Chatter detection for milling using novel p -leader multifractal features

Yun Chen¹, Huaizhong Li², Liang Hou^{1*}, Xiangjian Bu¹, Shaogan Ye¹ and Ding
Chen³

¹Department of Mechanical and Electrical Engineering, Xiamen University, Xiamen
361005, China

²School of Engineering and Built Environment, Gold Coast campus, Griffith
University, QLD 4222, Australia

³School of Electrical Engineering and Automation, Xiamen University of
Technology, Xiamen 361024, China

Abstract

Chatter in machining results in poor workpiece surface quality and short tool life. A reliable chatter detection method is needed to monitor this self-excited vibration before its complete development. This paper applies multifractal features extracted from a novel p -leader multifractal formalism for chatter detection in milling processes. This novel formalism is able to discover a complex singular behavior rising on unstable chatter signals, and improves chatter detection performance. The multifractal features are selected using a multivariate filter, and assessed in terms of their dynamic monitoring abilities and classification accuracies under wide cutting conditions. The results show that the multifractal features cannot only detect chatter with high accuracy and short computation time, but are also efficient to distinguish stable tests with large amplitudes from unstable tests. A small p is recommended to highlight dynamic variations and obtain high classification accuracies. For further verification, the

---

* Corresponding author: hliang@xmu.edu.cn.

proposed method is compared with an ensemble empirical mode decomposition method and a traditional wavelet decomposition method. The p -leader multifractal formalism gives a better performance than the other two methods, especially for detecting unstable tests. Additionally, the p -leader multifractal formalism takes a computation time about 10% of the ensemble empirical mode decomposition method for feature extraction.

Keywords: Chatter detection; milling processes; multifractal features; p -leader; feature selection.

1 Introduction

Chatter in machining is a self-excited vibration between cutting tool and workpiece, which is detrimental due to abnormal vibration increases. The increased vibration results in poor surface finish, severe tool wear, tool breakage, and even damage to machine tools. In order to avoid chatter, theoretical modelling of the machining process can be used to select cutting parameters from stability diagrams. The zeroth-order (Altintas and Budak, 1995) and semi-discrete (Insperger and Stepan, 2004) are typical methods to determine stability diagrams. However, a solid knowledge background about machining dynamics is needed for stability diagrams, and is not practical for industrial users to operate. Moreover, these models often do not account for the effect of the changing dynamics or highly complex machining operations (Yesilli et al., 2019). Given the current development of sensory techniques and advanced signal processing methods, early chatter detection becomes an alternative to efficiently monitor and control the machining condition.

Force, vibration and acoustic emission signals are often used for chatter detection. Among these signals, vibration signals are the most common for fault diagnosis in rotary machinery and machining processes. For an intelligent chatter detection system, signal processing is necessary for feature extraction in order to highlight chatter characteristics. The acquired signals are often processed in the time, frequency and

time-frequency domains (Benkedjouh et al., 2013; Tao et al., 2019). As chatter is a nonlinear and nonstationary process, time-frequency methods, such as short-time frequency transform (STFT) (Chen et al., 2019), empirical mode decomposition (EMD) (Fu et al., 2016), variational mode decomposition (VMD) (Liu et al., 2018) and wavelet-based analysis (Zhang et al., 2016), are popular to disclose chatter characteristics hidden in the time-domain signals. The STFT is based on a fixed window size of Fourier transform theory, and a proper balance between time and frequency resolution cannot be easily found (Albertelli et al., 2019). Wavelet-based time-frequency methods need experiences to determine the suitable wavelet base function. EMD, ensemble EMD and VMD are adaptive signal decomposition methods that decompose signals into intrinsic mode functions. However, they require the selection and/or the tuning of several parameters, and are time-consuming compared to STFT and wavelet-based methods (Li et al., 2020).

The underlying machining process is high dimensional and stochastic. Complexity arises in signals and result in different probability distributions (Ren et al., 2019). The commonly-used complexity measure of signals is fractal analysis. Ji et al. (2018) applied a single-fractal analysis for chatter detection, assuming scale invariance of signals independent on time. However, temporal variations in scale invariant structure often appear (Ihlen, 2012). The variations in scale invariant structure imply a multifractal structure which is an approximate probability distribution of local singularities. The wavelet transform modulus maxima (WTMM) and multifractal detrended fluctuation analysis (MF-DFA) are two typical multifractal methods often used for fault diagnosis. Du et al. (2019) indicated that multifractal analysis is efficient to analyze non-uniformity and singularity of nonstationary and nonlinear signals. It is noted that WTMM uses the continuous wavelet transform with high computational complexity and is not suitable for real-time diagnosis. MF-DFA also has the same problem when the considered scaling range is wide.

