Numerical Modelling of consolidation-induced solute transport in unsaturated soil with dynamic hydraulic conductivity and degree of saturation

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Abstract

The effects of the pore pressure related dynamic hydraulic conductivity and dynamic degree of saturation on the consolidation-induced solute transport in deformable unsaturated soils are investigated. The storage equation and solute transport equations are revised to account for the dependence of these two soil parameters on pore pressures. Three dynamic models were conducted for dynamic hydraulic conductivity, dynamic degree of saturation, and both. Compared with the conventional model, the simulation results showed that both hydraulic conductivity and degree of saturation increased near the soil surface where pore pressure exceeded air-entry value. Dynamic hydraulic conductivity results in a slightly slower solute transport while dynamic degree of saturation accelerates the migration of contaminants. Including both dynamic effects produced limited differences in solute concentration, while consolidation results were affected significantly. Although dynamic degree of saturation has some influences on consolidation-induced solute transport, the correlation of the dynamic path is less significant. The air-entry value is an important parameter to determine when soil parameters become dynamic, and it affects the soil consolidation process in certain extent. This study provides new methods to incorporate in-homogeneous soil parameters, which are not only spatiotemporal variables, but also dynamic variations with pore pressure.

Keywords: Dynamic hydraulic conductivity; dynamic degree of saturation; solute transport; consolidation; unsaturated soil; deformable soil.

1. Introduction

Contaminant mass transport through porous media is usually described by well-established conventional transport models [1, 2] with the ability to account for advection, dispersion, and sorption. Since the mid-20th century, researchers have studied the advection-dispersion equation (ADE) through analytical approximations [3], numerical simulations [4, 5], and experimental investigations [6]. These studies were based on the assumption of a rigid porous media where the volume of the porous media does not change. As a result, the advective flow was usually induced by external hydraulic gradient (ground water flow, rainfall infiltration, etc.), however, considerations of transient flow due to soil deformation was missing. In fact, soil volume change (i.e., soil consolidation) occurs simultaneously with solute transport in many cases. For example, it occurs where the field is under an applied load (self-weight, fill placement, finite size loading, etc.) or experiencing changes in the groundwater table (pumping, artesian wells, etc.). In such cases, the coupled effect of soil deformation and solute transport would be necessary to be included in modelling. Alshawabkeh *et al.*[7] showed that the excess pore pressure dissipation produced a transient advective flux of contaminants, that had a strong influence on overall flux. The first attempt at explaining consolidation-induced solute transport in a porous medium was formulated by Potter *et al.*[8]. Smith [9] derived a one-dimensional(1D) analytical solution for a steady-state concentration with a varying porosity in a fully saturated deformable porous medium. In this model, soil consolidation was incorporated into the solute transport model with the following steps: (1) computing the porosity variation during consolidation, and (2) introducing solute migration into the solid phase. To link two components of solute in both pore fluid and solid phases, a linear sorption relationship was postulated. Peter & Smith[10] extended the previous model [9] to accommodate the time-dependent solute migration process. Zhang *et al.*[12] using the consolidation theory [11], further developed the small strain model [10] to account for a degree of saturation and fluid compressibility. Pu & Fox[13] used a piecewise method to study containment transport in a saturated layered soil. To incorporate the partially saturated soil environment, Wu & Jeng [14] conducted a comprehensive parametric study to examine the effect of soil stratification on consolidation-induced solute transportation and demonstrated the importance of correctly modelling a multi-layered soil instead of simply assuming the averaged properties across the soil profile.

Previous studies have treated the hydraulic conductivity and the degree of saturation as constant in the entire soil layer. However, hydraulic conductivity has been reported to be a function of void ratio (e) [15, 16, 17]. Further, pore-water pressure also affects the hydraulic conductivity [18, 19, 20]. Changes in pore water pressure lead to changes in stresses and in turn soil deformation. Since the hydraulic properties of the soil, such as porosity, hydraulic conductivity and water storage capacity are affected by the changes in stresses, so too will the void flow. This coupled effect highlights the strong correlation between flow and stress-deformation, and the connection to the effects of pore pressure related hydraulic conductivity and the degree of saturation to the consolidation-induced solute transportation. Different expressions for the constitutive relationships of dynamic hydraulic conductivity and degree of saturation were available such as presented in [20, 21, 23].

The aim of this study is to investigate the effects of dynamic hydraulic conductivity and the degree of saturation on consolidation-induced solute transport in unsaturated deformable porous media. First, these effects are introduced when there is positive pore pressure, then new non-linear governing equations for the pore pressure field and solute concentration field are derived. Finally, parametric studies for solute transport in an unsaturated soil layer are conducted to investigate the influence of dynamic parameters on the consolidation-induced solute transport.

2. Dynamic hydraulic conductivity and degree of saturation as functions of pore pressures

Dynamic hydraulic conductivity and degree of saturation are considered to be functions of pore pressures in order to investigate the consolidation-induced solute transport in a deformable medium. Herein, some key previous theories are outlined and the modified forms of this dependence are proposed.

2.1. Dynamic hydraulic conductivity

Several different forms of the dependence of hydraulic conductivity ($K(p)$) on the pore pressure p (or equivalently suction s) have been proposed. Gardner [20] considered two forms for hydraulic conductivity, a power law of the form:

$$K(s) = \frac{a}{s^d + b}, \quad (1)$$

for various d , and an exponential law:

$$K(s) = ae^{-fs}, \text{ or } K(p) = ae^{fp}. \quad (2)$$

where a, b, d, f are constants and s is the suction.

Wu & Zhang [21] used the version of exponential relationship and extended into the form:

$$K(p) = \begin{cases} K_s & |p| \leq |\psi_{ae}|, \\ K_s e^{\alpha\psi_{ae} + \alpha p} & |p| > |\psi_{ae}|, \end{cases} \quad (3)$$

where $K(p)$ [m/s] is the dynamic hydraulic conductivity, K_s [m/s] is the coefficient of conductivity at full saturation and α [1/kPa] is the de-saturation coefficient, which is related to the grain size distribution of a soil [30]. Generally speaking, the greater the clay content, the lower the de-saturation rate (i.e., the smaller the value of α)[31]. For a fully saturated soil, α is set as zero, hence the hydraulic conductivity is a constant K_s . ψ_{ae} [kPa] is the air-entry constant that is determined by the radius of the largest pore of a specific soil type. A smaller pore size (such as for clay) produces a relatively larger air-entry value. The typical values of these parameters for several soil types are listed in Table 1.

Table 1: Coefficient of hydraulic conductivity at full saturation (K_s), air-entry value (ψ_{ae}) and desaturation coefficient (α) for some soils.

Soil type	k_s (m/s)	ψ_{ae} (kPa)	α (kPa ⁻¹)
Clay	10 ⁻¹⁰ —10 ⁻⁸	20–39	0.016 for Beit Netofa clay
Silt	10 ⁻⁸ —10 ⁻⁶	7–155	0.043 for a silt loam
Sand	10 ⁻⁵ —10 ⁻³	1–3	0.106 for a fine sand
Gravel	10 ⁻² —10 ⁻¹	Nil	0.081 for Hpgiene sandstone
Ref: [24]; [25]; [26]			

2.2. Dynamic degree of saturation

For the degree of saturation, Gallipoli[23], using experimental data, proposed a model of the form:

$$S_r(s) = \left(\frac{1}{1 + (\gamma s)^{m_1}} \right)^{m_2}, \quad (4)$$

75 in which γ, m_1, m_2 are soil constants.

Wu & Zhang [21] proposed an exponential relationship between degree of saturation and pore pressure in the same format as they used for hydraulic conductivity. The dynamic degree of saturation was determined with respect to suction:

$$S_r(s) = \begin{cases} 1 & |s| \leq |\psi_{ae}|, \\ e^{\alpha\psi_{ae} + \alpha p} & |s| > |\psi_{ae}|, \end{cases} \quad (5)$$

76

With loads acting on the soil matrix, air bubbles may be expelled out from the pore fluid, leading to an increase in the degree of saturation. However, $S_r(p)$ cannot exceed unity. To incorporate the positive pore pressure, the constitutive relationship for dynamic degree of saturation is proposed as:

$$S_r(p) = \begin{cases} S_r^0 & |p| \leq |\psi_{ae}|, \\ -e^{\alpha\psi_{ae} - \alpha p} + 1 + S_r^0 & |\psi_{ae}| < |p| \leq |P_{Sr=1}|, \\ 1 & |p| > |P_{Sr=1}|. \end{cases} \quad (6)$$

77 The degree of saturation ($S_r(p)$) is again picked to be related to the air-entry value ψ_{ae} [kPa]
 78 and the de-saturation value α [1/kPa]; S_r^0 refers to the initial degree of saturation (or commonly
 79 known as the residual saturation) before a load acts on the soil. Fig. 1 illustrates the relationship
 80 between dynamic degree of saturation and the pore pressure (6), but not to scale. In the figure,
 81 another important parameter $P_{Sr=1}$ is defined as that pore pressure at which S_r reaches unity.
 82 Beyond this point, S_r will stop increasing; i.e., the soil matrix remains fully saturated.

