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Distributed H_∞ -consensus filtering for target tracking over a wireless filter network with switching topology, channel fading and packet dropouts

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Abstract

This paper is concerned with the problem of distributed H_∞ -consensus filtering for target tracking over a wireless filter network. The wireless filter network consists of a large number of filter nodes. The network topology is supposed to be switching or changeable over time. Data communication between filter nodes is subject to some network-induced constraints including packet dropouts and channel fading. Different from some existing results, packet dropouts and channel fading are assumed to occur not only in the measurement process for sensors but also in the transmission process among filter nodes. Within such settings, a distributed H_∞ -consensus filtering method is developed to ensure both the estimation accuracy of filters and the robustness of the filtering error system against network-induced constraints. Criteria on designing desired distributed H_∞ filters are derived in terms of a set of linear matrix inequalities. Numerical examples are given to verify the effectiveness of the proposed target tracking filtering algorithm.

Keywords: distributed H_∞ -consensus estimation, target tracking, wireless filter network, switching network topology, packet dropouts, channel fading

1. Introduction

Tracking of a maneuvering target is one of most important applications of a wireless filter network (WFN) that is composed of a number of filters widely distributed in a large monitoring area [1–4]. The WFN possesses many significant advantages such as high tracking accuracy, easy installation and maintenance and low cost [5–7]. However, within communication channels of a WFN, unreliable communication phenomena (e.g., limited bandwidth, energy and communication capacity as well as the masking phenomenon of obstacles) inevitably cause some communication constraints [8–10]. For example, packet losses are well known to be a kind of communication constraints [11–15]. Another common communication constraint is channel fading [16–19]. It should be noted that most existing results suppose that packet losses and channel fading may occur only in the measurement process. Nevertheless, such an assumption cannot better reflect the practical scenarios in networked systems. In this paper, packet losses and channel fading are assumed to occur randomly not only in the data measurement process for sensors but also in the data transmission process among filters over different communication channels. On the other hand, recalling some existing results on distributed filtering, the filter network topology is usually assumed to be fixed. However, during data communication, due to limited communication energy of communication channels, failures or creations of links of filter nodes in the filter network may occur in a random way, which means that the network topology may switch/change over time [20, 21].

To deal with the issue of target tracking subject to network-induced constraints, the distributed filtering strategy without requiring the centralized node is adopted [22–27], where each filter estimates the system states on the basis of information not only from its corresponding sensor but also from its local adjacent filters. Such an approach is of higher estimation accuracy, better reliability and stronger robustness if compared with the centralized filtering approach [28]. Notice that the network-induced communication constraints might cause the estimation deviations between the neighboring filters. To reduce such estimation disagreements, various distributed consensus-based filtering

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algorithms have been proposed [23–27, 29–31]. It is worth pointing out that most existing distributed consensus-based filtering algorithms have been proposed by using classical Kalman filtering techniques [23, 24]. Kalman filtering is based on a common assumption that priori statistical information on external disturbances/noises should be known. However, in the presence of network-induced communication constraints, this assumption does not always hold. As such, H_∞ filtering, which does not need to know the exact priori information on the external disturbances/noises [32], becomes an effective alternative approach [25, 26, 33, 34]. For example, in [25], a threshold-parameter-dependent approach has been developed to handle the distributed H_∞ -consensus filtering issue over a sensor network in an event-triggered communication context. In [26], the robust distributed H_∞ -consensus filtering algorithm has been proposed to address the filtering problem in which packet dropouts occur in the measurement process. In [34], an event-triggered distributed H_∞ filtering problem is solved over a fixed sensor network subject to communication delays. However, to the best of the authors' knowledge, little attention has been paid to the distributed H_∞ -consensus target tracking filter design over the WFN with switching network topology as well as both channel fading and packet dropouts occurring randomly not only in the measurement process for sensors but also in the transmission process among filters, which serves as the motivation of the current study.

In this paper, we aim to deal with the target tracking problem over a multi-filter network subject to the simultaneous presence of random switching network topology, channel fading and packet dropouts. The main contributions of this paper are described as follows: (i) A novel target tracking model is constructed to account for the coexistence of switching network topology, channel fading and packet dropouts. Such a model reflects a practical scenario of the networked filtering system more closely; and (ii) A new distributed H_∞ -consensus filtering algorithm for solving the target tracking problem is presented. This algorithm can ensure both estimation accuracy of the filters and robustness of the filtering system with respect to the considered network-induced effects.

Notations: $\|x\|$ denotes the l_2 norm of the vector x , i. e., $\|x\| = (x^T x)^{1/2}$ where “ T ” represents the transpose of the vector x . Specifically, for a given $R = R^T$, $\|x\|_R = \sqrt{x^T R x}$. $Prob\{\cdot\}$ stands for the occurrence probability of the corresponding event. $E\{\cdot\}$ denotes the mathematical expectation operator. $\mathbb{R}^n$ and $\mathbb{R}^{n \times n}$ respectively represent the n -dimensional Euclidean space and the set of all $n \times n$ real matrices. In symmetric block matrices, the term determined by symmetry is denoted as the “*”. $diag\{\cdot\}$ stands for a block-diagonal matrix or a diagonal matrix. The vector of all entries being one is represented by the boldface character $\mathbf{1}$. I denotes the identity matrix with appropriate dimensions.

2. Problem formulation

In this paper, we will design a novel target tracking filtering model, formulate the distributed H_∞ -consensus target tracking filtering problem and construct the distributed H_∞ -consensus filter for addressing the target tracking filtering problem on the network induced communication constraints.

The dynamics of the moving target are assumed to be modeled by a discrete-time linear-invariant state-space equation of the form

$$x(k+1) = \mathcal{A}x(k) + \mathcal{B}v(k), \quad (1)$$

where $x(k) \in \mathbb{R}^{n_x}$ is the state vector; $\mathcal{A}$ and $\mathcal{B}$ are the known matrices with appropriate dimensions; it is assumed that the system disturbance $v(k) \in l_2[0, \infty)$, and $\|v\|_2^2 \leq v_{max} < \infty$. The i th sensor measurement output is as follows

$$\bar{y}_i(k) = C_i x(k) + \mathcal{D}_i \omega_i(k), \quad (i = 1, 2, \dots, N), \quad (2)$$

where $\bar{y}_i(k) \in \mathbb{R}^{p_y}$; C_i , and $\mathcal{D}_i$ are known real-valued constant matrices with appropriate dimensions; $\omega_i(k) \in \mathbb{R}^{n_\omega}$ denotes the noise signal of the output measurement at the i th sensor which is assumed to belong to $l_2[0, \infty)$.

In practice, the channel fading as well as the packet losses may arise in the data measurement process for i th sensor, simultaneously. Thus, according to Remark 1, (2) can be described as

$$y_i(k) = \sum_{\varrho=0}^{b_k} \xi_i^\varrho(k) \bar{y}_i(k - \varrho), \quad (\varrho = 1, 2, \dots, L; b_k = \min\{L, k\}), \quad (3)$$

where $y_i(k) \in \mathbb{R}^{p_y}$; $\bar{y}_i(k)$ is defined in (2); L is the order of the Rice fading model; $\xi_i^\varrho(k)$ represents a Bernoulli-distributed white sequence, and $Prob\{\xi_i^\varrho(k) = 1\} = 1 - Prob\{\xi_i^\varrho(k) = 0\} = \bar{\xi}_i^\varrho$.

Remark 1. In this paper, the values 0 and 1 taken by the Bernoulli distributed random variables respectively represent failure and success not only in the data measurement for the sensors but also in the data transmission between the neighboring filters at any time. When a signal is transmitted over a wireless communication channel, some special physical phenomena such as the shadowing effects from obstacles, the multi-path propagation and the path loss have a great impact on the signal power, which may lead to signal fading. In practice, since the channel coefficients of the L th-order Rice fading model typically take values in the interval $[0, 1]$, and thus such a model can characterize the effects of both packet dropouts and channel fading arising not only in the data measurement for the sensors but also in the data transmission among the filters. Thus, in this paper, the measurement information for the sensor/the transmission information for the filter is actually a weighted sum of the information from paths of the channel for occurring both packet dropouts and different time delays where the weights are random variables (channel coefficients of the L th-order Rice fading model are assumed to be random variables) taking values on the interval $[0, 1]$. Moreover, the measurement information for the sensors/the transmission information among the filters includes the traditional packet losses and the stochastic communication delays as special cases (e.g., $b = 0$ indicates packet dropouts with no time delay, and $b = 1$ means the simultaneous occurrence of the packet dropouts and the one-step communication delay).

Since the distributed filtering framework is taken into account, the target (1) is converted into

$$\begin{cases} x(k+1) = \mathcal{A}x(k) + \mathcal{B}v(k), & x(0) = x_0, \\ y_i(k) = \sum_{\varrho=0}^{b_k} \xi_i^{\varrho}(k) \bar{y}_i(k-\varrho), & (\varrho = 1, 2, \dots, L; b_k = \min\{L, k\}). \end{cases} \quad (4)$$

In the WFN, the data exchanges among filters can be described via a directed graph $\mathcal{S} = (\mathcal{Y}, \mathcal{J})$, where $\mathcal{Y} = \{1, 2, \dots, N\}$ and $\mathcal{J} \subseteq \mathcal{Y} \times \mathcal{Y}$ stand for the set of the filter nodes as well as the set of the edges, respectively. The set of neighbors of filter i plus the node itself is denoted by $\mathcal{X}_i = \{j \in \mathcal{Y} : (i, j) \in \mathcal{J}\}$. Let $\mathcal{O} = [a_{ij}] \in \mathbb{R}^{n \times n}$ be the nonnegative adjacency matrix with

$$a_{ij} = \begin{cases} 0, & (j, i) \notin \mathcal{X}_i, \\ 1, & (j, i) \in \mathcal{X}_i, \end{cases}$$

where $(j, i) \in \mathcal{X}_i$ represents a directed edge which means that there is a communication link from i to j . Also, it is also assumed that the filters have no self-connected edge. If $(j, i) \in \mathcal{X}_i$, then i filter is called one of the neighboring filters of j filter. In the distributed filtering framework, each filter transfers information from its local neighboring filters besides measurement information from the corresponding sensor on the basis of switching filter network topology. The information received by the j filter is transmitted via unreliable communication channels, and therefore might encounter the phenomena of the channel fading and random packet dropouts. Moreover, the data transmission is affected by the channel noises. Based on the Remark 1, the transmission information from j th filter received by i th filter can be described as

$$\hat{y}_{ij}(k) = \sum_{\varrho=0}^{b_k} \xi_j^{\varrho}(k) U_{ij} \hat{x}_j(k-\varrho) + T_{ij} v_{ij}(k), \quad (i = 1, 2, \dots, N; j \in \mathcal{X}_{ij}^{g_k}; b_k = \min\{L, k\}), \quad (5)$$

where $\hat{y}_{ij}(k) \in \mathbb{R}^p$; $v_{ij}(k) \in \mathbb{R}^{\hat{w}}$ is the random channel noise among the filters; U_{ij} , and T_{ij} are known constant matrices with appropriate dimensions. Bernoulli-distributed stochastic variables $\xi_j^{\varrho}(k)$, and $\xi_i^{\varrho}(k)$ are independent of each other.

