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Numerical study and parametric analysis of the propagation buckling behaviour of subsea pipe-in-pipe systems

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ABSTRACT

Propagation buckling mechanisms in pipe-in-pipe (PIP) systems with thin and moderately thin carrier pipes with a diameter-to-thickness (D_o/t_o) ratio in the range 26–40 are investigated using 2D analytical and 3D nonlinear (material and geometry) finite element (FE) models. The FE models are validated against hyperbaric chamber tests of a PIP system with D_o/t_o of 30. Using the validated FE model a parametric study is conducted and two distinct buckle propagation modes in PIPs are observed. Empirical expressions for each mode are proposed and are found to be different from previous expressions suggested for PIPs with (D_o/t_o) ratio in the range 15–25.

Nomenclature

D_o	diameter of the outer pipe
D_i	diameter of the inner pipe
D_s	diameter of a solid inner pipe
E	modulus of elasticity
E'	tangent modulus
m_p	plastic moment
P_p	buckle propagation pressure of a single pipe
P_i	buckle initiation pressure of a single pipe or outer pipe of the PIP system
P_{p2}	buckle propagation pressure of the PIP system
P_{ps}	buckle propagation pressure of the PIP system with a solid inner pipe
t_o	wall thickness of the outer pipe
t_i	wall thickness of the inner pipe
V_o	initial internal volume of the PIP system
W_{ex}	external work done by the net hydrostatic pressure
$W_{in(f)}$	internal work due to the circumferential flexure
$W_{in(m)}$	internal work due to the membrane effect
σ_{Y_o}	yield stress of the outer pipe
σ_{Y_i}	yield stress of the inner pipe
ν	Poisson's ratio

Δl	change in the circumferential length
ΔA	change in the cross-section area
ΔV	change in the volume of the PIP system
Ω	ovalization ratio

1. Introduction

Subsea pipe-in-pipe (PIP) systems are widely used due to their superior thermal insulation performance. A typical PIP system consists of a concentric inner pipe (also known as the product pipe) and the outer pipe (sometimes called the carrier pipe). Typically, the annulus space between the two pipes is filled with non-structural insulation materials such as polyurethane foam or water. In subsea applications the outer pipe is designed to provide the protection from external pressure and mechanical damage, while the inner pipe is designed to carry the high temperature and high pressure (HT/HP) of the transporting hydrocarbons inside the pipe. The HP/HT conditions can cause global upheaval [1] or lateral buckling [2,3] in the PIP system. The external pressure in the vicinity of local dents or ovality in the outer pipe-wall can cause a local collapse, which may catastrophically propagate along the PIP system, known as propagation buckling. The lowest pressure required to sustain such a buckle propagation, is known as the propagation pressure P_p , which is only 15–30% of the initiation pressure P_i of the intact

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pipe. The propagation buckling (or buckle propagation) has been extensively investigated in single pipelines using analytical [4,5], experimental [6,7] and numerical methods [8,9]. The possible interaction between lateral/upheaval buckling and propagation buckling was recently investigated by Karampour et al. [10,11]. A novel design for ultra-deep sub-sea pipelines was proposed and was shown to increase the propagation buckle capacity without increasing the wall thickness [12,13].

Unlike single pipelines, buckle propagation in PIPs has only been marginally addressed [14–17]. A thorough experimental study on propagation buckling of steel PIPs with carrier pipes with D_o/t_o values of 24.1, 21.1 and 16.7 and inner pipes of various D_i/t_i ratios ranging between 15 to 37 was conducted by Kyriakides [14]. Based on this experimental study, an empirical formula for buckle propagation pressure, P_{p2} , of PIPs was proposed by Kyriakides and Vogler [15]. An extensive finite element study of propagation buckling of PIPs using carrier pipes with D_o/t_o of 25, 20 and 15 and inner pipes having D_i/t_i of 15 and 20 was conducted by Gong and Li [16] and another empirical formula for buckle propagation of PIPs was proposed. Karampour et al. [17] investigated propagation buckling of PIPs with thinner carrier pipes ($26 < D_o/t_o < 40$) using ring squash tests, hyperbaric chamber tests and finite element analyses. They observed confined buckling mode shapes [18] in PIP systems with moderately thick carrier pipes which were not reported before. Their non-linear FE results suggested that the confined buckling modes were in accordance with different boundary conditions that the carrier pipe adopted. The empirical expressions reported in [15,16] were derived based on experimental and numerical results of buckle propagation pressures of PIPs with thick carrier pipes ($15 < D_o/t_o < 25$). Increasing the wall-thickness of the carrier pipe increases the installation cost of the PIP system. A thicker carrier pipe also amplifies the axial force developed in the PIP system due to high internal pressure and temperature which in turn increases the risk of upheaval and lateral buckling in the system [19,20]. It is rather beneficial to use thinner carrier pipes with buckle arrestors [21] to mitigate the risk of buckle propagation in the PIP system.