83 When S_r is dynamic, it will vary both spatially and temporally. The variation of degree of
 84 saturation is particular important in deriving the consolidation-induced solute transport theory.
 85 Herein, the partial derivative of S_r with respect to p will be used and is firstly introduced here:

$$\frac{dS_r(p)}{dp} = \begin{cases} 0 & |p| \leq |\psi_{ae}|, \\ \alpha e^{\alpha\psi_{ae} - \alpha p} & |\psi_{ae}| < |p| \leq |P_{Sr=1}|, \\ 0 & |p| > |P_{Sr=1}|. \end{cases} \quad (7)$$

Figure 1: Relationship between the dynamic degree of saturation and pore pressure (Type 1)

As shown in Fig. 1(b), the partial differentiation of degree of saturation function is discontinuous. Before pore pressure reaches ψ_{ae} or exceeds the value of $P_{Sr=1}$, $\frac{dS_r(p)}{dp}$ equals to 0 because S_r is constant.

Note that the details of the shape of $S_r(p)$ in Fig. 1(a) is a proposed shape only. The curve is concave and we designate it as a Type 1 shape. In order to study the effects of the concavity of the dynamic degree of saturation function, an alternative convex-shape formulation is proposed and named as 'Type 2':

$$S_r(p) = \begin{cases} S_r^0 & |p| \leq |\psi_{ae}|, \\ e^{\alpha\psi_{ae}+\alpha p} - e^{2\alpha\psi_{ae}} + S_r^0 & |\psi_{ae}| < |p| \leq |P_{Sr=1}|, \\ 1 & |p| > |P_{Sr=1}|. \end{cases} \quad (8)$$

and the co-responding derivative of S_r with respect to p becomes:

$$\frac{dS_r(p)}{dp} = \begin{cases} 0 & |p| \leq |\psi_{ae}|, \\ \alpha e^{\alpha\psi_{ae}+\alpha p} & |\psi_{ae}| < |p| \leq |P_{Sr=1}|, \\ 0 & |p| > |P_{Sr=1}|. \end{cases} \quad (9)$$

Figure 2: Relationship between the dynamic degree of saturation and pore pressure (Type 2)

The diagram for the dynamic degree of saturation relationship with pore pressure for Type 2 is shown in Fig. 2. As for Type 1, the degree of saturation increases from 0.85 to unity as the pore pressure increases. For Type 2 the S_r versus p curve is convex, which results in an increasing function $\frac{dS_r(p)}{dp}$.

3. Theoretical models

When an external load is applied to a deformable soil matrix, an excess pore pressure is generated. Under draining conditions, the excess pore pressures will gradually dissipate after the load stops increasing (post-loading stage). When there is a source of non-active pollution, the contaminants are carried by the transient advective flow (triggered by the excess pore pressure) and finally spread and cause pollution. To model this process, three governing equations are required to calculate the excess pore pressure, soil displacement and solute concentration fields.

In this study, four models are considered, with summary shown in Table 2. Note that, for both Model Srp and Model Kp+Srp, dynamic degree of saturation formulation Type 1 is used. The following sections explain the theories of the conventional model and modified dynamic models.

Here the work of Zhang *et al.*[12] is extended to consider the effects of dynamic hydraulic conductivity and degree of saturation. Although the original equilibrium equation is directly applicable to all dynamic models, the governing equations for both the excess pore pressures

Table 2: Summary of models

Name in paper	Label in figures	Description
Conventional model	CON	coupled consolidation-transport model, with constant K and S_r , similar as [12].
Dynamic K model	Kp	coupled consolidation-transport model, with constitutive law for $K(p)$, Eq (3), and constant S_r
Dynamic S_r model	Srp	coupled consolidation-transport model, with constitutive law for $S_r(p)$ (Type 1, Eq (6)) and constant K
Dynamic K & S_r model	Kp+Srp	coupled consolidation-transport model, with constitutive laws for $K(p)$, Eq (3) and $S_r(p)$ (Type 1, Eq (6))

and concentration fields need to be updated. This section will only present the final governing equations for each model while the detailed derivation is in Appendixes A & B.

According to [12], the force balance equation is:

$$G \frac{2(1-\nu)}{(1-2\nu)} \frac{\partial^2 u}{\partial z^2} + (1-n^0)(\rho_s - S_r \rho_w) g \frac{\partial u}{\partial z} = \frac{\partial p}{\partial z}; \quad (10)$$

where u [m] is the vertical displacement, p [Pa] is the excess pore pressure, G [Pa] is the shear modulus and assumed to be constant in this study, ν is the Poisson's ratio, and ρ_w [kg/m^3] & ρ_s [kg/m^3] denote the densities of the pore fluid and solid particles.

3.1. Dynamic K & S_r model

The spatial derivative of hydraulic conductivity for $|p| > |\psi_{ae}|$ is:

$$\frac{\partial K}{\partial z} = \frac{dK}{dp} \frac{\partial p}{\partial z} = \alpha K_s e^{\alpha \psi_{ae} + \alpha p} \frac{\partial p}{\partial z} = \alpha K \frac{\partial p}{\partial z}. \quad (11)$$

120 Similarly, the temporal and spatial derivations of degree of saturation (Type 1) are, for $|\psi_{ae}| <$
 121 $|p| \leq |P_{Sr=1}|$:

$$\frac{\partial S_r}{\partial t} = \frac{dS_r}{dp} \frac{\partial p}{\partial t} = \alpha e^{\alpha \psi_{ae} - \alpha p} \frac{\partial p}{\partial t}, \quad (12)$$

and

$$\frac{\partial S_r}{\partial z} = \frac{dS_r}{dp} \frac{\partial p}{\partial z} = \alpha e^{\alpha \psi_{ae} - \alpha p} \frac{\partial p}{\partial z}. \quad (13)$$

122 It is important to point out that, by using product law, the term $\frac{\partial K}{\partial t}$, $\frac{\partial S_r}{\partial t}$, $\frac{\partial S_r}{\partial z}$ could be solved
 123 as long as $\frac{dK}{dp}$ and $\frac{dS_r}{dp}$ are known. Therefore, other forms of dynamic constitutive relationships
 124 are also applicable.

125 Substitute (11)-(13) into (A.11), and replace porosity n with its initial value n^0 [12], then
 126 the governing equations for excess pore pressure are summarised as:

- for $|p| \leq |\psi_{ae}|$,

$$S_r n^0 \beta \frac{\partial p}{\partial t} + S_r \frac{\partial^2 u}{\partial z \partial t} = \frac{K}{\rho_w g} \left(\frac{\partial^2 p}{\partial z^2} \right), \quad (14)$$

127 where β is the fluid compressibility as defined in (A.5) [27].

- for $|\psi_{ae}| < |p| \leq |P_{Sr=1}|$,

$$\left(S_r n^0 \beta + n^0 \alpha e^{\alpha \psi_{ae} - \alpha p} \right) \frac{\partial p}{\partial t} + S_r \frac{\partial^2 u}{\partial z \partial t} = \frac{K}{\rho_w g} \left(\frac{\partial^2 p}{\partial z^2} \right) + \frac{\alpha K}{\rho_w g} \left(\frac{\partial p}{\partial z} \right)^2 - n^0 \alpha e^{\alpha \psi_{ae} - \alpha p} \frac{\partial u}{\partial t} \frac{\partial p}{\partial z}, \quad (15)$$

- for $|p| > |P_{Sr=1}|$,

$$S_r n^0 \beta \frac{\partial p}{\partial t} + S_r \frac{\partial^2 u}{\partial z \partial t} = \frac{K}{\rho_w g} \left(\frac{\partial^2 p}{\partial z^2} \right) + \frac{\alpha K}{\rho_w g} \left(\frac{\partial p}{\partial z} \right)^2. \quad (16)$$

129 The governing equations for solute transport with dynamic hydraulic conductivity and de-
 130 gree of saturation are given below, with the detailed derivations provided in Appendix B.

