true
27:        for all  $t_k \in T_c$  do
28:           $D_k = t_k \bullet - D_c$   $\triangleright$  uncommon outputs
29:          if place node  $xor \notin t_k \bullet$  then
30:            add place node  $xor$  to  $N_M$ ,  $t_k \mapsto xor$ 
31:          if  $t'_k = C^s(p \in D_k) \notin N_M$  then
32:            add  $t'_k$  to  $N_M$ ,  $xor \mapsto t'_k$ 
33:          for each  $p \in D_k$  do
34:            add place  $p$  to  $N_M$ , add  $t'_k \mapsto p$ , Proc[ $t_k \mapsto p$ ] = true
35:          if  $D_k = \emptyset$  then
36:            if place node  $p_{\star C^s0} \notin N_M$  then
37:              add place node  $p_{\star C^s0}$  to  $N_M$ 
38:               $t'_k \mapsto p_{\star C^s0}$ 
39:            if transition  $t_{CX} \notin N_M$  then
40:              add transition  $t_{CX}$  to  $N_M$ 
41:               $p_{\star C^s0} \mapsto t_{CX}$ 
42:          if place node  $p_{CS(p \in D_k)} \notin N_M$  then
43:            add place node  $p_{CS(p \in D_k)}$  to  $N_M$ ,  $p_{CS(p \in D_k)} \mapsto t'_k$ 
44:             $cs_x \mapsto p_{CS(p \in D_k)}$   $\triangleright$   $cs_x$  is the corresponding case transition
45:        add place  $p_\ominus$  to  $N_N$ 
46:         $p_\ominus \mapsto t_e$ 
47:        for all cases  $x$  do
48:          if  $t_{CX} \in N_M$  then
49:             $t_{CX} \mapsto p_\ominus$ 
50:          else
51:             $cs_x \mapsto p_\ominus$ 
52:        for each place  $p \in N_i$  do  $\triangleright$  add remaining places and arcs
53:        if  $p \notin N_M$  then
54:          add place  $p$  to  $N_M$ 
55:        for each  $t \in \bullet p$  do
56:          if Proc[ $t \mapsto p$ ] = false then
57:             $t \mapsto p$ , Proc[ $t \mapsto p$ ] = true
58:        for each  $t \in p \bullet$  do
59:          if Proc[ $p \mapsto t$ ] = false then
60:             $p \mapsto t$ , Proc[ $p \mapsto t$ ] = true
61:  return  $N_M$ 

```

an empty preset. This is a result of the skipped transitions C and E that are concurrent with some other transitions that are common between both cases.

This is resolved by adding a set of $p_{\star C^s}$ places to the postset of the affected C^s transitions (Line 35–38), which are synchronised by transition t_{CX} into place $p_\ominus$ (Line 39–

Fig. 9: Missing connections with skipped transitions.

51). Additionally, a place p_{C^j} is added to the preset of the unconnected C^j transitions (Line 20–23). The required additions are highlighted in purple in Figure 10.

Fig. 10: Additional connections to support skipped transitions.

Finally, after all Petri nets N_i are merged, we optimise the merged Petri net N_M by removing all redundant τ transitions using the reduction techniques described in [16], which preserve the properties of liveness and boundedness. That is, if (N_M, M_0) is live and 1-bounded, then the reduced net (N'_M, M'_0) is live and 1-bounded as well [16]. As the reduction only removes τ transitions, N'_M is behaviourally equivalent to N_M . The resulting optimised merged Petri net of N_M in Figure 7 is depicted graphically in Figure 11.

Fig. 11: Optimised Petri net N_M from Fig. 7.

D. Correctness properties

The Petri net N obtained from Alg. 2 represents exactly the behaviour as specified by a case-specific rule set R_C . That is, there exist no traces in N that are not specified by R_C , and there is no behaviour allowed by the rules in R_C that is not allowed by N . This property can be summarised as follows:

Lemma 1: Let R_C be a rule set, and N be the Petri net obtained by applying Alg. 2 to R_C . Then, there is a derivation $L_1, \dots, L_n$ in R_C iff there exists a sequence of

markings $m_0, m_1, \dots, m_{n'}$ in N obtained through firing transitions $t_{L_1}, \dots, t_{L_n}$, where m_i is reachable from m_{i-1} for $i \in \{1, \dots, n'\}$.

Proof. See Appendix A. $\square$

The merged Petri net merges the behaviour of the input nets, such that it allows only those traces that are allowed by one of its input nets. As such, it constitutes a correct merge of the behaviour of the input nets, which can be formulated formally as follows:

Lemma 2: Let N_{IN} be the set of input Petri nets to Alg. 3 (as obtained from Alg. 2), and N_M be the merged Petri net obtained by applying Alg. 3 to N_{IN} . Then, there is a trace $t_{L_1}, \dots, t_{L_n}$ in N_M iff there exists a trace $t_{L_1}, \dots, t_{L_n}$ in some $N_i \in N_{IN}$.

Proof. See Appendix A. $\square$

In addition to the behavioural properties, the generated Petri nets satisfy a number of important structural properties, irrespective of the process to be supported. First of all, each place in the merged Petri net N_M contains at most 1 token for every possible reachable marking. This property can be formulated as follows:

Lemma 3: Let R be a rule set, and N_M be the Petri net obtained by applying Alg. 1–3 to R . The merged Petri net N_M is 1-bounded.

Proof. See Appendix A. $\square$

Finally, for every reachable marking (that is at any state) there is an enabled transition that can fire. As such, the merged Petri net N_M is constructed in such a way that there exist no deadlocks and any possible execution can finish, irrespective of the path chosen:

Lemma 4: Let R be a rule set, and N_M be the Petri net obtained by applying Alg. 1–3 to R . The merged Petri net N_M is L_1 -live and deadlock-free.

Proof. See Appendix A. $\square$

As such, Alg. 1–3 produce a correct and sound Petri net N_M from rule set R . That is, the relation between the behaviour allowed by N_M and the behaviour allowed by the rules in R is bijective and N_M is bounded, live and deadlock-free.

