Influence of dispersion distribution on the propagation and compression of self-similar optical beam

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Abstract: The influence of different dispersion distribution in ND-DDF (dispersion-decreasing optical fiber with normal group-velocity dispersion) on the generation and compression of self-similar optical beam is investigated in this paper. The split-step Fourier numerical method is adopted and the amplitude of the pulses' envelope under different distributed dispersion is determined numerically. The generation and compression of the self-similar optical beam are simulated and analyzed based on the perspective of chirp feature. The result shows that chirp nonlinear region and chirp linearity level in different dispersion distribution fiber affect the generation and compression of the self-similar optical beam significantly. The quality of obtained self-similar and compressed optical beam in different tapered fiber increases in order of: cosinoidally tapered DDF, linearly tapered DDF, exponentially tapered DDF and hyperbolically tapered DDF, from the lowest to the highest.

Keywords: split-step Fourier method; ND-DDF; self-similar optical beam; dispersion distribution; chirp.

1. Introduction

The exact and asymptotically stable self-similarly evolving solutions, governed by the generalized nonlinear Schrödinger equation (NLSE)[1] have attracted considerable attention due to the particular advantages, such as resistance to optical wave breaking, self-similarity in shape, and enhanced chirp linearity. Many researches have focused on the Self-similar pulse generated in dispersion-decreasing optical fiber [2-3], fiber amplifier [4], nonlinear waveguides [5], highly nonlinear fibers[6], or gas-filled hollow-core fiber[7], etc. The advantage of self-similar pulses also leads to a wide-range practical significance, such as parabolic pulse laser generation[8], optical pulse synthesis[9], multi-wavelength sources[10]. Recently, more and more researches focus on self-similar pulses propagating in pairs. The interaction properties of two parabolic

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pulses propagating in a Raman fiber amplifier were presented in [11-12]. Ponomarenko and Agrawal studied the nonlinear interaction of two or more similaritons in loss- and dispersion-managed fibers[13]. Liu et al. researched parabolic pulse-pair interaction using dispersion-managed fiber links with non-zero dispersion[14]. Loomba et al. researched nonlinear tunneling of optical similaritons in a tapered graded-index nonlinear waveguide[15]. Kanchan Kumar De et al. demonstrated the self-similar waves in two-dimensional graded-index waveguide[16]. Zhao et al. reported on pre-chirping management of a self-similar Yb-fiber amplifier towards 80W average power with sub-40 fs pulse generation [17] and Liu et al. studied on a high-power third-order dispersion managed amplification system that generate high-quality pulses train [18]. Luo further discussed the fiber self-similar-amplification (SSA) comb system that delivers a pulse train with ultrashort pulse and wide spectrum [19]. Although there are more and more studies on the generation of self-similar pulses, to date, there are still limited studies on the propagation and compression of self-similar pulse beam in ND-DDF [20-21].

This paper further investigates the influence of different dispersion distribution on the generation and compression of self-similar optical beam based on the perspective of chirp feature. The ND-DDF is tapered hyperbolically, exponentially, linearly and cosinoidally respectively. The numerical simulation adopts split-step Fourier numerical method and the amplitude of the pulses' envelope under different distributed dispersion is determined numerically. We find that generation and compression of self-similar optical beam are affected significantly by the chirp nonlinear region and the chirp linearity level in different dispersion distribution fiber. The quality of obtained self-similar and compressed optical beam in different tapered fiber increases in order of: cosinoidally tapered DDF, linearly tapered DDF, exponentially tapered DDF and hyperbolically tapered DDF, from the lowest to the highest.