A wavelet leader multifractal method proposed by Lashermes et al. (2005) calculated the multifractal spectrum from discrete wavelet transform (DWT) and avoided long and complicated calculation as WTMM and MF-DFA. It achieved high performance for bearing fault diagnosis (Du et al., 2014) and analysis of heart rate variability (Abry et al., 2010). Recently, an improved formalization, namely, the p -leader multifractal formalism, has been developed by Leonarduzzi et al. (2016) and provided richer and more detailed information on characteristics of the singularities in signals. A thorough comparison of the p -leader multifractal formalism and MF-DFA indicated that the former significantly improves the multifractal spectrum estimation. Most importantly, when the p -leader multifractal spectra vary for different p , it indicates a complex singular behavior (i.e. lacunary singularities) on the signal. Such a dependence is a distinctive feature of the p -leader formalism and is not available with the original wavelet-leader method and MF-DFA. The p -leader multifractal analysis has been used in heart rate variability analysis for survival probability assessment (Wendt et al., 2014) and fetal heart rate variability in the first stage intrapartum (Leonarduzzi et al., 2015). It is found that some signals show great dependence on p , and therefore, proper selection of p can improve signal pattern recognition performance.

The above signal processing methods highlight chatter characteristics, but do not give the boundary between the stable and unstable tests. There are two common types of methods used for the boundary determination. The first method uses a threshold of individual chatter index or feature to distinguish chatter. For example, Fu et al. (2017) applied deep belief networks for chatter detection. Tao et al. (2019) determined the threshold through extensive tests. Additionally, machining learning techniques, such as support vector machine (SVM) (Chen et al., 2018), gradient tree boosting (Li et al., 2020) and hidden Markov model (Liu et al., 2017), are also used for intelligent chatter detection.

In the present paper, multifractal features based on the novel p -leader multifractal

formalism have been for the first time applied for chatter detection in milling. The p -leader multifractal formalism based on a discrete wavelet decomposition structure can identify complex singularities in chatter signals, which avoids long calculation and gives high detection accuracy. The rest of this paper is organized as follows. In section 2, the p -leader multifractal formalism and its corresponding multifractal features are explained in detail. Section 3 describes the experimental setup used for the verification, and critically analyzes the chatter detection performances in both numerical simulations and experimental analysis, and is followed by section 4 that outlines conclusions.

2 Methodology

Signals from the machining systems are nonlinear and nonstationary, and show complex and irregular fluctuations even under stable machining conditions. When chatter occurs, complexity and singularity of the signal rise. Single-fractal measures neglect local singularity properties of signals, whereas the multifractal spectrum, as an approximate probability distribution of local singularities, can reflect the overall singularity information (Ihlen, 2012). Due to the particular translation and dilation structure, DWT becomes an ideal tool for fast multifractal calculation. Moreover, the additional parameter p introduced in the p -leader multifractal formalism can provide more singularity properties of signals (Abry et al., 2015), and improve chatter detection performance.

2.1 p -leader multifractal formalism

Let $\psi(x)$ be a mother wavelet that consists of oscillating functions with fast decay and good joint time-frequency localization. A collection of dilated (to scales 2^j) and translated (to space/time location $2^j k$) templates is defined based on the mother wavelet $\psi(x)$

$$\{2^{-j/2}\psi(2^{-j}x-k), \text{ for } j, k \in \mathbb{Z}\} \quad (1)$$

which forms an orthonormal basis of $L^2(\mathbb{R})$. The mother wavelet $\psi(x)$ has a number

$N_\psi \geq 1$ of vanishing moments, which satisfies $\int P(x)\psi(x)dx = 0$ for any polynomial $P(x)$ of degree smaller than N_ψ , while the integral does not vanish for some polynomials of degree N_ψ . The coefficients of DWT for a signal X are defined as

$$c_{j,k} = \int X(x)2^{-j/2}\psi(2^{-j}x-k)dx \quad (2)$$

Define the dyadic interval $\lambda = \lambda_{j,k} = [k2^j, (k+1)2^j)$, and accordingly $c_\lambda = c_{j,k}$. 3λ is defined as three adjacent intervals $3\lambda = \lambda_{j,k-1} \cup \lambda_{j,k} \cup \lambda_{j,k+1}$, as shown in Figure 1. The p -leaders evaluated over 3λ are (Jaffard et al., 2016)

$$l^{(p)}(j,k) \equiv l_\lambda^{(p)} = \left(\sum_{j' \leq j, \lambda' \subset 3\lambda} |c_{\lambda'}|^p 2^{j'-j} \right)^{1/p} \text{ for } p > 0 \quad (3)$$

where $j' < j$ is the scale associated with sub-interval λ' of width $2^{j'}$ included in the interval 3λ . When $p = \inf$, p -leaders boil down to the classical wavelet leaders (Jaffard et al., 2016)

$$l^{(\inf)}(j,k) \equiv l_\lambda = \sup_{j' \leq j, \lambda' \subset 3\lambda} |c_{\lambda'}| \quad (4)$$

Figure 1. Illustration of the p -leaders from the DWT structure.