Theorem 1: Let R be a rule set and N_M be the Petri net obtained by applying Alg. 1–3 to R . N_M is a correct and sound representation of the rules in R .

Proof. Follows directly from Lemma 1–4. $\square$

E. Computation Complexity

To show that the method presented above does not have any tractability issues, we provide a formal discussion of the computational complexity of each proposed algorithm.

Given a finite, modal defeasible theory D , the size Σ of D is the number of occurrences of literals plus the number of the rules in D . For each literal, we implement a hash table with pointers to rules where the literal occurs in; thus, retrieving the set of rules containing a given literal requires constant time.

Lemma 5: Algorithm 1 terminates and its computational complexity is $O(\Sigma^2)$.

Proof. Σ is finite thus the cycles at Line 2 and 6 always end. The main and nested cycles examine at most Σ rules / literals, respectively, while removal (Line 4) and deletion (Line 9) require constant time. $\square$

Lemma 6: Algorithm 2 terminates and its computational complexity is $O(\Sigma^2)$.

Proof. Termination: same as before. Complexity: the main cycle (Line 9–30) is iterated at most Σ times, and so are the nested ones. The cycle at Line 22–26 produces 8 new symbols for each rule in $R' \in O(\Sigma)$, and thus $8 \cdot \Sigma \in O(\Sigma)$. As such, the complexity of the outer cycle is $O(\Sigma^2)$. All assignment operations and controls can be performed in constant time. $\square$

Theorem 2: Algorithm 3 is in $O(\Sigma^4)$. The overall complexity of Algorithm 2 and Algorithm 3 is $O(\Sigma^2) + O(\Sigma^4) = O(\Sigma^4)$.

Proof. Termination: trivial. There are no **repeat-until/while** cycles. Each **for** cycle iterates at most Σ times, with Σ finite. This also proves the complexity. (Note that structures such as the one at Line 7 of Algorithm 3 can be computed in linear time given the implementation proposed.) $\square$

V. EVALUATION

We implemented our approach in a tool called Rules2PetriNet² to assess the performance of the approach as well as the quality of the generated Petri nets. The tool takes as input a set of cases, each containing a set of rules. Its output is a merged Petri net N_M that represents the behaviour as specified by all rules in the input cases.

Using this tool, we conducted a two-pronged (quantitative and qualitative) evaluation. First, we executed a quantitative evaluation assessing the performance and correctness of our approach compared to a state-of-the-art technique, using a large set of synthetic models. Second we performed a qualitative evaluation of our method on a real-life case containing the rules of a customer support process, to show the feasibility of our approach in a real-life setting.

All performance tests were executed on a machine with a quad core Intel® Core™ i5-5300U 2.3GHz with 8GB memory, running Linux 14.04. To eliminate load times, we executed each test five times and recorded average time of three executions, removing the fastest and the slowest execution.

A. Quantitative evaluation with synthetic models

First, we created a set of Petri nets, each representing an input scenario from which we generated all cases and their corresponding rules. As such, we obtain rules for these different scenarios. This allowed us to have a structured set of input scenarios with an increasing complexity. The 21 synthetic Petri nets comprise increasing branch counts and branch lengths for both *and*-splits and *xor*-splits. Furthermore, the input models contain multiple successive *and*-constructs or *xor*-constructs, to maximise the amount of cases and, therefore, potential computational complexity.

We assessed the performance of our tool by generating a merged Petri net for each input scenario. Subsequently, we assessed the quality of the generated Petri nets in terms of the amount of nodes and compared the resulting Petri net with the input Petri net to check behavioural equivalence.

To the best of our knowledge, there exist no tools that are able to transform regulations into Petri nets while preserving case-specific dependencies among subsequent exclusive branches. However, an alternative approach could be

to generate all derivations from the rules and generate the Petri nets from the derivations. The *Inductive Miner (IM)* plugin [18] in the ProM 6.5.1 environment [19] is able to obtain a Petri net from an *event log*, which contains a set of traces (or derivations) that each consist of the sequence of events produced by a case [20]. As the approach of [18] requires an event log as input (instead of a set of rules), we generated an event log for each input net using the approach presented in [21], to simulate a scenario where we would have a set of derivations from rules.

Subsequently, we generated a Petri net from the event log (using IM) and assessed the performance in comparison with our approach. The evaluation approach for each scenario is shown graphically in Figure 12.

Fig. 12: Evaluation method for each scenario.

Table I provides an overview of the details of each synthetic Petri net. The two rightmost columns show the performance of our approach (i.e. Alg.1–3 and the optimisation) and ProM IM respectively.

For our approach, *xor*-branches naturally result in higher complexity, because they constitute different cases and are, therefore, represented by different case-specific Petri nets that need to be merged. Although concurrency combinatorially increases the amount of derivations, they have very little effect on the execution times for our approach, as they represent a single case and Petri nets are designed to model concurrent systems (hence interleaving is not an issue). For ProM IM, however, the amount of distinct traces (i.e. derivations) determines the execution time.

For each scenario, our approach outperforms ProM IM, with the exception of scenario 21 (42.85s vs 39.68s). Contrary to our approach, however, ProM IM provides a generalised Petri net that does not take into account dependencies among consecutive *xor*-constructs. As such, the resulting Petri net from ProM IM allows behaviour that is not allowed according to the rules and can, as a result, *not be guaranteed to be compliant with the rules*. In our approach, on the other hand, these dependencies are taken into account by default and can be removed automatically (and, therefore, generalised) when required. Consequently, for very large models the performance of our approach is comparable with ProM IM, as we are specifying consecutive branching constraints in the model as well, which naturally results in a larger number of transitions in case of our approach. Therefore, a direct comparison of the amount of nodes in both models is pointless.

There were three scenarios where ProM IM could not finish within 90s due to the large number of derivations. The large number of derivations in case of complex concurrent models (scenario 1 to 4) shows the infeasibility of generating all derivations prior to constructing the required model.

²<https://bitbucket.csiro.au/projects/BPLI/repos/rules2petrinet/>

TABLE I: Details of synthetic scenarios.

