Simulation of Punching and Post-Punching Shear Behaviours of RC Slab-Column

Connections

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Abstract

Punching and post-punching shear behaviours of reinforced concrete (RC) flat plate structures are often studied using a representative slab-column connection being isolated from the parent structure. A set of numerical modelling techniques is established in this study to create a competent 3D nonlinear model to simulate punching and post-punching shear behaviours of RC slab-column connections without shear reinforcement. With which, the punching shear failure featuring a critical punching shear surface and an abrupt drop of the applied force in the load-displacement response is able to be accurately reproduced. The post-punching shear behaviour, taking the form of an increased load-carrying capacity which is ceased by rebar fracture in the suspension stage, is also well captured. Using the proposed numerical model, typical punching and post-punching shear failure mechanisms are studied in some detail.

Keywords

Modelling; punching; structural analysis.

1. Introduction

Punching shear is a brittle failure occurring at slab-column connections which commonly governs the design of reinforced concrete (RC) flat plate structures. When the failure occurs, the column suddenly perforates the slab with an abrupt and considerable drop in the load-

carrying capacity. Precautionary design and construction measures must be taken to prevent such a fatal failure. However, there is no unified design method for punching shear strength prediction. Various prediction methods (Kinnunen and Nylander, 1960; Muttoni, 2008; Broms, 2016) to compute the punching shear resistance have been proposed based on a large amount of experimental results. Guandalini et al. (2009); Regan (1986); Taylor and Hayes (1965); Drakatos et al. (2016); Moe (1961); Kinnunen and Nylander (1960); Elstner and Hognestad (1956); Teng et al. (2018) experimentally studied the factors influencing punching shear strength including, concrete strength, flexural reinforcement ratio and arrangement, slab thickness, column shape and size, existence of unbalanced moments, and boundary restraints, etc. These prediction methods and experiments have helped to derive and advance the widely used empirical or semi-empirical equations in the design standards (BS EN 1992-1-1, 2004; AS 3600, 2009; fib, 2013; ACI 318M, 2014). Various numerical models of slab-column connections have also been created to further understand the stress states of the punching region in the slab, the failure mechanisms and the influences brought by some of the design factors mentioned above (Genikomsou and Polak, 2017; Guan and Polak, 2007; Polak, 1998; Shu et al., 2016; Navarro et al., 2018; Goh and Hrynyk, 2018; Liu et al., 2015). Although reasonable numerical predictions of the overall punching shear strength could be achieved, a truthful deformed shape or failure status featuring a critical punching shear surface was unable to be displayed and the contributions of the concrete and reinforcement to the overall punching shear strength could not be differentiated.

Although punching shear failure is localised at the slab-column connection region, concrete cracking and spalling may propagate to the surrounding slab areas and adjacent slab-column connections, leading to catastrophic progressive collapse. Several progressive collapse events of RC flat plate structures in the last decade or so (Wood, 2003; King and Delatte, 2004; Park,

2012; Schellhammer et al., 2013) have highlighted an urgent necessity to gain an in-depth understanding of the collapse mechanisms and resistance capacities of this type of structure, thereby developing effective design approaches. One important research finding, for instance, is that, after punching shear failure occurs, the provision of the slab bottom reinforcing bars continuously passing through the columns (also known as the “integrity bars”) is a favourable design practice, with which a post-punching shear mechanism can be efficiently developed as a secondary defence against progressive collapse (Hawkins and Mitchell, 1979; Mitchell and Cook, 1984; Mirzaei and Sasani, 2013).

