Seismic-induced dynamic responses in a poro-elastic seabed: solutions of different formulations

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ABSTRACT

The evaluation of dynamic response in a porous seabed has been studied since the 1980s. However, most previous analytical investigations focused on the loading from ocean-wave action. In this paper, an analytical method is presented to study the soil displacements, pore pressures and effective stresses in a nearly saturated porous seabed subjected to earthquake excitation. Based on Biot's theory and the concept of homogeneous pore fluid, a set of analytical solutions for various response are derived in terms of seawater depth, seabed parameters and seismic characteristics of bedrock. Numerical examples are given to demonstrate the significant influences of saturation degree, soil permeability and excitation frequency on the earthquake-induced soil displacement and pore pressure in a nearly saturated seabed. In addition, to investigate the effect of inertia force on the induced response in soils, two common-used formulations, i.e., fully dynamic (FD form) and partly dynamic (PD form), are considered, respectively. The regions of validity for these two formulations are identified and depicted to help engineers to select the appropriate solution for the seismic analysis in coastal engineering practice.

Keywords: Porous seabed; Dynamic response; Earthquake; Partial saturation; Pore pressure

1 Introduction

Over the past decades, the issue of the dynamic response of a porous seabed has attracted wide attention in coastal and offshore engineering. To date, most existing analytical approaches focused on the behavior of the seabed subjected to gravity waves propagating in the ocean environments. Based on Biot theory [1], Madsen [2] and Yamamoto et al. [3] firstly developed the analytical solutions for wave-induced stresses and pore pressure in the seabed. Okusa [4] adopted a simple analytical solution for wave-induced pore pressure and effective stresses in unsaturated sediments. Later, a semi-analytical approach for the wave-induced seabed response for a layered seabed was developed by Rahman et al. [5]. Jeng and Hsu [6] derived a closed-form analytical solution for the wave-induced response in an elastic seabed subjected to short-crested waves. After that, analytical studies were carried out on the effects of various influencing factors such as cross-anisotropic behavior, inertia term and wave non-linearity on the response variables in the seabed and offshore structures [7-11], as well as the numerical simulation [12-16]. All these investigations were focused on the soil response under the ocean wave loading.

Besides the wave loading, the seismic loading should also be considered for some important structures in active seismic zone. Until now, only a few investigations have been launched to analyze the seismic-induced response of the seabed [17-20] and marine structures in offshore area [21-26]. However, these works were mainly carried out through numerical methods. The analytical study for the seismic-induced response in a partly saturated porous seabed has not yet gained sufficient attention [18,19]. Note that Yang and Sato [27,28] presented analytical study on the saturation effect on the amplification of soil layer to vertical earthquake excitation. However, this study was not aimed at the marine environment and the ratio between the solid and relative fluid displacements was assumed in the separation analysis procedure, which may not be strict enough in mathematics deduction.

In general, the analytical models for the dynamic response of a porous seabed depend on the rate of loading and the characteristics of flow and deformation of soil [29-31]. In this study, two common formulations for the dynamic analysis of porous media are adopted:

- Fully dynamic (FD): The coupled Biot's equations of flow and deformation are formulated including inertial terms of both soil skeleton and average relative pore fluid displacements. This is

so-called u - w model.

- Partly dynamic (PD): For simplicity, the coupled equations are formulated considering only the acceleration of soil skeleton. This so-called u - p approximation was firstly proposed by Zienkiewicz et al.[29] and has shown efficiency and convenience in numerical analysis [24,31]. In this formulation, only soil displacement and pore pressure are used as field variables. Ulker and Rahman [10] is the first study that named these Biot formulations (PD and FD) and gave a comprehensive study of these formulation domains.

The inertial terms have been demonstrated to play an important role in the dynamic behavior of seabed [8, 9, 10, 30], and is also discussed for the seismic issue in present paper. In the analysis of wave-seabed interaction, the ocean wave loading is always be considered as a two-dimensional dynamic loading [16] or three-dimensional dynamic loading [32]. However, seismic action could be properly simplified as a one-dimensional problem in both analytical and numerical analysis under symmetrical conditions [27, 28, 33]. In this study, the generalized analytical solutions for the one-dimensional problem of earthquake-induced seabed response (for both FD and PD formulations) are developed. Compared with Yang and Sato [28], the derivative process clearly indicates the independent effect of each type of seismic waves and no imprecise assumption is introduced, which makes the analytical solutions more rigorous in mathematics. The parametric analysis is conducted to examine the influences of saturation degree, soil permeability, earthquake frequency on the seismic wave speeds and the soil response variables such as soil displacement and pore pressure. In addition, the comparison between FD and PD solutions is also presented and their respective applicable conditions are assessed, which is particularly important in estimating which one could be used in a particular engineering problem.

2 Governing equations

Marine sediment is a mixture consisting of three phases: a solid phase that forms skeletal frame, a liquid phase that occupies a major portion of pore space, and a gas phase that occupies a small portion of pore space. Both the pore fluid (a continuum comprising both liquid and gas) and soil skeleton are assumed to be compressible in this paper. These assumptions can be used in a nearly saturated seabed whose degree of saturation is sufficiently high that the gas bubbles are occluded and smaller than the normal void spaces, and the pore gas pressure and pore water pressure are

nearly equal. The dynamic Biot's equations [1, 34] proposed by Zienkiewicz et al. [29] are employed here to describe the dynamic response of the porous seabed.

Under condition of plane strain, the effective stress controlling the deformation of porous medium is defined by

$$\sigma_{ij} = \sigma'_{ij} - \delta_{ij} p \quad (i, j = x, z) \quad (1)$$

where σ_{ij} and σ'_{ij} are total stress and effective stress, respectively; δ_{ij} is Kronecker delta; p is the pore pressure, which is assumed to be positive in tension here.