Id	and-constr.	and-br	xor-constr.	xor-br	Act.	Rules	Cases	Trans.	Places	Arcs	Derivations	Our approach (s)	ProM IM (s)
1	7	4	0	0	148	148	1	150	172	342	$8.21 \cdot 10^8$	0.01	Timed out
2	1	16	0	0	322	322	1	324	340	678	$6.88 \cdot 10^{369}$	0.02	Timed out
3	16	2	5	4	164	176	16	182	189	433	24,192	0.03	62.40
4	32	2	9	$8 \times 4 + 1 \times 8$	328	352	32	362	373	861	48,384	0.09	Timed out
5	0	0	1	8	162	169	8	164	159	337	8	0.02	0.50
6	0	0	1	16	322	337	16	324	311	665	16	0.05	0.57
7	0	0	4	2	42	49	16	72	65	207	16	0.01	0.66
8	0	0	5	2	52	61	32	102	81	351	32	0.02	0.78
9	0	0	6	2	62	73	64	148	97	639	64	0.04	0.97
10	0	0	7	2	72	85	128	226	113	1,247	128	0.08	1.14
11	0	0	8	2	82	97	256	368	129	2,559	256	0.19	1.44
12	0	0	9	2	92	109	512	638	145	5,407	512	0.46	2.39
13	0	0	10	2	102	121	1,024	1,164	161	11,583	1,024	1.11	3.70
14	0	0	11	2	112	133	2,048	2,202	177	24,927	2,048	2.72	7.60
15	0	0	12	2	122	145	4,096	4,264	193	53,631	4,096	6.78	17.77
16	0	0	13	2	132	157	8,192	8,374	209	115,103	8,192	18.64	40.91
17	0	0	14	2	142	169	16,384	16,580	225	246,207	16,384	73.29	79.50
18	0	0	4	4	82	121	256	388	147	1,597	256	0.12	1.03
19	0	0	5	4	102	153	1,024	1,192	187	6,549	1,024	0.69	2.31
20	0	0	6	4	122	185	4,096	4,300	227	29,165	4,096	4.10	8.97
21	0	0	7	4	142	217	16,384	16,624	267	131,653	16,384	42.85	39.68

However, the most complicated scenario (17) with 14 consecutive *xor*-splits with 2 branches (16,384 cases in total and 246,207 arcs) still took only 73.3 seconds to complete, while scenario 21 with 7 consecutive *xor*-splits with 4 branches (16,384 cases in total and 131,653 arcs) took 42.9 seconds. The performance of both scenarios is still acceptable, particularly considering that model generation is a design-time exercise. For less complicated scenarios (and more realistic ones), the execution time is measured in milliseconds. This implies that the method could even be applied in a runtime setting, where on-the-fly reconfiguration is required.

Finally, the resulting Petri nets were compared to the original input Petri nets. The traces of each pair of nets (i.e. input vs. resulting) were compared. Each of the scenarios showed a one-on-one match, which implies that all of the produced nets were behaviourally equivalent to their respective input nets.

B. Qualitative evaluation with a real-life process

For the qualitative evaluation, we used a real-life process concerning a customer support process in a telecom company in Australia, as presented in [22]. The process contains 24 visible activities, 8 τ transitions and a start and end transition. The normative model is shown graphically in Figure 13.

Again, we derived the cases and corresponding rules from this process, that will be used as input for our tool. This resulted in 92 different cases having on average 19 rules, with the smallest case containing 4 rules and the largest case containing 24 rules. In total, the process contains 33 distinct rules among all cases. Subsequently, the 92 different case-specific Petri nets were obtained using Alg. 2. By using Alg. 3, these individual Petri nets were merged to obtain Petri net N_M .

The resulting net N_M contains 24 transitions representing task literals, 92 case-specific transitions and 18 additional silent transitions (C^s and C^j). As the successive *xor*-splits in the customer support process are independent in this process (there are no case-specific dependencies among *xor*-splits), the case-specific transitions can be removed along with the additional C^s and C^j transitions for which the preset and postset equals 1. This results in a total of 42 remaining transitions. An overview of the results is shown in Table II, where the details of the full model (i.e. including cases) are

Fig. 13: Customer support process [22].

shown between parentheses. The optimised resulting net is sound and behaviourally equivalent to the initial input process, as verified using the approach explained in Section V-A.

TABLE II: Performance of the customer support process.

Act.	Rules	Cases	Trans.	Places	Arcs	Our approach	ProM
24	33	92	42(134)	37(63)	92(934)	0.080s	1.090s

Consequently, for more a structurally complicated real-life process, the algorithms presented in this paper have shown to be very well able to generate a correct and sound process from a set of rules and is performing well (80ms) and considerably faster than ProM IM (1.090s).

VI. RELATED WORK

The approach we presented departs from the standard BDI architecture and agent programming languages implementing it (e.g., 3APL-2APL [23][24], JASON [25]), and extensions with norms in several respects (e.g., BOID by [26], while we refer the reader to [27] for an overview). While in these

approaches the agent has to select (partially) predefined plans from a plan library, we propose on-the-fly business process generation (corresponding to a set of plans) to meet the objectives without violating the norms it is subject to.

In [28], Alechina *et al.* present a BDI-based agent programming language based on 2APL for *norm-aware agents*; a norm-aware agent can deliberate on its goals, norms, and sanctions before deciding which plan to select and execute. Our agents are *de facto* norm-aware. A major issue of this work is that if a goal triggers two (or more) sanctions, each of which is lower in rank than the achievement of that goal, the agent will try to achieve that particular goal even if the sum of the two sanctions is higher in rank.

Automated planning is a technique to organise actions with the aim of achieving some pre-specified goal starting from the current state of the system [13]. Each action features a set of preconditions that have to be satisfied prior to its execution and a set of effects, which specify the state change resulting from its execution. There are frameworks to generate plans (e.g., KPG [29] and Golog [30]), but these are typically based on classical AI planning and do not consider norms. In addition, many automated planning approaches in the business process domain focus on runtime adaptation of pre-specified processes, concerning runtime repair instead of design time process generation based on rules (see e.g. [5][6]). Automated planning techniques require a goal to be specified along with an initial state. However, in case of multiple initial states, multiple possible plans have to be generated (i.e. all possible traces for each existing input case), that need to be merged in order to represent a full business process [7]. As such, providing a business process model that supports *all* possible traces as specified by the rules remains a challenge.