- for $|p| \leq |\psi_{ae}|$ or $|p| > |P_{Sr=1}|$,

$$\begin{aligned}
\left[S_r n^0 + (1 - n^0) \rho_s K_d \right] \frac{\partial c}{\partial t} &= \frac{\partial^2 c}{\partial z^2} \left(S_r n^0 D_m - \alpha_L \frac{K}{\rho_w g} \frac{\partial p}{\partial z} \right) \\
&+ \frac{\partial c}{\partial z} \left\{ -\alpha_L S_r n^0 \beta \frac{\partial p}{\partial t} - \alpha_L S_r \frac{\partial^2 u}{\partial z \partial t} \right. \\
&+ \frac{\alpha_L \beta K}{\rho_w g} \left(\frac{\partial p}{\partial z} \right)^2 + S_r D_m (1 - n^0) \frac{\partial^2 u}{\partial z^2} \\
&+ \left. \frac{K}{\rho_w g} \frac{\partial p}{\partial z} - \left[S_r n^0 + (1 - n^0) \rho_s K_d \right] \frac{\partial u}{\partial t} \right\} \\
&+ c \left[S_r n^0 \beta \frac{\partial p}{\partial t} - \beta \frac{K}{\rho_w g} \left(\frac{\partial p}{\partial z} \right)^2 + S_r n^0 \beta \frac{\partial u}{\partial t} \frac{\partial p}{\partial z} \right],
\end{aligned} \tag{17}$$

- for $|\psi_{ae}| < |p| \leq |P_{Sr=1}|$,

$$\begin{aligned}
\left[S_r n^0 + (1 - n^0) \rho_s K_d \right] \frac{\partial c}{\partial t} &= \frac{\partial^2 c}{\partial z^2} \left(S_r n^0 D_m - \alpha_L \frac{K}{\rho_w g} \frac{\partial p}{\partial z} \right) \\
&+ \frac{\partial c}{\partial z} \left\{ n^0 \alpha e^{\alpha \psi_{ae} - \alpha p} \left(D_m \frac{\partial p}{\partial z} - \alpha_L \frac{\partial p}{\partial t} - \alpha_L \frac{\partial u}{\partial t} \frac{\partial p}{\partial z} \right) \right. \\
&- \alpha_L S_r n^0 \beta \frac{\partial p}{\partial t} - \alpha_L S_r \frac{\partial^2 u}{\partial z \partial t} \\
&+ \frac{\alpha_L \beta K}{\rho_w g} \left(\frac{\partial p}{\partial z} \right)^2 + S_r D_m (1 - n^0) \frac{\partial^2 u}{\partial z^2} \\
&+ \left. \frac{K}{\rho_w g} \frac{\partial p}{\partial z} - \left[S_r n^0 + (1 - n^0) \rho_s K_d \right] \frac{\partial u}{\partial t} \right\} \\
&+ c \left[S_r n^0 \beta \frac{\partial p}{\partial t} - \beta \frac{K}{\rho_w g} \left(\frac{\partial p}{\partial z} \right)^2 + S_r n^0 \beta \frac{\partial u}{\partial t} \frac{\partial p}{\partial z} \right],
\end{aligned} \tag{18}$$

3.2. Dynamic Sr model

When the degree of saturation is dynamic (Type 1) and the hydraulic conductivity is constant, the governing equations for excess pore pressure field are:

- for $|p| \leq |\psi_{ae}|$ or $|p| > |P_{Sr=1}|$, same as (14),

- for $|\psi_{ae}| < |p| \leq |P_{Sr=1}|$,

$$\left(S_r n^0 \beta + n^0 \alpha e^{\alpha \psi_{ae} - \alpha p} \right) \frac{\partial p}{\partial t} + S_r \frac{\partial^2 u}{\partial z \partial t} = \frac{K}{\rho_w g} \left(\frac{\partial^2 p}{\partial z^2} \right) - n^0 \alpha e^{\alpha \psi_{ae} - \alpha p} \frac{\partial u}{\partial t} \frac{\partial p}{\partial z}. \tag{19}$$

The updated solute transport equations are:

- for $|p| \leq |\psi_{ae}|$ or $|p| > |P_{Sr=1}|$, same as (17)

- for $|\psi_{ae}| < |p| \leq |P_{Sr=1}|$, same as (18)

3.3. Dynamic K model

Under the assumption of dynamic hydraulic conductivity but constant degree of saturation, the governing equations for excess pore pressure are:

- for $|p| \leq |\psi_{ae}|$, same as (14),
- for $|p| > |\psi_{ae}|$, same as (16).

Although the hydraulic conductivity is dynamic, the solute transport equation is the same as (17). This is because the dynamic hydraulic conductivity term is only introduced into the solute transport equation when replacing the velocity terms. A detailed explanation is given in Appendix B.

3.4. The conventional model with constant K and S_r

When both the hydraulic conductivity and degree of saturation are constant, the governing equation for excess pore pressure is the same as (14), and for solute concentration field, it is written as the form of (17). It is important to point out that these two equations are the same as presented in [12].

3.5. Model validation

Since no experimental data for solute transport in a deformable porous medium with dynamic hydraulic conductivity and degree of saturation is available in the literature, a possible validation of the present model is to compare with the previous models with constant soil properties [10, 12]. To compare with those results, the new dynamic model (with both K & S_r dynamic) was run with a very high ψ_{ae} value (1000 kPa). Then, pore pressure will never exceed the air-entry value. That is, the dynamic model was actually calculating the conventional study case where both K and S_r remains unchanged. Furthermore, utilizing the piecewise function, the ramp load can be applied with different smoothing methods at the turning point. Here, a continuous second derivative was adopted and applied for a period of half a year. This smoothing method can be explained as follows: when the landfill site is about to reach capacity, less waste is disposed into this field, and more waste delivered to a new site. Therefore, the loading rate decreases. Fig. 3 is presented as validation of the newly added smoothing methods. Two

Figure 3: The comparison of the present dynamic model (solid lines) with Zhang et al. (2012) (dotted lines) and Peter and Smith (2002) (circled points). Note: case 1 (red) is presented for $c_v = 0.6 \text{ m}^2/\text{s}$, case 2 (blue) is presented for $c_v = 1 \text{ m}^2/\text{s}$

sets of results were compared with the control of c_v calculated from (25). Results in red and blue are for the coefficient of consolidation equals $0.6 \text{ m}^2/\text{s}$ and $1.0 \text{ m}^2/\text{s}$ respectively. It is clear that the dynamic model with the smoothing method (solid lines) is a good fit to both [12] (dotted lines) and [10] (circled points).

4. Numerical model for a landfill system

4.1. Boundary conditions and initial conditions

As an application, a landfill with one leachate collection system is assumed to be constructed on the bottom of a compacted clay layer (Fig. 4). The contaminant migration through the clay layer beneath the landfill is evaluated. The width of the landfill site is considered to be larger than the thickness of soil layer, and the load on the top surface is assumed to be uniform, therefore, 1D model will be applied with z -axis positive pointing downwards.

At the top boundary ($z = 0$), the impermeable geomembrane layer prevents Darcy's flow. Therefore, a zero excess pore pressure gradient can be postulated, as follows:

Figure 4: Sketch of a typical landfill site

$$\frac{\partial p}{\partial z}(0, t) = 0. \quad (20)$$

Furthermore, as the wastes is disposed to landfill gradually until reaching its capacity, a ramp load is proposed with a constant increasing rate. As shown in Fig. 5, the external load (Q) keeps increasing in a rate of 200 kPa annually for two years and remains constant of 400 kPa until the end of simulation period.

To derive the top boundary condition for soil deformation, an elastic deformation is taken into account, and a vertical force balance relationship is applied. This leads to the soil deformation at the top boundary to be [12]:

$$\frac{\partial u}{\partial z}(0, t) = \frac{1 - 2\nu}{2G(1 - \nu)}[-Q(t) + p]. \quad (21)$$

According to [12], considering the volatile organic compounds that diffuse through the geomembrane layer and dissolve into the pore water, the top boundary condition for the solute concentration is expressed as:

$$\frac{\partial c}{\partial z}(0, t) = \frac{D_G}{n^0 h D_m}(c - c_0), \quad (22)$$

Figure 5: Ramp load

188 where D_G is the mass transfer coefficient of geomembrane, and c_0 is the reference solute con-
 189 centration in the waste.