The relationship between the effective stress and strain in terms of Lamé's parameters, λ and μ , is written as

$$\sigma'_{ij} = \lambda \varepsilon_{kk} \delta_{ij} + 2\mu \varepsilon_{ij} \quad (2)$$

where μ is also called shear modulus of soil. ε_{kk} is volumetric strain and strain ε_{ij} is defined as

$$\varepsilon_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}) \quad (3)$$

where $u_{i,j}$ and $u_{j,i}$ are the derivatives of the soil skeleton displacement with respect to spatial coordinates.

2.1 Fully dynamic (FD) form

Considering the inertial forces associated with both soil skeleton and pore fluid, the governing equations in their fully dynamic (FD) form are derived in terms of solid skeleton and relative pore fluid displacements.

The overall equilibrium equation for a porous medium is written as

$$\sigma_{ij,j} + \rho g_i = \rho \ddot{u}_i + \rho_f \ddot{w}_i \quad (4)$$

where g_i is the appropriate component of the gravitational acceleration, ρ is total density of the porous medium, $\rho = (1 - n)\rho_s + n\rho_f$, ρ_s is the density of the solid skeleton, n is the soil porosity; ρ_f is the fluid density, $\ddot{u}_i$ is the acceleration of solid skeleton, $\ddot{w}_i$ is the average pore fluid acceleration relative to solid frame. The average pore fluid relative displacement is denoted as $w = n(\bar{w} - u)$, where $\bar{w}$ and u are the displacements of the pore fluid and the solid skeleton, respectively.

The flow in the porous seabed is assumed to be governed by Darcy's law, the equation for the equilibrium of fluid phase can be written as

$$-p_{,j} + \rho_f g_i = \rho_f \ddot{u}_i + \frac{\rho_f}{n} \ddot{w}_i + \frac{\rho_f g_i}{k_f} \dot{w}_i \quad (5)$$

where k_f denotes the hydraulically isotropic permeability of seabed.

Based on conservation of mass, the continuity equation can be expressed as

$$\dot{\epsilon}_{kk} + \dot{w}_{i,i} = -n\beta \dot{p} \quad (6)$$

where β is the compressibility of the pore fluid which can be expressed by the apparent bulk modulus of the pore fluid K_f and the saturation degree S_r as [35]

$$\beta = \frac{1}{K_f} = \frac{1}{K_w} + \frac{1 - S_r}{p_{w0}} \quad (7)$$

where K_w is the true bulk modulus of water and p_{w0} is the absolute pore water pressure, d is the water depth.

2.2 Partly dynamic (PD) form

An alternative simplification is the partly dynamic form which ignores the inertial forces associated with the relative pore fluid displacement. This approximation has been widely used to describe the dynamic response of the porous seabed under wave loading [9]. In partly dynamic (PD) formulation, Eqs. (4) and (5) degenerate to

$$\sigma_{ij,j} + \rho g_i = \rho \ddot{u}_i \quad (8)$$

$$-p_{,j} + \rho_f g_i = \rho_f \ddot{u}_i + \frac{\rho_f g_i}{k_f} \dot{w}_i \quad (9)$$

3 Analytical solutions for seismic response

The seawater-seabed-bedrock system considered in this study is shown in Fig. 1. This system comprises seawater, poro-elastic sediment layer, and underlying impermeable rigid bedrock. The depth of overlying seawater is assumed to be d and the seabed is modeled as linear porous medium with finite thickness L . The z axis is directed vertically upwards and the x axis is

assumed along horizontal direction. In order to simulate the seismic ground motion, the steady-state displacement excitation is acting at the base of the seabed in vertical and horizontal directions, respectively. It should be noted that the steady-state excitation could not accurately simulate the real seismic response. However, this paper focuses on effect of inertial terms on the seismic-induced dynamic responses such as the amplification factor of displacement, rather than the variation of seismic response with time. In addition, assuming the seismic excitation to take the form of steady state is an effective way to solve the coupled governing equations [28].

3.1 FD form for compressional wave

For the compressional wave, the vibration direction of the particles is consistent with the direction of wave propagation. Accordingly, the variables u and w in this case represent the solid skeleton displacement and average fluid displacement in the compression direction (i.e., z axis direction). Neglecting the body force terms, the governing equations reduce to

$$\frac{\partial \sigma}{\partial z} - \rho \frac{\partial^2 u}{\partial t^2} - \rho_f \frac{\partial^2 w}{\partial t^2} = 0 \quad (10)$$

$$\frac{\partial p}{\partial z} + \rho_f \frac{\partial^2 u}{\partial t^2} + \frac{\rho_f}{n} \frac{\partial^2 w}{\partial t^2} + \frac{\rho_f g}{k_f} \frac{\partial w}{\partial t} = 0 \quad (11)$$

$$\sigma = (\lambda + 2\mu) \frac{\partial u}{\partial z} - p \quad (12)$$

$$p = -\frac{1}{n\beta} \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial z} \right) \quad (13)$$

Substituting Eqs. (12) and (13) into Eqs. (10) and (11), the dynamic equations are obtained

$$\left(\lambda + 2\mu + \frac{1}{n\beta} \right) \frac{\partial^2 u}{\partial z^2} + \frac{1}{n\beta} \frac{\partial^2 w}{\partial z^2} - \rho \frac{\partial^2 u}{\partial t^2} - \rho_f \frac{\partial^2 w}{\partial t^2} = 0 \quad (14)$$

$$\frac{1}{n\beta} \left(\frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 w}{\partial z^2} \right) - \frac{\rho_f g}{k_f} \frac{\partial w}{\partial t} - \rho_f \frac{\partial^2 u}{\partial t^2} - \frac{\rho_f}{n} \frac{\partial^2 w}{\partial t^2} = 0 \quad (15)$$