DECLARE provides an approach for declarative specifications of business processes by means of constraints [31] and graphical representations are available to visualise the constraints and the activities in the model [32]. However, the graphical representation shows the exact constraints and does not provide an actual process model imposed by the rules (i.e. a model showing the possible traces). In [8], an approach is presented to convert a DECLARE model to a behaviourally equivalent Petri net. However, this approach leads to multiple transitions representing the same activity and, therefore, a highly complicated model. Our approach, on the other hand, provides a model without additional duplication of transitions. In [33], an extension is developed on top of BPMN, BPMN-D, which supports declarative process modelling. It allows to transform DECLARE models into readable BPMN-D models. This approach, however, does not support concurrency. Additionally, DECLARE is based on Linear Temporal Logic (LTL), which is not able to represent certain complex norms [34] and can, as such, not be used for this purpose.

Sardina *et al.* [35] provide an account of goals in the view of declarative aspects by integrating BDI failure mechanisms with Hierarchical Task Network (HTN) planning techniques. HTN planning is notoriously undecidable even if no variables are allowed, or PSPACE-hard if restrictions are given either on non-primitive tasks or on the ordering of tasks. The main feature of their CAN^ℳ is its detailed operational semantics where, if a plan fails, alternative plans for achieving the goal are tried. Compared to theirs, the approach presented in this

paper has the advantage that we generate all possible plans compactly at design time.

Finally, in [36][37], a declarative approach is used to create compliant business processes on the fly using BPMN notation. Although their approach has been proven sound, a formal proof of completeness is lacking. Moreover, our choice of using Petri nets instead of BPMN notation facilitates us to prove the formal verification of soundness and completeness.

VII. CONCLUSION

Enterprises spend a significant effort to maintain their business processes to be efficient and compliant with the regulations affecting their operational environment. Declarative formalisms are able to describe both the means an organisation uses to reach its business objectives and the regulations impacting the organisational environment. Although extremely important, it is difficult to adhere to such rules, as the amount of rules is usually very large. In addition, understandability of such large sets of rules is generally very low.

This paper addresses these problems by presenting a new method to automatically and efficiently transform a set of business rules and regulations to a behaviourally equivalent corresponding Petri net. As such, this paper presents advanced formal, complete, automated algorithms to build a compliant by design business process starting from such declarative specifications. The approach presented in this paper is particularly useful for organisations as it ensures that their business processes are compliant by design and resilient to changes in regulations or corporate policies. More specifically, rule changes can be automatically adopted in the design of the affected business processes and their effect on the actual process flow can be immediately visualised.

We implemented the approach in a tool and subsequently evaluated the approach on a set of synthetic scenarios and a real-life process, to assess the performance and the correctness of the resulting Petri net. The results show that the algorithms perform very well, even for highly complex scenarios with 16,384 cases. Furthermore, in almost all scenarios, our approach significantly outperforms a state-of-the-art technique for process synthesis from event logs.

The resulting Petri nets provide an exact representation of the behaviour as specified by the rules and serve, therefore, as an accurate visualisation of those rules. The application to the real-life process showed that the approach is feasible in real-life scenarios and produces an understandable Petri net.

In this paper, loops are not considered in the rules. Although the rules themselves allow to specify loops, the use of derivations prevented us from including such behaviour in the final model. Furthermore, the coverage of the applicability of the resulting Petri net is limited to the cases explicitly given by the designers. However, the constructions could be used at runtime to generate a Petri net corresponding to a set of rules and an input case on the fly. Additionally, future lines of research will lead us to an elaborate empirical evaluation to study the understandability and usability of the resulting Petri net compared to the rule set as perceived by end users.

APPENDIX: COMPLETE PROOFS

Lemma 1: Let R_C be a rule set, and N be the Petri net obtained by applying Algorithm 2 to R_C . Then, there is

a derivation $L_1, \dots, L_n$ in R_C iff there exists a sequence of markings $m_0, m_1, \dots, m_{n'}$ in N obtained through firing transitions $t_{L_1}, \dots, t_{L_n}$, where m_i is reachable from m_{i-1} for $i \in \{1, \dots, n'\}$.

Proof. Proof of *only if* part by induction:

Base case: When $n = 1$, we have the derivation L_1 which means that there is a rule $r : l_1, \dots, l_n \Rightarrow L_1$ where all l_i s, $i \in \{1, \dots, n\}$ are true in the initial state. In the preprocessing step (Algorithm 1), these literals are deleted from the body of the rule and since the body is empty (Line 10–11), there will be arcs $t_{L_1} \mapsto t_{L_1} : t_{L_1} \mapsto p \mapsto t_{L_1}$. Considering markings $m_0 = \{p_{L_1}\}$, $m_1 = \{p\}$, the sequence m_0, m_1 is the marking sequence that fires t_{L_1} .