The pointwise p -exponent $h^{(p)}(x_0)$ at x_0 is defined as follows (Leonarduzzi et al., 2016)

$$h^{(p)}(x_0) = \lim_{j \rightarrow -\infty} \inf \frac{\log(I_{\lambda_j(x_0)}^{(p)})}{\log(2^j)} \quad (5)$$

where $\lambda_j(x_0)$ is the interval of width 2^j that contains x_0 . $h^{(p)}(x_0)$ defined above does not depend on the wavelet as long as the wavelet is smooth enough (Leonarduzzi et al., 2016) and is the singularity strength around x_0 (Chhabra et al., 1989). However, multifractal analysis does not estimate $h^{(p)}(x_0)$ for all x_0 . Instead, a global description of the fluctuations of $h^{(p)}(x_0)$ is considered across all points. This global information is the so-called multifractal or singularity spectrum $D^{(p)}(h)$ that is defined as the fractal dimension of the set of points x_0 where the p -exponent $h^{(p)}(x_0)$ takes the value h .

According to the general multifractal formalism (Jaffard et al., 2016; Leonarduzzi et al., 2016), the p -leader multifractal spectrum $D^{(p)}(h)$ is constructed based on the p -leader structure function $S_l^{(p)}(j, q)$ which is defined as the sample moment of the p -leaders

$$S_l^{(p)}(j, q) = 2^j \sum_k (l^{(p)}(j, k))^q \text{ for } \forall q \in \mathbb{R} \quad (6)$$

The p -leader scaling function is defined as

$$\zeta^{(p)}(q) = \lim_{j \rightarrow -\infty} \inf \frac{\log(S_l^{(p)}(j, q))}{\log(2^j)} \quad (7)$$

$\zeta^{(p)}(q)$ does not depend on the wavelet basis when the mother wavelets are C^∞ (Leonarduzzi et al., 2016). The relationship between $D^{(p)}(h)$ and $\zeta^{(p)}(q)$ can be obtained via the Legendre transform $\mathcal{L}^{(p)}(h)$

$$D^{(p)}(h) \leq \mathcal{L}^{(p)}(h) := \inf_{q \in \mathbb{R}} (1 + qh - \zeta^{(p)}(q)) \text{ for } \forall h \quad (8)$$

Eq. (8) provides an upper bound for $D^{(p)}(h)$. In the most cases, the inequality in Eq.

(8) turns out to be an equality, and therefore $\mathcal{L}^{(p)}(h)$ is often used to replace $D^{(p)}(h)$ (Du et al., 2014). According to Eq. (4), when $p = \inf$, $D^{(\inf)}(h)$ becomes the classical wavelet leader multifractal spectrum (Jaffard et al., 2016; Leonarduzzi et al., 2016).

Eqs.(1)-(8) define a theoretical definition of p -leader multifractal spectrum. In fact, only values of signal X sampled at finite resolution are available, and the infinite sum is truncated at the available scale induced by the resolution of data (Leonarduzzi et al., 2016). The $\liminf_{j \rightarrow \infty}$ for $\zeta^{(p)}(q)$ cannot be evaluated. Therefore, the equivalents or estimators for Eqs. (1) - (8) should be computed in the discrete and finite domain. The p -leader scaling function $\zeta^{(p)}(q)$ is estimated by means of linear regression in $\log(2^j)$ versus $\log(S_l^{(p)}(j, q))$. The multifractal spectrum $D^{(p)}(h)$ and singularity strength h are estimated using direct determination proposed by Chhabra et al. (1989).

2.2 Chatter detection using p -leader multifractal formalism

This paper uses the novel p -leader multifractal features for chatter detection. A detailed algorithm flow chart for the proposed method is given in Figure 2. A numerical simulation of the stable and unstable tests is firstly conducted in order to examine the signals' dependence on p . Based on the simulation analysis, the multifractal feature set $[\zeta^{(p)}(q), D^{(p)}(h), h]$ extracted from the p -leader multifractal formalism is applied for chatter detection. The features are selected using ReliefF in order to reduce the feature dimension. The performance of the selected features to monitor the dynamic variations is examined. Two classifiers, support vector machine (SVM) and k-nearest neighbors (KNN), are employed for chatter detection using the selected features. Hereto, SVM and KNN are implemented using the functions `fitsvm` and `fitcknn` in Matlab2019a, respectively. The parameter C for penalizing slack variables and the scale parameter σ for the Gaussian kernel in the nonlinear SVM are optimized using Bayesian

optimization algorithm (MathWorks, 2020). The optimal number of k-nearest neighbors and the optimal distance metric for KNN are also obtained using Bayesian optimization algorithm.

Figure 2. Proposed chatter detection method

3 Results and discussions

3.1 Numerical simulation analysis

Numerical simulations are carried out to examine dependence on p of stable and unstable tests. The vibration signal is simulated as

$$y = 4 \sin(90\pi t) + 4 \sin(60\pi t) + 4 \sin(30\pi t) + A(1 + 0.6 \sin(30\pi t)) \cos(420\pi t + 1.5 \sin(15\pi t)) \quad (9)$$

The first three items of Eq. (9) are sinusoidal waves representing the tooth passing frequency and its harmonics. The last item is an amplitude and phase modulation component, which always appears in unstable test signals (Cao et al., 2013). If A ($A \geq 0$) in Eq. (9) is zero, the simulated test y is stable, and vice versa. Figure 3 gives the simulated tests and their multifractal spectra under different p . The spectra are obtained within the range $q \in [-2, 2]$. The simulated unstable test has wide multifractal spectra, indicating more singularity (Leonarduzzi et al., 2015). All spectra for the stable test are to the right of the unstable test. As p decreases, spectra for both stable and unstable tests

become wider. More importantly, the spectra for the unstable test show a significantly movement towards the negative h when p becomes smaller, indicating its more obvious dependence on p exponents. According to Ref. (Leonarduzzi et al., 2016), the variation of p -spectra with p has a profound meaning. It evidences that data have a special sort of oscillating singularities called lacunary singularities. In the next section, the effect of such a dependence on chatter detection performance is discussed.