The current study aims to provide insight on buckle propagation mechanisms of PIPs with moderately thin and thin carrier pipes using simplified analytical solutions and non-linear FE analyses. The FE re-

sults are validated against hyperbaric chamber tests of a 1.6 m long aluminium (Al-6060-T5) PIP system. Using the validate FE model, two distinctive buckle propagation modes in PIPs with thin carrier pipes are presented and separate expressions for buckle propagation pressure P_{p2} , of each mode are proposed. Through combining the previous experimental [14] and numerical [16] results with current FE results, a more accurate expression of P_{p2} of PIPs with thick and moderately thick carrier pipes is proposed.

2. Propagation buckling of pipe-in pipe systems

2.1. Analytical solution

Previous studies have proposed analytical solutions to the collapse of a single pipeline based on initial and final configurations of the cross-section of the pipe [4,5]. In such 2D models, the material is assumed to be rigid-perfectly plastic and deformation is limited to development of four plastic hinges in the pipe wall shown in Fig. 1a–c. The propagation pressure is calculated by equating the external work done by the pressure due to the change in the volume, to the plastic flexural work depleted in the four hinges [22]. Albermani et al. [7] accounted for the membrane and flexural effects in the wall of a single pipe during the deformation and arrived at an expression for propagation pressure of a single pipe:

$$\bar{P}_p = \frac{3\pi\sigma_y}{2.515} \left(\frac{t}{D} \right)^2 \quad (1)$$

The schematic deformation stages in propagation buckling of a pipe-in-pipe system are shown in Fig. 1d–f. Kyriakides and Vogler [15] used plane strain conditions and strain hardening behaviour and proposed the following expression for the propagation pressure of the PIP system based on development of four plastic hinges in each of the carrier and the inner pipes (Fig. 1d–f).

$$\hat{P}_{p2} = \frac{2\pi}{\sqrt{3}} \sigma_{Yo} \left(\frac{t_o}{D_o} \right)^2 \left[1 + \frac{\sigma_{Yi}}{\sigma_{Yo}} \left(\frac{t_i}{t_o} \right)^2 \right] \quad (2)$$

Fig. 1. Schematic of deformation stages in propagation buckling; (a–c) single pipe and (d–f) pipe-in-pipe system.

Accounting for the circumferential membrane as well as flexural effects in the outer and inner pipe walls:

$$W_{ex} = W_{in(f)} + W_{in(m)} \quad (3)$$

where W_{ex} is the external work done by the net hydrostatic pressure and W_{in} is the internal work due to the circumferential flexure, f , and membrane, m , effects. Based on the experimental observations from the hyperbaric chamber and ring squash tests, the initially circular cross-section of the outer pipe (Fig. 1d) has found to deform into the shape shown in (Fig. 1e). Further increase in the external pressure causes the pipe-in-pipe system to eventually deform into the dog-bone shape (Fig. 1f). Thus Eq. (3) can be written as:

$$\bar{P}_{p2}(\Delta A) = 3\pi(m_{po} + m_{pi}) + \bar{P}_{p2}(r_o \Delta l_o + r_i \Delta l_i) \quad (4)$$

where the subscript “o” denotes the outer pipe, and “i” represents the inner pipe, ΔA is the change in the cross-section area, Δl is the change in the circumferential length and m_p is the plastic moment [7]. These are given by:

$$\Delta A = \pi r_o^2 \quad (5a)$$

$$\Delta l_o = (2\pi - 4\sqrt{2})r_o; \Delta l_i = (2\pi - 4\sqrt{2})r_i \quad (5b)$$

$$m_{po} = \sigma_{Yo} \frac{t_o^2}{4}; m_{pi} = \sigma_{Yi} \frac{t_i^2}{4} \quad (5c)$$

Substituting Eqs. (5a)–(5c) into (4), the propagation pressure, $\bar{P}_{p2}$, and the propagation pressure normalized by the propagation pressure (Eq. (1)) of the single outer pipe, $\bar{P}_{p2}/\bar{P}_p$, of the PIP system are obtained as:

$$\bar{P}_{p2} = \left[\frac{3\pi\sigma_{Yo}}{2.515} \left(\frac{t_o}{D_o} \right)^2 \right] \left[1 + \frac{\sigma_{Yi}}{\sigma_{Yo}} \left(\frac{t_i}{t_o} \right)^2 \right] \left[\frac{1}{1 - (D_i/2D_o)^2} \right] \quad (6a)$$