190 At the exit boundary ($z = L$), with a leachate collection system, a free drainage condition is
 191 considered, which implies a zero pore pressure, and no deformation is allowed at that point. In
 192 addition, the concentration gradient is assumed to be zero. Hence, only advective flow occurs
 193 at the outlet boundary. The lower boundary conditions can be expressed as:

$$\begin{cases} p(L, t) = 0, \\ u(L, t) = 0, \\ \frac{\partial c}{\partial z}(L, t) = 0. \end{cases} \quad (23)$$

To simplify the model, all initial values for pore pressure field, soil deformation and solute transport are set to zero, i.e.,

$$\begin{cases} p(z, 0) = 0, \\ u(z, 0) = 0, \\ c(z, 0) = 0. \end{cases} \quad (24)$$

The initial values of dynamic hydraulic conductivity and degree of saturation are also critical in this study, and the initial values (K_s & S_r^0) will be introduced in the following section along with other input parameters.

4.2. Input parameters and numerical approach

In this study, all models simulates the same landfill case but with different considerations of the hydraulic conductivity and degree of saturation. Table 3 summarises all parameters used in Model CON and parameters only used in the dynamic models.

The combination of the parameters is selected to ensure that the coefficient of consolidation (c_v) stays within the range of 1×10^{-8} to 3×10^{-7} m²/s [28, 29]. The coefficient of consolidation (c_v) can be calculated as

$$c_v = \frac{2GK(1 - \nu)}{\rho_w g(1 - 2\nu)}. \quad (25)$$

Additionally, while making selection of the parameters, soil deformation was kept less than 20% to satisfy the small deformation assumption.

Note that in the conventional model, S_r is assumed to be equal to S_r^0 and K is set to be equal to K_s through the whole simulation.

All models are simulated by COMSOL Multiphysics 5.0, which is a popular finite element solver. Some settings such as parametric studies are achieved through COMSOL with Matlab 5.0.

5. Results and discussions

In Section 5.1, 5.2 and 5.3, the results from the dynamic Models Kp , Srp and $Kp + Srp$ are presented together with the conventional model, CON. Section 5.4 then summarizes the averaged flow velocity and the advective emission for all models. The results in Sections 5.1 - 5.4 are all for the Type 1 $S_r(p)$ shape. A comparison for the Type 2 $S_r(p)$ shape is given in Section 5.5

Table 3: Parameters

Parameters for Model CON		
Parameter	Value	Description
$Q(t)$	As introduced in Section 4.1	Waste loading
h	0.0015 m	Thickness of geomembrane
L	3 m	Thickness of clay layer
S_r^0	0.85	Initial degree of saturation
n^0	0.33	Initial porosity
G	5×10^5 Pa	Shear modulus
ν	0.33	Poisson's ratio
K_s	1×10^{-9} m/s	Initial hydraulic conductivity
α_L	0.1 m	Longitudinal dispersion factor
ρ_w	1×10^3 kg/m ³	Initial density of the pore fluid, varied due to fluid compressibility
ρ_s	2.6×10^3 kg/m ³	Density of the solid phase
K_d	0	Partitioning coefficient
r_h	0.02 m	Volumetric fraction of dissolved air within pore water
D_G	1.5×10^{-4} m ² /y	Mass transfer coefficient of geomembrane
D_m	5×10^{-9} m ² /s	Molecular diffusion coefficient in the clay
c_0	0.1 kg/m ³	Reference solute concentration
g	9.8 m/s ²	Gravity acceleration
Parameters for Model Kp, Srp & Kp+Srp		
ψ_{ae}	30 kPa	Air-entry value for clay
α	0.016 kPa ⁻¹	De-saturation coefficient for clay

Figure 6: Distribution of dynamic hydraulic conductivity vs time and depth for Model Kp

5.1. Dynamic hydraulic conductivity model (Model Kp)

The results presented in this section are from the dynamic hydraulic conductivity model while keeping the degree of saturation as a constant value.

The most important result of Model Kp is the form of the evolution of the dynamic conductivity (K). Fig. 6 shows changes in K at different locations. At the bottom (the contaminant outlet in the clay layer) K is constant and always equal to K_s . This is attributed to the zero pore pressure at that point allowing for the pore fluid to flow freely into the leachate collection pipe, since the pore pressure at the outlet will never exceed the air entry value. However, the values of K at both the top point (the contaminant inlet in the clay layer) and the middle point (where $z = L/2$ in the clay layer) experience increasing and then decreasing periods. The excess pore pressure increases as the ramp load is gradually applied to the clay layer, when the pore fluid does not have time to drain out. Once the pore pressure rises up to the air entry level, K starts to increase. After the load reaches the post-loading period (load stops increasing) in around 2 years, the excess pore pressure starts to dissipate with the pore fluid gradually expelling out from soil matrix. According to the dynamic conductivity model, the pore pressure recovers to the air-entry level, when K then maintains the level of K_s . The largest K value of around $3.5 \times 10^{-9} \text{m/s}$ occurs at the inlet in 1.5 years. The dynamic conductivity curve at the top point is higher than that at the middle point, implying that the pore pressure is generally a maximum at the top. Fig. 6 also shows the distributions of K along the clay layer were plotted at different times. The dark blue line labelled '1yr' indicates values after 1 year of loading. K curves for 1

(a) Excess pore pressure

(b) Vertical displacement

(c) Solute concentration

Figure 7: Distribution of (a) excess pore pressure, (b) vertical displacement and (c) solute transport vs time and depth for Model Kp and Model CON

year and 2 years confirm the maximum increase at the upper layer, where the pore pressure is harder to dissipate compared to the lower layer. After 3 years, K returns to initial value of K_s and remains unchanged through the whole layer to the end of the simulation.

Spatial and temporal changes in K affect the corresponding excess pore pressure (p), as indicated in Fig.7(a). The solid lines describe the results of Model Kp while the dashed lines are the results of Model CON, where $K = K_s$. Comparing the two models, some similar trends can be found. For example, p in the upper layer is higher than in the lower layer; p rises rapidly during the loading period then drops gradually after entering the post-loading stage. However, their increasing/dissipation rate and amount are all affected by dynamic K . The peak excess pore pressures for Model Kp at the top point and middle point are 50kPa & 40kPa respectively, while for Model CON, the highest p is almost double (100kPa & 75kPa) at the two locations. Moreover, the dynamic conductivity also influences the dissipation rate during the post-loading period. At the top boundary of the clay layer, p has been nearly fully dissipated after 5 years for Model Kp while a remainder of around 8 kPa exists for the conventional model. These features can be explained by the definition of hydraulic conductivity ; a larger K means the pore fluid is easier to travel within the void, hence accelerates the drainage at the outlet. The dynamic hydraulic conductivity and pore pressure relationship that results in a higher K on the top layer, further leads to a smaller excess pore pressure build up and quicker dissipation.

Compared to the pore pressure, the vertical soil deformation (u) was less influenced by the dynamic conductivity and the final level of soil deformation is not affected at all. The displacement versus time figure (Fig. 7(b)) shows that the largest vertical displacement is 0.6m measured from the top and 0.3m at the middle point for both Model Kp and Model CON. The bottom point is assumed to be rigid so no vertical displacement is allowed. However, the rate of soil deformation is affected when K becomes dynamic. The displacement versus depth figure indicates that after 2 years, the top surface has sunk 0.53 m with dynamic conductivity. However, the conventional model only shows a 0.48 m downward settlement. Furthermore, the Model Kp predicts that the top surface reaches its final deformation level after 3 years, but Model CON is still 5 cm short of its final value at that time.

For the volatile pollutant to spread in the soil matrix, part of the solute dissolves in the transient advective flow, which is produced from the soil consolidation process. Meanwhile,

hydrodynamic dispersion, which is a combined process of molecular diffusion and longitudinal dispersion, takes place. Molecular diffusion is mainly manifested as particles move from an area of high concentration to an area of low concentration and longitudinal diffusion refers to the process whereby solutes are mechanically mixed by velocity variations. Generally speaking, in relatively low speed environments, molecular diffusion dominates the process. For the numerical examples considered here, during the loading period, there is no significant difference at the top point (due to the boundary condition constrain) or the bottom point (since the contaminants have not reached bottom). Nevertheless, a faster contaminant migration can be discovered at the middle point (Fig. 7(c)). Detailed results can be seen from the plot of concentration (c/c_0) versus depth. After 2 years, at a depth of $1.7m$ the contaminant level has reached 10% ($c/c_0 = 0.1$) for Model Kp, while for Model CON, the same contaminant level was reached 10 cm higher. On the post-loading stage, K recovers to its initial value and the transient excess flow carrying contaminants continues spreading. Due to pore pressure dissipation, the rate of solute transportation reduces in both models. After the pore pressure is fully dissipated, only hydrodynamic dispersion contributes to contaminant transportation. Moreover, according to the outlet boundary condition, the contaminant cannot escape the soil layer, which results in an accumulation at the bottom. Compared to Model CON, Fig. 7(c) reveals that a dynamic K will eventually reduce pollution. For example, the breakthrough time, the time required for the solute concentration to reach 10% of the referenced concentration at the outlet, takes half a year longer with dynamic conductivity.