Figure 3. Dependence on p of simulated tests (a) simulated tests (b) p -leader multifractal spectra. A is 5 for the unstable test.

3.2 Experimental results

3.2.1 Experimental setup

The proposed chatter detection method was assessed with experimental data from shoulder milling of titanium alloy Ti6Al4V under dry cutting conditions. The experimental setup is shown in Figure 4, and the feed direction is along the x direction. A PCB 356A16 accelerometer was mounted near the workpiece to measure the vibration signals. The sample rate was 20 kHz. The cutters were Sutton E5060800 end mills with two flutes, and had diameters of 8 mm and helix angles of 30° . The

overhanging length of cutters was 38 mm for all investigated tests. Table 1 lists the natural frequencies of the machine tool-workpiece system obtained by impact testing in Ref. (Chen et al., 2018). The experimental tests were conducted under wide combinations of parameter settings including spindle speeds n_p from 1100 to 8100 rpm, feed per tooth $f_t = 0.08, 0.1, 0.12$ mm/tooth, axial depth of cut $a_p = 4, 8, 12$ mm, and radial depth of cut $a_e = 0.5, 1, 1.5, 4$ mm. The total number of tests is 86. Similar to Ref. (Yesilli et al., 2019), test labels were tagged using both frequency and time-frequency analysis.

Figure 4. Experimental setup for shoulder milling of titanium alloy Ti6Al4V.

Table 1. Natural frequencies for the workpiece and tool (Chen et al., 2018).

	Direction	Natural frequency (Hz)
Workpiece	x	1107
	y	1220
	z	1111, 2945
Tool	x	1355, 1900, 2780, 4912
	y	1308, 1875, 2719, 5011

3.2.2 Scale range selection

The theoretical assumption that the signal X in Eqs. (6)-(8) is exactly self-similar would imply an infinite scaling range. In practice, the scaling range must be considered limited (Leonarduzzi et al., 2014). Firstly, finite scaling range may result from physical mechanisms whose dynamics involve a large bounded range of scales, while other competing mechanisms may become dominant at finer or coarser scales. Also, the

signal X is digitally collected at a given rate, and the sampling devices inevitably destroy the power law behaviors at fine and coarse scales. Further, noise can be superimposed to data, often leading to a substantial narrowing of the range of scales. These different mechanisms imply that scale invariance in practice holds only within a finite range of scales.

Before the p -leader multifractal analysis, the scale invariance is firstly examined to identify the finite range of scales with linear regression. Figure 5 shows two sample tests and their FFT plots. Abry et al. (2010) stated that q 's range can be estimated by $q = \pm\sqrt{2/|c_2|}$, where c_2 can be estimated by Eq. (30) in Ref. (Leonarduzzi et al., 2016). For the 344 data samples used in Section 3.2.3, $|c_2|$ ranges from 0.0136 to 0.8772, and hence, q from ± 12 to ± 1.5 . Therefore, a small range $q = \pm 2$ close to the low limit $q = \pm 1.5$ is selected for analysis of all data samples. The parameter p is selected according to the values used by Leonarduzzi et al. (2015). The finite scaling range analysis results are given in Figure 6. The mother wavelet is selected according to the criterion that a vanishing moment N_ψ should be larger than the largest singularity exponent h (Wendt et al., 2009). Hence, Daubechies 3 function with $N_\psi = 3$ is used. Both tests show a linear scaling regression over the first three scales $j=1, 2, 3$ for $p = 0.5, 1, 2$ and $\inf$.

According to discrete wavelet decomposition structure, the scales $j=1, 2, 3$ correspond to the frequency bands 5-10 kHz, 2.5-5 kHz and 1.25 kHz-2.5 kHz, respectively. High frequencies above 1.25 kHz contain most of natural frequencies of the tool-workpiece machine systems, and chatter frequencies often rise around dominant natural frequencies. The estimated $\zeta^{(p)}(q)$ is related to the ratio between low frequencies and high frequencies (LF/HF) in the selected scales $j=1, 2, 3$ (Abry et al., 2010). When chatter occurs, the energy around certain frequency bands rises, resulting in the LF/HF varying and therefore $\zeta^{(p)}(q)$ and the multifractal spectrum also varying. Thus, the

selected set of scales $j=1, 2, 3$ has a strong physical background for chatter detection, and is used for the p -leader multifractal formalization in the following sections.

Figure 5. Time-domain signals and FFT plots (a) stable at n_p 3100 rpm, a_p 8 mm, f_t 0.1 mm/tooth and a_e 1 mm and (b) unstable at n_p 1600 rpm, a_p 8 mm, f_t 0.1 mm/tooth and a_e 1 mm.