$$\frac{\bar{P}_{p2}}{\bar{P}_p} = \left[1 + \frac{\sigma_{Yi}}{\sigma_{Yo}} \left(\frac{t_i}{t_o} \right)^2 \right] \left[\frac{1}{1 - (D_i/2D_o)^2} \right] \quad (6b)$$

when $D_i = t_i = 0$, Eq. (6a) yields the propagation pressure of a single pipe given in Eq. (1). Unlike Eqs. (2), (6a) accounts for the effect of D_i/D_o as well as that of t_i/t_o and σ_i/σ_o . When compared with the experimental results, a better prediction of $\bar{P}_{p2}$ for PIPs was reported [17] using Eq. (6a) rather than Eq. (2) and irrespective of D_o/t_o ratio.

2.2. Hyperbaric chamber test

A concentric aluminium (Al-6060-T5) PIP system with parameters represented in Table 1 and a length of 1.6 m i.e. $L/D_o > 20$, was end-sealed and pressurized inside the hyperbaric chamber shown in Fig. 2.

Table 1
Geometric and material parameters of PIP system.

	D/t	D (mm)	t (mm)	L (mm)	E (MPa)	E'/E (%)	σ_Y (MPa)
Outer pipe	30	60	2	1600	66,680	1.0	139
Inner pipe	25	40	1.6	1600	66,680	1.0	172

The chamber has an inner-diameter of 173 mm and a length of 4 m and is rated for working pressure of 20 MPa (2000 m water depth). To end-seal the PIP system, thick aluminium discs were glued to the ends, ensuring that the inner and outer pipes were concentric and that the inner pipe was completely sealed from the outer pipe. A set of two valves were connected to the end of each of the outer and inner pipes of the PIP. One valve was used for bleeding the pipe while filling it with water. The second valve was utilized to vent each of the carrier and inner pipes, as well as to separately collect water from the inner pipe and the cavity between the inner and outer pipes during the buckle propagation. The pressure inside the chamber was incremented using a high pressure pump (shown in Fig. 2) until the collapse of PIP system occurred under quasi-static steady-state conditions. The total change in volume of the PIP system (ΔV) during the test was calculated by adding the weight of water being discharged from the inner pipe and the cavity between the pipes which were measured using digital weighing scales. Control tests using a single pipe (outer pipe) were conducted first.

The experimental results of the buckle propagation response of the PIP system of Table 1 are shown in Fig. 3a. The pressure inside the chamber was increased until collapse of the outer pipe of the PIP system was triggered at the initiation pressure P_i . The initiation pressure is very sensitive to imperfections of the pipe cross-section [14] which is defined in terms of ovalization ratio Ω :

$$\Omega = \frac{D_{max} - D_{min}}{D_{max} + D_{min}} \quad (7)$$

where D_{max} and D_{min} are the maximum and minimum diameters of the outer pipe. The ovalization of the outer pipe cross-section of the PIP system was measured at different points along the length of the PIP and maximum ratio of $\Omega = 0.5\%$ was obtained. Following the initial collapse, the pressure in the system suddenly dropped to a much lower pressure highlighted as P_{p2} in Fig. 3a. By maintaining a low rate of pressurization, the chamber pressure was stabilized at propagation pressure, P_{p2} , with buckling longitudinally propagating along the PIP system sample accompanied by water flow from the vents. The deformed configuration and dog-bone cross-section of the PIP system after the test are depicted in Fig. 3b. The propagation pressures of the control test using a single outer pipe (P_p) and the PIP system (P_{p2}) are presented in Table 2. These results are the average of three PIP tests.

The modulus of elasticity (E) of the specimens listed in Table 1 were obtained from two compressive stub tests (Fig. 4a) conducted for each D/t . The yield stress listed in Table 1 was calculated based on results of two ring squash tests (RST) shown in Fig. 4b. The ring squash test [7,17,23] is conducted on a short segment of the pipe specimen compressed between two rigid indenters of the same diameter as the pipe specimen (Fig. 4b). The yield stress, σ_Y , is calculated from

$$\sigma_Y = \frac{F_0 D}{2L_{RST} t^2} \quad (8)$$

where F_0 is the RST load at which the four plastic hinges shown in Fig. 1a–c are developed in the pipe wall. L_{RST} is the length of the RST sample which is 150 mm [17]. The material tangent modulus of $E'/E = 1\%$ was adopted for the inner and outer pipes.

Fig. 2. The hyperbaric chamber used in the test showing; sample, high-pressure pump, pressure gauge and vents.