It is assumed that $\text{Prob}\{\xi_j^{\varrho}(k) = 1\} = 1 - \text{Prob}\{\xi_j^{\varrho}(k) = 0\} = \bar{\xi}_j^{\varrho}$. Since white noise is a common type of channel noise, $v_{ij}(k)$ represents the mean-square l_2 summable random channel noise, and it is satisfied that $\mathbb{E}[v_{ij}(k)] = 0$. The channel noise $v_{ij}(k)$ and the random variables $\xi_j^{\varrho}(k)$ and $\xi_i^{\varrho}(k)$ are independent for all $k \geq 0$.

According to both the measurement output of the system (4) and the information exchanging of among the filters in (5), the full-order distributed consensus-based filter can be designed as follows

$$\hat{x}_i(k+1) = \mathcal{A} \hat{x}_i(k) + \mathcal{L}_i^{g_k} \left(y_i(k) - \sum_{\varrho=0}^{b_k} \xi_i^{\varrho}(k) C_i \hat{x}_i(k-\varrho) \right) + \sum_{j \in \mathcal{X}_i^{g_k}} \mathcal{K}_{ij}^{g_k} \left(\hat{y}_{ij}(k) - \sum_{\varrho=0}^{b_k} \xi_j^{\varrho}(k) U_{ij} \hat{x}_i(k-\varrho) \right), \quad (6)$$

where $\hat{x}_i(k) \in \mathbb{R}^n$ is the estimation of the i th local filter; $\mathcal{K}_{ij}^{g_k}$ and $\mathcal{L}_i^{g_k}$ are parameters of the filter i which are all

to be determined; Let g_k ($k \geq 0$) be a Markov chain taking values in a finite set $g_k \in \mathfrak{S} = \{1, 2, \dots, f_0\}$ with the probability transfer matrix $\Pi(k) = (\pi^{ab}(k))_{f_0 \times f_0}$ given by $\text{Prob}\{g_{k+1} = w \mid g_k = q\} = \pi^{qw}(k)$, $\forall a, b \in \mathfrak{S}$. $v_{ij}(\cdot)$ in (5) is independent of the Markov chain g_k . The considered topology evolves over time which is assumed to be subject to a nonhomogeneous Markov chain. Assume that $\Pi(k)$ evolves in a polyhedron with z vertices Π^z , where $\Pi^z = [(\pi^{cd})^z]$, ($z = 1, 2, \dots, z_0$) are given matrices. Particularly, assume that

$$\Pi(k) = \sum_{z=1}^{z_0} \tilde{h}_z(k) \Pi^z,$$

where $\tilde{h}_z(k) \geq 0$, and $\sum_{z=1}^{z_0} \tilde{h}_z(k) = 1$. Let $\bar{\pi}^{cd} = \max\{(\pi^{cd})^z\}$.

Let $e_i(k) = x(k) - \hat{x}_i(k)$ be local estimation error at i th filter. It is worth noting that the time evolution of communication graphs of the WFN is modeled by a stochastic graph-valued process $\mathcal{S}^{g_k}$ with the same set of filters as $\mathcal{S}$. In terms of such a process, the adjacent matrix is prescribed as $\mathcal{O}^{g_k} = [a_{ij}^{g_k}] \in \mathbb{R}^{n \times n}$ with

$$a_{ij}^{g_k} = \begin{cases} 0, & (j, i) \notin \mathcal{X}_i, \\ 1, & (j, i) \in \mathcal{X}_i. \end{cases}$$

By (4) and (6), the local estimation errors evolve in terms of

$$\begin{aligned} e_i(k+1) &= \mathcal{A}e_i(k) + \mathcal{B}v(k) - \mathcal{L}_i^{g_k} \sum_{\varrho=0}^{b_k} \xi_i^{\varrho}(k) (C_i e_i(k-\varrho) + \mathcal{D}_i \omega_i(k-\varrho)) \\ &\quad - \sum_{j \in \mathcal{X}_i^{g_k}} \mathcal{K}_{ij}^{g_k} (\sum_{\varrho=0}^{b_k} \xi_j^{\varrho}(k) U_{ij} (e_i(k-\varrho) - e_j(k-\varrho) + T_{ij} v_{ij}(k))) \\ &= \mathcal{A}e_i(k) + \mathcal{B}v(k) - \mathcal{L}_i^{g_k} \sum_{\varrho=0}^{b_k} \xi_i^{\varrho}(k) (C_i e_i(k-\varrho) + \mathcal{D}_i \omega_i(k-\varrho)) \\ &\quad - \sum_{j=1}^N a_{ij}^{g_k} \mathcal{K}_{ij}^{g_k} (\sum_{\varrho=0}^{b_k} \xi_j^{\varrho}(k) U_{ij} (e_i(k-\varrho) - e_j(k-\varrho))) + \Theta_i^{g_k}, \end{aligned} \quad (7)$$

where $\Theta_i^{g_k} \triangleq \sum_{j=1}^N a_{ij}^{g_k} \mathcal{K}_{ij}^{g_k} T_{ij} v_{ij}(k)$.

For filter (6) of system (4), the deviation function is defined by [36]

$$\Psi^{g_k}(\hat{x}(k)) = \frac{1}{N} \sum_{i=1}^N \sum_{j \in \mathcal{N}_i^{g_k}} \|\hat{x}_j(k) - \hat{x}_i(k)\|^2. \quad (8)$$

Such a function stands for the average of the total consensus-based estimation deviations between the filters.

The local estimation error dynamics in (7) can be converted to the following compact form:

$$\begin{aligned} e(k+1) &= (I \otimes \mathcal{A})e(k) - \sum_{\varrho=0}^{b_k} (\bar{\mathcal{L}}\Lambda_{\bar{\xi}^{\varrho}} - (\bar{\mathcal{L}}\Lambda_{\bar{\xi}^{\varrho}} - \bar{\mathcal{L}}(k)\Lambda_{\xi^{\varrho}(k)}))Ce(k-\varrho) - \sum_{\varrho=0}^{b_k} (\bar{\mathcal{K}} - (\bar{\mathcal{K}} - \bar{\mathcal{K}}(k)))e(k-\varrho) \\ &\quad + (I \otimes \mathcal{B})v(k) - \sum_{\varrho=0}^{b_k} (\bar{\mathcal{L}}\Lambda_{\bar{\xi}^{\varrho}} - (\bar{\mathcal{L}}\Lambda_{\bar{\xi}^{\varrho}} - \bar{\mathcal{L}}(k)\Lambda_{\xi^{\varrho}(k)}))\mathcal{D}\omega(k-\varrho) - \Theta^{g_k}, \quad e(0) = \mathbf{1} \otimes x_0, \end{aligned} \quad (9)$$

where

$$\begin{aligned} \bar{\mathcal{K}} &= \bar{\mathcal{K}} + \bar{\mathcal{K}}, \quad \bar{\mathcal{K}}(k) = \bar{\mathcal{K}}(k) + \bar{\mathcal{K}}(k), \quad \Lambda_{\xi^{\varrho}(k)} = \text{diag}\{\xi_1^{\varrho}(k), \xi_2^{\varrho}(k), \dots, \xi_N^{\varrho}(k)\}, \quad \Lambda_{\bar{\xi}^{\varrho}} = \text{diag}\{\bar{\xi}_1^{\varrho}, \bar{\xi}_2^{\varrho}, \dots, \bar{\xi}_N^{\varrho}\}, \\ C &= \text{diag}\{C_1, C_2, \dots, C_N\}, \quad D = \text{diag}\{\mathcal{D}_1, \mathcal{D}_2, \dots, \mathcal{D}_N\}, \quad \bar{\mathcal{L}}(k) = \text{diag}\{\bar{\mathcal{L}}_1^{g_k}(k), \bar{\mathcal{L}}_2^{g_k}(k), \dots, \bar{\mathcal{L}}_N^{g_k}(k)\}, \\ \bar{\mathcal{L}} &= \text{diag}\{\bar{\mathcal{L}}_1^{g_k}, \bar{\mathcal{L}}_2^{g_k}, \dots, \bar{\mathcal{L}}_N^{g_k}\}, \quad v(k) = [v_1^T(k), v_2^T(k), \dots, v_N^T(k)]^T, \quad \Theta^{g_k} = [(\Theta_1^{g_k})^T, (\Theta_2^{g_k})^T, \dots, (\Theta_N^{g_k})^T]^T, \\ e(k-\varrho) &= [e_1^T(k-\varrho), e_2^T(k-\varrho), \dots, e_N^T(k-\varrho)]^T, \quad \omega(k-\varrho) = [\omega_1^T(k-\varrho), \omega_2^T(k-\varrho), \dots, \omega_N^T(k-\varrho)]^T, \end{aligned}$$

with $\Theta_i^{g_k}$ ($i = 1, 2, \dots, N$) defined in (7) and

$$\begin{aligned} \bar{\mathcal{K}} &= [\bar{\mathcal{K}}_{ij}]_{N \times N}, \quad \bar{\mathcal{K}}(k) = [\bar{\mathcal{K}}_{ij}(k)]_{N \times N}, \quad \bar{\mathcal{K}}(k) = [\bar{\mathcal{K}}_{ij}(k)]_{N \times N}, \quad \bar{\mathcal{K}} = [\bar{\mathcal{K}}_{ij}]_{N \times N}, \\ \bar{\mathcal{K}}_{ij} &= -a_{ij}^{g_k} \mathcal{K}_{ij}^{g_k} \bar{\xi}_j^0 U_{ij}, \quad (j \neq i; i, j = 1, 2, \dots, N), \quad \bar{\mathcal{K}}_{ii} = \sum_{j \neq i, j=1}^N a_{ij}^{g_k} \mathcal{K}_{ij}^{g_k} \bar{\xi}_j^0 U_{ij}, \quad (i = 1, 2, \dots, N), \\ \bar{\mathcal{K}}_{ij}(k) &= -a_{ij}^{g_k} \mathcal{K}_{ij}^{g_k} \bar{\xi}_j^0(k) U_{ij}, \quad (j \neq i; i, j = 1, 2, \dots, N), \quad \bar{\mathcal{K}}_{ii}(k) = \sum_{j \neq i, j=1}^N a_{ij}^{g_k} \mathcal{K}_{ij}^{g_k} \bar{\xi}_j^0(k) U_{ij}, \quad (i = 1, 2, \dots, N), \end{aligned}$$

$$\begin{aligned}\bar{\mathcal{K}}_{ij} &= - \sum_{\varrho=1}^{b_k} \alpha_{ij}^{g_k} \mathcal{K}_{ij}^{g_k} (\tilde{\xi}_j^{\varrho} - \xi_j^{\varrho}(k)) U_{ij}, (j \neq i; i, j = 1, 2, \dots, N), \\ \bar{\mathcal{K}}_{ii} &= - \sum_{j \neq i, j=1}^N \sum_{\varrho=1}^{b_k} \alpha_{ij}^{g_k} \mathcal{K}_{ij}^{g_k} (\tilde{\xi}_j^{\varrho} - \xi_j^{\varrho}(k)) U_{ij}, (i = 1, 2, \dots, N), \\ \bar{\mathcal{K}}_{ij}(k) &= - \sum_{\varrho=1}^{b_k} \alpha_{ij}^{g_k} \mathcal{K}_{ij}^{g_k}(k) (\tilde{\xi}_j^{\varrho}(k) - \xi_j^{\varrho}(k)) U_{ij}, (j \neq i; i, j = 1, 2, \dots, N), \\ \bar{\mathcal{K}}_{ii}(k) &= - \sum_{j \neq i, j=1}^N \sum_{\varrho=1}^{b_k} \alpha_{ij}^{g_k} \mathcal{K}_{ij}^{g_k}(k) (\tilde{\xi}_j^{\varrho}(k) - \xi_j^{\varrho}(k)) U_{ij}, (i = 1, 2, \dots, N).\end{aligned}$$