Figure 6. Relationship between p -leader structure function $S_l^{(p)}(j, q)$ and scales j for tests in Figure 5 (a) stable (b) unstable.

3.2.3 Feature assessment

The multifractal spectra for the tests in Figure 5 are given in Figure 7. **Whatever p ,** spectra for the stable tests are to the right of those for unstable ones, which agrees with

the simulated results in Figure 3. It can be seen that the multifractal spectra depend on p . As p decreases from $\inf$ to 0.5, the spectra are shifted to large values of h for the stable test, and small values for the unstable test. Dependence on p is more obvious and significant for the unstable test. Therefore, the unstable test is characterized by a rich oscillatory singularity behavior with potentially lacunary singularities. This broadens the description provided by Holder exponent based fractal analysis (Leonarduzzi et al., 2015), which again indicates outperformance of the multifractal features over single-fractal measures.

Figure 7. Estimated multifractal spectra for Figure 5 (a) stable (b) unstable.

To obtain a larger group of data samples, the 86 tests investigated is divided into 4 segments, and each segment contains 20 revolutions of sample points. Therefore, a group with 344 data samples is constructed. The feature selection method, ReliefF, is a multivariate filter independent of the classification method, and outperforms over 10 other feature selection methods (Bolon-Canedo et al., 2013). The larger the importance weight from ReliefF, the more useful the feature. Figure 8 shows the ReliefF results for all the multifractal features. The multifractal feature numbers and their notations are given in Table 2. The features evaluated from negative q (e.g. $f_9 - f_{12}$) have small weights for all p values, which indicates their less usefulness to distinguish the stable and unstable tests and monitor dynamic variations. According to Ihlen (2012), multifractal features evaluated from negative q are used to describe small amplitude variations. Unstable tests are often coupled with large amplitude variations, whereas the stable

tests show gentle variations. This results in the less usefulness of the multifractal features based on negative q . It can be seen that f_{15}, f_{16} and f_{24} are always the top three multifractal features for any p . A small p generally gives high importance weights for the three top features. As p decreases from 1 to 0.5, the importance weight for each feature almost does not change.

Figure 8. Feature importance analysis using ReliefF (a) $p = \inf$ (b) $p = 2$ (c) $p = 1$ (d) $p = 0.5$.

Table 2. Feature number and its notation.

Feature #	Notation
$f_1 - f_8$	$\zeta_{q=-2}^{(p)}, \zeta_{q=-1.5}^{(p)}, \zeta_{q=-1}^{(p)}, \zeta_{q=-0.5}^{(p)}, \zeta_{q=0.5}^{(p)}, \zeta_{q=1}^{(p)}, \zeta_{q=1.5}^{(p)}, \zeta_{q=2}^{(p)}$
$f_9 - f_{16}$	$D_{q=-2}^{(p)}, D_{q=-1.5}^{(p)}, D_{q=-1}^{(p)}, D_{q=-0.5}^{(p)}, D_{q=0.5}^{(p)}, D_{q=1}^{(p)}, D_{q=1.5}^{(p)}, D_{q=2}^{(p)}$
$f_{17} - f_{24}$	$h_{q=-2}^{(p)}, h_{q=-1.5}^{(p)}, h_{q=-1}^{(p)}, h_{q=-0.5}^{(p)}, h_{q=0.5}^{(p)}, h_{q=1}^{(p)}, h_{q=1.5}^{(p)}, h_{q=2}^{(p)}$

A moving window is applied to signals to assess the ability of the top three multifractal features to monitor the real-time dynamic variations. Sensitivity analysis is conducted in order to select an appropriate window size. The two stable tests with the smallest and highest spindle speeds 1100 rpm and 8100 rpm respectively are selected for sensitivity

analysis. Figure 9 gives the time-domain signals for the two tests. Segments extracted from the middle of the time-domain signals are used to calculate the multifractal features. The variations of the top 3 features with different window sizes are shown in Figure 10. The top 3 features for the test with 1100 rpm become stable when the window size is above 1,000, whereas the features for the test with 8100 rpm level off at about 150. The numbers of sampling points per spindle revolution are around 1090 and 148 for 1100 and 8100 rpm, respectively. It can be seen that the proper window size depends on the spindle speed. Therefore, the window size is selected according to the number of points per spindle revolution (NPSR), instead of a fixed size for all tests.

Figure 9. Time-domain signal for (a) stable at n_p 1100 rpm, a_p 4 mm, f_t 0.1 mm/tooth and a_e 1 mm and (b) stable at n_p 8100 rpm, a_p 8 mm, f_t 0.1 mm/tooth and a_e 1 mm.

Figure 10. Effect of the window size on the multifractal features for Figure 9.