Combining Eqs (14) and (15), the governing equation for the soil skeleton displacement can be derived after eliminating the variable w

$$\frac{\partial^4 u}{\partial t^4} + C_1 \frac{\partial^3 u}{\partial t^3} + C_2 \frac{\partial^4 u}{\partial t^2 \partial z^2} + C_3 \frac{\partial^3 u}{\partial t \partial z^2} + C_4 \frac{\partial^4 u}{\partial z^4} = 0 \quad (16)$$

in which

$$C_1 = -\frac{n\rho g}{k_f(n\rho_f - \rho)} \quad C_2 = \frac{n\rho^+ \rho_f [(\lambda + 2\mu)n\beta + 1 - 2n]}{\rho_f n\beta(n\rho_f - \rho)}$$

$$C_3 = \frac{g[(\lambda + 2\mu)n\beta + 1]}{\beta k_f(n\rho_f - \rho)} \quad C_4 = -\frac{\lambda + 2\mu}{\beta \rho_f(n\rho_f - \rho)}$$

For steady state problem, since all the equations concerned are linear, the seismic response variables F (such as u and w in this case) take the complex form as

$$F = F_0 \exp[i(\omega t - zk)] \quad (17)$$

where F_0 represents the amplitude of the corresponding response variable, “ $\exp[i(\omega t - zk)]$ ” denotes the spatial and temporal variations of response variables F , where i is the root of -1, t is time, ω is the angular frequency, k is a complex number.

Then Eq. (16) can be further reduced to

$$C_4 k^4 + (C_2 \omega^2 - iC_3 \omega) k^2 + \omega^4 - iC_1 \omega^3 = 0 \quad (18)$$

The roots for four-order equation (18) are assumed to be $\pm k_{p1}$ and $\pm k_{p2}$, and satisfy $|\operatorname{Re}(k_{p1})| < |\operatorname{Re}(k_{p2})|$, where $\operatorname{Re}()$ denotes the real part of the corresponding complex root. Here k_{p1} and k_{p2} are the wave numbers for the two types of compressional waves, namely P1 wave and P2 wave in the saturated soils [34]. The general solution for u in Eq. (16) can be written as

$$u(z, t) = [a_1 \exp(-ik_{p1}z) + a_2 \exp(ik_{p1}z) + a_3 \exp(-ik_{p2}z) + a_4 \exp(ik_{p2}z)] \exp(i\omega t) \quad (19)$$

By the same operations, the average relative fluid displacement w takes the similar form as

$$w(z, t) = [a_1 \eta_1 \exp(-ik_{p1}z) + a_2 \eta_2 \exp(ik_{p1}z) + a_3 \eta_3 \exp(-ik_{p2}z) + a_4 \eta_4 \exp(ik_{p2}z)] \exp(i\omega t) \quad (20)$$

where

$$\eta_1 = \eta_2 = \frac{\left(\lambda + 2\mu + \frac{1}{n\beta} \right) k_{p1}^2 - \omega^2 \rho}{\omega^2 \rho_f - \frac{1}{n\beta} k_{p1}^2} \quad (21)$$

$$\eta_3 = \eta_4 = \frac{\left(\lambda + 2\mu + \frac{1}{n\beta} \right) k_{p2}^2 - \omega^2 \rho}{\omega^2 \rho_f - \frac{1}{n\beta} k_{p2}^2} \quad (22)$$

Then, the response pore pressure and effective normal stress can be further expressed as

$$p(z, t) = \frac{i}{n\beta} \left[a_1(1 + \eta_1)k_{p1} \exp(-ik_{p1}z) - a_2(1 + \eta_2)k_{p1} \exp(ik_{p1}z) \right. \\ \left. + a_3(1 + \eta_3)k_{p2} \exp(-ik_{p2}z) - a_4(1 + \eta_4)k_{p2} \exp(ik_{p2}z) \right] \exp(i\omega t) \quad (23)$$

$$\sigma'_{zz}(z, t) = (\lambda + 2\mu)i \left[a_2k_{p1} \exp(ik_{p1}z) - a_1k_{p1} \exp(-ik_{p1}z) \right. \\ \left. + a_4k_{p2} \exp(ik_{p2}z) - a_3k_{p2} \exp(-ik_{p2}z) \right] \exp(i\omega t) \quad (24)$$

It is demonstrated from Eqs. (19) ~ (24) that the earthquake-induced displacements and stresses of the seabed consist of two main parts, i.e., independent terms from P1 wave and P2 wave. In fact, these two types of compressional waves have a combined influence on the dynamic characteristics of the soils.

The boundary conditions in this case, as shown in Fig. 1, are: at $z = L$, the vertical effective stress vanishes and the induced pore pressure p in the soils equals to the induced dynamic pressure of the overlying seawater; at $z = 0$, the displacement excitation with frequency ω and amplitude u_0 is acting at the impermeable and rigid base, they are

$$z = 0 : u = u_0 \exp(i\omega t), \quad w = 0 \quad (25)$$

$$z = L : p = -p_w, \quad \sigma'_{zz} = 0 \quad (26)$$

It is worth noting that, at the surface of the seabed ($z = L$), the compatibility between the normal pore fluid movement in and out of the skeletal frame and the normal displacement of upper seawater w_u is represented by

$$w_u = (1 - n)u + n\bar{w} = u + w \quad (27)$$

The dynamic water pressure at the seabed surface induced by the vibration of seabed takes the following form [36]

$$p_w = \rho_f d\ddot{w}_u \quad (28)$$

Then the coefficients a_j ($j = 1-4$) in Eqs. (19) and (20) can be solved by enforcing the boundary conditions (25) and (26) (see Appendix).