Induction step: We assume that for a derivation $L_1, \dots, L_k$, there is a sequence of markings $m_0, m_1, \dots, m_{k'}$ that fires transitions $t_{L_1}, \dots, t_{L_k}$ and we show that for a derivation $L_1, \dots, L_k, L_{k+1}$ there is a sequence of markings $m_0, \dots, m_{k'}, \dots, m_{k''}$ that fires transitions $t_{L_1}, \dots, t_{L_k}, t_{L_{k+1}}$. When we have a derivation $L_1, \dots, L_k, L_{k+1}$, it means that at the last step of the derivation we use a rule $r : l_1, \dots, l_n \Rightarrow L_{k+1}$ where $l_1, \dots, l_n$ are in $\{L_1, \dots, L_k\}$. According to Algorithm 2 (Line 14–15), we have arcs $t_{L_i} \mapsto t_{L_i} : t_{L_i} \mapsto p_i \mapsto t_{L_i}$. By the induction assumption, all transitions t_{L_i} have fired; we thus have a token in each place p_i in $m_{k'}$. This implies t_{L_i} can fire. There are two cases in the algorithm based on the number of rules with head L_{k+1} . If there is only a single rule (Line 16–17), we have arcs $t_{L_i} \mapsto t_{L_{k+1}} : t_{L_i} \mapsto p' \mapsto t_{L_{k+1}}$. This means marking sequence $m_0, \dots, m_{k'}, m_{k''}$ where $m_{k''} = m_{k'} - \{p_1, \dots, p_n\} + \{p'\}$ can fire $t_{L_{k+1}}$ and this proves the induction step. In the case that we have several rules $R'_C = \{r_1, \dots, r_m\}$ with head L_{k+1} where $r_i : l_1^i, \dots, l_n^i \Rightarrow L_{k+1}$ and $r = r_y \in R'_C$ (Line 18–26), besides the arcs $t_{L_j^i} \mapsto p_j^i \mapsto t_{L_j^i}$, we also have arcs $t_{L_i} \mapsto x_i \mapsto t_{L_i} \mapsto p_x \mapsto t_{L_{k+1}}$ and also $t_{L_i} \mapsto p_{\oplus} \mapsto t_{L_i}$. As we argued in the previous case, marking $m_{k'}$ can fire $t_{L_i} = t_{L_y}$ and leave a token in place x_y . In addition, since t_{L_i} has fired, we have a token in place $p_{\oplus}$. These places are the only input places of transition t_{L_y} so it can fire. The marking sequence $m_1, \dots, m_{k'}, m_{k''}, m_{k''}$ where $m_{k''} = m_{k'} - \{p_1^y, \dots, p_n^y\} + \{x_y\}$, $m_{k''} = m_{k'} - \{x_y, p_{\oplus}\} + \{p_x\}$ can fire $t_{L_{k+1}}$, which proves the induction step.

Proof of *if* part by induction:

Base case: Suppose there is a sequence of markings $m_0, m_1, \dots, m_{n'}$ that fires transition t_{L_1} . According to Algorithm 2, every transition for literals has exactly one input place (Line 11, 17 and 20). This means that there should be a token at input place p of transition t_{L_1} in $m_{n'}$. Since no other transition is fired, the only transition that can fire and put a token in this place is t_{L_1} , meaning that we should have arcs $t_{L_1} \mapsto p \mapsto t_{L_1}$. We have these arcs only when we have a rule $r : l_1, \dots, l_n \Rightarrow L_1$ where all l_i s, $i \in \{1, \dots, n\}$ are true at the initial state. This means we have a derivation for L_1 .

Induction step: We assume that for a sequence of markings $m_0, m_1, \dots, m_{k'}$ that fires transitions $t_{L_1}, \dots, t_{L_k}$, there is a derivation $L_1, \dots, L_k$ and we show that for a sequence of markings $m_0, \dots, m_{k'}, \dots, m_{k''}$ that fires transitions $t_{L_1}, \dots, t_{L_k}, t_{L_{k+1}}$, there exists a derivation $L_1, \dots, L_k, L_{k+1}$. For transition $t_{L_{k+1}}$ to fire, there needs to be a token in the input place of $t_{L_{k+1}}$ in marking $m_{k''}$. According to the algorithm, there are two cases where we can have a token at that place. In the first case (Line 17), we have

arcs $t_{L_i} \mapsto t_{L_{k+1}} : t_{L_i} \mapsto p' \mapsto t_{L_{k+1}}$ and p' has only t_{L_i} as input which means t_{L_i} should have fired. Since $r : l_1, \dots, l_n \Rightarrow L_{k+1}$ and according to the algorithm (Line 15), we have arcs $t_{L_i} \mapsto t_{L_i} : t_{L_i} \mapsto p_i \mapsto t_{L_i}$, all p_i s must have a token in some marking m before $m_{k''}$, meaning that all t_{L_i} s are fired before. Since $t_{L_1}, \dots, t_{L_k}$ are the only transitions fired, $\{l_1, \dots, l_n\} \subseteq \{L_1, \dots, L_k\}$. Therefore, by the induction assumption, we have a derivation for all l_i s in body of rule r and we can use this rule to derive L_{k+1} . We thus have a derivation $L_1, \dots, L_k, L_{k+1}$ and this proves the induction step. In the second case, we have arcs $t_{L_i} \mapsto p_x \mapsto t_{L_{k+1}}$ (Line 20 and 25) which means one of t_{L_i} s should have fired to put a token in place p_x . All transitions t_{L_i} s in the algorithm have two input places (Line 25–26): one coming from t_{L_i} ($t_{L_i} \mapsto p_{\oplus} \mapsto t_{L_i}$) and one from the corresponding t_{L_i} ($t_{L_i} \mapsto x_i \mapsto t_{L_i}$). Since t_{L_i} fires at the first step, place $p_{\oplus}$ always has a token and there needs to be a token at place x_i for t_{L_i} to fire. Accordingly, t_{L_i} must have fired and we can use the same argument of the first case to prove that we have derivations for all l_j s in body of rule r_i and we can use this rule to derive L_{k+1} . Therefore we have a derivation $L_1, \dots, L_k, L_{k+1}$, which proves the induction step. $\square$

Lemma 2: Let N_{IN} be the set of Petri nets obtained through Algorithm 2, and N_M be the merged Petri net obtained by applying Algorithm 3 to N_{IN} . Then, there is a trace $t_{L_1}, \dots, t_{L_n}$ in N_M iff there exists a trace $t_{L_1}, \dots, t_{L_n}$ in some $N_i \in N_{IN}$.