2. Theoretical analysis

The propagation of optical pulses in a dispersion-decreasing optical fiber with normal group-velocity dispersion is modeled by an NLS equation of the form [1]

$$i \frac{\partial A}{\partial z} - \frac{\beta_2(0)}{2} D(z) \frac{\partial^2 A}{\partial T^2} + \gamma |A|^2 A = 0 \quad (1)$$

Where $A(z, T)$ is the slowly varying amplitude of the pulse envelope and T is measured in a frame of reference moving with the pulse at the group velocity v_g ($T=t-z/v_g$), $D(z)$ represents the variation in the GVD (Group-Velocity Dispersion) due to dispersion tapering and is normalized such that $D(0)=1$. $\beta_2(0) = 1.25\text{ps}^2/\text{km}$ and $\gamma=3.5 \text{ W}^{-1}\text{km}^{-1}$

are the GVD value at $z = 0$ and the nonlinearity coefficient respectively.

We launch a pair of Gaussian pulses separated by a time-delay $\Delta T=4.5\text{ps}$ into ND-DDF, which can be expressed in the follow equation.

$$A(0,T)=\sqrt{P_0}\exp\left[-\frac{(T+\Delta T/2)^2}{2T_0^2}\right]+\sqrt{P_0}\exp\left[-\frac{(T-\Delta T/2)^2}{2T_0^2}\right] \quad (2)$$

Each pulse has a full width at half maximum (FWHM) of 1.0ps, a pulse energy of 50pJ and the center wavelength of 1550nm. These conditions lead to the half-width at 1/e-intensity point T_0 , the peak power P_0 , the dispersion length L_D and nonlinearity length L_{NL} as follows.

$$T_0=\frac{T_{FWHM}}{2\sqrt{\ln 2}}=0.6006\text{ps}, \quad P_0=\frac{E_0}{2T_0}=41.6277\text{W}, \quad L_D=\frac{T_0^2}{|\beta_2|}=0.2885\text{km}, \quad L_{NL}=\frac{1}{\gamma P_0}=0.0069\text{km} \quad (3)$$

The numerical method adopts the split-step Fourier method which has been used extensively to solve the pulse-propagation problem in nonlinear dispersive media[22]. According to Eq (1), we obtain the expressions using split-step Fourier method as follows.

$$\begin{cases} \frac{\partial A}{\partial Z}=(\hat{D}+\hat{N})A \\ \hat{D}=-\frac{i}{2}\beta_2(0)D(z)\frac{\partial^2}{\partial T^2} \\ \hat{N}=i\gamma|A|^2 \end{cases} \quad (4)$$

where $\hat{D}$ is a differential operator that accounts for dispersion and absorption in a linear medium and $\hat{N}$ is a nonlinear operator that governs the effect of fiber nonlinearities on pulse propagation.

The split-step Fourier method obtains an approximate solution by assuming that in propagating the optical field over a small distance h , the dispersive and nonlinear effects can be pretended to act independently. Mathematically,

$$A(z+h,T)\approx\exp\left[\int_z^{z+h}\hat{D}(z')dz'\right]\exp\left[\int_z^{z+h}\hat{N}(z')dz'\right]A(z,T) \quad (5)$$

The integral in the middle exponential is useful to include the z dependence of the nonlinear operator $\hat{N}$. If the step size h is small enough, it can be approximated by $\exp(h\hat{N})[1]$,

$$A(z+h,T)\approx\exp\left[\int_z^{z+h}\hat{D}(z')dz'\right]\exp(h\hat{N})A(z,T) \quad (6)$$

By substituting Eq. (4) in Eq. (6), the amplitude of the pulse envelope is given by

$$\begin{cases} A(z+h, T) \approx \exp\left[-\frac{i}{2}\beta_2(0)\frac{\partial^2}{\partial T^2}F(z)\right]\exp(ih\gamma|A|^2)A(z, T) \\ F(z) = \int_z^{z+h} D(z')dz' \end{cases} \quad (7)$$

Assuming that:

$$z = (g-1)dL + mh \quad g = 1, 2, \dots, \text{fix}(L/dL); \quad m = 1, 2, \dots, \text{fix}(dL/h) \quad (8)$$

where dL and h are the step size. From Eq. (7), the amplitude of the pulse envelope is influenced by the different dispersion distribution.