3.2 FD form for shear wave

In the case of shear wave (also known as equivoluminal wave), different from compressional wave, the motion direction of the particle in the soil is perpendicular to the direction of wave propagation. The seismic-induced displacement occurs in the shear direction (i.e., x axis direction

in Fig. 1). The FD form of differential governing equations for shear wave is rewritten as

$$\mu \frac{\partial^2 u}{\partial z^2} - \rho \frac{\partial^2 u}{\partial t^2} - \rho_f \frac{\partial^2 w}{\partial t^2} = 0 \quad (29)$$

$$\rho_f \frac{\partial^2 u}{\partial t^2} + \frac{\rho_f}{n} \frac{\partial^2 w}{\partial t^2} + \frac{\rho_f g}{k_f} \frac{\partial w}{\partial t} = 0 \quad (30)$$

Combining Eqs. (29) and (30), the expression for the soil displacements is derived as

$$\frac{\partial^3 u}{\partial t^3} + C_5 \frac{\partial^2 u}{\partial t^2} + C_6 \frac{\partial^3 u}{\partial t \partial z^2} + C_7 \frac{\partial^2 u}{\partial z^2} = 0 \quad (31)$$

with

$$C_5 = -\frac{n \rho g}{k_f (n \rho_f - \rho)} \quad C_6 = \frac{\mu}{n \rho_f - \rho} \quad C_7 = \frac{n \mu g}{k_f (n \rho_f - \rho)}$$

Following the same steps as Eq. (17), the Eq. (31) can be further reduced to

$$(i\omega C_6 + C_7)k^2 + i\omega^3 + C_5\omega^2 = 0 \quad (32)$$

The roots for Eq. (32) are $\pm k_s$, which indicate the complex wave number for the shear wave (namely, S wave) in the soils. Then the displacement of solid skeleton can be written as

$$u(z, t) = [a_5 \exp(-ik_s z) + a_6 \exp(ik_s z)] \exp(i\omega t) \quad (33)$$

The shear stress in the seabed can be further obtained as

$$\sigma_{xz}(z, t) = i\mu[a_6 k_s \exp(ik_s z) - a_5 k_s \exp(-ik_s z)] \exp(i\omega t) \quad (34)$$

This case has the boundary conditions expressed as

$$z = 0 : u = u_0 \exp(i\omega t) \quad (35)$$

$$z = L : \sigma_{xz} = 0 \quad (36)$$

Then the coefficients a_j ($j = 5, 6$) can be solved after the substitution of Eqs. (33) and (34) into the above boundary Eqs. (35) and (36) (see Appendix).

3.3 PD form for compressional wave

Ignoring the inertial forces associated with the relative fluid displacement, an alternative partly dynamic (PD) form, so-called u - p form, can be obtained. Similar to FD form discussed above, the displacement of solid skeleton u is designated to be in the direction of compressional wave propagation (i.e., z axis direction). The other primary field variable is pore pressure p . Thus the governing equations can be further simplified to

$$(226) \quad (\lambda + 2\mu) \frac{\partial^2 u}{\partial z^2} - \frac{\partial p}{\partial z} - \rho \frac{\partial^2 u}{\partial t^2} = 0 \quad (37)$$

$$(227) \quad \frac{k_f}{\rho_f g} \frac{\partial^2 p}{\partial z^2} - \frac{\partial^2 u}{\partial t \partial z} - n\beta \frac{\partial p}{\partial t} + \frac{k_f}{g} \frac{\partial^3 u}{\partial t^2 \partial z} = 0 \quad (38)$$

(228) Then, the four-order governing equation for the soil skeleton displacement is derived as

$$(229) \quad \frac{\partial^3 u}{\partial t^3} + C'_1 \frac{\partial^4 u}{\partial z^2 \partial t^2} + C'_2 \frac{\partial^3 u}{\partial z^2 \partial t} + C'_3 \frac{\partial^4 u}{\partial z^4} = 0 \quad (39)$$

(230) in which

$$(231) \quad C'_1 = \frac{k_f(\rho_f - \rho)}{n\beta\rho\rho_f g} \quad C'_2 = -\frac{(\lambda + 2\mu)n\beta + 1}{n\beta\rho} \quad C'_3 = \frac{k_f(\lambda + 2\mu)}{n\beta\rho\rho_f g}$$

(232) Assuming the field variables u and p yield the same form as Eq. (17), the Eq. (39) becomes

$$(233) \quad C'_3 k^4 + (C'_1 \omega^2 - iC'_2 \omega) k^2 - i\omega^3 = 0 \quad (40)$$

(234) The roots for Eq. (40) are assumed to be $\pm k'_{p1}$ and $\pm k'_{p2}$ and satisfy $|\text{Re}(k'_{p1})| < |\text{Re}(k'_{p2})|$. Thus

(235) the general solutions for u in Eq. (39) and p in Eq. (37) can be written as

$$(236) \quad u(z, t) = [a'_1 \exp(-ik'_{p1} z) + a'_2 \exp(ik'_{p1} z) + a'_3 \exp(-ik'_{p2} z) + a'_4 \exp(ik'_{p2} z)] \exp(i\omega t) \quad (41)$$

$$(237) \quad p(z, t) = [a'_1 \eta'_1 \exp(-ik'_{p1} z) + a'_2 \eta'_2 \exp(ik'_{p1} z) + a'_3 \eta'_3 \exp(-ik'_{p2} z) + a'_4 \eta'_4 \exp(ik'_{p2} z)] \exp(i\omega t) \quad (42)$$

(238) where

$$(239) \quad \eta'_1 = -\eta'_2 = \frac{(\lambda + 2\mu)k'^2_{p1} - \rho\omega^2}{ik'_{p1}} \quad (43)$$

$$(240) \quad \eta'_3 = -\eta'_4 = \frac{(\lambda + 2\mu)k'^2_{p2} - \rho\omega^2}{ik'_{p2}} \quad (44)$$