Proof. Proof of *if* part by induction:

Base case: If L_1 is a trace in N_i , then we have arcs $t_{L_1} \mapsto p \mapsto t_{L_1}$ and markings sequence $\{m_0 = \{p_{L_1}\}, m_1 = \{p\}\}$ fires t_{L_1} (Line 11, Alg. 2). If $\bullet t_{L_1}$ is the same $\forall N_j \in N_{IN}$ that contain t_{L_1} , we will have two cases depending on $t_{L_1} \bullet$ in N_j . In the first case, we have arcs $t_{L_1} \mapsto p \mapsto t_{L_1}$ in N_M so the same marking sequence $\{m_0, m_1\}$ will fire t_{L_1} (Line 24–26, Alg. 3). In the second case, we have arcs $t_{L_1} \mapsto xor \mapsto C^s$, and $C^s \mapsto p \mapsto t_{L_1}$ along with arcs $t_{L_1} \mapsto xor' \mapsto cs_1 \mapsto p' \mapsto C^s$ (Line 28–34, Alg. 3). Therefore, marking sequence $\{m_0 = \{p_{L_1}\}, m_1 = \{xor, xor'\}, m_2 = \{xor, p'\}, m_3 = \{p\}\}$ fires t_{L_1} . If $\bullet t_{L_1}$ is different in one or more $N_j \in N_{IN}$ that contains t_{L_1} (Line 13–19, Alg. 3), we will also have two cases based on $t_{L_1} \bullet$. In the first case, we have arcs $t_{L_1} \mapsto p \mapsto C^j \mapsto xor \mapsto t_{L_1}$ so the mapping sequence $\{m_0 = \{p_{L_1}\}, m_1 = \{p\}, m_2 = \{xor\}\}$ will fire t_{L_1} . In the second case, we have arcs $t_{L_1} \mapsto xor_1, xor_1 \mapsto C^s, C^s \mapsto p, p \mapsto C^j, C^j \mapsto xor_2 \mapsto t_{L_1}$ along with arcs $t_{L_1} \mapsto xor' \mapsto cs_1 \mapsto p' \mapsto C^s$. Therefore, marking sequence $\{m_0 = \{p_{L_1}\}, m_1 = \{xor_1, xor'\}, m_2 = \{xor_1, p'\}, m_3 = \{p\}, m_4 = \{xor_2\}\}$ fires t_{L_1} .

Induction step: we assume that for a trace $t_{L_1}, \dots, t_{L_k}$ in N_i , we have a trace $t_{L_1}, \dots, t_{L_k}$ in N_M . Then we show that for a trace $t_{L_1}, \dots, t_{L_k}, t_{L_{k+1}}$ in N_i , we will have a trace $t_{L_1}, \dots, t_{L_k}, t_{L_{k+1}}$ in N_M . If $t_{L_1}, \dots, t_{L_k}, t_{L_{k+1}}$ is a trace in N_i , then for $t_{L_{k+1}}$ to fire, there exists a token in the input place of $t_{L_{k+1}}$ in N_i . This place will have a token if there exists a rule $r : l_1, \dots, l_n \Rightarrow L_{k+1}$ where all $l_m \in \{l_1, \dots, l_n\}$ are among $L_1, \dots, L_k$ so that transition t_{L_i} will fire and lead a token to the input place of $t_{L_{k+1}}$ directly or via transition t_{L_i} . If $\bullet t_{L_{k+1}}$ is the same in all Petri nets $N_j \in N_{IN}$ that contain $t_{L_{k+1}}$, the same structure of N_i exists in N_M (Line

8–11, Alg. 3). Therefore, the same arcs that lead a token from rule transition t_r to fire $t_{L_{k+1}}$ will be in N_M . Based on the induction assumption, since $t_{L_1}, \dots, t_{L_k}$ is a trace in N_M , transition t_r and consequently $t_{L_{k+1}}$ can fire in N_M , that is $t_{L_1}, \dots, t_{L_k}, t_{L_{k+1}}$ is a trace in N_M . Note that the transitions for rules will be the same in N_M as in N_{IN} , because their inputs and outputs are the same in all Petri nets. In the case that inputs of transition $t_{L_{k+1}}$ are different in one or more Petri nets $N_j \in N_{IN}$ that contain $t_{L_{k+1}}$ (Line 13–19, Alg. 3), there exist arcs from these different inputs to different transitions C^j . Therefore, we have arcs $p \mapsto C^j$ where p is an input place of $t_{L_{k+1}}$ in N_i , which is different from the input place of $t_{L_{k+1}}$ in N_j . Additionally, we have arcs $C^j \mapsto xor \mapsto t_{L_{k+1}}$. As argued before, place p will have a token, because $L_1, \dots, L_k$ is a trace in N_M and transition t_r will fire and lead a token to this place. Consequently, C^j will fire and so will $t_{L_{k+1}}$.

Proof of *only if* part by induction:

Base case: If L_1 is a trace in N_M , it means we either have (1) $t_s \mapsto p \mapsto t_{L_1}$, or (2) $t_s \mapsto xor \mapsto C^s \mapsto p \mapsto t_{L_1}$, or (3) $t_s \mapsto p \mapsto C^j \mapsto xor \mapsto t_{L_1}$, or (4) $t_s \mapsto xor \mapsto C^s \mapsto p \mapsto C^j \mapsto xor \mapsto t_{L_1}$. In (1), t_{L_1} is a transition in all N_i s and we have trace L_1 in all Petri nets. In (2), transition C^s will have an input xor place from some of case transitions cs_x . Trace L_1 is a trace in all the corresponding Petri nets. In (3), L_1 is a trace in all Petri nets corresponding to the cases related to C^j meaning all cases in which the input of L_1 is the same in their Petri nets as the inputs of C^j . Finally in (4), L_1 is a trace in all Petri nets corresponding to input cases of C^s which are the same as corresponding cases for C^j .