3. Numerical simulation and analysis

Due to the special engineering requirements and different processes, DDF can be tapered with various dispersion profile structures [23-25]. In this paper, different types of dispersion tapering in a passive fiber is considered respectively, which are hyperbolic dispersion, exponential dispersion, linear dispersion and cosine dispersion. The input Gaussian pulses are assumed to propagate in DDF over a dispersion length L_D . Supposing that the GVD value at $z = L_D$ is $\beta_2(L_D) = 0.15\text{ps}^2/\text{km}$, then it yields a ratio

$$K = \beta_2(0) / \beta_2(L_D) = 8.333.$$

3.1 hyperbolic dispersion

When DDF is tapered hyperbolically, $D(z)$ is expressed as

$$\begin{cases} D(z) = \frac{1}{1+g_0z} \\ g_0 = \frac{K-1}{L} = \frac{K-1}{L_D} = 25.4154\text{m}^{-1} \end{cases} \quad (9)$$

Here g_0 is a constant gain coefficient.

Using Eq. (7), Eq. (8) and Eq. (9), $F(z)$ is given by

$$\begin{aligned} F(z) &= \int_z^{z+h} D(z')dz' = \int_z^{z+h} \frac{1}{1+g_0z'} dz' \\ &= \frac{1}{g_0} \ln \left\{ 1 + \frac{g_0h}{1+g_0[(g-1)dL+mh]} \right\} \end{aligned} \quad (10)$$

Fig. 1 Output self-similar optical beam generated in the hyperbolically tapered ND-DDF at $z = L_D$. (a) frequency chirp and Waveform of the output pulses; (b) Output spectrum.

In Fig. 1 (a), the linearly chirp in overlap region of self-similar optical beam becomes nonlinear, so the different frequency between the overlapping falling and raising edges of the pulses induces a beating on the waveform. In Fig.1(b), SPM (Self-Phase Modulation)-induced spectral broadening is accompanied by an oscillatory structure covering the entire frequency range. The oscillatory spectrum consists of 50 peaks, which is symmetrical over the 1550nm wavelength and the intense of the peaks decreases from central to outermost. As the chirp becomes nonlinear over the central region, most of the energy remains in the central peaks. Meanwhile, the linear frequency chirp occurs mainly near the leading and trailing edges. As these edges become steeper, the tails in Fig. 1(b) extend over a longer frequency range but, at the same time, carry less energy because chirping occurs over a small-time duration. The output spectral width in Fig. 1(b) exhibits a value of 90 nm, from 1508nm to 1598nm.

Then, the self-similar optical beam is compressed by using the dispersion compensation technology, and the GVD is compensated in the dispersion compensated fiber. Provided the length of ND-DDF is L_1 and the length of compensated fiber is L_2 , the necessary condition for GVD to be compensated is:

$$\int_0^{L_1} \beta_2(z')dz' + \int_0^{L_2} \beta_{2c}(z')dz' = 0 \quad (11)$$

Where $\beta_2(z)$ and $\beta_{2c}(z)$ represent the GVD of ND-DDF and compensated optical fiber, respectively. From the above simulation, we already know that:

$$\begin{cases} L_1 = L_D \\ \beta_2(z) = \beta_2(0)D(z) \end{cases} \quad (12)$$

Now let us see how the compression of the self-similar optical beam, using these dispersion profiles, is theoretically expected to evolve. The self-similar optical beam is launched into a compensated fiber with a constant $\beta_{2c}(z) = -1.25 \text{ ps}^2 / \text{km}$.

By substituting Eq. (9) and Eq. (12) into Eq. (11), the theoretically expected optimum length L_2 of the compensated fiber for the self-similar optical beam obtained from hyperbolic dispersion profiles ND-DDF can be determined as

$$L_2 = \frac{\ln(1 + g_0 L_D)}{g_0} = 0.0834 \text{ km} \quad (13)$$

Fig. 2. (a) Self-similar optical beam evolution in the compensated fiber (b) Waveform of the compressed pulses, the input pulses, and the self-similar pulses before compression plotted on a log scale.