(241) In the PD formulation, the field variables are soil skeleton displacement and pore pressure, so

(242) the boundary conditions (25) and (26) are modified as

$$(243) \quad z = 0 : \quad u = u_0 \exp(i\omega t), \quad \frac{\partial p}{\partial z} = 0 \quad (45)$$

$$(244) \quad z = L : \quad p = -p_w, \quad \sigma'_{zz} = 0 \quad (46)$$

(245) It is worthwhile to note that dynamic water pressure p_w can also be calculated by Eq. (28), and its

(246) second-time derivative of the relative fluid displacement is also neglected in PD formulation. Then,

(247) the same procedure as FD form is used here to obtain the undetermined coefficients a'_j ($j=1-4$) in

(248) the Eqs. (41) and (42) (see Appendix).

3.4 PD form for shear wave

In this case, the PD form of governing equation for the solid skeleton displacement in the shear direction is rewritten as

$$\mu \frac{\partial^2 u}{\partial z^2} - \rho \frac{\partial^2 u}{\partial t^2} = 0 \quad (47)$$

The propagation velocity of shear wave can be solved as $V_s = \sqrt{\frac{\mu}{\rho}}$. Unlike the shear wave speed calculated using Eq. (32), the shear waves obtained using PD form is frequency- independent.

4 Effects of inertial terms on seismic waves

Since the seismic response of a seabed is associated with the seismic waves, the effects of inertial terms on the behavior of seismic wave in a partially saturated seabed are first discussed in this section. In general, the propagation characteristics of seismic waves can be described by the corresponding wave numbers obtained in the last section. The speeds of P1, P2 and S waves can be calculated as the ratios of angular frequency to the real part of complex wave numbers, and can be respectively represented as

$$V_{p1} = \frac{\omega}{\text{Re}(k_{p1})} \quad V_{p2} = \frac{\omega}{\text{Re}(k_{p2})} \quad V_s = \frac{\omega}{\text{Re}(k_s)} \quad (48)$$

It is also instructive to examine the effects of inertial terms on the seismic waves calculated from these two formulations in different limiting cases (see Table 1). When no relative movement of fluid is permitted (i.e., $\omega \rightarrow 0$, $k_f \rightarrow 0$), P2 wave disappears and the FD formulations for P1 and S waves take the same simplified forms as PD form. Physically, this situation corresponds to the fully undrained case in which all the solid skeleton and pore fluids move together. On the contrary, in fully drained case, as $\omega \rightarrow \infty$ and $k_f \rightarrow \infty$, the degenerated expressions for wave velocities could be derived from Eq. (18). However, based the PD approximation (40) in this extreme case, the velocity of P1 wave approaches to infinity and the shear wave speed is equivalent to the fully undrained situation.

The influence of of inertial terms on the wave speeds of P1, P2 and S waves at various saturation degrees is illustrated in Figs. 2 and 3. To have a basic understanding of different seabed, we discuss three typical marine soils, coarse gravel, fine sand and silty sand, which are associated

with distinct values of permeability. The values of model parameters are selected from Table 2. In particular, the saturation degree for the nearly saturated seabed is assumed to range from 0.95 to 1.0, and the excitation frequencies are 10Hz and 2 Hz in Figs. 2 and 3, respectively. It can be seen from both Figs. 2 and 3 that, for different seabed, the velocity of P1 wave increases substantially with even a slight increase below complete saturation and reaches the maximum in the case of full saturation. The influence of air inclusion on the P2 wave is also noticeable. The velocity of P2 wave increases steadily with the decreasing saturation, except for the PD solution in coarse gravel seabed when $f = 10$ Hz. For S wave, as expected, the influence of saturation is negligible. It is concluded from these six cases that the effects of inertial terms on seismic waves could not be ignored for coarse gravel seabed at different frequencies (i.e., case 1 and case 4).

5 Seismic-induced displacement and pore pressure

Under the earthquake loading, the induced displacement and pore pressure in seabed always play an important role around and beneath the offshore structures such as submarine pipeline and tunnel. To obtain reliable results in this section, the fully dynamic solutions, i.e., FD solutions, are employed to investigate the effects of major seabed parameters (e.g., saturation degree and soil permeability) and earthquake parameter (e.g., excitation frequency) on the induced soil displacement and pore pressure in a nearly saturated seabed of finite thickness. It should be noted that this study is conducted within the framework of poro-elastic theory, and focuses on the amplitude of seismic responses such as the amplification factor of displacement. Therefore, the elasto-plastic behavior, the rearrangement of particles under cyclic action, and the development of pore pressure are not considered here. The absolute values of vertical displacement ($|u|$) and pore pressure ($|p|$) versus the relative soil depth (z/L) are presented in Figs. 4 and 5, respectively. The soil displacement u is normalized with the excitation displacement u_0 while the pore pressure p is non-dimensionalized with respect to the absolute hydrostatic water pressure p_{w0} at the surface of seabed. The excitation frequency is taken to be 2 Hz and 5 Hz and the saturation degree is 0.98 and 1.0. The values of other parameters used here are also selected from Table 2.