Regarding post-punching shear behaviour, limited experiments were conducted to investigate the post-punching shear capacity (Melo and Regan, 1998; Diao et al., 2019) and the influencing design variables such as slab thickness, amount and arrangement of integrity bars, and inclusion of shear reinforcement (Habibi et al., 2012; Ruiz et al., 2013). Along with these experimental investigations, several analytical studies have also been conducted. For example, Melo and Regan (1998) suggested lower bound approaches to calculate the post-punching shear capacity, governed by either dowel action or strength of the integrity bars. Ruiz et al. (2013) further considered the contribution of flexural reinforcing bars, with its upper limit being controlled by dowel actions resisted by the concrete. Habibi et al. (2014) also proposed a comprehensive, yet complicated, solution to predict the post-punching shear behaviour. Besides these experimental and analytical studies, a simplified macro numerical model (Keyvani et al., 2014; Mirzaei and Sasani, 2013) was also proposed to replicate the post-punching shear behaviour of slab-column connections, in which the punching shear strength was pre-determined based on a well-accepted critical shear crack theory (Muttoni, 2008). Nevertheless, the pre-assumed spring/connector stiffness may require further investigation, suggested by Sagaseta et al. (2017), who also pointed out the benefits and necessity of numerically simulating the post-punching

response using detailed 3D models, yet no satisfactory models have been reported to date.

In this study, a set of numerical finite element modelling techniques is established using LS-DYNA finite element software (LSTC, 2016) to simulate the punching and post-punching shear behaviours of slab-column connections (referred to as “SCCs” thereafter) without shear reinforcement. Firstly, the establishment of the numerical model, incorporating material properties and element failure criteria calibrated by eight SCC test results, is introduced. Secondly, the accuracy of the proposed model is further validated by simulating four specimens reported in the post-punching shear study of Ruiz et al. (2013). Thirdly, the punching shear failure mechanism of a typical SCC is discussed by studying the stress states of the concrete elements on the critical punching shear surface and the punching shear strength contributions by both the concrete and reinforcement. Post-punching shear behaviour is discussed based on the deformed configuration of reinforcing bars.

2. Research Significance

The 3D nonlinear model offers a numerical solution to predict punching and post-punching shear capacities of a SCC and help to understand its failure mechanisms. Further detailed analysis of this model will help to establish a simplified numerical model to facilitate collapse simulations of entire RC flat plate structures

3. Overview of SCC Experiments

Test results of eight 1/3-scaled SCC specimens (Fig. 1) (Yang et al., 2018; Diao et al., 2018; Diao et al., 2019) were used to develop and calibrate the proposed numerical model. The punching and post-punching shear behaviours of these specimens, considering different punching shear directions (denoted as “DPS” and “UPS”), slab thicknesses and reinforcement

ratios were investigated. The DPS and UPS specimens were differentiated by downward and upward punching shear failure modes, respectively, likely occurring at different SCCs in an RC flat plate structure under an interior column loss scenario and damage propagation (Xue et al., 2018; Diao et al., 2019).

The typical DPS specimen (Fig. 2) had a $2,000 \times 2,000$ -mm slab and two 150×150 -mm column stubs above and below the slab surfaces. The UPS specimens had identical dimensions and reinforcement layout to their counterpart DPS specimens except that the slab top and bottom reinforcements were swapped. Table 1 summarises the design variables of all the eight specimens. For the DPS (and similar to UPS) specimens, DPS-2 had a reduced slab thickness of 70 mm relative to the 90 mm thickness of DPS-1 while its reinforcement layout remained the same. DPS-3 only differed from DPS-1 by having an increased reinforcement ratio. To examine the influences of bond development length of flexural reinforcement, DPS-4 had its slab top reinforcement extended to the supports, with other variables being identical to DPS-1. Concrete with a nominal compressive strength of 25 MPa and 8-mm diameter reinforcing bars with a nominal yield strength of 335 MPa, were used in the slab design. The tested concrete cylinder compressive strength for each specimen is given in Table 1 and the stress-strain curve of the slab reinforcement obtained from the uniaxial tensile test is shown in Fig. 3. During the tests, each specimen with surrounding boundary beams, was fully fixed at its four corners to steel columns and quasi-statically loaded under displacement control by a servo hydraulic actuator at the top column stub.

4. Numerical Modelling Techniques

4.1 Element and Geometric Considerations

The numerical modelling techniques were established based on the general purpose finite

element program LS-DYNA (LSTC, 2016) which is powerful in simulating nonlinear material behaviours in large deformation problems. The concrete was modelled using eight-node 3D solid Lagrangian elements. The default reduced integration (one point at the element centre) rule was selected to save computational time. Using such a constant stress element required a reasonably fine mesh to achieve a sufficiently accurate solution. A characteristic mesh size, defined as the cubic root of the volume of the solid element, will be determined in Section 4.2.2.