The distributed H_∞ -consensus target tracking filtering problem regarding the network induced communication constraints to be handled is then described as follows: For known scalars $\epsilon_0 > 0$ with $\epsilon_0 \in (0, 1)$, and $\lambda_1 = \max\{\lambda_{\min}(P^{g_k})\}$ such that

(i) In the absence of the external disturbances/noises, the system (10) is stable such that

$$\mathbb{E}^{g_0} \{ \|e(k)\|^2 \} \leq V(0) = e^T(0) P^{g_0} e(0) \leq \frac{\lambda_{\max}(P^{g_0})}{\lambda_1} \epsilon_0^k \|e(0)\|^2;$$

(ii) In the presence of the external noises/disturbances, the estimation errors of the filters (6) satisfy the following mean-square H_∞ -consensus performance:

$$\mathbb{E}^{g_0} \left\{ \sum_{k=0}^{\infty} \sum_{\varrho=0}^{b_k} \Psi^{g_k}(e(k - \varrho)) \right\} \leq \gamma \left(\|x_0\|_R^2 + \sum_{\varrho=0}^{b_k} \|\bar{\omega}(k - \varrho)\|_2^2 + \|\bar{v}(k)\|_2^2 + \gamma \sum_{i=1}^N \sum_{j=1}^N \mathbb{E}^{g_0} \{ \|\alpha_{ij}^{g_k} v_{ij}(k)\|_2^2 \} \right)$$

where γ is a positive constant.

3. Distributed H_∞ consensus-based filtering performance analysis

This section will establish a criterion of the performance analysis about the constructed filtering error system (9) by constructing a Lyapunov-Krasovskii functional. In order to facilitate performance analysis, a necessary Lemma is introduced firstly.

Lemma 1. Let $p_i^{g_k}$ and $q_i^{g_k}$ be the in-degree and out-degree of i th filter, respectively, and $\phi^{g_k} = \text{diag}\{p_1^{g_k} + q_1^{g_k}, p_2^{g_k} + q_2^{g_k}, \dots, p_N^{g_k} + q_N^{g_k}\}$, then,

$$\Psi^{g_k}(\hat{x}(k)) = \Psi^{g_k}(e(k)) = \frac{1}{N} e^T(k) ((\phi^{g_k} - 2O^{g_k}) \otimes I) e(k) = \frac{1}{N} e^T(k) ((\phi^{g_k} - O^{g_k} - (O^{g_k})^T) \otimes I) e(k).$$

Theorem 1. Let $g_k \in \mathbb{S}$ be the Markov chain, and γ be a given constant scalar. Assume that the local filter's gains $\mathcal{L}_i^{g_k}$ and the coupling gains $\mathcal{K}_{ij}^{g_k}$ ($i = 1, 2, \dots, N; j \in \mathcal{X}_{ij}^{g_k}$) be known constant matrices. Then, for $m = 1, 2, \dots, m_0$, the distributed filtering error dynamics in (10) is exponentially stable in mean square, and the H_∞ -consensus disturbance rejection attenuating performance level γ with $R = \frac{1}{N} \sum_{i=1}^N P_i^{g_0}$ is achieved, if there exist positive definite symmetric matrices $P^{g_k} = \text{diag}\{P_1^{g_k}, P_2^{g_k}, \dots, P_N^{g_k}\}$, and $Q^{g_k} = \text{diag}\{Q_1^{g_k}, Q_2^{g_k}, \dots, Q_N^{g_k}\}$ in such a way that

$$\begin{bmatrix} -P_i^{g_k} & \alpha_{ij}^{g_k} \mathcal{L}_i^{g_k} \mathcal{K}_{ij}^{g_k} T_{ij}^{g_k} \\ \alpha_{ij}^{g_k} T_{ij}^{g_k} \mathcal{K}_{ij}^{g_k} \mathcal{L}_i^{g_k} & -\gamma I \end{bmatrix} < 0, \quad (10)$$

$$\sum_{\varrho=0}^{b_k} (\Phi^{g_k - \varrho})^z = (\Phi^{g_k})^z + \sum_{\varrho=1}^{b_k} (\Phi^{g_k - \varrho})^z < 0, \quad (11)$$

where

$$(\Phi^{g_k})^z = \begin{bmatrix} -P^{g_k} + \Xi^{g_k} + T_0 & * & * & * & * \\ 0 & -I & * & * & * \\ 0 & 0 & -I & * & * \\ T_1 & T_2 & T_3 & -(P^{g_k})^z & * \\ T_4 & 0 & T_5 & 0 & -(P^{g_k})^z \end{bmatrix}, \quad (\Phi^{g_{k-\varrho}})^z = \begin{bmatrix} \Xi^{g_{k-\varrho}} - Q^{g_k} & * & * & * & * \\ 0 & 0 & * & * & * \\ 0 & 0 & -I & * & * \\ T_6 & 0 & T_7 & -(P^{g_k})^z & * \\ T_8 & 0 & T_9 & 0 & -(P^{g_k})^z \end{bmatrix},$$

with

$$\begin{aligned} T_0 &= (-1 + b_k(\bar{\varrho}_M - \bar{\varrho}_z))Q, & T_1 &= (P^{g_k})^z(I \otimes \mathcal{A} - \bar{\mathcal{L}}\Lambda_{\bar{\xi}^0}C - \bar{\mathcal{K}}), & T_2 &= (P^{g_k})^z(\mathbf{1} \otimes \mathcal{B}), \\ T_3 &= -(P^{g_k})^z(\bar{\mathcal{L}}\Lambda_{\bar{\xi}^0}\mathcal{D}), & T_4 &= (P^{g_k})^z(\bar{\mathcal{L}}\Lambda_{\bar{\xi}^0(1-\Lambda_{\bar{\xi}^0})}C + \bar{\mathcal{K}}), & T_5 &= (P^{g_k})^z(\bar{\mathcal{L}}\Lambda_{\bar{\xi}^0(1-\Lambda_{\bar{\xi}^0})}\mathcal{D}), \\ T_6 &= -\sum_{\varrho=1}^{b_k} (P^{g_k})^z(\bar{\mathcal{L}}\Lambda_{\bar{\xi}^{\varrho}}C + \bar{\mathcal{K}}), & T_7 &= -\sum_{\varrho=1}^{b_k} (P^{g_k})^z(\bar{\mathcal{L}}\Lambda_{\bar{\xi}^{\varrho}}\mathcal{D}), & T_8 &= \sum_{\varrho=1}^{b_k} (P^{g_k})^z(\bar{\mathcal{L}}\Lambda_{\bar{\xi}^{\varrho}(1-\Lambda_{\bar{\xi}^{\varrho}})}C + \bar{\mathcal{K}}), \\ T_9 &= \sum_{\varrho=1}^{b_k} (P^{g_k})^z(\bar{\mathcal{L}}\Lambda_{\bar{\xi}^{\varrho}(1-\Lambda_{\bar{\xi}^{\varrho}})}\mathcal{D}), & \Lambda_{\bar{\xi}^{\varrho}(1-\bar{\xi}^{\varrho})} &= \mathbb{E}[(\Lambda_{\bar{\xi}^{\varrho}} - \Lambda_{\bar{\xi}^{\varrho}(k)})^2] = \text{diag}\{\Lambda_{\bar{\xi}_1^{\varrho}(1-\bar{\xi}_1^{\varrho})}, \dots, \Lambda_{\bar{\xi}_N^{\varrho}(1-\bar{\xi}_N^{\varrho})}\}, \\ \Xi^{g_k} &= \frac{1}{\gamma}(\phi^{g_k} - \mathcal{O}^{g_k} - (\mathcal{O}^{g_k})^T) \otimes I, & \Lambda_{\bar{\xi}^{\varrho}(1-\bar{\xi}^{\varrho})} &= \mathbb{E}[(\Lambda_{\bar{\xi}^{\varrho}} - \Lambda_{\bar{\xi}^{\varrho}(k)})^2] = \text{diag}\{\Lambda_{\bar{\xi}_1^{\varrho}(1-\bar{\xi}_1^{\varrho})}, \dots, \Lambda_{\bar{\xi}_N^{\varrho}(1-\bar{\xi}_N^{\varrho})}\}. \end{aligned}$$

Moreover, $\bar{\mathcal{L}}, \bar{\mathcal{K}}$ are defined by (9) and $\bar{\bar{\mathcal{K}}} = \bar{\mathcal{K}} + \bar{\mathcal{K}}, \bar{\mathcal{K}} = [\bar{\mathcal{K}}_{ij}]_{N \times N}, \bar{\bar{\mathcal{K}}} = [\bar{\bar{\mathcal{K}}}_{ij}]_{N \times N}$, where

$$\begin{aligned} \bar{\mathcal{K}}_{ii} &= \sum_{j \neq i, j=1}^N \alpha_{ij}^{g_k} \mathcal{K}_{ij}^{g_k} (\bar{\xi}_j^0 (1 - \bar{\xi}_j^0)) U_{ij}; (i = 1, 2, \dots, N), & \bar{\mathcal{K}}_{ij} &= -\alpha_{ij}^{g_k} \mathcal{K}_{ij}^{g_k} (\bar{\xi}_j^0 (1 - \bar{\xi}_j^0)) U_{ij}, (i \neq j, \{i, j\} = \{1, 2, \dots, N\}), \\ \bar{\bar{\mathcal{K}}}_{ii} &= \sum_{j \neq i, j=1}^N \sum_{\varrho=1}^{b_k} \alpha_{ij}^{g_k} \mathcal{K}_{ij}^{g_k} (\bar{\xi}_j^{\varrho} (1 - \bar{\xi}_j^{\varrho})) U_{ij}; (i = 1, 2, \dots, N), & \bar{\bar{\mathcal{K}}}_{ij} &= -\sum_{\varrho=1}^{b_k} \alpha_{ij}^{g_k} \mathcal{K}_{ij}^{g_k} (\bar{\xi}_j^{\varrho} (1 - \bar{\xi}_j^{\varrho})) U_{ij}, (i \neq j, \{i, j\} = \{1, 2, \dots, N\}). \end{aligned}$$