As shown in Fig. 4(a), compared with other two cases, smaller displacement is generated in coarse gravel seabed due to its larger permeability. With the increasing of frequency in Figs 4(a)

and 4(b), the displacement changes more significant from the seabed bottom to its upper surface. This finding is certainly reasonable from an intuitive point of view, since the case of sufficiently low frequency will approach to the quasi-static state and the whole seabed will have the same displacement. Compared with Fig. 4(a), the induced displacement in saturated seabed in Fig. 4(b) bears a quite different behavior. This is probably relevant to the fact that even small amount of air in soil will drastically increase the speed of P1 wave (see Fig. 2) and the corresponding wavelength will decrease. Generally, the predominant frequencies are usually not high in the case of earthquake loading. Compared with the results of full saturation in Fig. 4(b), the amplification in the case of incomplete saturation in Fig. 4(a) is usually much greater in earthquake frequency range. Thus it should be noted that a saturated model will always underestimate the motion amplification in the seabed.

Fig. 5 illustrates the vertical distributions of pore pressure versus the relative soil depth for various seabed. For the nearly saturated state, as shown in Fig. 5(a), the pore pressure decreases with depth, attains the minimum and then increases to the maximum at the seabed surface. In most depth of seabed, greater pore pressure is generated under the earthquake loading with higher frequency. From Figs. 4 and 5, it is also noted that the influence of permeability on displacement and pore pressure becomes insignificant when the permeability is sufficiently low (e.g., fine sand and silty sand).

The motion amplification factor is defined herein as the ratio of the amplitude of solid displacement at any depth to that at the bottom of seabed. Fig. 6 shows the amplification factors at different locations of a coarse gravel seabed as a function of frequency under nearly and fully saturated conditions. As the frequency increases, the displacement amplification factors in the situations of both complete and incomplete saturation will reach peak values at different frequencies (denoted as peak frequency). Furthermore, these peak values of amplification factors are shown to decrease with the increasing frequency. A noticeable difference can also be observed between nearly and fully saturated cases. For the incomplete saturation in Fig. 6(a), the intervals between the peak frequencies are similar. However, for the full saturation in Fig. 6(b), at the location closer to the seabed surface, the interval becomes smaller.

There are few available references that can be directly used to verify the results in this study.

Fig. 7 shows the variation of amplification factor with frequency when the seawater is removed (i.e., $d = 0$ m). This reduced solutions in the limiting case could be compared with the results of Yang and Sato [27] for both partly and fully saturated cases. The discrepancy between the present study and Yang and Sato [27] may come from different derivation process of solutions and the compressibility of grains which is neglected here.

6. Domains of applicability of two formulations

In engineering practice, it is particularly important to investigate the relative difference of the seismic-induced seabed response between FD and PD formulation (the former with fluid inertia force and the later without) and determine when the fluid inertia force need to be considered in the evaluation of earthquake-induced seabed response. Ulker and Rahman [10] have studied various conditions and the regions of validity for various formulations in a parametric space, in which the solutions for different formulations are developed in terms of non-dimensional parameters. Similarly, the domains of applicability of FD and PD formulations for the seismic responses in the seabed are given in this section.

As demonstrated in previous sections, the analytical solutions for the response variables depend on the individual contributions from P1 wave and P2 wave (see Eqs. (19) and (41)). In other words, it can be concluded that the inertia force of pore fluid can be ignored if the discrepancies in the P1 and P2 waves evaluated from two formulations becomes insignificant. Thus the parameters affecting the difference between the P1 wave (or P2 wave) calculated from FD formulation and that from PD formulation always determine the domains of applicability of these two formulations.

As illustrated in Eq. (48), the velocities of wave propagation are determined by the frequency and wave numbers which are computed using Eqs. (18) and (40). Since Lamé constants nearly remain almost constant for most marine sediments, the main parameters need to be considered are earthquake parameter (e.g., frequency) and soil parameters (e.g. porosity, saturation degree, permeability). Additionally, as demonstrated in previous section, saturation degree is not the dominant factor to distinguish FD and PD solutions (see Figs. 2 and 3). Herein the characteristic frequency is adopted as $f_c = ng / (2\pi k_f)$ [37], which includes the influence of both permeability and

porosity. Fig. 8 demonstrates the influence of non-dimensional frequency on wave speeds of P1, P2 and S waves. Four cases with different saturation degrees and porosities ($S_r = 0.98$, $n = 0.35$; $S_r = 1.0$, $n = 0.35$; $S_r = 0.98$, $n = 0.55$; $S_r = 1.0$, $n = 0.55$) are shown in Figs. 8(a) ~ 8(d). The calculation results are shown to be independent of soil permeability and it is worthy to note that the wave speeds obtained using FD and PD formulations are almost identical when $\lg(f/f_c) \leq -1$.

Based on the discrimination criterion $f \leq ng/(20\pi k_f)$, Fig. 9 depicts the regions of applicability of FD and PD formulations for seismic waves in seabed. This is more simplified than that proposed by Ulker and Rahman [10] in which the related non-dimensional parameters Π_1 and Π_2 depend on more parameters such as the thickness of the soil layer. The permeability range covers the typical values for various ocean soils and the shaded area indicates the representative frequency in geotechnical earthquake engineering [38]. Based on the values of porosity, different boundary lines can be obtained to separate the region where only FD formulation need be employed and the region where the simplified formulation, i.e. PD form, is applicable with adequate accuracy. Cases 1–6 in Figs 2 and 3 are adopted here again as the examples, which are represented by different symbols in Fig. 9. Note that for the points of cases 1 and 4 which locate above the boundary line of $n = 0.35$, the dynamic of pore fluid must be considered and only FD formulation can ensure the accuracy of calculation. For other points below the line (e.g., Cases 2, 3, 5 and 6), the results of PD formulation are acceptable. This is consistent with the results shown in Figs. 2 and 3, in which the difference between FD and PD solutions in cases 1 and 4 could not be ignored. According to the regions of applicability in Fig. 9, in the frequency range of seismic interest ($0.1\text{Hz} \leq f \leq 30\text{Hz}$), when $k_f \leq 5 \times 10^{-4}$ m/s (for most clay, silt, clayed or silty sand), PD formulation has sufficient accuracy for practical engineering. Such a similar extrapolation to earthquake response of soil has also been made by Ulker and Rahman [10], which demonstrates that the formulation to be used depends on the loading frequency as well as the permeability in the case of sandy soil. Moreover, the FD solution is needed for accurate results in the case of coarse sand under high frequency loading [10]. This is consistent with the proposed domains of applicability for the two formulations in Fig. 9.