2.3. FE analyses and comparison with experimental results

Finite-element analyses of 1.6-m-long samples of the single carrier pipe and the PIP system described in Table 1 and tested in the hyperbaric chamber were carried out using ANSYS 17.0 [24]. Thin four-node shell elements (181) were used to model the carrier and the inner pipes. Frictionless contact elements (174 and 170) were used in three pairs to define the non-linear contact between the carrier and inner pipes and the inner surfaces of the inner-pipe wall. Because of symmetry, a one-half model of the pipe wall (180°) was discretised using 24 elements in the circumferential direction with seven integration points through the thickness of the carrier and inner pipes. Ovalization ratio of $\Omega = 0.5\%$ (same as that measured in the test sample) was intro-

duced at mid-length on the carrier pipe in the FE model. The nodes at either end of the PIP system were restrained from translation in all directions. A von-Mises elastoplastic (bilinear) material definition with isotropic hardening and parameters shown in Table 1 was adopted.

The FE response of the PIP system is plotted in Fig. 3a and deformed configurations are shown in Fig. 3c and show good agreement with the experimental results. The propagation pressures of the single pipe and PIP system from the FE models are presented in Table 2 together with the experimental results. The FE prediction is nearly 96% of the experimental results for both P_p and P_{p2} .

3. Propagation buckling mechanisms in PIPs

3.1. Propagation buckling of the PIP system with a solid inner pipe

Subsea pipe-in-pipe systems are designed to safely operate at pressures larger than the propagation pressure P_{p2} . The buckle propagation of the system due to the collapse of the outer pipe is controlled by using buckle arrestors at intermittent locations along the length of the PIP system [21]. It is of interest to find the propagation pressure of a PIP system with a solid inner pipe (P_{ps}) as it provides a bounding value of P_{p2} in the PIP system. Finite element buckle propagation response of a thin carrier pipe of $D_o/t_o = 40$ with a solid insert of $D_s/D_o = 0.5$ is shown in Fig. 5a. The pressure is normalized to the propagation pressure of a single outer pipe (P_p) and is plotted against the normalized ovalization of the carrier pipe ($\Delta D_o/D_o$). In the FE model, the modulus of elasticity of the solid inner pipe is substantially larger than that of the carrier pipe ($E_i/E_o = 100$) to account for its rigid behaviour. Deformed shapes of the pipe at different pressure levels are shown in Fig. 5b. Following the initial collapse, the carrier pipe touches the solid insert at configuration (II). Then the pressure is maintained at P_{ps}/P_p and the buckle propagates along the length of the carrier pipe. At configuration (IV), the buckle is extended to the end bulk-heads and the pressure level needs to increase to cause further deformation of the cross-section.

Fig. 3. (a) Pressure versus normalized change in volume for pipe-in-pipe sample in Table 1, (b) deformed configuration of the pipe-in-pipe sample after test, (c) deformed shape of the FE model.

Table 2
Comparison between experimental, numerical and analytical results.

	Experimental	FEA	Analytical
P_p (kPa)	1020	976	578 (Eq. (1))
P_{p2} (kPa)	1620	1504	1165 (Eq. (6a))
	1570		
	1550		
P_{p2}/P_p	1.539	1.541	2.015 (Eq. (6b))

A set of FE analyses are conducted for a range of moderately thick to thin outer pipes ($D_o/t_o = 26.7, 30, 40$) and solid inserts with diameter ratios of ($0.4 < D_s/D_o < 0.75$) and the results are shown in Fig. 6a. Using the Levenberg-Marquardt algorithm of non-linear least squares, best fit of all data from $D_o/t_o = 30, 40$ and 26.7 of Fig. 6a is obtained with a correlation factor of $R^2 = 0.9835$ as:

$$\frac{P_{ps}}{P_p} = 1 + 2.097 \left(\frac{D_s}{D_o} \right)^{2.507} \quad (9)$$

The proposed expression (Eq. (9)) is plotted against reported experimental [15] and numerical [16] empirical equations in Fig. 6b. It should be noted that those studies [15,16] only considered propagation pressure of thick to moderately thick carrier pipes with solid inner pipes. Current expression (Eq. (9)) appears to be in good agreement with that reported in [15] which was derived based on extensive experimental results. With respect to Fig. 6a, the differences between P_{ps} of moderately thin and moderately thick pipes are constant (about 20%) for various D_s/D_o . This suggests that Eq. (9) which is derived based on non-linear least squares be independent of D_o/t_o ratio and thus Eq. (9) and that proposed by Kyriakides [15] yield similar results.