Proof. For simplifying notation, we denote $\eta(k) = [e^T(k) \quad v^T(k) \quad \omega^T(k)]^T$, and

$$\begin{aligned} \mathcal{H}^{g_k} &= [I \otimes \mathcal{A} - \bar{\mathcal{L}}\Lambda_{\bar{\xi}^0}C - \bar{\mathcal{K}} \quad \mathbf{1} \otimes \mathcal{B} \quad -\bar{\mathcal{L}}\Lambda_{\bar{\xi}^0}\mathcal{D}], \\ \tilde{\mathcal{H}}^{g_k} &= [\bar{\mathcal{L}}\Lambda_{\bar{\xi}^0(1-\bar{\xi}^0)}C + \bar{\mathcal{K}} \quad 0 \quad \bar{\mathcal{L}}\Lambda_{\bar{\xi}^0(1-\bar{\xi}^0)}\mathcal{D}], \\ \mathcal{H}^{g_{k-\varrho}} &= [-\bar{\mathcal{L}}\Lambda_{\bar{\xi}^{\varrho}}C - \bar{\mathcal{K}} \quad 0 \quad -\bar{\mathcal{L}}\Lambda_{\bar{\xi}^{\varrho}}\mathcal{D}], \\ \tilde{\mathcal{H}}^{g_{k-\varrho}} &= [\bar{\mathcal{L}}\Lambda_{\bar{\xi}^{\varrho}(1-\bar{\xi}^{\varrho})}C + \bar{\mathcal{K}} \quad 0 \quad \bar{\mathcal{L}}\Lambda_{\bar{\xi}^{\varrho}(1-\bar{\xi}^{\varrho})}\mathcal{D}]. \end{aligned}$$

To prove that the distributed estimation error dynamics (10) is exponentially stable in mean square and satisfies the H_∞ -consensus performance requirements, the Lyapunov-Krasovskii functional [37, 38] is constructed

$$V(e(k+1), g_{k+1}, k+1) = \sum_{i=1}^3 V_i(k). \quad (12)$$

This paper assumes that the nonhomogenous Markov chain takes values at instant $k+1$ under the condition that it takes a given constant value ($g_k \in \mathfrak{S}$) at instant k . By calculating the mathematical expectation of the difference of $V(k)$ [39, 40], it turns out that

$$\mathbb{E}\{V(e(k+1), g_{k+1}, k+1) - V(e(k), g_k, k)\} = \mathbb{E}\{\Delta V_1(k) + \Delta V_2(k) + \Delta V_3(k)\}, \quad (13)$$

where

$$\begin{aligned} \mathbb{E}\{\Delta V_1(k)\} &= \mathbb{E}^{g_{k+1}}\{e^T(k+1)P^{g_{k+1}}e(k+1)\} - e^T(k)P^{g_k}e(k) \\ &= \eta^T(k) \left(\mathcal{H}^{g_k T} \sum_{t=1}^{n_0} \pi^{g_{kt}}(k) P^t \mathcal{H}^{g_k} + \tilde{\mathcal{H}}^{g_k T} \sum_{t=1}^{n_0} \pi^{g_{kt}}(k) P^t \tilde{\mathcal{H}}^{g_k} \right) \eta(k) + \mathbb{E}^{g_k} \left\{ \Theta^{g_k T} \sum_{t=1}^{n_0} \pi^{g_{kt}}(k) P^t \Theta^{g_k} \right\} \\ &\quad + \sum_{\varrho=1}^{b_k} \eta^T(k-\varrho) \left(\mathcal{H}^{g_{k-\varrho} T} \sum_{t=1}^{n_0} \pi^{g_{kt}}(k) P^t \mathcal{H}^{g_{k-\varrho}} + \tilde{\mathcal{H}}^{g_{k-\varrho} T} \sum_{t=1}^{n_0} \pi^{g_{kt}}(k) P^t \tilde{\mathcal{H}}^{g_{k-\varrho}} \right) \eta(k-\varrho) - e^T(k)P^{g_k}e(k) \\ &= \eta^T(k) \left(\mathcal{H}^{g_k T} \sum_{t=1}^{n_0} \sum_{z=1}^{z_0} \bar{h}_z(k) (\pi^{g_{kt}})^z P^t \mathcal{H}^{g_k} + \tilde{\mathcal{H}}^{g_k T} \sum_{t=1}^{n_0} \sum_{m=1}^{m_0} \bar{h}_m(k) (\pi^{g_{kt}})^z P^t \tilde{\mathcal{H}}^{g_k} \right) \eta(k) + \sum_{\varrho=1}^{b_k} \\ &\quad \times \eta^T(k-\varrho) \left(\mathcal{H}^{g_{k-\varrho} T} \sum_{t=1}^{n_0} \sum_{z=1}^{z_0} \bar{h}_z(k) (\pi^{g_{kt}})^z P^t \mathcal{H}^{g_{k-\varrho}} + \tilde{\mathcal{H}}^{g_{k-\varrho} T} \sum_{t=1}^{n_0} \sum_{z=1}^{z_0} \bar{h}_z(k) (\pi^{g_{kt}})^z P^t \tilde{\mathcal{H}}^{g_{k-\varrho}} \right) \eta(k-\varrho) \\ &\quad + \mathbb{E}^{g_k} \left\{ \Theta^{g_k T} \sum_{t=1}^{n_0} \pi^{g_{kt}}(k) P^t \Theta^{g_k} \right\} - e^T(k)P^{g_k}e(k) \end{aligned}$$

$$\begin{aligned}
 &= \sum_{z=1}^{z_0} \tilde{h}_z(k) \eta^T(k) \left(\mathcal{H}^{g_k T} (P^{g_k})^z \mathcal{H}^{g_k} + \tilde{\mathcal{H}}^{g_k T} (P^{g_k})^z \tilde{\mathcal{H}}^{g_k} \right) \eta(k) + \sum_{\varrho=1}^{b_k} \sum_{z=1}^{z_0} \tilde{h}_z(k) \eta^T(k - \varrho) (\mathcal{H}^{g_{k-\varrho} T} (P^{g_k})^z \\
 &\quad \times \mathcal{H}^{g_{k-\varrho}} + \tilde{\mathcal{H}}^{g_{k-\varrho} T} (P^{g_k})^z \tilde{\mathcal{H}}^{g_{k-\varrho}}) \eta(k - \varrho) + \mathbb{E}^{g_k} \left\{ \Theta^{g_k T} \sum_{t=1}^{n_0} \pi^{g_k t}(k) P^t \Theta^{g_k} \right\} - e^T(k) P^{g_k} e(k) \\
 &= \eta^T(k) \left(\mathcal{H}^{g_k T} (P^{g_k})^z \mathcal{H}^{g_k} + \tilde{\mathcal{H}}^{g_k T} (P^{g_k})^z \tilde{\mathcal{H}}^{g_k} \right) \eta(k) + \sum_{\varrho=1}^{b_k} \eta^T(k - \varrho) \left(\mathcal{H}^{g_{k-\varrho} T} (P^{g_k})^z \mathcal{H}^{g_{k-\varrho}} + \tilde{\mathcal{H}}^{g_{k-\varrho} T} (P^{g_k})^z \right. \\
 &\quad \left. \times \tilde{\mathcal{H}}^{g_{k-\varrho}} \right) \eta(k - \varrho) + \mathbb{E}^{g_k} \left\{ \Theta^{g_k T} \sum_{t=1}^{n_0} \pi^{g_k t}(k) (P^{g_k})^t \Theta^{g_k} \right\} - e^T(k) P^{g_k} e(k),
 \end{aligned}$$

$$\begin{aligned}
 \mathbb{E}^{g_k} \{\Delta V_2(k)\} &= \mathbb{E}^{g_k} \left\{ \sum_{\varrho=0}^{b_k} \left(\sum_{i=k-\varrho+1}^k e^T(i) Q^{g_k} e(i) - \sum_{i=k-\varrho}^{k-1} e^T(i) Q^{g_k} e(i) \right) \right\} \\
 &= \mathbb{E}^{g_k} \left\{ \sum_{\varrho=0}^{b_k} \left(e^T(k) Q^{g_k} e(k) - e^T(k - \varrho) Q^{g_k} e(k - \varrho) + \sum_{i=k-\varrho+1}^{k-1} e^T(i) Q^{g_k} e(i) - \sum_{i=k-\varrho}^{k-1} e^T(i) Q^{g_k} e(i) \right) \right\} \\
 &= \mathbb{E}^{g_k} \left\{ \sum_{\varrho=0}^{b_k} \left(e^T(k) Q^{g_k} e(k) - e^T(k - \varrho) Q^{g_k} e(k - \varrho) + \sum_{i=k-\bar{\varrho}_m+1}^{k-1} e^T(i) Q^{g_k} e(i) + \sum_{i=k-\varrho+1}^{k-\bar{\varrho}_m} e^T(i) Q^{g_k} e(i) \right. \right. \\
 &\quad \left. \left. - \sum_{i=k-\varrho+1}^{k-1} e^T(i) Q^{g_k} e(i) \right) \right\} \\
 &\leq \mathbb{E}^{g_k} \left\{ \sum_{\varrho=0}^{b_k} \left(e^T(k) Q^{g_k} e(k) - e^T(k - \varrho) Q^{g_k} e(k - \varrho) + \sum_{i=k-\bar{\varrho}_m+1}^{k-\bar{\varrho}_m} e^T(i) Q^{g_k} e(i) \right) \right\}, (b_k = \min\{L, k\}; 0 \leq \bar{\varrho}_z \leq \varrho \leq \bar{\varrho}_Z),
 \end{aligned}$$

$$\begin{aligned}
 \mathbb{E}^{g_k} \{\Delta V_3(k)\} &= \mathbb{E}^{g_k} \left\{ \sum_{\varrho=0}^{b_k} \left(\sum_{j=k-\bar{\varrho}_Z+2}^{k-\bar{\varrho}_Z+1} \sum_{i=j}^k e^T(i) Q^{g_k} e(i) - \sum_{i=k-\bar{\varrho}_Z+1}^{k-\bar{\varrho}_m} \sum_{i=j}^{k-1} e^T(i) Q^{g_k} e(i) \right) \right\} \\
 &= \mathbb{E}^{g_k} \left\{ \sum_{\varrho=0}^{b_k} \left(\sum_{j=k-\bar{\varrho}_Z+1}^{k-\bar{\varrho}_Z} \sum_{i=j+1}^k e^T(i) Q^{g_k} e(i) - \sum_{j=k-\bar{\varrho}_Z+1}^{k-\bar{\varrho}_Z} \sum_{i=j}^{k-1} e^T(i) Q^{g_k} e(i) \right) \right\} \\
 &= \mathbb{E}^{g_k} \left\{ \sum_{\varrho=0}^{b_k} \left(\sum_{j=k-\bar{\varrho}_Z+1}^{k-\bar{\varrho}_Z} (e^T(k) Q^{g_k} e(k) - e^T(j) Q^{g_k} e(j)) \right) \right\} \\
 &= \mathbb{E}^{g_k} \left\{ \sum_{\varrho=0}^{b_k} \left((\bar{\varrho}_Z - \bar{\varrho}_z) e^T(k) Q^{g_k} e(k) - \sum_{j=k-\bar{\varrho}_Z+1}^{k-\bar{\varrho}_Z} e^T(j) Q^{g_k} e(j) \right) \right\}, (b_k = \min\{L, k\}).
 \end{aligned}$$