To further validate the reliability of the applicable conditions for two formulations in Fig. 9, Fig.

10 illustrates the influence of normalized frequency on the discrepancy of induced displacement u between FD and PD formulations. Different curves represent the values of displacements at different positions of coarse gravel seabed, i.e., $z = L, 2L/3, L/3$. Both the partly and fully saturated states are considered. It is shown from Fig. 10 that the proposed discrimination criterion is also applicable for the seismically induced displacement with less than 3% of relative error, which is generally acceptable in geotechnical engineering practice.

7 Conclusions

In this paper, a series of analytical solutions are presented on the seismic-induced responses of nearly saturated seabed. To examine the effect of inertial terms of pore fluid, two formulations which are commonly used in the analysis of dynamic response of porous seabed are respectively adopted: fully dynamic (FD) and partial dynamic (PD). The influences of saturation degree, excitation frequency and soil permeability on the propagation speeds of seismic waves in the seabed are firstly discussed. The slight unsaturation has significant effect on the propagation characteristics of P1 and P2 waves in the seabed. Then, the significant influences of saturation degree, soil permeability and excitation frequency on the earthquake-induced soil displacement and pore pressure in the seabed are investigated. Compared with soil permeability, the effects of saturation degree and excitation frequency on the seismic-induced response are more notable.

An applicable condition for FD and PD formulations is further proposed and depicted in a parametric space. When the discrimination criterion $f \leq ng / (20\pi k_f)$ is satisfied, the simplified formulation, i.e. PD form, is applicable with adequate accuracy (with less than 3% of relative error for seismic-induced displacement). Otherwise, FD formulation need to be employed in the analysis for the seismic-induced response of seabed. The proposed domain of applicability is

410 useful in selecting the appropriate solution for the seismic analysis in geotechnical engineering
411 practice.

412

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497 **Appendix :**

498 The explicit expressions of the elements a_j ($j=1-4$) in Eqs. (19) and (20) are given as follows

499
$$a_1 = \Delta_1/\Delta, \quad a_2 = \Delta_2/\Delta, \quad a_3 = \Delta_3/\Delta, \quad a_4 = \Delta_4/\Delta \quad (48)$$

500 in which

501
$$\Delta_1 = u_0 \begin{vmatrix} \eta_2 & \eta_3 & \eta_4 \\ m_{32} & m_{33} & m_{34} \\ m_{42} & m_{43} & m_{44} \end{vmatrix} \quad \Delta_2 = -u_0 \begin{vmatrix} \eta_1 & \eta_3 & \eta_4 \\ m_{31} & m_{33} & m_{34} \\ m_{41} & m_{43} & m_{44} \end{vmatrix} \quad \Delta_3 = u_0 \begin{vmatrix} \eta_1 & \eta_2 & \eta_4 \\ m_{31} & m_{32} & m_{34} \\ m_{41} & m_{42} & m_{44} \end{vmatrix}$$

502
$$\Delta_4 = -u_0 \begin{vmatrix} \eta_1 & \eta_2 & \eta_3 \\ m_{31} & m_{32} & m_{33} \\ m_{41} & m_{42} & m_{43} \end{vmatrix} \quad \Delta = \begin{vmatrix} 1 & 1 & 1 & 1 \\ \eta_1 & \eta_2 & \eta_3 & \eta_4 \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{vmatrix}$$

503 where

504
$$m_{31} = -k_{p1} \exp(-ik_{p1}L) \quad m_{32} = k_{p1} \exp(ik_{p1}L) \quad m_{33} = -k_{p2} \exp(-ik_{p2}L) \quad m_{34} = k_{p2} \exp(ik_{p2}L)$$

505
$$m_{41} = [n\beta\rho_f d\omega^2 - i(1+\eta_1)k_{p1}] \exp(-ik_{p1}L) \quad m_{42} = [n\beta\rho_f d\omega^2 + i(1+\eta_2)k_{p1}] \exp(ik_{p1}L)$$

506
$$m_{43} = [n\beta\rho_f d\omega^2 - i(1+\eta_3)k_{p2}] \exp(-ik_{p2}L) \quad m_{44} = [n\beta\rho_f d\omega^2 + i(1+\eta_4)k_{p2}] \exp(ik_{p2}L)$$

507 The explicit expressions of the elements a_j ($j=5, 6$) in Eq. (33) are

508
$$a_5 = \frac{\exp(ik_s L)}{\exp(ik_s L) + \exp(-ik_s L)} u_0 \quad a_6 = \frac{\exp(-ik_s L)}{\exp(ik_s L) + \exp(-ik_s L)} u_0 \quad (49)$$

509 The explicit expressions of the elements a'_j ($j=1-4$) in Eqs. (41) and (42) are given as follows

510
$$a'_1 = \Delta'_1/\Delta', \quad a'_2 = \Delta'_2/\Delta', \quad a'_3 = \Delta'_3/\Delta', \quad a'_4 = \Delta'_4/\Delta' \quad (50)$$

511 in which

512
$$\Delta'_1 = u_0 \begin{vmatrix} \eta'_2 k'_{p1} & -\eta'_3 k'_{p2} & \eta'_4 k'_{p2} \\ m'_{32} & m'_{33} & m'_{34} \\ m'_{42} & m'_{43} & m'_{44} \end{vmatrix} \quad \Delta'_2 = -u_0 \begin{vmatrix} -\eta'_1 k'_{p1} & -\eta'_3 k'_{p2} & \eta'_4 k'_{p2} \\ m'_{31} & m'_{33} & m'_{34} \\ m'_{41} & m'_{43} & m'_{44} \end{vmatrix}$$