Fig.2(a) shows the self-similar optical beam propagating in the compensated fiber, which indicates that the optimum compressed pulses are obtained exactly at the theoretical value $z=0.0834\text{km}$, and the peak power is 315.81W . The waveform of the compressed pulses, as well as the input pulses and the self-similar pulses before compression, are also plotted in Fig. 2(b) on a log scale. From this figure, the compressed pulses have a FWHM of 120.5fs , corresponding to the compression factor of 8.3. Additionally, the pedestal of the obtained compressed pulses is less than -10.92dB from the peak, demonstrating high-quality femtosecond optical beams generation.

3.2 exponential dispersion

When the DDF is tapered exponentially, the $D(z)$ can be expressed as

$$\begin{cases} D(z) = \exp(-g_0 z) \\ g_0 = \frac{\ln K}{L} = \frac{\ln K}{L_D} = 7.3483 \text{ m}^{-1} \end{cases} \quad (14)$$

Using Eq. (7), Eq. (8) and Eq. (14), $F(z)$ is given by

$$F(z) = \int_z^{z+h} D(z') dz' = \int_z^{z+h} \exp(-g_0 z') dz' \\ = \frac{1}{g_0} \exp\{-g_0[(g-1)dL + mh]\} - \frac{1}{g_0} \exp\{-g_0[(g-1)dL + (m+1)h]\} \quad (15)$$

Fig. 3 Output self-similar optical beam generated in the exponential tapered ND-DDF at $z=L_D$. (a) frequency chirp and Waveform of the output pulses; (b) Output spectrum.

In Fig. 3 (a), the nonlinear region of the chirp increases slightly, resulting in the oscillating region of waveform increasing correspondingly. In Fig.3(b), due to the decreased linear chip region, spectra exhibit 32 peaks and the output spectral width exhibits a value of 70nm, from 1516nm to 1586nm. Compared to the hyperbolic dispersion distributed fiber, the spectrum broaden is reduced by 22.2%.

The interacting of the linear chirp also influences the dispersion compensation. By substituting Eq. (12) and Eq. (14) into Eq. (11), the theoretically expected optimum compensated fiber length L_2 considering exponential tapered ND-DDF can be determined as

$$L_2 = \frac{1 - \exp(-g_0 L_D)}{g_0} = 0.1198 \text{ km} \quad (16)$$

From Fig 4(a), the optimum compensated fiber length is exactly equal to the theoretical value $z=0.1198\text{km}$, and the peak power is 227.97W. The compressed pulses have a FWHM of 169fs, corresponding to the compression factor of 5.92, which is a 28.7% reduction from Fig.2. Additionally, the pedestal of the obtained compressed pulses is less than -9.6dB from the peak.

Fig. 4. (a) Self-similar optical beam evolution in the compensated fiber (b) Waveform of the compressed pulses, the input pulses, and the self-similar pulses before compression plotted on a log scale.

3.3 linear dispersion

When the DDF is tapered linearly, the $D(z)$ is expressed as

$$\begin{cases} D(z) = 1 + g_0 z \\ g_0 = \frac{1}{L} \left(\frac{1}{K} - 1 \right) = \frac{1}{L_D} \left(\frac{1}{K} - 1 \right) = -3.0498 m^{-1} \end{cases} \quad (17)$$

Using Eq. (7), Eq. (8) and Eq. (17), $F(z)$ is given by

$$\begin{aligned} F(z) &= \int_z^{z+h} D(z') dz' = \int_z^{z+h} (1 + g_0 z') dz' \\ &= \frac{g_0 h^2}{2} + \{1 + g_0 [(g-1)dL + mh]\} h \end{aligned} \quad (18)$$

Fig. 5 Output self-similar optical beam generated in the linear tapered ND-DDF at $z = L_D$. (a) frequency chirp and Waveform of the output pulse; (b) Output spectrum.