Since $\pi^{g_k t}(k) \leq \bar{\pi}^{g_k t}$, by (13) it turns out that

$$\begin{aligned}
 \mathbb{E}^{g_k} \left\{ \Theta^{g_k T} \sum_{t=1}^{n_0} \pi^{g_k t}(k) P^t \Theta^{g_k} \right\} &= \sum_{t=1}^{n_0} \pi^{g_k t}(k) \sum_{i=1}^N \sum_{j=1}^N \alpha_{ij}^{g_k} \mathbb{E}^{g_k} \left\{ v_{ij}^T(k) T_{ij}^T \mathcal{K}_{ij}^{g_k T} (P_i^{g_k})^t \mathcal{K}_{ij}^{g_k} \mathcal{G}_{ij} v_{ij}(k) \right\} \\
 &\leq \sum_{t=1}^{n_0} \bar{\pi}^{g_k t} \sum_{i=1}^N \sum_{j=1}^N \alpha_{ij}^{g_k} \mathbb{E}^{g_k} \left\{ (v_{ij}^T(k) T_{ij}^T \mathcal{K}_{ij}^{g_k T}) P^t (\mathcal{K}_{ij}^{g_k} U_{ij} v_{ij}(k)) \right\} \leq \gamma \sum_{i=1}^N \sum_{j=1}^N \alpha_{ij}^{g_k} \mathbb{E}^{g_k} \{ \|v_{ij}(k)\|^2 \}.
 \end{aligned} \tag{14}$$

From (11), (14) and Remark 1, it turns out that

$$\begin{aligned}
 &\mathbb{E}^{g_{k+1}} \{V(e(k+1), g_{k+1}, k+1) - V(e(k), g_k, k)\} + \sum_{\varrho=0}^{b_k} \frac{1}{\gamma} \Psi^{g_{k-\varrho}}(e(k - \varrho)) - \sum_{\varrho=0}^{b_k} \|\omega(k - \varrho)\|^2 + \|\nu(k)\|^2 \\
 &\leq \eta^T(k) \left(\mathcal{H}^{g_k T} (P^{g_k})^z \mathcal{H}^{g_k} + \tilde{\mathcal{H}}^{g_k T} (P^{g_k})^z \tilde{\mathcal{H}}^{g_k} + \text{diag}\{\Xi^{g_k} - P^{g_k} + \Gamma_0, -I, -I\} \right) \eta(k) + \sum_{\varrho=1}^{b_k} \eta^T(k - \varrho) \left(\mathcal{H}^{g_{k-\varrho} T} \right. \\
 &\quad \left. \times (P^{g_k})^z \mathcal{H}^{g_{k-\varrho}} + \tilde{\mathcal{H}}^{g_{k-\varrho} T} (P^{g_k})^z \tilde{\mathcal{H}}^{g_{k-\varrho}} + \sum_{\varrho=1}^{l_k} \text{diag}\{\Xi^{g_{k-\varrho}} - Q^{g_k}, 0, -I\} \right) \eta(k - \varrho) + \gamma \sum_{i=0}^N \sum_{j=1}^N \alpha_{ij}^{g_k} \mathbb{E}^{g_k} \{ \|v_{ij}(k)\|^2 \}.
 \end{aligned} \tag{15}$$

Based on the Schur Complement [41], and the condition (15), the inequality (11) is equal to

$$\begin{aligned}
 &\mathcal{H}^{g_k T} (P^{g_k})^z \mathcal{H}^{g_k} + \tilde{\mathcal{H}}^{g_k T} (P^{g_k})^z \tilde{\mathcal{H}}^{g_k} + \text{diag}\{\Xi^{g_k} - P^{g_k} + \Gamma_0, -I, -I\} + \sum_{\varrho=1}^{b_k} \left(\mathcal{H}^{g_{k-\varrho} T} (P^{g_k})^z \mathcal{H}^{g_{k-\varrho}} \right. \\
 &\quad \left. + \tilde{\mathcal{H}}^{g_{k-\varrho} T} (P^{g_k})^z \tilde{\mathcal{H}}^{g_{k-\varrho}} + \text{diag}\{\Xi^{g_{k-\varrho}} - Q^{g_k}, 0, -I\} \right) < 0, (z = 1, 2, \dots, z_0).
 \end{aligned}$$

Let $\lambda_0 = \max \left\{ \lambda_{\max} \left(\mathcal{H}^{g_k T} (P^{g_k})^z \mathcal{H}^{g_k} + \tilde{\mathcal{H}}^{g_k T} (P^{g_k})^z \tilde{\mathcal{H}}^{g_k} + \text{diag}\{\Xi^{g_k} - P^{g_k} + \Gamma_0, -I, -I\} + \sum_{\varrho=1}^{b_k} \left(\mathcal{H}^{g_{k-\varrho} T} (P^{g_k})^z \mathcal{H}^{g_{k-\varrho}} + \tilde{\mathcal{H}}^{g_{k-\varrho} T} (P^{g_k})^z \tilde{\mathcal{H}}^{g_{k-\varrho}} + \text{diag}\{\Xi^{g_{k-\varrho}} - Q^{g_k}, 0, -I\} \right) \right) \right\}$. Obviously, $\lambda_0 < 0$. From (15) it turns out that

$$\begin{aligned}
 &\mathbb{E}^{g_k} \{V(e(k+1), g_{k+1}, k+1) - V(e(k), g_k, k)\} + \sum_{\varrho=0}^{b_k} \frac{1}{\gamma} \Psi^{g_{k-\varrho}}(e(k - \varrho)) - \sum_{\varrho=0}^{b_k} \|\omega(k - \varrho)\|^2 - \|\nu(k)\|^2 \\
 &\leq \lambda_0 \left(\sum_{\varrho=0}^{b_k} \|\eta(k - \varrho)\|^2 \right) + \gamma \sum_{i=1}^N \sum_{j=1}^N \alpha_{ij}^{g_k} \mathbb{E}^{g_k} \{ \|v_{ij}(k)\|^2 \}.
 \end{aligned} \tag{16}$$

Noticing that $\mathbb{E}^{g_k}\{\Psi^{g_k-\varrho}(e(k-\varrho))\} \geq 0$, ($\varrho = 0, 1, \dots, b_k$), (16) implies that

$$\begin{aligned} & \mathbb{E}^{g_k}\{V(e(k+1), g_{k+1}, k+1)\} - \mathbb{E}^{g_k}\{V(e(k), g_k, k)\} - \sum_{\varrho=0}^{b_k} \|\omega(k-\varrho)\|^2 - \|\nu(k)\|^2 \\ & \leq \lambda_0 \left(\sum_{\varrho=0}^{b_k} \mathbb{E}^{g_k}\{\|\eta(k-\varrho)\|^2\} \right) + \gamma \sum_{i=1}^N \sum_{j=1}^N \alpha_{ij}^{g_k} \mathbb{E}^{g_k}\{\|v_{ij}(k)\|^2\}. \end{aligned} \quad (17)$$

If $\omega(k-\varrho) = 0$, $\nu(k) = 0$, and $v_{ij}(k) = 0$, then $\|\eta(k-\varrho)\| = \|e(k-\varrho)\|$, ($\varrho = 0, 1, \dots, g_k$). Hence (17) is simplified as

$$\mathbb{E}^{g_k}\{V(e(k+1), g_{k+1}, k+1)\} - \mathbb{E}^{g_k}\{V(e(k), g_k, k)\} \leq \lambda_0 \left(\sum_{\varrho=0}^{b_k} \mathbb{E}^{g_k}\{\|e(k-\varrho)\|^2\} \right). \quad (18)$$

Letting $\epsilon > 0$, by (18) one has

$$\begin{aligned} & \frac{1}{\epsilon^{k+1}} \mathbb{E}^{g_k}\{V(e(k+1), g_{k+1}, k+1)\} - \frac{1}{\epsilon^k} \mathbb{E}^{g_k}\{V(e(k), g_k, k)\} = \frac{1}{\epsilon^{k+1}} \left(\mathbb{E}^{g_k}\{V(e(k+1), g_{k+1}, k+1)\} \right. \\ & \left. - \mathbb{E}^{g_k}\{V(e(k), g_k, k)\} \right) + \frac{1}{\epsilon^k} \left(\frac{\lambda_0}{\epsilon} + \left(\frac{1}{\epsilon} - 1 \right) \lambda \right) \left(\sum_{\varrho=0}^{b_k} \mathbb{E}^{g_k}\{\|e(k-\varrho)\|^2\} \right), \end{aligned} \quad (19)$$

where $\lambda = \max\{\lambda_{\max}(P^{g_k})\}$.

Noticing that $\frac{1}{\epsilon^k} \left(\frac{\lambda_0}{\epsilon} + \left(\frac{1}{\epsilon} - 1 \right) \lambda \right) = \lambda_0 < 0$, when $\epsilon = 1$, then, by the continuity, one can find a constant $0 < \epsilon_0 < 1$

such that $\frac{1}{\epsilon_0^k} \left(\frac{\lambda_0}{\epsilon_0} + \left(\frac{1}{\epsilon_0} - 1 \right) \lambda \right) < 0$.