Four sample tests, i.e., stable Test 1 with small amplitudes, stable Test 2 with large amplitudes, moderately unstable Test 3 and fully unstable Test 4 in Table 3, are examined in Figure 11. The multifractal features are calculated over a moving

rectangular window with twice number of points per spindle revolution. Each pair of successive windows has an overlap of one NPSR. The unstable tests show energy rises at frequencies around 5 kHz as shown in Figure 12. When energy rises around 5 kHz, the top 3 multifractal features decrease at the same time. Thus, those features can well track the dynamic spectral variation during the machining process. As the multifractal spectrum is an approximation of the probability distribution of local singularities (Ihlen, 2012), the multifractal features are independent on the size of vibration amplitudes (e.g., either stable tests with large amplitudes or unstable with small amplitudes), but depend on the intrinsic chaotic patterns of the machining process. Therefore, **the stable Test 2 with large amplitudes has multifractal feature values similar to the stable Test 1 with small amplitudes**, and is easy to be distinguished from the unstable tests.

Table 3. Cutting parameters for four selected tests.

Test #	Axial depth of cut a_p (mm)	Radial depth of cut a_e (mm)	Spindle speed n_p (rpm)	Feed per tooth f_t (mm/tooth)
1	2	4	2600	0.08
2	12	1	6100	0.10
3	8	1	2600	0.10
4	12	1	2600	0.12

Figure 13 is a zoomed view of Figure 11. As expected, the variation of p values has less influence on the stable tests as shown in Figure 13(a) and (b), whereas p has **an obvious effect** on the unstable tests. Figure 13(c) and (d) show that a small p results in more obvious variations of the three features, which highlights the dynamic transition and improves the classification accuracies as discussed in the next section.

Figure 11. Assessment of dynamic monitoring abilities of multifractal features (a) stable Test 1 (b) stable Test 2 with large amplitudes (c) moderately unstable Test 3 (d) unstable Test 4 in [Table 3](#).

Figure 12. STFT plots (a) stable Test 1 (b) stable Test 2 with large amplitudes (c) moderately unstable Test 3 (d) unstable Test 4 in Table 3. The left colorbar unit is m/s^2 .

Figure 13. Zoomed views of Figure 11 (a) stable Test 1 (b) stable Test 2 with large amplitudes (c) moderately unstable Test 3 (d) unstable Test 4 in Table 3.

3.2.4 Cutting state classification

Two pattern recognition methods, SVM and KNN are used to classify the 344 data samples constructed in Section 3.2.3. The procedures to apply SVM and KNN are explained in section 2.2. The performance of the two methods is assessed using the accuracies AC_{sd} , AC_{usd} and AC defined as the percentages of stable tests classified as stable, unstable tests classified as unstable and total tests classified correctly, respectively. The AC_{sd} , AC_{usd} and AC are averaged over 20 cross fold evaluation. As the multifractal spectrum is defined by a pair of two variables $D_q^{(p)}$ and $h_q^{(p)}$, two pairs of multifractal features (f_{15}, f_{23}) and (f_{16}, f_{24}) , i.e., $(h_{q=1.5}^{(p)}, D_{q=1.5}^{(p)})$ and $(h_{q=2}^{(p)}, D_{q=2}^{(p)})$, respectively, are selected for chatter classification. The classification results using (f_{15}, f_{23}) and (f_{16}, f_{24}) are listed in Table 4. Generally, AC_{usd} for the unstable tests increases with the decrease of p . When $p = 0.5$, the classification accuracy AC_{usd} using both SVM and KNN reaches 97.00% and the total classification accuracies AC is the highest 97.14%. Therefore, it is recommended to use a small $p = 0.5$ for chatter

detection in this study. The computation time to calculate all the multifractal features of the 344 samples ranges from 15 to 16 seconds using Matlab2019a on a desktop computer with CPU 3.40 GHz and RAM 8 GB. Obviously, the p -leader multifractal spectrum based on DWT is a potential fast tool for online chatter detection.

Table 4. Cutting state classification results.

Method	p values	AC_{sd}	AC_{usd}	AC
SVM	inf	98.04%	95.00%	97.12%
	2	97.63%	95.00%	96.83%
	1	97.24%	96.00%	96.85%
	0.5	97.24%	97.00%	97.14%
KNN	inf	98.43%	95.00%	97.40%
	2	97.24%	94.00%	96.26%
	1	97.24%	97.00%	97.14%
	0.5	97.24%	97.00%	97.14%

Comparison with other methods

The proposed method for chatter detection is also compared with other two methods. The first method applies features based on EEMD to describe the dynamic variation when chatter occurs (Ji et al., 2018). Those features are the fractal dimension (E_1), standard deviation (E_2) and power spectral entropy (E_3). Figure 14 shows the ReliefF results for the EEMD-based features, which have smaller importance weights than the multifractal features in Figure 8. Figure 15 assesses the three features in terms of their dynamic monitoring abilities. E_1 and E_3 are normalized features and can well track dynamic transitions due to spectral variations. On the other hand, the standard deviation E_2 is a measure of the amount of variation of a set of values. Therefore, E_2 is large for the stable Test 2, which cause that tests similar to Test 2 with large amplitude are hard to be distinguished from the unstable tests. The classification accuracies obtained from the 344 data samples are lower than the proposed method, as shown in Table 5. Moreover, the EEMD-based method takes about 174 seconds to extract features for the 344 samples, which is not suitable for a timely online chatter detection system.

Figure 14. EEMD-based feature importance analysis using ReliefF.