3.2. Propagation buckling modes in pipe-in-pipe systems

In Section 4 of this article, a comprehensive parametric study is conducted using the validated FE model to find the buckle propagation pressures of PIP systems with various wall thickness t_i/t_o , diameter D_i/D_o , and the material yield stress σ_{yi}/σ_{yo} ratios. Prior to reviewing results of the parametric dependency of propagation buckling of PIPs, it is worth discussing the buckling modes observed in the FE simulations. Based on results of FE analyses with various parameters adopted for the outer and inner pipes, two dominant modes of failure under external pressure were observed in the PIPs. The buckling response of a thin PIP with D_o/t_o of 40 is shown in Fig. 7. The thickness ratio is $t_i/t_o = 0.6$ and the materials properties of outer and inner pipes are identical.

By increasing the external pressure, the carrier pipe collapses and gradually deforms from the undeformed shape (I) into the deformed shape (II). At this stage the outer and inner pipes come into contact. Following the touchdown (II), the pressure is slightly increased and the collapse is propagated along the lengths of the outer and inner pipes simultaneously as shown in stages (III) to (IV). The buckle propagation mode shown in Fig. 7 is referred to as Mode A herein. Fig. 8 shows the pressure response and the deformed shape of a moderately thin PIP with D_o/t_o of 30 and t_i/t_o of 0.8. The mechanical properties of the inner and outer pipes are alike. Following the initiation of collapse in the outer pipe, the pressure in the system is dropped and the buckle is propagated in the carrier pipe as shown in deformed shapes of II and III in Fig. 8. The buckle propagation in the outer pipe is eventually arrested by the end-caps as shown in the deformed shape (III). In the vicinity of the end-caps, a higher pressure is required to perpetuate the collapse in the outer pipe. However the increase in pressure causes a collapse in the inner pipe at the pressure level (IV) and initiates a buckle which is propagated through the length (V). This buckle propagation mode is referred to as Mode B in this study.

4. Parametric study on buckle propagation pressure of PIPs

The buckle propagation pressure of the PIP system is related to geometric and material parameters of the outer and inner pipes

$$P_{p2} = F \left(D_o, t_o, \sigma_{yo}, E_o, E'_o, D_i, t_i, \sigma_{yi}, E_i, E'_i \right). \quad (10)$$

In the parametric study carried out herein, both outer and inner pipes are assumed to be of the same material i.e. aluminium (Al-6060-T5) with the same modulus of elasticity (E) and tangent modulus (E'), shown in Table 1 and Poisson's ratio of ($\nu = 0.33$). Gong and Li [16] reported that the strain hardening modulus has little effect on the propagation pressure of PIP systems and is thus not considered in this study. Using the dimensional analysis, Eq. (10) can be written in terms of non-dimensional geometric and material parameters in the following non-dimensional power law format:

$$\frac{P_{p2}}{P_p} = 1 + A_1 \left(\frac{\sigma_{yi}}{\sigma_{yo}} \right)^a \left(\frac{D_i}{D_o} \right)^b \left(\frac{t_i}{t_o} \right)^c \quad (11)$$

Two significant experimental [15] and numerical [16] studies on buckle propagation of PIP systems with carrier pipes of $D_o/t_o < 25$ were carried out and empirical Eqs. (12a) and (12b) were derived respectively. Although both studies [15,16] covered similar D_o/t_o range of the carrier pipes, the empirical expressions suggested in Eqs. (12a) and (12b) are different.

Fig. 4. (a) Compression stub test; (b) ring squash test RST.

Fig. 5. Finite element results showing (a) pressure against normalized ovality and (b) corresponding deformed shapes for a PIP with a solid inner pipe.

Fig. 6. (a) FE results of propagation pressure P_{pi} of PIPs with solid inner pipes; (b) comparison between current expression and those proposed in previous studies.

Fig. 7. Finite element results showing pressure against normalized ovality and corresponding deformed shapes of PIP system exhibiting buckle propagation Mode A.

Fig. 8. Finite element results showing pressure against normalized ovality and corresponding deformed shapes of PIP system exhibiting buckle propagation Mode B.

$$\frac{P_{p2}}{P_p} = 1 + 1.095 \left(\frac{\sigma_{yi}}{\sigma_{yo}} \right)^{0.4} \left(\frac{D_i}{D_o} \right) \left(\frac{t_i}{t_o} \right)^2 \quad (12a)$$

$$\frac{P_{p2}}{P_p} = 1 + 0.970 \left(\frac{\sigma_{yi}}{\sigma_{yo}} \right)^{0.8} \left(\frac{D_i}{D_o} \right)^{0.3} \left(\frac{t_i}{t_o} \right)^2 \quad (12b)$$

Extensive FE simulations are conducted in the following sections to find best estimates for unknown power coefficients of Eq. (11) in PIP systems with thin and moderately thin carrier pipes ($D_o/t_o > 25$). In addition, the raw data from both previous studies [15,16] are combined with the current FE results for PIP system with $D_o/t_o = 26.7$ to establish an empirical expression that covers the buckle propagation pressure of PIPs with thick and moderately thick carrier pipes.