5.2. Dynamic degree of saturation model (Model Srp)

Here we vary the dynamic degree of saturation (Model Srp), but keep the hydraulic conductivity constant.

Fig. 8 shows that the degree of saturation (S_r) varies between S_r^0 (0.85) and 1. The dynamic S_r happens mainly during the loading period and in the upper layer. As the load acts on the soil layer, the air bubbles are expelled out from the pore fluid and result in a change in the degree of saturation. Fig. 8(a) presents the evolution of S_r in the three cut points. Except for the bottom point, degree of saturation at both top point and middle point are greatly changed. Specifically, soil has become fully saturated for nearly 3 years at the top and around 2 years at middle point.

(a) Dynamic degree of saturation vs time (b) Dynamic degree of saturation distribution

Figure 8: Distribution of dynamic degree of saturation vs time and depth for Model Srp (Type 1)

Due to the constrain of pore pressure at the Outlet S_r remains unchanged at the level of S_r^0 . After 3 years, with the excess pore pressure gradually dissipating, S_r reduced. Moreover, Fig. 8(b) shows the vertical distribution of the dynamic S_r with different colour lines indicating each time step. After 2 years, the fully saturation zone has reached 2.4 m down, however after 4 years, the whole clay layer returns to unsaturated status.

The effect of dynamic degree of saturation on the excess pore pressure is plotted in Fig. 9(a), and it is very different from Model Kp. With Model Srp the peak excess pore pressure shows almost no differences compared with Model CON at both top and middle points. However, both excess pore pressure accumulation rate and dissipation rate are affected especially during the period when S_r is experiencing either sudden rise (0.5 year– 1 year) and drop (3 years - 6 years). This implies that although the change of S_r magnitude is small (from 0.85 to 1), the rate of change (the additional terms in excess pore pressure governing equations due to $\frac{\partial S_r}{\partial t}$) contributes to the consolidation process to a certain extent. For Model CON, the excess pore pressure is almost fully dissipated after 6 years, while due to dynamic S_r , a residue of 10 kPa exits at the middle point. With excess pore pressure expelled out of the soil matrix, the soil deforms. Similar to Model Kp, dynamic S_r does not affect the final deformation, but the inlet will eventually drop 0.6m (Fig. 9(b)). Due to the reduction of excess pore pressure dissipation rate during Year 3 to Year 6, the soil deforms slightly slower at the this period. For the solute transport, the Model Srp results in a slightly slower solute transport during the loading period

(a) Excess pore pressure

(b) Vertical displacement

(c) solute concentration

Figure 9: Distribution of (a) excess pore pressure, (b) vertical displacement and (c) solute transport vs time and depth for Model Srp and Model CON

and a faster spreading on the post-loading stage. Eventually, higher solute concentration can be observed. According to Fig. 9(c), it takes 7.5 years for the outlet to be 20% contaminated for Model Srp, while with constant S_r it takes 1.5 years longer to reach the same contaminated level. At the end of 15 years, the outlet has been polluted to 38% with dynamic S_r , while Model CON suggests the final contamination level to be lower than 35%.

5.3. Dynamic hydraulic conductivity and degree of saturation model (Model Kp+Srp)

In the dynamic hydraulic conductivity and degree of saturation model (Model Kp+Srp), both hydraulic conductivity and dynamic degree of saturation vary with pore pressure.

Fig. 10 present Model Kp+Srp results for dynamic conductivity and dynamic degree of saturation versus time and depth. According to the dynamic theory, K , p and S_r are mutually interacting. As a result, although the same parameters were used in Model Kp+Srp, Model Kp and Model Srp, the evolution of both K and S_r are quite different from the previous two dynamic models. Specifically, the peak value of K for the current model is greater than the maximum K (3.5×10^{-9} m/s) in Model Kp. Moreover, the dynamic S_r evolution shows several differences compared with Model Srp, For instance, the fully saturated condition lasts shorter at both top and middle points; after 2 years the fully saturated zone shrinks to 1.5m while Model Sp obtained a 2.4m saturated zone; when S_r recovers back to the level of S_r^0 after 3 years, the top point in Model Srp remains fully saturated.

The dynamic effects of K and S_r have been discussed separately in previous sections. Under the dynamic effects of both, compared with Model CON, a reduction of excess pore pressure (Fig. 11(a)) and a slightly faster soil deformation (Fig. 11(b)) can be observed. This is reasonable since the dynamic S_r effect is less significant than the dynamic K in term of the soil consolidation process. However, since dynamic K results in a slower solute transport while dynamic S_r accelerates the contaminant migration process, for our numerical study case, concentration results for Model $Kp + Srp$ eventually shows limited differences with Model CON (Fig. 11(c)).

(a) Dynamic hydraulic conductivity

(b) Dynamic degree of saturation

Figure 10: Distribution of (a) dynamic hydraulic conductivity and (b) dynamic degree of saturation vs time and depth for Model Kp+Srp

(a) Excess pore pressure

(b) Vertical displacement

(c) solute concentration

Figure 11: Distribution of (a) excess pore pressure, (b) vertical displacement and (c) solute transport vs time and depth for Model Kp+Srp and Model CON

Figure 12: the advective emission and the averaged flow velocity for all models

5.4. Average flow velocity and advective emission

Here the average flow velocity (v_f) and the advective emission (E_{adv}) for all models are compared to examine the effects of dynamic K and dynamic S_r on the consolidation-induced solute transport model.

The average flow velocity at the bottom boundary for each model is plotted in Fig. 12(a). The average flow velocity can be calculated as the sum of Darcy's velocity and the solid phase velocity:

$$v_f = -\frac{K}{S_r n^0 \rho_w g} \frac{\partial p}{\partial z} + \frac{\partial u}{\partial t}. \quad (26)$$

The peak v_f for all models occurs at around two years, when the post-loading stage begins. The conventional model and dynamic degree of saturation model shows faster transient advective flow, with a peak flow velocity to be around 2.5×10^{-8} m/s (Fig. 12(a)). Further, the v_f curve for Model Kp+Srp does not show much difference to the dynamic K model, but is considerable lower than the other two models. Meanwhile, Model Srp and Model CON generate similar V_f trends. Hence, the transient flow reduces speed with the dynamic K , however, the varying degree of saturation has less influence on the average advective flow velocity.

Transient excess flows triggered by soil consolidation shows considerable influence on the solute transport. This effect can be observed from the advective emission (E_{adv}) as summarized in Fig. 12(b). E_{adv} can be calculated as [12]

$$E_{adv} = \int_0^t -\frac{\partial p(\tau)}{\partial z} c(\tau) d\tau, \quad (27)$$

where τ is the independent variable of integration.

With the assumption of zero concentration gradient at the outlet, no diffusion takes place and only advective flow is allowed. Thus, the advective emission at the bottom boundary refers to the cumulative contaminant mass outflow. Fig. 12(b) presents the advective emission at the bottom boundary for each model. Clearly the advective emissions are significantly different for each dynamic model compared to the conventional model. Specifically, dynamic hydraulic conductivity reduces the advective emission while dynamic degree of saturation tends to increase the emission by a larger amount. However, considering dynamic K and S_r simultaneously, Model Kp+Srp model suggest a mitigation of E_{adv} . This feature reveals a coupled effect of dynamic conductivity and degree of saturation so that their effects cannot be simply added. Further, the largest advective emission occurs in Model Srp, which is consistent with the solute concentration result and averaged flow velocity result, that dynamic S_r produces a faster solute transport while keeping a relatively fast transient flow rate.

5.5. Concavity of dynamic degree of saturation function

In Section 2.2, two expressions were proposed to describe the relationship between dynamic degree of saturation and pore pressure. For both expressions, the degree of saturation increases from S_r^0 to 1 as pore pressure increased. The only difference is that Type 1 function is concave downwards while the Type 2 function is concave up. To study the effect of the concavity to consolidation-induced solute transport, assuming constant hydraulic conductivity, a modified dynamic S_r model is used and compared with previous Model Srp.