The reinforcement was explicitly modelled using two-node Hughes-Liu integration beam elements with 2×2 integration points across their circular sections. Each beam element was of the same length as the characteristic size of the concrete elements. The reinforcement elements were assumed to be rigidly coupled to the concrete elements using the *CONSTRAINED_BEAM_IN_SOLID keyword, implying that the bond-slip failure was not considered. Compared to the commonly used *CONSTRAINED_LAGRANGE_IN_SOLID keyword (Bao et al., 2014; Yu et al., 2018), this coupling method corrects the false internal energy accumulation (Chen, 2016), which is crucial to the concrete material property calibration in the modelling process.

By virtue of symmetry, only a quarter of each specimen was created (Fig. 4 for DPS-1). For the slab top reinforcement net, the upper beam elements aligned in the x -direction were modelled 8 mm (rebar diameter) above the lower beam elements aligned in the y -direction (see enlarged beam element configurations in Fig. 4). The same treatment was followed for the slab bottom reinforcement, ensuring that the upper and lower beam elements were just “touching” without initial element penetrations which violate the software beam-to-beam contact algorithm. The keyword *CONTACT_AUTOMATIC_GENERAL was added to the model to generate beam-to-beam contacts (LSTC, 2016), which enabled the formation of the actual top

and bottom reinforcement nets within the slab, so that the true load-carrying mechanism due to the suspension action developed in the post-punching stage could be effectively reproduced.

The bottom nodes on the corner areas of the boundary beams were fully restrained to mimic their connection to the steel columns. Moreover, all the nodes of the concrete and steel elements on the symmetric planes were restrained accordingly. Finally, a thin rigid body plate only translating in the z -direction was created above the top column stub, to facilitate the application of a vertically displacement-controlled loading.

4.2 Material Considerations

4.2.1 Steel

The multilinear elasto-plastic material model (*MAT_024/*MAT_PIECEWISE_LINEAR_PLASTICITY) was adopted for the reinforcement, through which the tested stress-strain curve was entered. The engineering stress-strain curve of reinforcement (solid line in Fig. 3) was transformed into the true stress-strain curve (dashed line in Fig. 3). The plastic stress-strain curve (dotted line in Fig. 3) with flattened softening segment was used as input. The strain rate effect was not considered as the specimens were loaded quasi-statically in the experiments where no material strength enhancement was anticipated.

4.2.2 Concrete

The continuous surface cap model (*MAT_159/*MAT_CSCM) was used to replicate the concrete material behaviour. This model was successfully used to simulate RC framed structures under column loss scenarios exhibiting flexural/axial failures (Pham et al., 2017; Elsanadedy et al., 2017; Yu et al., 2018). When using the CSCM model, its specific built-in

element deletion function using the failure criterion (maximum principal strain and/or the damage index) was switched off, because this function was unable to model shear-type failures. Given dominant shear failure in the connection tests, another failure criterion using the *MAT_ADD_EROSION keyword was employed to accommodate shear-type failures with specially determined characteristic strain values (Section 4.3). Further, the strain rate effect option in the CSCM model was also turned off for the concrete material for the same reason described above. The elastic parameters including the bulk modulus K and the shear modulus G were calculated using the Poisson's ratio $\nu = 0.2$ and the elastic modulus E given by the CEB-FIP Model Code 1990 (CEB-FIP, 1993). The yield surface was defined using three sets of shear surface parameters for the low pressure (negative hydrostatic pressure) range and another set of cap parameters for the high pressure range (Murray et al., 2007). All relevant material parameters used are reproduced in Appendix for reference.