In Fig. 5 (a), the oscillating region of the waveform also increases with the increase of the nonlinear chirp region, which indicates the enhancing interaction effect between self-similar optical beams. In Fig.5(b), due to the decreased linear chirp region, spectra exhibit 30 peaks and the value of output spectral width is 62nm, from 1520nm to 1582nm. Compared to the hyperbolic dispersion distributed fiber, the spectrum broaden is reduced by 31.1%.

By substituting Eq. (12) and Eq. (17) into Eq. (11), the theoretically expected optimum compensated fiber length L_2 considering linearly tapered ND-DDF can be expressed as

$$L_2 = L_D + \frac{g_0 L_D^2}{2} = 0.1616 \text{ km} \quad (19)$$

Fig. 6. (a) Self-similar optical beam evolution in the compensated fiber (b) Waveform of the compressed pulses, the input pulses, and the self-similar pulses before compression plotted on a log scale.

From Fig. (6), the best compressed pulses are achieved at the ideal theoretical value obtained from Eq. (19), and the peak power is 161.83W. The compressed pulses have a FWHM of 173fs, corresponding to the compression factor of 5.78, which is a 30.4% reduction from Fig.2. Additionally, the pedestal of the obtained compressed pulses is less than -7.675 dB from the peak.

3.4 cosine dispersion

When the DDF is tapered cosinoidally, the $D(z)$ is expressed as

$$\begin{cases} D(z) = \cos(g_0 z) \\ g_0 = \frac{1}{L \cos(1/K)} = \frac{1}{L_D \cos(1/K)} = 3.4908 m^{-1} \end{cases} \quad (20)$$

Using Eq. (7), Eq. (8) and Eq. (20), $F(z)$ is given by

$$\begin{aligned}
F(z) &= \int_z^{z+h} D(z') dz' = \int_z^{z+h} \cos(g_0 z') dz' \\
&= \frac{2}{g_0} \sin \frac{g_0 h}{2} \cos \left[\frac{g_0 h}{2} + g_0 [(g-1)dL + mh] \right]
\end{aligned} \tag{21}$$

Fig. 7 Output self-similar optical beam generated in the cosinoidal tapered ND-DDF at $z=L_D$. (a) frequency chirp and Waveform of the output pulse; (b) Output spectrum.

From Fig. 7(a), due to the large nonlinear chirp region, the self-similar optical beam oscillates strongly. In Fig.7(b), spectra exhibit 28 peaks and the value of output spectral width is 55nm, from 1523nm to 1578nm. Compared to the hyperbolic dispersion distributed fiber, the spectrum broaden is reduced by 38.9%.

By substituting Eq. (12) and Eq. (20) into Eq. (11), the theoretically expected optimum compensated fiber length L_2 considering cosinoidally tapered ND-DDF can be expressed as

$$L_2 = \frac{\sin(g_0 L_D)}{g_0} = 0.2422 \text{ km} \tag{22}$$

From Fig 8, the best compressed pulses are obtained at the theoretical compensated fiber length, and the peak power is 144.02W. The compressed pulses have a FWHM of 210fs, corresponding to the compression factor of 4.76, which is a 42.7% reduction from Fig.2. Additionally, the pedestal of the obtained compressed pulses is less than -4.345 dB from the peak, demonstrating low-quality femtosecond optical beams generation.

Fig. 8. (a) Self-similar optical beam evolution in the compensated fiber (b) Waveform of the compressed pulses, the input pulses, and the self-similar pulses before compression plotted on a log scale.

4. Results and discussion

The influence of different dispersion on the evolution and compression of self-similar optical beam can be analyzed from the point of chirp. In Fig.9, the chirps in different dispersion distribution tapered fiber are illustrated simultaneously.