Using the definition (8), the modified storage equation for dynamic S_r Type 2 becomes:

- for $|p| \leq |\psi_{ae}|$ or $|p| > |P_{Sr=1}|$, same as (14),

- for $|\psi_{ae}| < |p| \leq |P_{Sr=1}|$,

$$\left(S_r n^0 \beta + n^0 \alpha e^{\alpha \psi_{ae} + \alpha p}\right) \frac{\partial p}{\partial t} + S_r \frac{\partial^2 u}{\partial z \partial t} = \frac{K}{\rho_w g} \left(\frac{\partial^2 p}{\partial z^2}\right) - n^0 \alpha e^{\alpha \psi_{ae} + \alpha p} \frac{\partial u}{\partial t} \frac{\partial p}{\partial z}. \quad (28)$$

The modified solute transport equation are:

- for $|p| \leq |\psi_{ae}|$ or $|p| > |P_{Sr=1}|$, same as (17)

- for $|\psi_{ae}| < |p| \leq |P_{Sr=1}|$,

$$\begin{aligned} \left[S_r n^0 + (1 - n^0) \rho_s K_d\right] \frac{\partial c}{\partial t} = & \frac{\partial^2 c}{\partial z^2} \left(S_r n^0 D_m - \alpha_L \frac{K}{\rho_w g} \frac{\partial p}{\partial z}\right) \\ & + \frac{\partial c}{\partial z} \left\{ n^0 \alpha e^{\alpha \psi_{ae} + \alpha p} \left(D_m \frac{\partial p}{\partial z} - \alpha_L \frac{\partial p}{\partial t} - \alpha_L \frac{\partial u}{\partial t} \frac{\partial p}{\partial z}\right) \right. \\ & - \alpha_L S_r n^0 \beta \frac{\partial p}{\partial t} - \alpha_L S_r \frac{\partial^2 u}{\partial z \partial t} \\ & + \frac{\alpha_L \beta K}{\rho_w g} \left(\frac{\partial p}{\partial z}\right)^2 + S_r D_m (1 - n^0) \frac{\partial^2 u}{\partial z^2} \\ & + \frac{K}{\rho_w g} \frac{\partial p}{\partial z} - \left[S_r n^0 + (1 - n^0) \rho_s K_d\right] \frac{\partial u}{\partial t} \left. \right\} \\ & + c \left[S_r n^0 \beta \frac{\partial p}{\partial t} - \beta \frac{K}{\rho_w g} \left(\frac{\partial p}{\partial z}\right)^2 + S_r n^0 \beta \frac{\partial u}{\partial t} \frac{\partial p}{\partial z} \right], \end{aligned} \quad (29)$$

Note that the force balance equation remains the same as previous models.

The distribution of dynamic degree of saturation versus time and depth are shown in Fig.

13. Solid lines indicate dynamic S_r Type 2 results while dashed lines are from the previous Model Srp (Type 1 dynamic S_r theory). In generally, the Type 2 shows a more pronounced change in degree of saturation dynamics. Specifically, as shown in Fig. 13(b), at year 5, the whole soil layer returns the original S_r level of 0.85 for Type 1, while Type 2 shows a higher S_r in the upper layer (around $z = 0.3m$). The fully saturated zone extents deeper with dynamic S_r Type 2, so that, for example, the fully saturated zone is nearly half meter thicker in year 2 and 1 meter thicker in year 3. Moreover, Type 2 dynamic S_r results in a slightly longer fully-saturated period at both top and middle points.

Fig. 14 (a) indicates that, while the Type 2 model has an impact on pore pressure distribution for a short period (during year 5 to year 9), there are no significant differences on the peak pore pressure values. Regardless of the path how S_r reaches unity, it takes 9 years for pore pressure to

(a) Dynamic degree of saturation vs time (b) Dynamic degree of saturation distribution

Figure 13: Distribution of dynamic degree of saturation vs time and depth for Model Srp, comparison between Type 1(dashed lines) and Type 2(solid line)

be fully dissipated for both models. Further, Fig. 14(b) indicates similar vertical displacement distribution for the two models showing that the soil deformation is hardly influenced by the concavity of dynamic S_r formulation. Consequently, the effect of concavity of dynamic S_r on solute concentration is limited (Fig. 14(c)).

5.6. Parametric study for varied air-entry value

As discussed in Section 3, the air-entry value (ψ_{ae}) is an important parameter of the criterion for judging whether hydraulic conductivity and degree of saturation are dynamic. Only in the situation where excess pore pressure is greater than the air-entry value, K and S_r are considered to be varying with respect to the pore pressure. Consequently, the effects of varied ψ_{ae} to the consolidation-induced solute transport is particularly of interest and presented in this section. Fig. 15-17 show the parametric study results for Model Kp, Model Srp and Model Kp+Srp respectively. In each figure, solid lines, dashed lines and dotted lines denote the simulation results when ψ_{ae} equals 20kPa, 30kPa and 39kPa assuming a clayey type of soil.

In general, ψ_{ae} alters the time when K and S_r become dynamic and affects the magnitudes of these parameters. Smaller ψ_{ae} implies that pore pressure is more easier to excess the air-entry value, so that the longer dynamic period can be observed. For Model Kp, with smaller ψ_{ae} (20 kPa), Fig. 15(a) indicates the hydraulic conductivity starts to increasing from 0.25 year, and

(a) Excess pore pressure

(b) Vertical displacement

(c) Solute concentration

Figure 14: Distribution of (a) excess pore pressure, (b) vertical displacement and (c) solute transport vs time and depth for Model Srp, comparison between Type 1(dashed lines) and Type 2(solid line)

(a) Dynamic hydraulic conductivity vs time

(b) Excess pore pressure vs time

(c) Solute concentration vs time

Figure 15: Distribution of (a) dynamic hydraulic conductivity (b) excess pore pressure and (c) solute concentration vs time for Model Kp with varied ψ_{ae}

the dynamic period ends at 3.5 year at the top point and 2.5 year at the middle point. With the largest air entry value of 39 kPa, the hydraulic conductivity dynamic period is shorten to around 1.8 years for middle point and around 2 years at top. It is important to point out that, the outlet boundary (bottom) is assumed to allow free drainage of the excess pore fluid ($p^e = 0$) so that pore pressure will never exceed the air entry value, and this feature further leads to constant K and S_r at bottom through out the whole simulation time. In addition to the influence of dynamic period, another impact on the magnitude of the dynamic K can be conclude that larger ψ_{ae} results in lower K at both top and middle points. With the combined effects of both, the excess pore pressure distribution (Fig. 15(b)) shows the lowest peak value occurs when $\psi_{ae} = 30$ kPa,

421 which is neither the smallest not largest air-entry value. This interesting finding highlights the
 422 importance of numerically conducting dynamic model rather than qualitative analysis. Further
 423 more, according to Fig. 15(c) for Model Kp, the solute concentration is less affected by air-
 424 entry value. It is also contributed by the joint effects that with increased ψ_{ae} , K shows shorter
 425 dynamic period but larger magnitude.

Figure 16: Distribution of (a) dynamic degree of saturation (b) excess pore pressure and (c) solute concentration vs time for Model Srp with varied ψ_{ae}

426 Fig. 16 presents the dynamic degree of saturation, excess pore pressure and solute concen-
 427 tration distribution for Model Srp with varied ψ_{ae} . When conducting the parametric study, the
 428 value $P_{Sr=1}$ needs to be adjusted for each air-entry value ($P_{Sr=1}=30\text{kPa}$ & 49kPa for $\psi_{ae}=20\text{kPa}$
 429 & 39kPa). Although the starting point of dynamic degree of saturation is less affected, longer

dynamic period as well as longer fully saturated period can be observed from Fig. 16(a). Specifically, the dynamic period for largest air-entry value is 1.5 years shorter than the result with $\psi_{ae} = 20kPa$. Further more, different from the trending of Model Kp, the peak value for S_r is not affected. As a result, conclusion can be drawn that compared with Model Kp, Model Srp is less influenced by the air-entry value.

Figure 17: Distribution of (a) dynamic hydraulic conductivity (b) dynamic degree of saturation (c) excess pore pressure and (d) solute concentration vs time for Model Kp+Srp with varied ψ_{ae}

In addition, the parametric study is also conducted for Model Kp+Srp, parameter $\mathbb{I}_{Sr=1}$ is set to be as same as listed in parametric study of Model Srp. As shown in Fig. 17(a)&(b), dynamic period for both hydraulic conductivity and degree of saturation is reduced compared to Model Kp and Model Srp. The peak values of K are similar as in Model Kp, while compared to Model

Srp, differences can be observed. For example, at the middle point when $\psi_{ae}=39\text{kPa}$, the peak degree of saturation only reaches 0.88, while in previous dynamic degree of saturation model, a 1.5 year fully saturation period exists at the same location. The lowest excess pore pressure occurs when $\psi_{ae}=39\text{kPa}$, and the peak value exists when $\psi_{ae}=39\text{kPa}$ at around 60kPa (Fig. 17(c)). This peak excess pore pressure is slightly higher than Model Kp and much lower than peak p^e in Model Srp, which implies that the hydraulic conductivity plays a more significant role in governing the excess pore pressure dissipation compared with degree of saturation. Lastly, according to Fig. 17(d), larger air-entry value generally contributes to a slower contaminant migration and leads to a later break through time. Recall that both Model Kp and Model Srp show limited impact on solute concentration, the fully dynamic model finally demonstrates the effects of varied air-entry value to the consolidation-induced solute migration.