Thus, (19) implies that

$$\frac{1}{\epsilon_0^k} \mathbb{E}^{g_0}\{V(e(0), g_0, 0)\} \leq V(e(0), g_0, 0) = e^T(0) P^{g_0} e(0) \leq \lambda_{\max}(P^{g_0}) \|e(0)\|^2. \quad (20)$$

Let $\lambda_1 = \max\{\lambda_{\min}(P^{g_k})\}$, and $g_k \in \mathfrak{S}$, by (20) we have

$$\mathbb{E}^{g_0}\{\|e(k)\|^2\} \leq V(e(0), g_0, 0) = e^T(0) P^{g_0} e(0) \leq \frac{\lambda_{\max}(P^{g_0})}{\lambda_1} \epsilon_0^k \|e(0)\|^2. \quad (21)$$

Consequently, the distributed estimation error dynamics (9) is exponentially stable in mean square.

On the other side, let γ be a positive constant, since $\lambda_0 < 0$, $\mathbb{E}^{g_0}\{\|\eta(k)\|^2\} \geq 0$, $\mathbb{E}^{g_0}\{V(k)\} \geq 0$, $x(0) - \hat{x}_i(0) = x_0$, and $\mathbf{R}$ be a positive definite symmetric matrix, by summing up both sides of (18) from 0 to t with respect to k and letting $t \rightarrow \infty$, one has

$$\begin{aligned} & \mathbb{E}^{g_0}\left\{ \sum_{k=0}^{\infty} \sum_{\varrho=0}^{b_k} \Psi^{g_k}(e(k-\varrho)) \right\} \\ & \leq \gamma \left(\gamma \sum_{i=1}^N \sum_{j=1}^N \sum_{k=0}^{\infty} \mathbb{E}^{g_0}\{\|\alpha_{ij}^{g_k} v_{ij}(k)\|^2\} + \sum_{k=0}^{\infty} \sum_{\varrho=0}^{b_k} \|\omega(k-\varrho)\|^2 + \|\nu(k)\|^2 + \mathbb{E}^{g_0}\{V(e(0), g_0, 0)\} \right) \\ & \leq \gamma \left(\|x_0\|_{\mathbf{R}}^2 + \sum_{\varrho=0}^{b_k} \|\omega(k-\varrho)\|_2^2 + \|\nu(k)\|_2^2 + \gamma \sum_{i=1}^N \sum_{j=1}^N \mathbb{E}^{g_0}\{\|\alpha_{ij}^{g_k} v_{ij}(k)\|_2^2\} \right). \end{aligned} \quad (22)$$

Therefore, the constructed filter satisfies a mean-square H_∞ -consensus performance requirements. Now, the proof about Theorem 1 is completed. ■

Remark 2. If the Markov chain is a homogeneous one, i.e., the case of $\Pi^m = \Pi$ ($1 \leq m \leq m_0$) with $\Pi = [\pi^{g_{kt}}]_{n_0 \times n_0}$, the relevant results follow by replacing $(\bar{P}^{g_k}(k))^m$, in (11), with $P^{g_k}(k) = \sum_{t=1}^{n_0} \pi^{g_{kt}} P^t(k)$.

According to Theorem 1, if γ is a known H_∞ -consensus disturbance rejection attenuation performance index, then the matrix inequality (11) is linear. Thus, it is necessary to find an γ in many practical cases. Then the solvable filtering problem will be transferred to the following optimization problem: Find an γ in such a way that

$$\min_{\gamma, P_i^{g_k}, Q_i^{g_k}, X_i^{g_k}, Y_{ij}^{g_k}} \gamma \quad (23)$$

satisfying (11) with $\gamma > 0$, and $P_i^{g_k} > 0$, $Q_i^{g_k} > 0$, ($i = 1, 2, \dots, N$).

Note that the optimal problem (23) may not be directly handled by using the Matlab toolbox in the case that (11) is a nonlinear matrix inequality in which γ is an unknown variable. The following theorem will give the solution of the optimal problem that transforms (11) into a linear matrix inequality (LMI).

The following theorem 2 will give a criterion for designing a desired distributed H_∞ -consensus filter through which filter gain matrices $L_i^{g_k}(k)$ and $K_{ij}^{g_k}(k)$, ($i = 1, 2, \dots, N$; $j \in \mathcal{X}_{ij}^{g_k}$) can be solved out.

4. Distributed H_∞ -consensus filters design

This section will give an LMIs-based sufficient condition regarding the existence of the designed distributed H_∞ -consensus filters. According to such a condition, the optimal filter parameters will be determined, and the described H_∞ -consensus disturbance/noise attenuation performance index will be obtained.

Theorem 2. Let $g_k \in \mathfrak{S}$ be the Markov chain, and γ be a given constant scalar. If the optimal problem (24) satisfying the following LMI constraints (25) and (26), then the filter (6) is a distributed H_∞ -consensus filter for the system (4).

$$\min_{\gamma, P_i^{g_k}, Q_i^{g_k}, X_i^{g_k}, Y_{ij}^{g_k}} \gamma, \quad (24)$$

$$\begin{bmatrix} -\bar{P}_i^{g_k} & a_{ij}^{g_k} X_i^{g_k} Y_{ij}^{g_k} T_{ij} \\ a_{ij}^{g_k} T_{ij}^T (Y_{ij}^{g_k})^T (X_i^{g_k})^T & -\gamma I \end{bmatrix} < 0, \quad (25)$$

$$\sum_{\varrho=0}^{b_k} (\Phi^{g_{k-\varrho}})^z = (\Phi^{g_k})^z + \sum_{\varrho=1}^{b_k} (\Phi^{g_{k-\varrho}})^z < 0, \quad (26)$$

where

$$(\hat{\Phi}^{g_k})^z = \begin{bmatrix} -P^{g_k} + T_0 & * & * & * & * & * \\ 0 & -I & * & * & * & * \\ 0 & 0 & -I & * & * & * \\ \bar{T}_1 & \bar{T}_2 & \bar{T}_3 & -\bar{P}^{g_k} & * & * \\ \bar{T}_4 & 0 & \bar{T}_5 & 0 & -\bar{P}^{g_k} & * \\ \bar{T}_0 & 0 & 0 & 0 & 0 & -\gamma I \end{bmatrix}, \quad (\hat{\Phi}^{g_{k-\varrho}})^z = \begin{bmatrix} -Q^{g_k} & * & * & * & * & * \\ 0 & 0 & * & * & * & * \\ 0 & 0 & -I & * & * & * \\ \bar{T}_6 & 0 & \bar{T}_7 & -\bar{P}^{g_k} & * & * \\ \bar{T}_8 & 0 & \bar{T}_9 & 0 & -\bar{P}^{g_k} & * \\ \bar{T}_{00} & 0 & 0 & 0 & 0 & -\gamma I \end{bmatrix},$$

with

$$\begin{aligned} \mathcal{L}_i^{g_k} &= (\sum_{t=1}^{n_0} \bar{\pi}^{g_k t} (P_i^{g_k})^t)^{-1} \mathcal{X}_i^{g_k}, & \mathcal{K}_{ij}^{g_k} &= (\sum_{t=1}^{n_0} \bar{\pi}^{g_k t} (k) (P_i^{g_k})^t)^{-1} \mathcal{Y}_{ij}^{g_k}, & \bar{r}_0 &= (\phi^{g_k} - \mathcal{O}^{g_k} - (\mathcal{O}^{g_k})^T)^{\frac{1}{2}} \otimes I, \\ \bar{r}_{00} &= (\phi^{g_{k-\varrho}} - \mathcal{A}^{g_{k-\varrho}} - (\mathcal{O}^{g_{k-\varrho}})^T)^{\frac{1}{2}} \otimes I, & \bar{r}_1 &= \bar{P}^{g_k} (I \otimes \mathcal{A}) - \bar{L} \Lambda_{\bar{\xi}_0} C - \bar{\mathcal{K}}, & \bar{r}_2 &= \bar{P}^{g_k} (\mathbf{1} \otimes \mathcal{B}), \\ \bar{r}_3 &= -\bar{L} \Lambda_{\bar{\xi}_0} \mathcal{D}, & \bar{r}_4 &= \bar{L} \Lambda_{\bar{\xi}_0(1-\bar{\xi}_0)} C + \bar{\mathcal{K}}, & \bar{r}_5 &= \bar{L} \Lambda_{\bar{\xi}_0(1-\bar{\xi}_0)} \mathcal{D}, \\ \bar{r}_6 &= -\sum_{\varrho=1}^{b_k} \bar{L} \Lambda_{\bar{\xi}_0} C - \bar{\mathcal{K}}, & \bar{r}_7 &= -\sum_{\varrho=1}^{b_k} \bar{L} \Lambda_{\bar{\xi}_0} \mathcal{D}, & \bar{r}_8 &= \sum_{\varrho=1}^{b_k} \bar{L} \Lambda_{\bar{\xi}_0(1-\bar{\xi}_0)} C + \bar{\mathcal{K}}, \\ \bar{r}_9 &= \sum_{\varrho=1}^{b_k} \bar{L} \Lambda_{\bar{\xi}_0(1-\bar{\xi}_0)} \mathcal{D}, & \bar{P}^{g_k} &= \text{diag}\{\bar{P}_1^{g_k}, \bar{P}_2^{g_k}, \dots, \bar{P}_N^{g_k}\} = \sum_{t=1}^{n_0} (\bar{\pi}^{g_k})^t (P^{g_k})^t, \end{aligned}$$

where $\mathcal{L}_i^{g_k}$ and $\mathcal{K}_{ij}^{g_k}$ are the i th filter parameters that are all determined; the matrix $(\phi^{g_{k-\varrho}} - \mathcal{O}^{g_{k-\varrho}} - (\mathcal{O}^{g_{k-\varrho}})^T)^{\frac{1}{2}}$ ($\varrho = 0, 1, \dots, b_k$) are the unique square root of the positive semi-definite symmetric matrix $\phi^{g_{k-\varrho}} - \mathcal{O}^{g_{k-\varrho}} - (\mathcal{O}^{g_{k-\varrho}})^T$; $\mathbf{R}$, P^{g_k} and Q^{g_k} are defined in Theorem 1.