4.1. Effect of D_i/D_o

To investigate the effect of D_i/D_o on the propagation pressure of PIPs, the thickness ratios are varied while the material properties of the two pipes are kept identical. Fig. 9 shows P_{p2}/P_p versus D_i/D_o for two different D_o/t_o ratios of 40 and 30 in four sets of t_i/t_o . The relationship between P_{p2}/P_p and D_i/D_o is linear with positive slope when $t_i/t_o \leq 0.6$, which corresponds to collapse propagation Mode A (see Fig. 7). However, by increasing the thickness ratio and at $t_i/t_o \geq 0.8$, the relationship becomes nonlinear with a decreasing negative slope for both D_o/t_o ratios. The decreasing trend at $t_i/t_o \geq 0.8$ is associated with collapse propagation Mode B (see Fig. 8) and was not reported in the previous studies [15,16].

A comparison between current FE and analytical results and those predicted by Eqs. (12a) and (12b) are depicted in Fig. 10. For D_o/t_o of 40 and t_i/t_o of 0.6 in Fig. 10a, the ascending linear trend obtained from the current FE results is similar to the previous predictions. This was expected because all failure modes are of mode A. However, as shown

Fig. 9. Propagation pressure as a function of D_i/D_o in (a) thin PIPs and (b) moderately thin PIPs.

Fig. 10. Comparison between current FE results and previous studies for (a) thin PIPs and (b) moderately thin PIPs.

in Fig. 10b, for D_o/t_o of 30 and t_i/t_o of 1.0, the previously proposed equations are incapable of predicting the correct propagation pressure of thin to moderately thin PIPs. This is due to the fact that Eqs. (12a) and (12b) are based on buckling mode A only, whereas the failure mode in Fig. 10b is mode B. The analytical results obtained from Eq. (6b) are shown with solid lines and provide upper bounds of P_{p2}/P_p . Similar to the FE results of mode A, the analytical normalized pressures in Fig. 10, increase with the corresponding increase in the diameter ratios. This agrees with the buckle propagation mode (Mode A) assumed in derivation of the analytical solution shown in Fig. 1d–f.

For a fixed value of t_i/t_o and assuming identical yield stresses in the outer and inner pipes, Eq. (11) is reduced to

$$\frac{P_{p2}}{P_p} = 1 + A_2 \left(\frac{D_i}{D_o} \right)^{b_i} \quad (13)$$

Based on the non-linear least-squares fit of eight sets of data in Fig. 9, the power exponents $b_1 = 0.4$ corresponding to failure mode A and $b_2 = -0.8$ corresponding to failure mode B, and the coefficients $A_2 = 0.394$ and 0.168 , respectively are calculated.

4.2. Effect of t_i/t_o

To understand the effect of t_i/t_o on the propagation pressure of PIPs, plots of normalized pressures against thickness ratios for two sets of D_i/D_o are presented in Fig. 11 and the collapse modes are indicated next to the corresponding numerical data. The thickness ratios cover a practical range used in offshore PIPs. Nonlinear ascending relationships are observed for both D_i/D_o ratios. The results show that in PIPs with $D_i/D_o = 0.4$ and at $t_i/t_o < 0.7$, the collapse propagation mode is Mode (A), and in PIP with $D_i/D_o = 0.7$ a distinction between modes A and B is observed at $t_i/t_o = 0.8$. It is also evident from Fig. 11 that at larger thickness ratios, previous expressions (Eqs. (12a) and (12b)) underestimate the propagation pressure of the PIP system significantly. The analytical solution gives the upper bound pressures of buckle propagation mode A. Given the material properties of the outer and inner pipes are

Fig. 11. Comparison between current FE results and previous studies for (a) thin and (b) moderately thin PIPs with different t_i/t_o ratios. The failure modes are labelled for corresponding data point.

identical, Eq. (11) can be written as

$$\frac{P_{p2}}{P_p} = 1 + A_3 \left(\frac{D_i}{D_o} \right)^{b_i} \left(\frac{t_i}{t_o} \right)^c \quad (14)$$

Non-linear least-squares fit of the data in Fig. 11a and b yield the same power exponent $c = 2.4$ and coefficient $A_3 = 0.461$ corresponding to failure mode A and $A_3 = 0.137$ corresponding to failure mode B.