Induction Step: We assume that for a trace $t_{L_1}, \dots, t_{L_k}$ in N_M , we have a trace $t_{L_1}, \dots, t_{L_k}$ in some $N_i \in N_{IN}$, then we show that for a trace $t_{L_1}, \dots, t_{L_k}, t_{L_{k+1}}$ in N_M , we will have a trace $t_{L_1}, \dots, t_{L_k}, t_{L_{k+1}}$ in some $N_i \in N_{IN}$. We prove this by contraposition, that is we show that if $L_1, \dots, L_k, L_{k+1}$ is not a trace in any $N_i \in N_{IN}$, it will not be a trace in N_M . Based on the induction assumption, if $t_{L_1}, \dots, t_{L_k}$ occurs in a trace in N_M , it will be a trace in some Petri nets $N_i \in N_{IN}$. If $t_{L_1}, \dots, t_{L_k}, t_{L_{k+1}}$ is not a trace in any $N_i \in N_{IN}$, then no rule exists in the corresponding rule sets where $r : l_1, \dots, l_n \Rightarrow L_{k+1}$ and all $l_m \in \{l_1, \dots, l_n\}$ are among $L_1, \dots, L_k$. Accordingly, there will be no path to lead a token to fire $t_{L_{k+1}}$ in N_M after trace $t_{L_1}, \dots, t_{L_k}$. This means $t_{L_1}, \dots, t_{L_k}, t_{L_{k+1}}$ is not a trace in N_M . $\square$

Lemma 3: Let R be a rule set, and N_M be the Petri net obtained by applying Algorithm 1–3 to R . The merged Petri net N_M is 1-bounded.

Proof. We prove by strong induction on the maximum number of transitions we need to traverse to reach a place.

Base case: When the maximum number of transitions needed to traverse to reach a place is 0, we have only place p_s which, by definition, has one token being a start place.

Induction step: We assume that for every place where the maximum number of transitions needed to traverse to reach is k , $k < n$, the place has at most one token and we prove that for a place where the maximum number of transitions needed to traverse to reach is n , it has at most one token. To prove this, we know that for places which have just one input, this is true. We consider places where $|\bullet p| > 1$ and show that they can have at most one token. These places are:

- 1) When we have different rules with the same task as their heads, we have a place p_x with more than one input from transitions t_r 's (Line 25, Alg. 2). However, transitions t_r 's have a common input from place $p_\oplus$ and therefore only one of them can fire (Line 26, Alg. 2). This proves that place p_x can have at most one token.
- 2) The other place with $|\bullet p| > 1$ is $p_\blacksquare$, which has more than one input from transitions t_r 's (Line 30, Alg. 2). Since transitions t_r 's have a common input $p_\sim$, only one of them can fire and thus place $p_\blacksquare$ can have at most one token.
- 3) The xor places after C^j transitions are the other places with more than one input (Line 17, Alg. 3). We have these places whenever the common transition in Petri nets have different inputs in those Petri nets. In this case, we have arcs $C^j \mapsto xor$ from different C^j transitions. When the inputs of a transition t are different in the Petri nets that we merged, this means there are different derivations in those Petri nets leading to t . This implies that there exists at least one common transition (which can be t_s) in these Petri nets with different outputs. Based on the algorithm, we have an xor place after this transition along with arcs $t \mapsto xor$ and $xor \mapsto C^s$ (Line 30–32, Alg. 3). Since one of the inputs of transitions C^s is from the corresponding case transitions cs_x (Line 43–44, Alg. 3) and only one of these transitions can fire (they share a common input x) (Line 3, Alg. 3), we can have only one of these derivations in the merged Petri net. Accordingly, it is not possible for more than one transition C^j to fire at the same time and, as a result, the xor place after them has at most one token.
- 4) The places p_{C^s} may also have more than one input (Line 44, Alg. 3). These inputs are from different case transitions and since only one of these transitions can fire at the same time, these places can have at most one token.
- 5) If the input places of a common transition in one Petri net are subset of the input of that transition in some other Petri net, we have places $p_\ominus$ which have more than one input (Line 45–51, Alg. 3). The inputs of this place are from case transitions cs_x or from transitions t_{C^x} which, again, have inputs from C^s transition with inputs from case transitions cs_x . Since only one of these case transitions fire, place $p_\ominus$ has only one token. $\square$

Lemma 4: Let R be a rule set, and N_M be the Petri net obtained by applying Algorithm 1–3 to R . The merged Petri net N_M is L_1 -live and deadlock-free.

Proof. To prove that the merged Petri net N_M is 1-live, we show that for every transition in the Petri net, there is a sequence of markings that can fire it. Furthermore, we show that the N_m is deadlock-free (i.e. there will be no remaining tokens such that no transition is enabled).

Since every task in the rule sets is contained in at least one derivation, based on Lemma 1, we have a sequence of markings that enables it and hence, based on Lemma 2, that task transition can fire in the merged N_M . We prove such a claim for different silent transitions that we have in N_M .

- 1) Start transition t_s can fire because the marking $m = \{p_s\}$ enables it (Line 8, Alg. 2).
- 2) Case transitions cs_x can also fire because we have arcs $t_s \mapsto x \mapsto cs_x$ (Line 3, Alg. 3). Thus, the marking sequence $m_0 = \{p_s\}, m_1 = \{x\}$ can fire transition cs_x .
- 3) Transition t_r corresponds to the rule $r : l_1, \dots, l_n \Rightarrow l_{n+1}$. If this is the only rule with the head l_{n+1} , we know that we

have a derivation $L_1, \dots, L_k$ where $l_1, \dots, l_n$ are in $L_1, \dots, L_k$. By Lemma 2, we thus have a sequence of markings that can fire $t_{l_1}, \dots, t_{l_n}$. After these transitions fire, since these are the only inputs of transition t_r with arcs $t_{l_i} \mapsto p \mapsto t_r$, transition t_r fires (Line 14–15, Alg. 2). Notice that if any of these transitions has different outputs in different Petri nets, we may have arcs $t_{l_i} \mapsto xor \mapsto C^s \mapsto p \mapsto t_r$ (Line 30–34, Alg. 3). Transition C^s has another input from corresponding case transition cs_x and since this transition can fire, we can have a sequence of markings that fires t_r (Line 42–44, Alg. 3).