6. Conclusions

In this study, three dynamic models were established, one for dynamic hydraulic conductivity (Model Kp), one for dynamic degree of saturation (Model Srp) and the third dynamic model for both (Model Kp+Srp). Due to the lack of field measurements and experimental data, model validation was conducted between previous numerical studies [10, 12] and present dynamic model while control the excess pore pressure to be always less than air entry value. The dynamic model results were compared with the conventional model to see the dynamic effects on the consolidation-induced solute transport. Following that, an alternative dynamic S_r model is proposed and compared with Model Srp to study the effects of the concavity of pore pressure related dynamic degree of saturation function on consolidation-induced solute transport. Additionally, parametric study was performed to investigate the model sensitivity to the air-entry value.

In general, the consideration of dynamic hydraulic conductivity greatly affects the consolidation process and slightly reduces the consolidation-induced solute transport speed. the dynamic degree of saturation, on the other hand, results in an acceleration of contaminant migration. It is important to include the dynamic soil parameters especially when pore pressure is large and the air-entry value is low. Furthermore, based on the simulation results, more conclusions can be drawn:

(1) Considering K and S_r to be dynamic and varying with respect to pore pressure, both K and S_r increased, but only during the period when pore pressure exceeding the air-entry value. Dynamic K and S_r may only happen for certain depth within the clay layer.

(2) The Dynamic hydraulic conductivity model (Model Kp) results in less excess pore pressure build up, faster soil deformation and slightly slower solute transport.

(3) The Dynamic degree of saturation model (Model Srp) does not affect to peak excess pore pressure but would accelerate contaminant migration.

(4) The dynamic hydraulic conductivity and degree of saturation model (Model Kp+Srp) combines Model Kp and Model Srp. The numerical simulation is compared with the conventional model and the dynamic model suggests less excess pore pressure build up, but shows no significant differences on solute transport.

(5) Dynamic K and dynamic S_r are coupled effects and can not be simply added up. To correctly consider dynamic effects for both simultaneously, Model Kp+Srp is applicable.

(6) Although the dynamic degree of saturation has certain effects on consolidation-induced solute transport, the dynamic path of S_r has limited influence on contaminant migration.

(7) Higher air-entry value results in shorter dynamic periods for both K and S_r . Although varied ψ_{ae} shows less influence on the dynamic degree of saturation model, higher ψ_{ae} leads to slower solute transport when considering dynamic constitutive relationship of hydraulic conductivity.

This study introduces novel ideas by considering dynamic changes of soil parameters. By extending the conventional solute transport in deformable unsaturated porous media, the fully dynamic model now is able to incorporate the fluid compressibility, dynamic fluid density, dynamic soil porosity, dynamic hydraulic conductivity as well as the dynamic degree of saturation. The application of incorporating dynamic soil parameters in soil response is not limited to the landfill case as presented in this study, more examples could be made such as seabed response or marine structure stability [32].

Appendix A. Derivation of fluid storage equation with dynamic hydraulic conductivity and degree of saturation

Mass conservation for pore fluid is presented as:

$$\frac{\partial}{\partial t}(S_r n \rho_w) = -\frac{\partial}{\partial z}(S_r n \rho_w v_f). \quad (\text{A.1})$$

in which S_r refers degree of saturation, n is the porosity, v_f denotes the flow velocity in the voids and ρ_w is the density of pore fluid.

The well known Darcy's Law can be written as:

$$S_r n (v_f - v_s) = -\frac{K}{\rho_w g} \frac{\partial p}{\partial z} \quad (\text{A.2})$$

where p represents the excess pore pressure K indicates hydraulic conductivity and v_s refers to the velocity of the solid.

Keeping in mind that K , S_r and ρ_w are all dynamic and substituting (A.2) into (A.1) gives the relationship shown as:

$$\rho_w \frac{\partial S_r n}{\partial t} + S_r n \frac{\partial \rho_w}{\partial t} = -\frac{\partial}{\partial z} \left[\rho_w \left(-\frac{K}{\rho_w g} \frac{\partial p}{\partial z} + S_r n v_s \right) \right] \quad (\text{A.3})$$

Expanding (A.3) leads to:

$$\rho_w \frac{\partial S_r n}{\partial t} + S_r n \frac{\partial \rho_w}{\partial t} = \frac{1}{g} \frac{\partial}{\partial z} \left(K \frac{\partial p}{\partial z} \right) - S_r n v_s \frac{\partial \rho_w}{\partial z} - \rho_w \frac{\partial}{\partial z} (S_r n v_s). \quad (\text{A.4})$$

Although the soil is partially saturated, the degree of saturation is assumed to be relatively high, so that the air is embedded in pore water in the form of bubbles. The fluid compressibility (β) can then be expressed in terms of degree of saturation as [27]:

$$\beta = \frac{S_r}{K_{w0}} + \frac{1 - S_r + r_h S_r}{P_a + P_0}, \quad (\text{A.5})$$

508 in which K_{w0} is the pore water bulk modulus (1000 MPa), r_h is the volumetric fraction of
 509 dissolved air within pore water (0.02), P_a and P_0 are the gauge air pressure and the atmosphere
 510 pressure ($P_a + P_0 = 100$ kPa).

511 The fluid density(ρ_w) is considered to be changing with pore pressure, which implies the
 512 spatial and time variation of fluid density as:

$$\frac{\partial \rho_w}{\partial t} = \beta \rho_w \frac{\partial p}{\partial t}, \quad (\text{A.6})$$

$$\frac{\partial \rho_w}{\partial z} = \beta \rho_w \frac{\partial p}{\partial z}. \quad (\text{A.7})$$

513 Hence, (A.4) becomes to:

$$S_r \frac{\partial n}{\partial t} + n \frac{\partial S_r}{\partial t} + S_r n \beta \left(\frac{\partial p}{\partial t} + v_s \frac{\partial p}{\partial z} \right) = \frac{1}{\rho_w g} \frac{\partial}{\partial z} \left(K \frac{\partial p}{\partial z} \right) - S_r \frac{\partial n v_s}{\partial z} - n v_s \frac{\partial S_r}{\partial z}. \quad (\text{A.8})$$

514 Considering the mass conservation of solid phase, the temporal derivative of porosity can
 515 be expressed as [12]:

$$\frac{\partial n}{\partial t} = \frac{\partial v_s}{\partial z} - \frac{\partial n v_s}{\partial z}. \quad (\text{A.9})$$

516 Further, the assumption of relatively small deformation implies the relationship that:

$$\left| \frac{\partial p}{\partial t} \right| \gg \left| v_s \frac{\partial p}{\partial z} \right|, \quad (\text{A.10})$$

517 where the velocity of solid phase can be calculated as $\frac{\partial u}{\partial t}$.

518 Combining (A.8), (A.9) and (A.10), the governing equation for excess pore pressure field
 519 that is applicable to the condition of dynamic hydraulic conductivity and dynamic degree of
 520 saturation is proposed as:

$$n \frac{\partial S_r}{\partial t} + S_r n \beta \frac{\partial p}{\partial t} + S_r \frac{\partial^2 u}{\partial z \partial t} = \frac{K}{\rho_w g} \left(\frac{\partial^2 p}{\partial z^2} \right) + \frac{1}{\rho_w g} \frac{\partial K}{\partial z} \frac{\partial p}{\partial z} - n \frac{\partial u}{\partial t} \frac{\partial S_r}{\partial z}. \quad (\text{A.11})$$

Appendix B. Derivation of solute transport equation with dynamic hydraulic conductivity and degree of saturation

For the governing equation of solute concentration field, conservation of solute mass in fluid and solid phase were considered, so that the solute transport equation can be written as:

$$\frac{\partial}{\partial t}(S_r n c) + \frac{\partial}{\partial t}[(1-n)c_s] = -\frac{\partial}{\partial z} \left[S_r n \left(-D \frac{\partial c}{\partial z} + v_f c \right) + (1-n)v_s c_s \right] \quad (\text{B.1})$$

where c and c_s refer to the solute concentration in fluid and solid phase, respectively, and D denotes the hydrodynamic dispersion coefficient.