4.3. Effect of σ_{yi}/σ_{yo}

To examine the effect of σ_{yi}/σ_{yo} ratio on the propagation pressure, two values of t_i/t_o with two values of D_o/t_o are assumed and displayed in Fig. 12. Comparison between current FE results and those of Eqs. (12a) and (12b) demonstrates similar trends. To identify the power exponent of σ_{yi}/σ_{yo} , Eq. (11) can be represented as

$$\frac{P_{p2}}{P_p} = 1 + A_4 \left(\frac{\sigma_{yi}}{\sigma_{yo}} \right)^a \quad (15)$$

Based on two different values of t_i/t_o for each value of D_o/t_o , four sets of data are generated. The least squares fit of the data gives the value of power exponent $a = 0.2$ and the coefficient $A_4 = 0.218$. The linear relationship between normalized pressure and yield ratios predicted in the analytical solution (Eq. 6b), provides an upper bound of the FE results.

5. Empirical expressions for P_{p2}

5.1. Empirical expressions for buckle propagation of PIPs with thin and moderately thin carrier pipes

The parametric study carried out in Section 4 ascertained the dependency of the propagation pressure of the PIP systems on geometric and material parameters of the outer and inner pipes. Moreover, cur-

Fig. 12. Propagation pressure as a function of σ_{yi}/σ_{yo} in (a) thin PIPs and (b) moderately thin PIPs the failure modes are labelled for corresponding data point.

rent FE results proved that the buckle propagation modes of PIPs with large D_o/t_o ratios are not essentially similar to mode A predicted in previous studies [15,16]. Since proposed Eqs. (12a) and (12b) are incapable of predicting proper estimates of propagation pressure of PIPs that exhibit buckle propagation mode B, it is sensible to propose expressions for buckle propagation modes A and B separately. Based on the results of the parametric study in the previous section and using non-linear square fits of sets of data taken from the FE results, the following expressions are derived for the propagation pressure of PIPs with thin and moderately thin carrier pipes for buckle propagation modes A and B

$$\begin{aligned} \frac{P_{p2}}{P_p} &= 1 + 1.047 \left(\frac{\sigma_{yi}}{\sigma_{yo}} \right)^{0.2} \left(\frac{D_i}{D_o} \right)^{0.4} \left(\frac{t_i}{t_o} \right)^{2.4} \quad \text{Mode (A)} \end{aligned} \quad (16a)$$

$$\begin{aligned} \frac{P_{p2}}{P_p} &= 1 + 0.596 \left(\frac{\sigma_{yi}}{\sigma_{yo}} \right)^{0.2} \left(\frac{D_i}{D_o} \right)^{-0.8} \left(\frac{t_i}{t_o} \right)^{2.4} \quad \text{Mode (B)} \end{aligned} \quad (16b)$$

The coefficients in Eqs. (16a) and (16b) are determined using the Leven-berg-Marquardt algorithm and correspond to correlation factors (R^2) of 0.9827 and 0.9860 respectively. Comparison between the FE results and the proposed expressions are shown in Fig. 13a and b for buckle propagation modes A and B respectively. The maximum differences between FE results and empirical expressions are less than 6.0%.

Since Eq. (9) is a special case of Eqs. (16a) and (16b), it is meaningful to compare buckle propagation pressure predictions obtained from those expressions. Assuming mode A of buckle propagation, Eq. (16a) can be written as:

$$\frac{P_{p2}}{P_p} = 1 + 1.047 \left(\frac{\sigma_{yi}}{\sigma_{yo}} \right)^{0.2} \left(\frac{D_i}{D_o} \right)^{2.8} \left(\frac{D_o/t_o}{D_i/t_i} \right)^{2.4} \quad (16a-1)$$

Fig. 13. Comparison between FE results and those predicted by Eqs. (16a) and (16b) of buckle propagation pressures of PIP with buckle propagation (a) Mode A; and (b) Mode B.

Assuming outer and inner pipes of similar D/t , and inner pipe with yield stress 100 times larger than the outer pipe ($\sigma_{yi}/\sigma_{yo} = 100$);

$$\frac{P_{ps}}{P_p} = 1 + 1.497 \left(\frac{D_s}{D_o} \right)^{1.6} \quad (16a-s)$$

Using same analogy, Eq. (16b) can be written as:

$$\frac{P_{ps}}{P_p} = 1 + 2.617 \left(\frac{D_s}{D_o} \right)^{2.8} \quad (16b-s)$$

For a range of D_s/D_o between 0.4 and 0.8, the difference between results of P_{ps}/P_p from Eqs. (9), (16a-s) and (16b-s) are less than 10%.

Figs. 14 and 15 show plots of normalized propagation pressures of thin and moderately thin PIP systems with different D_i/D_o and t_i/t_o ratios respectively. Buckle propagation modes A and B are distinguished with different data markers on the figures. Buckle propagation pressures of PIP systems with solid inner pipes (Eq. (9)) are shown with solid lines in Fig. 14. It is clear from Fig. 14 that propagation pressures of PIPs with solid inner pipes yield upper bounds of propagation pressures for mode A. In other words, Eq. (9) can be used to obtain a maximum of Eq. (16a).