Figure 15. Assessment of dynamic monitoring ability of the first method (a) stable Test 1 (b) stable Test 2 with large amplitudes (c) moderately unstable Test 3 (d) unstable Test 4 in Table 3.

Table 5. Classification results using EEMD and wavelet based methods.

Refs.	Method	AC_{sd}	AC_{usd}	AC
Ji et al. (2018)	SVM	98.01%	90.00%	95.72%
	KNN	97.21%	90.00%	95.13%
Yao et al. (2010)	SVM	98.01%	84.00%	94.00%
	KNN	98.81%	81.00%	93.71%

The second method uses features from the traditional wavelet decomposition (Yao et al., 2010). Two wavelet-based features, standard variation (T_1) and energy ratio (T_2), from the most sensitive component obtained by using discrete wavelet and wavelet package decomposition, respectively. The feature performance is examined in Figure 16 and Figure 17. This method gives much lower classification accuracies for unstable tests than the proposed and first methods. The non-adaptivity of the wavelet-based decomposition and improper features may result in low classification accuracies. However, the second method takes only about 5 seconds to extract features for the 344

samples.

Figure 16. Wavelet-based feature importance analysis using ReliefF.

Figure 17. Assessment of dynamic monitoring ability of the second method (a) stable Test 1 (b) stable Test 2 with large amplitudes (c) moderately unstable Test 3 (d) unstable Test 4 in Table 3.

4 Conclusions

This paper uses for the first time a novel p -leader multifractal formalism for chatter detection in milling operations. The multifractal features are assessed using both numerical simulation and experimental analysis. Due to the sensitivity of the unstable tests to the parameter p , the selected multifractal features can well monitor the dynamic variations during the machining process, and improves the classification accuracies of the unstable samples. A small p is recommended for better chatter detection results. Moreover, the multifractal features can well distinguish stable tests with large amplitudes from unstable tests. The proposed method outperforms over two existing methods, an EEMD-based method and a traditional wavelet decomposition method. The computation time of the p -leader multifractal formalism is about 10% of that of the EEMD-based method. To summarize, features from p -leader multifractal formalism can well monitor dynamic variations in signals and give high classification accuracies with a short computation time. As the experiments are conducted in a lab environment, this work will be further explored in a real industrial environment in the future.

Acknowledgments

This study was jointly supported by Natural Science Foundation Council of China (No. 51905461), National Key Research and Development Program of China (No. 2018YFB2001101 and No. 2019YFB2005101). The authors would like to express their acknowledgments to the Advanced Manufacturing Laboratory, UNSW, for the support of the experimental work. Comments and suggestions from reviewers are greatly appreciated.

References

Abry, P., Jaffard, S., & Wendt, H. (2015). Irregularities and scaling in signal and image processing: multifractal analysis. *Benoit Mandelbrot: A Life in Many Dimensions* (pp. 31-116). World Scientific.

- Abry, P., Wendt, H., Jaffard, S., Helgason, H., Gonçalves, P., Pereira, E., et al. (2010). 'Methodology for multifractal analysis of heart rate variability: From LF/HF ratio to wavelet leaders' *2010 Annual International Conference of the IEEE Engineering in Medicine and Biology*. IEEE, pp. 106-109.
- Albertelli, P., Braghieri, L., Torta, M., & Monno, M. (2019). Development of a generalized chatter detection methodology for variable speed machining. *Mechanical Systems and Signal Processing*, 123, 26-42.
- Altintas, Y., & Budak, E. (1995). Analytical prediction of stability lobes in milling. *CIRP Annals - Manufacturing Technology*, 44(1), 357-362.
- Benkedjouh, T., Medjaher, K., Zerhouni, N., & Rechak, S. (2013). Health assessment and life prediction of cutting tools based on support vector regression. *Journal of Intelligent Manufacturing*, 26(2), 213-223.
- Bolon-Canedo, V., Sanchez-Marono, N., & Alonso-Betanzos, A. (2013). A review of feature selection methods on synthetic data (Review). *Knowledge and Information Systems*, 34(3), 483-519.
- Cao, H., Lei, Y., & He, Z. (2013). Chatter identification in end milling process using wavelet packets and Hilbert–Huang transform. *International Journal of Machine Tools and Manufacture*, 69(0), 11-19.
- Chen, Y., Li, H., Hou, L., & Bu, X. (2019). Feature extraction using dominant frequency bands and time-frequency image analysis for chatter detection in milling. *Precision Engineering*, 56, 235-245.
- Chen, Y., Li, H., Hou, L., Wang, J., & Bu, X. (2018). An intelligent chatter detection method based on EEMD and feature selection with multi-channel vibration signals. *Measurement*, 127, 356-365.
- Chhabra, A. B., Meneveau, C., Jensen, R. V., & Sreenivasan, K. (1989). Direct determination of the $f(\alpha)$ singularity spectrum and its application to fully developed turbulence. *Physical Review A*, 40(9), 5284.
- Du, W., Kang, M., & Pecht, M. (2019). Fault Diagnosis Using Adaptive Multifractal Detrended Fluctuation Analysis. *IEEE Transactions on Industrial Electronics*, 1-1.
- Du, W., Tao, J., Li, Y., & Liu, C. (2014). Wavelet leaders multifractal features based fault diagnosis of rotating mechanism. *Mechanical Systems and Signal Processing*, 43(1-2), 57-75.
- Fu, Y., Zhang, Y., Gao, H., Mao, T., Zhou, H., Sun, R., et al. (2017). Automatic feature constructing from vibration signals for machining state monitoring. *Journal of Intelligent Manufacturing*, 30(3), 995-1008.
- Fu, Y., Zhang, Y., Zhou, H., Li, D., Liu, H., Qiao, H., et al. (2016). Timely online chatter detection in end milling process. *Mechanical Systems and Signal Processing*, 75, 668-688.
- Ihlen, E. A. (2012). Introduction to multifractal detrended fluctuation analysis in matlab. *Front Physiol*, 3, 141.
- Inspurger, T., & Stepan, G. (2004). Updated semi-discretization method for periodic delay-differential equations with discrete delay. *International Journal for Numerical Methods in Engineering*, 61(1), 117-141.
- Jaffard, S., Melot, C., Leonarduzzi, R., Wendt, H., Abry, P., Roux, S. G., et al. (2016).