4) To handle the situation where we have more than one rule with the same head, say $r_1, \dots, r_m, r_i : l_1^i, \dots, l_n^i \Rightarrow l_{n+1}$, we have transitions $t_{r'}$ and $t_{r''}$ in the Petri net (Line 18–30, Alg. 2). In this case, we have a derivation $L_1, \dots, L_k$ where $l_1^i, \dots, l_n^i$ are in $L_1, \dots, L_k$. Therefore, by Lemma 2, we have a sequence of markings that can fire all $t_{l_j^i}$ s. After all $t_{l_j^i}$ s have fired, all transitions t_{r_i} can fire and lead to a marking m . In this marking, we have $x_i \in m$. Accordingly, marking $m' = m \cup \{p_\oplus\}$ can fire transition $t_{r'_i}$ (but only one). Furthermore, marking $m'' = m - \{p_\oplus, x_i\} + \{p_\sim\}$ can fire transition $t_{r''_i}$. Notice that it is also possible for transitions $t_{l_j^i}$ to have different outputs in different Petri nets. The argument is the same as in (3).

5) C^s transitions are added after task transitions during the merge when the outputs of those transitions are different in the Petri nets. As mentioned, we have a derivation for each task t , hence, by Lemma 2, we have a sequence of markings that fires t . If the outputs of t are different in different Petri nets, we have arcs $t \mapsto xor \mapsto C^s$ (Line 30–32, Alg. 3). Transition C^s has another input from corresponding case transition cs_x with arcs $cs_x \mapsto p' \mapsto C^s$ (Line 42–44, Alg. 3). After transition t fires, we have a token at place xor and since the case transition cs_x corresponding to the derivation can fire, we have a token at place p' too. Therefore, the corresponding C^s can fire.

6) C^j transitions are also added before task transitions during merge when the inputs of those transitions are different in the Petri nets (Line 12–19, Alg. 3). Given that we have a derivation for every task t , we have a marking sequence that fires transition t . As we argued in the proof of Lemma 2, the token in the input of this task comes from the C^j transition corresponding to the case that derivation belongs to. Therefore, this sequence of markings fires that C^j as well. Since every C^j transition is for a derivation in different rule set, there is a marking sequence for each of them. However, considering inputs of t_e in different Petri nets, the inputs of it in one Petri net may be a subset of the inputs of it in other Petri nets. (Notice that this situation only occurs for t_e because all other task transitions have only one input and the inputs of other silent transition are always the same in all Petri nets). In this case, we may have C^j before t_e with just one input from the corresponding case transition (Line 20–23, Alg. 3). C^j can fire because the corresponding case transition can fire.

7) The other silent transitions that we have are transitions named t_{C^x} (Line 35–41, Alg. 3). Whenever the outputs of a transition in one Petri net N is subset of its outputs in other Petri nets, we have C^s transitions with no output. These transitions are the inputs of t_{C^x} transitions. Since we have a derivation including all tasks in the rule sets corresponding to Petri net N , all task transitions fire and thus all the C^s transitions fire. Consequently, the corresponding t_{C^x} can fire.

8) The inputs of t_e in each Petri net are from places $p_\blacksquare$ for each set of rules with common head and also from task transitions t that do not occur in the body of any rule (Line 31–34, Alg. 2). Since we have a derivation $L_1, \dots, L_k$ in the rule set which includes all tasks t , there is a marking sequence that fires all tasks and also as we argued for transitions $t_{r''}$, it fires one of them. The place $p_\blacksquare$ is an xor place with inputs from all $t_{r''}$ transitions (Line 29–30, Alg. 2); accordingly, it has a token and so do all the places after tasks which do not have any following transition to consume them (those which are not in the body of any rule). As a result t_e can fire in that Petri net. Since we have a trace in each Petri net that fires t_e , we have a trace that fires t_e in N_M . Note that in N_M , t_e may have an input from $p_\ominus$. This place is an xor place with inputs from transitions t_{C^x} or directly from case transitions (Line 45–51, Alg. 3). Based on the case the trace belongs to, the corresponding t_{C^x} (if it exists) fires and puts a token in place $p_\ominus$. Otherwise, if t_{C^x} does not exist, the token comes directly from the corresponding case transition. Therefore t_e can fire.

We now prove that there are no remaining tokens. Tokens remain in the inputs of transitions with more than one input. The only transitions with this condition are:

1) Transitions for rules. After t_e has fired, if we have a rule transition t_r with one or more remaining tokens, then the rule has not fired and, as a consequence, has not been used in the derivation corresponding to the current case x firing t_e . This remaining token comes from a task transition t that has fired and this means that t was in the derivation. Since t is in the Petri net for case x , it has an output either to one of the rules other than r or an output to t_e in that Petri net. Therefore, the outputs of t are different in this Petri net compared to the Petri net in which it has an output to rule r . Accordingly, there should be an xor place after t with arcs $t \mapsto xor \mapsto C^s \mapsto p \mapsto t_r$ (Line 30–34, Alg. 3). Since the transition C^s has an input from a case transition other than x , it cannot fire and thus the input place of r from t can never have a token.

2) Transitions $t_{r'}$ and $t_{r''}$ for handling a set of rules with common head (Line 18–30, Alg. 2). Whenever we have a set of rules with common head in one case, in all derivations that fire t_e , all these rules fire and consequently they all fire in Petri net N_M as well. We thus have tokens in all places x_i s and also we have a token in place $p_\oplus$. Now, one of transitions $t_{r'_i}$ can fire and consume the token in $p_\oplus$ and a token from corresponding x_i . The tokens in other x_i s are consumed by a transition $t_{r''}$. The inputs of that $t_{r''}$ transition is from all x_i s except the one for which its corresponding $t_{r'}$ has fired together with an input from $p_\sim$. Because the transition in the head of these rules fire and $p_\sim$ has a token, $t_{r''}$ fires. Therefore, no tokens remain in inputs of $t_{r'}$ and $t_{r''}$ transitions.

3) t_{C^x} transitions (Line 35–41, Alg. 3). The inputs of t_{C^x} are from C^s transition with no output in case x . These transitions have an input from case transition cs_x ; thus they all fire. Therefore, transition t_{C^x} fires and consumes all the tokens in its input.

4) Transition t_e fires in all Petri nets and does so in N_M . Since the N_M is 1-bounded (Lemma 3), whenever t_e fires, it consumes all input tokens and no tokens remain. $\square$

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