Proof. By defining $\bar{\mathcal{X}} = \text{diag}\{\mathcal{X}_1^{g_k}, \mathcal{X}_2^{g_k}, \dots, \mathcal{X}_N^{g_k}\}$, $\bar{\mathcal{Y}} = \bar{\mathcal{Y}} + \bar{\mathcal{Y}}$, $\bar{\mathcal{Y}} = [\bar{\mathcal{Y}}_{ij}]_{N \times N}$, $\bar{\mathcal{Y}} = [\bar{\mathcal{Y}}_{ij}]_{N \times N}$, $\bar{\mathcal{Y}} = \bar{\mathcal{Y}} + \bar{\mathcal{Y}}$, $\bar{\mathcal{Y}} = [\bar{\mathcal{Y}}_{ij}]_{N \times N}$, and $\bar{\mathcal{Y}} = [\bar{\mathcal{Y}}_{ij}]_{N \times N}$, where

$$\begin{aligned} \bar{\mathcal{Y}}_{ii} &= \sum_{j=1, j \neq i}^N a_{ij}^{g_k} \mathcal{Y}_{ij}^{g_k} \bar{\xi}_j^0 U_{ij}, (i = 1, 2, \dots, N), & \bar{\mathcal{Y}}_{ij} &= -\sum_{\varrho=1}^{b_k} a_{ij}^{g_k} \mathcal{Y}_{ij}^{g_k} \left(\bar{\xi}_j^0 (1 - \bar{\xi}_j^0) \right) U_{ij}, (i \neq j; i, j = 1, 2, \dots, N), \\ \bar{\mathcal{Y}}_{ij} &= -a_{ij}^{g_k} \mathcal{Y}_{ij}^{g_k} \bar{\xi}_j^0 U_{ij}, (i \neq j; i, j = 1, 2, \dots, N), & \bar{\mathcal{Y}}_{ii} &= \sum_{j=1, j \neq i}^N \sum_{\varrho=1}^{b_k} a_{ij}^{g_k} \mathcal{Y}_{ij}^{g_k} \left(\bar{\xi}_j^0 (1 - \bar{\xi}_j^0) \right) U_{ij}, (i = 1, 2, \dots, N), \\ \bar{\mathcal{Y}}_{ii} &= \sum_{j=1, j \neq i}^N \sum_{\varrho=1}^{b_k} a_{ij}^{g_k} \mathcal{Y}_{ij}^{g_k} \bar{\xi}_j^0 U_{ij}, (i = 1, 2, \dots, N), & \bar{\mathcal{Y}}_{ij} &= -\sum_{\varrho=1}^{b_k} a_{ij}^{g_k} \mathcal{Y}_{ij}^{g_k} \bar{\xi}_j^0 U_{ij}, (i \neq j; i, j = 1, 2, \dots, N), \\ \bar{\mathcal{Y}}_{ii} &= \sum_{j=1, j \neq i}^{b_k} a_{ij}^{g_k} \mathcal{Y}_{ij}^{g_k} \left(\bar{\xi}_j^0 (1 - \bar{\xi}_j^0) \right) U_{ij}, (i = 1, 2, \dots, N), & \bar{\mathcal{Y}}_{ij} &= -a_{ij}^{g_k} \mathcal{Y}_{ij}^{g_k} \left(\bar{\xi}_j^0 (1 - \bar{\xi}_j^0) \right) U_{ij}, (i \neq j; i, j = 1, 2, \dots, N), \end{aligned}$$

and

$$\begin{aligned}\bar{\mathcal{H}}^{g_k} &= \begin{bmatrix} I \otimes \mathcal{A} - (\bar{P}^{g_k})^{-1} \bar{\mathcal{X}} \Lambda_{\bar{\xi}^0} C - \bar{\mathcal{Y}} & \mathbf{1} \otimes \mathcal{B} & -(\bar{P}^{g_k})^{-1} \bar{\mathcal{X}} \Lambda_{\bar{\xi}^0} \mathcal{D} \end{bmatrix}, \\ \bar{\mathcal{H}}^{g_k} &= \begin{bmatrix} (\bar{P}^{g_k})^{-1} \bar{\mathcal{X}} \Lambda_{\bar{\xi}^0(1-\bar{\xi}^0)} C + \bar{\mathcal{Y}} & 0 & (\bar{P}^{g_k})^{-1} \bar{\mathcal{X}} \Lambda_{\bar{\xi}^0(1-\bar{\xi}^0)} \mathcal{D} \end{bmatrix}, \\ \bar{\mathcal{H}}^{g_{k-\varrho}} &= \begin{bmatrix} -(\bar{P}^{g_k})^{-1} \bar{\mathcal{X}} \Lambda_{\bar{\xi}^0} C - \bar{\mathcal{Y}} & 0 & -(\bar{P}^{g_k})^{-1} \bar{\mathcal{X}} \Lambda_{\bar{\xi}^0} \mathcal{D} \end{bmatrix}, \\ \bar{\mathcal{H}}^{g_{k-\varrho}} &= \begin{bmatrix} (\bar{P}^{g_k})^{-1} \bar{\mathcal{X}} \Lambda_{\bar{\xi}^0(1-\bar{\xi}^0)} C(k) + \bar{\mathcal{Y}} & 0 & (\bar{P}^{g_k})^{-1} \bar{\mathcal{X}} \Lambda_{\bar{\xi}^0(1-\bar{\xi}^0)} \mathcal{D} \end{bmatrix},\end{aligned}$$

one can have

$$\begin{aligned}\bar{\mathcal{H}}^{g_k} &= \begin{bmatrix} I \otimes \mathcal{A} - \bar{\mathcal{L}} \Lambda_{\bar{\xi}^0} C - \bar{\mathcal{K}} & \mathbf{1} \otimes \mathcal{B} & -\bar{\mathcal{L}} \Lambda_{\bar{\xi}^0} \mathcal{D} \end{bmatrix} = \mathcal{H}^{g_k}, \\ \bar{\mathcal{H}}^{g_k} &= \begin{bmatrix} \bar{\mathcal{L}} \Lambda_{\bar{\xi}^0(1-\bar{\xi}^0)} C + \bar{\mathcal{K}} & 0 & \bar{\mathcal{L}} \Lambda_{\bar{\xi}^0(1-\bar{\xi}^0)} \mathcal{D} \end{bmatrix} = \tilde{\mathcal{H}}^{g_k}, \\ \bar{\mathcal{H}}^{g_{k-\varrho}} &= \begin{bmatrix} -\bar{\mathcal{L}} \Lambda_{\bar{\xi}^0} C - \bar{\mathcal{K}} & 0 & -\bar{\mathcal{L}} \Lambda_{\bar{\xi}^0} \mathcal{D} \end{bmatrix} = \mathcal{H}^{g_{k-\varrho}}, \\ \bar{\mathcal{H}}^{g_{k-\varrho}} &= \begin{bmatrix} \bar{\mathcal{L}} \Lambda_{\bar{\xi}^0(1-\bar{\xi}^0)} C + \bar{\mathcal{K}} & 0 & \bar{\mathcal{L}} \Lambda_{\bar{\xi}^0(1-\bar{\xi}^0)} \mathcal{D} \end{bmatrix} = \tilde{\mathcal{H}}^{g_{k-\varrho}},\end{aligned}\tag{27}$$

where $\mathcal{H}^{g_k}$, $\tilde{\mathcal{H}}^{g_k}$, $\mathcal{H}^{g_{k-\varrho}}$, and $\tilde{\mathcal{H}}^{g_{k-\varrho}}$ are defined in Theorem 1.

On the basis of the Schur Complement, the inequality (27) is equivalent to

$$\begin{aligned}\bar{\mathcal{H}}^{g_k T} \bar{P}^{g_k} \bar{\mathcal{H}}^{g_k} + \bar{\mathcal{H}}^{g_k T} \bar{P}^{g_k} \bar{\mathcal{H}}^{g_k} + \text{diag}\{-\bar{P}^{g_k} + \Gamma_0, -I, -I\} + \gamma^{-1}(\bar{\phi}^{g_k})^T \bar{\phi}^{g_k} + \sum_{\varrho=1}^{b_k} \left(\bar{\mathcal{H}}^{g_{k-\varrho} T} \bar{P}^{g_k} \bar{\mathcal{H}}^{g_{k-\varrho}} + \right. \\ \left. \bar{\mathcal{H}}^{g_{k-\varrho} T} \bar{P}^{g_k} \bar{\mathcal{H}}^{g_{k-\varrho}} + \text{diag}\{-Q^{g_k}, 0, -I\} + \gamma^{-1}(\bar{\phi}^{g_{k-\varrho}})^T \bar{\phi}^{g_{k-\varrho}} \right) < 0,\end{aligned}\tag{28}$$

where $\bar{\phi}^{g_{k-\varrho}} = [(\phi^{g_{k-\varrho}} - O^{g_{k-\varrho}} - (O^{g_{k-\varrho}})^T)^{\frac{1}{2}} \otimes I, 0, 0, 0, 0]$.

According to the Schur Complement, the inequality (28) is equivalent to

$$\begin{aligned}\bar{\mathcal{H}}^{g_k T} \bar{P}^{g_k} \bar{\mathcal{H}}^{g_k} + \bar{\mathcal{H}}^{g_k T} \bar{P}^{g_k} \bar{\mathcal{H}}^{g_k} + \text{diag}\{\Xi^{g_k} - \bar{P}^{g_k} + \Gamma_0, -I, -I\} + \sum_{\varrho=1}^{b_k} \left(\bar{\mathcal{H}}^{g_{k-\varrho} T} \bar{P}^{g_k} \bar{\mathcal{H}}^{g_{k-\varrho}} + \right. \\ \left. \bar{\mathcal{H}}^{g_{k-\varrho} T} \bar{P}^{g_k} \bar{\mathcal{H}}^{g_{k-\varrho}} + \text{diag}\{\Xi^{g_{k-\varrho}} - Q^{g_k}, 0, -I\} \right) < 0,\end{aligned}\tag{29}$$

where Ξ^{g_k} is defined in Theorem 1.