Under the assumption of compressible fluid and incompressible soil particle and combining with mass conservation of pore fluid and soil particles, Zhang *et al.* [12] suggested the transport equation to be written as:

$$S_r n \frac{\partial c}{\partial t} + (1-n) \frac{\partial c_s}{\partial t} = \frac{\partial}{\partial z} \left(S_r n D \frac{\partial c}{\partial z} \right) - S_r n v_f \frac{\partial c}{\partial z} - (1-n) v_s \frac{\partial c_s}{\partial z} + S_r n \beta \left(\frac{\partial p}{\partial t} + v_f \frac{\partial p}{\partial z} \right) c. \quad (\text{B.2})$$

By considering linear sorption:

$$c_s = \rho_s K_d c, \quad (\text{B.3})$$

in which K_d describes the partitioning of the contaminant, and substitute v_f and v_s as introduced in Appendix A into (B.2), the governing equation can be expressed as:

$$\begin{aligned} [S_r n + (1-n)\rho_s K_d] \frac{\partial c}{\partial t} &= S_r n D \frac{\partial^2 c}{\partial z^2} \\ &+ \frac{\partial c}{\partial z} \left[\frac{\partial}{\partial z} (S_r n D) + \frac{K}{\rho_w g} \frac{\partial p}{\partial z} - S_r n \frac{\partial u}{\partial t} - (1-n)\rho_s K_d \frac{\partial u}{\partial t} \right] \\ &+ c \left[S_r n \beta \frac{\partial p}{\partial t} - \beta \frac{K}{\rho_w g} \left(\frac{\partial p}{\partial z} \right)^2 + S_r n \beta \frac{\partial u}{\partial t} \frac{\partial p}{\partial z} \right]. \end{aligned} \quad (\text{B.4})$$

To incorporate the dynamic degree of saturation in the derivation process, chain rule is applied so that:

$$\frac{\partial}{\partial z} (S_r n D) = S_r n \frac{\partial D}{\partial z} + S_r D \frac{\partial n}{\partial z} + D n \frac{\partial S_r}{\partial z}. \quad (\text{B.5})$$

535 The hydrodynamic dispersion coefficient (D) can be calculated as the summation of molec-
536 ular diffusion coefficient (D_m) and longitudinal dispersion(α_L). i.e.,

$$D = D_m + \alpha_L (v_f - v_s). \quad (\text{B.6})$$

Furthermore, the spatial variation can be calculated as [12] :

$$\frac{\partial n}{\partial z} = \frac{(1 - n)^2}{1 - n^0} \frac{\partial^2 u}{\partial z^2}, \quad (\text{B.7})$$

537 where n^0 refers to the initial soil porosity.

538 Substituting (B.5), (B.6) and (B.7) into (B.4), the governing equation becomes to:

$$\begin{aligned} [S_r n + (1 - n) \rho_s K_d] \frac{\partial c}{\partial t} &= S_r n D_m \frac{\partial^2 c}{\partial z^2} + S_r n \alpha_L (v_f - v_s) \frac{\partial^2 c}{\partial z^2} \\ &+ \frac{\partial c}{\partial z} \left\{ S_r n \alpha_L \frac{\partial (v_f - v_s)}{\partial z} + S_r D_m \frac{(1 - n)^2}{1 - n^0} \frac{\partial^2 u}{\partial z^2} \right. \\ &+ S_r \alpha_L (v_f - v_s) \frac{\partial n}{\partial z} + D_m n \frac{\partial S_r}{\partial z} + n \alpha_L (v_f - v_s) \frac{\partial S_r}{\partial z} \\ &+ \frac{K}{\rho_w g} \frac{\partial p}{\partial z} - [S_r n + (1 - n) \rho_s K_d] \frac{\partial u}{\partial t} \left. \right\} \\ &+ c \left[S_r n \beta \frac{\partial p}{\partial t} - \beta \frac{K}{\rho_w g} \left(\frac{\partial p}{\partial z} \right)^2 + S_r n \beta \frac{\partial u}{\partial t} \frac{\partial p}{\partial z} \right] \end{aligned} \quad (\text{B.8})$$

539 Utilising the produce rule, the following relationship can be obtained, i.e.,

$$S_r n \frac{\partial (v_f - v_s)}{\partial z} + S_r (v_f - v_s) \frac{\partial n}{\partial z} = \frac{\partial}{\partial z} [S_r n (v_f - v_s)] - n (v_f - v_s) \frac{\partial S_r}{\partial z}. \quad (\text{B.9})$$

540 Combining with Darcy's law (A.2), (B.8) now becomes:

$$\begin{aligned}
[S_r n + (1 - n) \rho_s K_d] \frac{\partial c}{\partial t} &= \frac{\partial^2 c}{\partial z^2} \left(S_r n D_m - \alpha_L \frac{K}{\rho_w g} \frac{\partial p}{\partial z} \right) \\
&+ \frac{\partial c}{\partial z} \left\{ \alpha_L \frac{\partial}{\partial z} \left(-\frac{K}{\rho_w g} \frac{\partial p}{\partial z} \right) + S_r D_m \frac{(1 - n)^2}{1 - n^0} \frac{\partial^2 u}{\partial z^2} \right. \\
&+ D_m n \frac{\partial S_r}{\partial z} + \frac{K}{\rho_w g} \frac{\partial p}{\partial z} - [S_r n + (1 - n) \rho_s K_d] \frac{\partial u}{\partial t} \Big\} \\
&+ c \left[S_r n \beta \frac{\partial p}{\partial t} - \beta \frac{K}{\rho_w g} \left(\frac{\partial p}{\partial z} \right)^2 + S_r n \beta \frac{\partial u}{\partial t} \frac{\partial p}{\partial z} \right].
\end{aligned} \tag{B.10}$$

Now, recall the updated storage equation (A.11) and apply the product rule, the only term which might be affected by the dynamic hydraulic conductivity can be replaced as:

$$\begin{aligned}
\frac{\partial}{\partial z} \left(-\frac{K}{\rho_w g} \frac{\partial p}{\partial z} \right) &= -\frac{1}{\rho_w g} \frac{\partial}{\partial z} \left(K \frac{\partial p}{\partial z} \right) + K \frac{\partial p}{\partial z} \left[\frac{\partial}{\partial z} \left(-\frac{1}{\rho_w g} \right) \right] \\
&= -S_r \frac{\partial^2 u}{\partial z \partial t} - n \frac{\partial S_r}{\partial t} - S_r n \beta \frac{\partial p}{\partial t} - n \frac{\partial u}{\partial t} \frac{\partial S_r}{\partial z} + \beta \frac{K}{\rho_w g} \left(\frac{\partial p}{\partial z} \right)^2.
\end{aligned} \tag{B.11}$$

Moreover, based on the discussion in [12], it is reasonable to approximate n to a constant initial soil porosity (n^0) at this stage. Hence, the final governing equation for solute concentration field is presented as:

$$\begin{aligned}
[S_r n^0 + (1 - n^0) \rho_s K_d] \frac{\partial c}{\partial t} &= \frac{\partial^2 c}{\partial z^2} \left(S_r n^0 D_m - \alpha_L \frac{K}{\rho_w g} \frac{\partial p}{\partial z} \right) \\
&+ \frac{\partial c}{\partial z} \left\{ -n^0 \alpha_L \frac{\partial S_r}{\partial t} - n^0 \alpha_L \frac{\partial u}{\partial t} \frac{\partial S_r}{\partial z} + D_m n^0 \frac{\partial S_r}{\partial z} \right. \\
&- \alpha_L S_r n^0 \beta \frac{\partial p}{\partial t} - S_r \alpha_L \frac{\partial^2 u}{\partial z \partial t} + S_r D_m (1 - n^0) \frac{\partial^2 u}{\partial z^2} \\
&+ \frac{\alpha_L \beta K}{\rho_w g} \left(\frac{\partial p}{\partial z} \right)^2 + \frac{K}{\rho_w g} \frac{\partial p}{\partial z} - [S_r n^0 + (1 - n^0) \rho_s K_d] \frac{\partial u}{\partial t} \Big\} \\
&+ c \left[S_r n^0 \beta \frac{\partial p}{\partial t} - \beta \frac{K}{\rho_w g} \left(\frac{\partial p}{\partial z} \right)^2 + S_r n^0 \beta \frac{\partial u}{\partial t} \frac{\partial p}{\partial z} \right].
\end{aligned} \tag{B.12}$$

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