Propagation pressures of moderately thick to thin PIP systems with $\sigma_{yi}/\sigma_{yo} = 1.0$ and thickness ratios between 0.3 and 1.2 are presented in Fig. 15. According to the figure the buckle propagation modes A and B correspond to $t_i/t_o < 0.7$ and $t_i/t_o > 0.8$ respectively. It can be deduced that in PIPs with outer and inner pipes of similar material prop-

Fig. 14. Buckle propagation pressures of Mode A and B compared against buckle propagation pressures of PIP with solid inner pipe for (a) $D_o/t_o = 30$, and (b) $D_o/t_o = 40$.

Fig. 15. Buckle propagation pressures of thin to moderately thin PIPs, showing separation between buckle propagation modes A and B.

erties, the separation between buckle propagation modes A and B occurs at thickness ratios between 0.7 and 0.8. Thus, given $\sigma_{yi}/\sigma_{yo} = 1.0$, Eqs. (16a) and (16b) can be used to predict the propagation pressures of PIPs with $t_i/t_o < 0.7$ and $t_i/t_o > 0.8$ respectively.

5.2. Empirical expressions for buckle propagation of PIPs with thick and moderately thick carrier pipes

The expressions (Eqs. (16a) and (16b)) derived in Section 5.1 can be used to predict the propagation pressure of PIP systems with thin and moderately thin carrier pipes. In order to come up with an expression to predict the propagation pressure of PIPs with thick and moderately thick carrier pipes, a total of 254 data points were collected from the raw data reported in [14,16], and the current FE results for PIPs with $D_o/t_o = 26.67$. Using the Levenberg-Marquardt algorithm of non-linear least squares the following expression was derived for the propagation pressure, P_{p2} , of PIPs with $D_o/t_o < 27$

$$\frac{P_{p2}}{P_p} = 1 + 0.803 \left(\frac{\sigma_{yi}}{\sigma_{yo}} \right)^{0.4} \left(\frac{D_i}{D_o} \right)^{0.13} \left(\frac{t_i}{t_o} \right)^{1.8} \quad (17)$$

with multiple correlation factor (R^2) of the fit is 0.9781. To derive Eq. (17), same procedure as explained in derivation of Eqs. (16a) and (16b) is followed and the interaction between non-dimensional variables are incorporated. For sake of brevity, the procedure is not shown here. Current FE results, the FE results of [16] and experimental results of [14] are plotted in Fig. 16 against the proposed expression (Eq. (17)) and form a nice linear band. It should be noted that all of the data points in Fig. 16 and thus those derived from Eq. (17) correspond to buckle propagation mode A.

6. Conclusions

Buckling propagation mechanisms of subsea pipe-in-pipe (PIP) systems under external pressure in quasi-static steady-state conditions were investigated using 2D analytical solutions and 3D FE analyses considering non-linear material and geometric behaviour. The FE results were validated against experimental results of propagation buckling response of a PIP system tested in a hyperbaric chamber. Using the validated FE model a parametric study was conducted and two major buckle propagation modes in PIPs with thin and moderately thin carrier pipes were observed. In Mode A the buckle propagated simultaneously in the outer and inner tubes and in Mode B the buckle propagated in the outer pipe and the collapse in the inner pipe was delayed.

It was shown that compared to FE results, the analytical solution provides upper bound for normalized buckle propagation pressures (P_{p2}/P_p) of the PIP systems. Based on the FE results, a new expression for the propagation buckling pressure of pipe with a solid inner pipe was proposed (Eq. (9)). It was found that the proposed expression provides a lower bound for buckling pressure of PIPs in mode B. Current FE results of propagation pressures of thin PIPs demonstrated a nonlinearly decreasing trend in the normalized pressure with corresponding

Fig. 16. Comparison between buckle propagation pressures of thick to moderately thick PIP systems from previous studies and current expression. (All results correspond to the buckle propagation mode A).

increase in D_i/D_o ratios. This behaviour is associated with buckle propagation mode B and was not reported in the previous studies. The FE results showed that in PIPs with outer and inner pipes of the same modulus of elasticity and yield stress, mode A occurs when $t_i/t_o < 0.7$ and mode B happens at $t_i/t_o > 0.7$. Two separate expressions (Eqs. (16a) and (16b)) for buckle propagation pressures in modes A and B of PIP systems with thin and moderately thin carrier pipes are proposed. Based on the combined data from previous studies and current FE results, a more comprehensive empirical expression (Eq. (17)) was proposed to predict the propagation pressure P_{p2} of PIPs with thick and moderately thick carrier pipes.

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