513
$$\Delta'_3 = u_0 \begin{vmatrix} -\eta'_1 k'_{p1} & \eta'_2 k'_{p1} & \eta'_4 k'_{p2} \\ m'_{31} & m'_{32} & m'_{34} \\ m'_{41} & m'_{42} & m'_{44} \end{vmatrix} \quad \Delta'_4 = -u_0 \begin{vmatrix} -\eta'_1 k'_{p1} & \eta'_2 k'_{p1} & -\eta'_3 k'_{p2} \\ m'_{31} & m'_{32} & m'_{33} \\ m'_{41} & m'_{42} & m'_{43} \end{vmatrix}$$

514

$$\Delta' = \begin{vmatrix} 1 & 1 & 1 & 1 \\ -\eta'_1 k'_{p1} & \eta'_2 k'_{p1} & -\eta'_3 k'_{p2} & \eta'_4 k'_{p2} \\ m'_{31} & m'_{32} & m'_{33} & m'_{34} \\ m'_{41} & m'_{42} & m'_{43} & m'_{44} \end{vmatrix}$$

515

where

516

$$m'_{31} = -k'_{p1} \exp(-ik'_{p1}L) \quad m'_{32} = k'_{p1} \exp(ik'_{p1}L) \quad m'_{33} = -k'_{p2} \exp(-ik'_{p2}L) \quad m'_{34} = k'_{p2} \exp(ik'_{p2}L)$$

517

$$m'_{41} = (\rho_f d \omega^2 - \eta'_1) \exp(-ik'_{p1}L) \quad m'_{42} = (\rho_f d \omega^2 - \eta'_2) \exp(ik'_{p1}L)$$

518

$$m'_{43} = (\rho_f d \omega^2 - \eta'_3) \exp(-ik'_{p2}L)$$

519

Table 1. Expressions of wave velocities in the limiting cases.

Seismic waves	$\omega \rightarrow 0, k_f \rightarrow 0$		$\omega \rightarrow \infty, k_f \rightarrow \infty$	
	FD form	PD form	FD form	PD form
P1 wave	$V_{p1} = \sqrt{\frac{(\lambda + 2\mu)n\beta + 1}{\rho n\beta}}$	$V_{p1} = \sqrt{\frac{(\lambda + 2\mu)n\beta + 1}{\rho n\beta}}$	$V_{p1} = \sqrt{\frac{-C_2 + \sqrt{C_2^2 - 4C_4}}{2}}$	∞
P2 wave	–	–	$V_{p2} = \sqrt{\frac{-C_2 - \sqrt{C_2^2 - 4C_4}}{2}}$	$V_{p2} = \sqrt{\frac{\lambda + 2\mu}{\rho - \rho_f}}$
S wave	$V_s = \sqrt{\frac{\mu}{\rho}}$	$V_s = \sqrt{\frac{\mu}{\rho}}$	$V_s = \sqrt{\frac{\mu}{\rho - n\rho_f}}$	$V_s = \sqrt{\frac{\mu}{\rho}}$

Table 2 Values of the parameters used in computation.

Parameter, Notation (unit)	Value
Water depth, d (m)	30
Thickness of porous seabed, L (m)	15
Porosity, n	0.35
Density of solid grain, ρ_s (kg/m ³)	2650
Density of water, ρ_f (kg/m ³)	1000
Permeability, k_f (m/s)	10^{-1} (coarse gravel) 10^{-4} (fine sand) 10^{-6} (silty sand)
Lame constant, μ (MPa)	10
Lame constants, λ (MPa)	19.4
Bulk modulus of water, K_w (GPa)	2.0
Excitation amplitude at bedrock, u_0 (m)	0.1

Fig. 1. A porous seabed subjected to earthquake excitation at rigid base

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530

531 Fig. 2 Influence of saturation degree on the wave speeds in (a) case 1: coarse gravel seabed, (b)
 532 case 2: fine sand seabed, (c) case 3: silty sand seabed (when $f = 10 \text{ Hz}$)

Fig. 3 Influence of saturation degree on the wave speeds in (a) case 4: coarse gravel seabed, (b) case 5: fine sand seabed, (c) case 6: silty sand seabed (when $f = 2 \text{ Hz}$)

Fig. 4 Vertical distributions of amplitude of $|u|/u_0$ versus the relative soil depth (z/L) for various seabeds: (a) $S_r = 0.98$, (b) $S_r = 1.0$

Fig. 5 Vertical distributions of amplitude of $|p|/p_{w0}$ versus the relative soil depth (z/L) for various seabed: (a) $S_r = 0.98$, (b) $S_r = 1.0$

Fig. 6 Variation of amplification factor of displacement with frequency in a coarse gravel seabed:

(a) $S_r = 0.98$, (b) $S_r = 1.0$

Fig. 7 Variation of amplification factor at seabed surface with frequency when the seawater is removed (i.e., $d = 0$ m)

Fig. 8 Influence of non-dimensional frequency on speeds of P1, P2 and S waves: (a) $S_r = 0.98$ and $n = 0.35$, (b) $S_r = 1.0$ and $n = 0.35$, (c) $S_r = 1.0$ and $n = 0.55$, (d) $S_r = 0.98$ and $n = 0.55$

Fig. 9 Regions of applicability of FD and PD formulations for seismic waves in seabed

Fig. 10 Influence of non-dimensional frequency on the error of induced displacements at different seabed locations between FD and PD formulations (a) $S_r = 0.98$, (b) $S_r = 1.0$