- p-exponent and p-leaders, Part I: Negative pointwise regularity. *Physica a-Statistical Mechanics and Its Applications*, 448, 300-318.
- Ji, Y., Wang, X., Liu, Z., Wang, H., Jiao, L., Wang, D., et al. (2018). Early milling chatter identification by improved empirical mode decomposition and multi-indicator synthetic evaluation. *Journal of Sound and Vibration*, 433, 138-159.
- Lashermes, B., Jaffard, S., & Abry, P. 4 (2005) 'Wavelet leader based multifractal analysis' *IEEE International Conference on Acoustics, Speech, and Signal Processing*. 23-23 March 2005. pp. 161-164.
- Leonarduzzi, R., Wendt, H., Abry, P., Jaffard, S., Melot, C., Roux, S. G., et al. (2016). p-exponent and p-leaders, Part II: Multifractal analysis. Relations to detrended fluctuation analysis. *Physica a-Statistical Mechanics and Its Applications*, 448, 319-339.
- Leonarduzzi, R. F., Spilka, J., Wendt, H., Jaffard, S., Torres, M. E., Abry, P., et al. (2015). p-Leader Based Classification of First Stage Intrapartum Fetal HRV. (Paper presented at the VI Latin American Congress on Biomedical Engineering CLAIB 2014, Paraná, Argentina)
- Leonarduzzi, R. F., Torres, M. E., & Abry, P. (2014). Scaling range automated selection for wavelet leader multifractal analysis. *Signal Processing*, 105, 243-257.
- Li, K., He, S., Li, B., Liu, H., Mao, X., & Shi, C. (2020). A novel online chatter detection method in milling process based on multiscale entropy and gradient tree boosting. *Mechanical Systems and Signal Processing*, 135, 106385.
- Liu, C., Zhu, L., & Ni, C. (2018). Chatter detection in milling process based on VMD and energy entropy. *Mechanical Systems and Signal Processing*, 105, 169-182.
- Liu, J., Hu, Y., Wu, B., Wang, Y., & Xie, F. (2017). A Hybrid Generalized Hidden Markov Model-Based Condition Monitoring Approach for Rolling Bearings. *Sensors (Basel)*, 17(5).
- MathWorks (2020). Bayesian Optimization Algorithm. <https://www.mathworks.com/help/stats/bayesian-optimization-algorithm.html#bvaz8tr-1>. Accessed.
- Ren, M., Zhang, Q., & Zhang, J. (2019). An introductory survey of probability density function control. *Systems Science & Control Engineering*, 7(1), 158-170.
- Tao, J., Qin, C., Xiao, D., Shi, H., Ling, X., Li, B., et al. (2019). Timely chatter identification for robotic drilling using a local maximum synchrosqueezing-based method. *Journal of Intelligent Manufacturing*.
- Wendt, H., Kiyono, K., Abry, P., Hayano, J., Watanabe, E., & Yamamoto, Y. (2014) 'Multiscale wavelet p-leader based heart rate variability analysis for survival probability assessment in CHF patients' *2014 36th Annual International Conference of the IEEE Engineering in Medicine and Biology Society*. IEEE, pp. 2809-2812.
- Wendt, H., Roux, S. G., Jaffard, S., & Abry, P. (2009). Wavelet leaders and bootstrap for multifractal analysis of images. *Signal Processing*, 89(6), 1100-1114.
- Yao, Z., Mei, D., & Chen, Z. (2010). On-line chatter detection and identification based on wavelet and support vector machine. *Journal of Materials Processing Technology*, 210(5), 713-719.

- Yesilli, M. C., Khasawneh, F. A., & Otto, A. (2019). On transfer learning for chatter detection in turning using wavelet packet transform and ensemble empirical mode decomposition. *CIRP Journal of Manufacturing Science and Technology*.
- Zhang, Z., Li, H., Meng, G., Tu, X., & Cheng, C. (2016). Chatter detection in milling process based on the energy entropy of VMD and WPD. *International Journal of Machine Tools and Manufacture*, 108, 106-112.