From simulation, the chirp linearity level and chirp linear region are influenced by different dispersion. Fig. 9 shows that the chirp linearity level and chirp linear region decrease in different DDF in an order of: hyperbolically tapered DDF, exponentially tapered DDF, linearly tapered DDF and cosinoidally tapered DDF. For Gaussian pulse, the SPM-induced chirp is linear and positive (up-chirp) over a large central region, and the GVD-induced chirp in the normal-dispersion regime ($\beta_2 > 0$), is also negative at the leading edge and increases linearly across the pulse, which means that the GVD-induced chirp adds with the SPM-induced chirp, resulting in an enhanced chirp. As the increase of the chirp linearity level extends the spectral broaden, the spectral broadens with dispersion type in an order of: cosine dispersion, linear dispersion, exponential dispersion and hyperbolic dispersion, while the spectral peak number also increases in the same order. Compared with the hyperbolically tapered DDF, the spectrum broadens reduced by 22.2%, 31.1% and 38.9% respectively when the fiber is tapered exponentially, linearly and cosinoidally.

(a) chirp plotted in the same figure

(b) chirp plotted in separated figures

Fig.9 frequency chirp of the self-similar optical beam obtained in different dispersion distribution tapered fiber at $z = L_D$

Meanwhile, as the chirp for single self-similar pulse is nonlinear at the edge, so when two self-similar pulses propagate simultaneously in the fiber by a time-delay, the nonlinear chirp of the trailing edge of the left-hand side pulse and the leading edge of the right-hand side pulse interact. Because of the nonlinear nature of the composite chirp, different parts of the pulse propagate at different speeds, the leading and trailing regions of the pulses contain light at two different frequencies that interfere, results in oscillating in the overlap region. From the simulation, the increase of the chirp linearity level leads to a narrow region of the nonlinear chirp. As a result, the increase of the chirp linearity level fasts the evolution speed of the self-similar optical beam while the decrease of the nonlinear chirp region slows down the interacting of the self-similar optical beam. That is why the hyperbolically tapered DDF can generate the best quality of the self-similar pulses output with minimal interaction effect.

The chirp linearity level and the chirp linear region also influence the quality of pulses compression. The further simulation on the chirp compensation illustrates that the low level of chirp linearity and the large region of the nonlinear chirp lead to the increase of the compensated fiber length. The length of the compensated fiber increases with different dispersion tapered fiber in an order of: hyperbolically tapered DDF, exponentially tapered DDF, linearly tapered DDF and cosinoidally tapered DDF. As the fiber losses relates to the fiber length, the longer fiber length will cause more fiber losses, so the peak power of the compressed pulses also decreases with different dispersion tapered fiber in an order of: hyperbolically tapered DDF, exponentially tapered DDF, linearly tapered DDF and cosinoidally tapered DDF. The simulation results also indicate that the absolute value of the pedestal of obtained compressed pulses and the compression factor decrease with different dispersion type in an order of: hyperbolic dispersion, exponential dispersion, linear dispersion and cosine dispersion. Compared with the hyperbolic dispersion, the compression factor reduced

by 28.7%, 30.4% and 42.7% respectively when the fiber is tapered exponentially, linearly and cosinoidally, meaning the hyperbolically tapered DDF can generate the best quality of self-similar and ultra-short femtosecond optical beams.

5 Conclusion

In conclusion, we have analyzed the dynamic of the evolution of a pair of identical pulses in ND-DDF with different distributed dispersion. The split-step Fourier method is adopted as the numerical method and the amplitude of the pulses' envelope propagate in ND-DDF with different distributed dispersion is determined numerically. A complete characterization of the nonlinear region and the linearity level of chirp under different dispersion distribution fiber prove the quality of the obtained self-similar and compressed optical beam increasing with the dispersion distributed in an order of: cosine distributed dispersion, linear distributed dispersion, exponential distributed dispersion and hyperbolic distributed dispersion. The results may facilitate the development of Dense Wavelength Division Multiplexing transmission system which is in heavy demands of light source in wide-range wavelength.

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Author contribution statement

QF Zhang prepared the initial draft. Both LM Wu and J Gao discussed the results. HZ Li took part in English writing of the manuscript. All authors contributed to manuscript preparation.

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