Noticing that $\mathcal{H}^{g_k T} (P^{g_k})^z \mathcal{H}^{g_k} \leq \mathcal{H}^{g_k T} \bar{P}^{g_k} \mathcal{H}^{g_k} = \bar{\mathcal{H}}^{g_k T} \bar{P}^{g_k} \bar{\mathcal{H}}^{g_k}$, $\tilde{\mathcal{H}}^{g_k T} (P^{g_k})^z \tilde{\mathcal{H}}^{g_k} \leq \tilde{\mathcal{H}}^{g_k T} \bar{P}^{g_k} \tilde{\mathcal{H}}^{g_k} = \tilde{\mathcal{H}}^{g_k T} \bar{P}^{g_k} \tilde{\mathcal{H}}^{g_k}$, and $\mathcal{H}^{g_{k-\varrho} T} (P^{g_k})^z \mathcal{H}^{g_{k-\varrho}} \leq \mathcal{H}^{g_{k-\varrho} T} \bar{P}^{g_k} \mathcal{H}^{g_{k-\varrho}} = \bar{\mathcal{H}}^{g_{k-\varrho} T} \bar{P}^{g_k} \bar{\mathcal{H}}^{g_{k-\varrho}}$, $\tilde{\mathcal{H}}^{g_{k-\varrho} T} (P^{g_k})^z \tilde{\mathcal{H}}^{g_{k-\varrho}} \leq \tilde{\mathcal{H}}^{g_{k-\varrho} T} \bar{P}^{g_k} \tilde{\mathcal{H}}^{g_{k-\varrho}} = \tilde{\mathcal{H}}^{g_{k-\varrho} T} \bar{P}^{g_k} \tilde{\mathcal{H}}^{g_{k-\varrho}}$. By (29) it follows that

$$\begin{aligned}\mathcal{H}^{g_k T} (P^{g_k})^z \mathcal{H}^{g_k} + \tilde{\mathcal{H}}^{g_k T} (P^{g_k})^z \tilde{\mathcal{H}}^{g_k} + \text{diag}\{\Xi^{g_k} - P^{g_k} + T_0, -I, -I\} + \sum_{\varrho=1}^{b_k} \left(\mathcal{H}^{g_{k-\varrho} T} (P^{g_k})^z \mathcal{H}^{g_{k-\varrho}} + \right. \\ \left. \tilde{\mathcal{H}}^{g_{k-\varrho} T} (P^{g_k})^z \tilde{\mathcal{H}}^{g_{k-\varrho}} + \text{diag}\{\Xi^{g_{k-\varrho}} - Q^{g_k}, 0, -I\} \right) < 0,\end{aligned}\tag{30}$$

which is equivalent to LIM (11). According to Theorem 1, the filter (6) is a distributed H_∞ -consensus filter for the system (4). Now the proof regarding the Theorem 2 is completed. ■

Remark 3. It is well known that the distributed estimation scheme offers several advantages such as low communication and computation costs and high robustness against network induced influences. However, the network induced influences may cause the estimation disagreements among filters. The aim of the paper is to develop a distributed H_∞ -consensus target tracking filtering scheme which promises the following two outstanding advantages: (1) the robustness of the target tracking filtering system against the network-induced effects is increased; (2) in the unreliable network communication environment, the accuracy of the target tracking system state estimation is improved.

Remark 4. It can be seen from the main result of Theorem 2 that the proposed distributed H_∞ consensus-based filtering approach can deal with a combination of network induced influences including packet losses, channel fading and switching topology. Hence, such a modeling framework has great potential to be adopted in several practical scenarios such as multi-agent coordination [42–44], distributed secondary control in islanded microgrids [45], security of cyber-physical systems [46], synchronization of neural networks [47–49].

5. An illustrative example

This section will demonstrate a maneuvering target tracking application on the armored fighting vehicles installed the fire control system [50] for illustrating the effectiveness of the proposed filter design approach. In the experiment design, the five vehicles are spatially deployed in a very large war zone, where each vehicle is equipped with passive infrared/temperature/vibration/sound/magnetic filter and sensor. These sensors and filters have limited energy, and collect energy from the environment including mechanical vibrations, thermal or solar. Moreover, these passive filters can self organize themselves to build up a network. When the filters' energies are dissipated, existing network links may vanish, and when their energies are collected, new links may appear. This means that the filter network topology may switch/change over time, and the packet dropouts and the channel fading phenomena may randomly occur in the data communication.

The motion model of the target [51] is described as follows:

$$\begin{bmatrix} \dot{r}(t) \\ \ddot{r}(t) \\ \ddot{r}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -a \end{bmatrix} \begin{bmatrix} r(t) \\ \dot{r}(t) \\ \ddot{r}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v(t),$$

where the state vector of the target $x(t) = (x_1(t) \ x_2(t) \ x_3(t))^T = (r(t) \ \dot{r}(t) \ \ddot{r}(t))^T$ where $r(t)$, $\dot{r}(t)$ and $\ddot{r}(t)$ represent location, velocity and acceleration, respectively; the maneuvering frequency of target motion $a = 0.05 \text{m} \cdot \text{s}^{-1}$; The random disturbance is defined as $v(t) = (v_1(t) \ v_2(t))^T$ with $v(t) \in l_2[0, \infty)$; The deterministic input

$$u(t) = -Kx(t) = \begin{bmatrix} -1.0000 & -1.7246 & -0.9866 \end{bmatrix} x(t)$$

where K is the optimal state feedback gain matrix; the control input is taken as $u(t) = (u_1(t) \ u_2(t))^T$.

Let the sampling interval $T = 0.1\text{s}$. In a filter network, the continuous data on the above system are not suitable due to digital implementation of filters. Therefore, discretization of the above system is necessary. According a zero-order holder and a time-domain solution approach, the discrete-time model is given in (1). The system parameters of the target (1) can be expressed by as

$$\mathcal{A} = \begin{bmatrix} 0.9953 & 0.0917 & 0.0001 \\ -0.0918 & 0.8370 & 0.0012 \\ -0.0870 & -0.1545 & 0.9014 \end{bmatrix}, \mathcal{B} = \begin{bmatrix} 0.0047 \\ 0.0918 \\ 0.0870 \end{bmatrix}.$$

The system disturbance/noise $v(k)$ is randomly given by $g_0^{(g_1 k)} e^{(g_2 k)} \cos(k)$ where g_0 , g_1 , and g_2 are known constants.

In this example, the measurement noise $\omega_i(k)$ is randomly given by $e^{(u_0(k+u_1))} \cos(k)$ where u_0 and u_1 are known constants, and the channel noise from filter j to i is expressed as $v_{ij} = N(0, 1)$, which is normally distributed random variable with mean being zero and variance being 1. The parameters of measurement output in (3) of the sensors are given as follows: $C_1 = [1 \ 1 \ 1]$, $C_2 = [1 \ 0.6 \ 0.5]$, $C_3 = [0.7 \ 0.8 \ 0.7]$, $C_4 = [0.9 \ 0.6 \ 0.5]$, $C_5 = [1 \ 1 \ 0.9]$, $\mathcal{D}_1 = [0.4 \ 0.5]$, $\mathcal{D}_2 = [0.5 \ 1]$, $\mathcal{D}_3 = [1 \ 1]$, $\mathcal{D}_4 = [0.6 \ 0.3]$, $\mathcal{D}_5 = [0.2 \ 0.2]$. The fading model order $\varrho = 1$, and the fading probabilities are $\tilde{\xi}_1^0(k) = 0.1$, $\tilde{\xi}_2^0(k) = 0.2$, $\tilde{\xi}_3^0(k) = 0.3$, $\tilde{\xi}_4^0(k) = 0.4$, $\tilde{\xi}_5^0(k) = 0.5$, $\tilde{\xi}_1^1(k) = 0.2$, $\tilde{\xi}_2^1(k) = 0.4$, $\tilde{\xi}_3^1(k) = 0.6$, $\tilde{\xi}_4^1(k) = 0.8$, $\tilde{\xi}_5^1(k) = 0.9$, $\tilde{\xi}_1^2(k) = 0.1$, $\tilde{\xi}_2^2(k) = 0.3$, $\tilde{\xi}_3^2(k) = 0.5$, $\tilde{\xi}_4^2(k) = 0.7$, $\tilde{\xi}_5^2(k) = 0.8$, $\tilde{\xi}_1^3(k) = 0.2$, $\tilde{\xi}_2^3(k) = 0.1$, $\tilde{\xi}_3^3(k) = 0.3$, $\tilde{\xi}_4^3(k) = 0.6$, $\tilde{\xi}_5^3(k) = 0.7$, respectively.

Fig. 1. (a) Markov chain and filter error (b) System state evolution (c) Evolution of x_1 and $\hat{x}_{11}$ (d) Evolution of x_2 and $\hat{x}_{12}$ (e) Evolution of x_3 and $\hat{x}_{13}$ (f) Filter network topology.

The randomly connection models of the filter network are described by the switching topologies shown in Fig. 2(e). Therefore, it can easily be found that the related adjacency matrices are, respectively,

$$\mathcal{O}^1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \mathcal{O}^2 = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}, \mathcal{O}^3 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

Case 1. Let initial state $x_0 = [5.5 \ 0 \ 0]^T$. Suppose that the probability transition matrix $\Pi(k)$ of the Markov chain is characterized by a polytope with three vertices:

$$\Pi^1 = \begin{bmatrix} 0.56 & 0.43 & 0.02 \\ 0.01 & 0.64 & 0.35 \\ 0.09 & 0.01 & 0.90 \end{bmatrix}, \Pi^2 = \begin{bmatrix} 0.45 & 0.51 & 0.02 \\ 0.01 & 0.74 & 0.25 \\ 0.20 & 0.01 & 0.79 \end{bmatrix}, \Pi^3 = \begin{bmatrix} 0.56 & 0.43 & 0.01 \\ 0.01 & 0.60 & 0.39 \\ 0.17 & 0.01 & 0.82 \end{bmatrix}.$$

Let $g_0 = 1$. By resort to Theorem 2, the optimal problem (26) satisfying (27), and (28) is solved, and the described

Fig. 2. (a) Markov chain and filter error (b) System state evolution (c) Evolution of x_1 and $\hat{x}_1$ (d) Evolution of x_2 and $\hat{x}_2$ (e) Evolution of x_3 and $\hat{x}_3$.

H_∞ noise/disturbance rejection attenuation performance level $\gamma = 0.6423$ is obtained. Figs. 1(a)-1(f) show that the dynamics of the Markov chain and the filter error norm, the trajectories of plant states and its estimates, respectively.

Case 2. In this case, we attempt to make the switching of the passive filter network topology much slower in an "average" way. For this we increase the weight of diagonal entries for the probability transition matrices. Let

$$\Pi^1 = \begin{bmatrix} 0.75 & 0.25 & 0.00 \\ 0.00 & 0.75 & 0.25 \\ 0.10 & 0.00 & 0.90 \end{bmatrix}, \Pi^2 = \begin{bmatrix} 0.70 & 0.30 & 0.00 \\ 0.00 & 0.85 & 0.15 \\ 0.20 & 0.00 & 0.80 \end{bmatrix}, \Pi^3 = \begin{bmatrix} 0.76 & 0.24 & 0.00 \\ 0.00 & 0.80 & 0.2 \\ 0.18 & 0.00 & 0.82 \end{bmatrix}.$$

Take $x_0 = [6 \ 0 \ 0]^T$, $r_0 = 2$, and keep other parameters as in Case 1. Then the optimal problem (26) subject to (27), and (28) can be solved with $\gamma = 0.6463$. The experiment results are shown in Figs. 2(a)-2(e).

The simulation results of the two cases show that the designed filters estimate the system state accurately.

6. Conclusions

In this paper, distributed H_∞ -consensus filtering for target tracking over a wireless filter network have been investigated. The switching network topology has been assumed to be switching over time. Two kinds of communication constrains, i.e. channel fading and packet dropouts occurring not only in the measurement process for the sensors but also in the transmission process among the filters, have been considered. Sufficient conditions on the designed filters have been derived to ensure both the optimization of filter parameters and the described H_∞ -consensus disturbance/noise rejection attenuation performance index. Simulation examples have been finally used to verify effectiveness of the proposed target tracking filtering algorithm.

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