In the CSCM model, the strain-softening parameters are determined based on a damage accumulation approach using the default damage model which can minimise mesh size dependency (Murray et al., 2007; Murray, 2007). However, this study showed otherwise. For instance, the DPS-1 specimen was modelled using six characteristic element sizes, viz. 31.6 mm, 24.1 mm, 18.5 mm, 15.0 mm, 12.1 mm, and 9.2 mm, keeping all the other material parameters unchanged. In Fig. 5, the experimental applied load versus the centre column stub displacement (referred to as " Δ_c " thereafter) curve of DPS-1 is compared with the numerical results of the six mesh sizes. The test specimen went through a flexural behaviour stage up until $\Delta_c = 41$ mm at which the punching shear failure occurred, a phenomenon to be modelled in Section 4.3. Fig. 5 demonstrates that the numerical results of different mesh sizes disperse after the initial elastic stage. The smaller the element size, the more the flexural strength is deviated from the test result for a given displacement. To minimise this deviation, the strain-

softening parameters, determined using concrete material fracture energies, need to be further modified (Murakami, 2012) based on the characteristic element size.

For each of the six mesh sizes, the total internal energy accumulated within the concrete is plotted in Fig. 6 against Δ_c . As anticipated, the internal energy curves diverge significantly and more energy is accumulated in models with smaller elements. It is also observed that the internal energy increases in an almost linear manner in the flexural stage up to $\Delta_c = 41$ mm. A further observation of Fig. 5 reveals that the model with 24.1 mm mesh size bears the closest resemblance to the experimental load-displacement response during the flexural stage, suggesting that this model predicted the closest energy accumulation to the experiment. Therefore, regulating the concrete internal energy accumulations in other models to be identical to that in the 24.1 mm model can further reduce the mesh size effect. To achieve this, the slopes of the energy accumulation curves in the flexural stage (see Fig. 6) for all the models were obtained through best fit (see insert table on Fig. 7). The slope ratios, also given in the table, are defined as the slope of a specific model relative to that of the 24.1 mm model. The relationship between the slope ratios and the element sizes (Fig. 7) led to Eq. (1), where C_{ms} denotes the mesh size coefficient and l_e is the mesh size. The coefficient C_{ms} is then multiplied to the three input fracture energies in three stress states, G_{fs} (pure shear), G_{fc} (uniaxial compression), and G_{ft} (uniaxial tension), to calculate the strain-softening parameters. Note that $G_{fc} = 100 G_{fs}$ and $G_{ft} = G_{fs}$ (see Appendix), suggested by Murray (2007), were also suitable for this simulation.

$$C_{ms} = 0.04212l_e + 0.02638 \quad (1)$$

Six mesh size models were analysed again using their corresponding regulated fracture energies,

with the results (Fig. 8) showing that the mesh size effect on the damage model was reasonably reduced in the flexural stage. In the subsequent numerical simulations, the 15-mm characteristic element size was used because six elements along the slab thickness (90 mm) were appropriate to display through-thickness stress and strain variations with reasonable computation time.

4.3 Failure Criteria

Fig. 8 demonstrates that the flexural behaviour of DPS-1 in the early loading stage was well captured except the sudden load drop due to punching shear failure at $\Delta_c = 41$ mm. To simulate such a brittle failure, the element deletion function was activated through the keyword *MAT_ADD_EROSION with the maximum effective strain $\bar{\varepsilon}_{c,max}$ as the concrete failure criterion. The effective strain $\bar{\varepsilon}$ is expressed as (LSTC, 2016):

$$\bar{\varepsilon} = \sqrt{\frac{2}{3} e_{ij} e_{ij}} \quad (2)$$

where e_{ij} is the deviatoric strain tensor signifying the distortional deformation which is more critical for the elements on the inclined punching shear surface. With this failure criterion, the element is deleted from the numerical model if its effective strain reaches a characteristic value. Further, as the punching shear failure is not a pure material failure but a compound failure, key structural factors must also be considered. Especially, the punching shear failure is dependent on the flexural deformation capacity of the specimen (Drakatos et al., 2016; Drakatos et al., 2018; Muttoni, 2008) which may be affected by such factors including concrete strength, column shape and size, slab thickness, amount and arrangement of reinforcement, boundary conditions, shear reinforcement, and slab in-plane force, etc. With no existing theory nor

previous experience to acquire an appropriate value of $\bar{\varepsilon}_{c,max}$, a trial and error was performed herein to correlate the deformation capacities of eight DPS/UPS specimens to several key factors. The “correct” $\bar{\varepsilon}_{c,max}$ value was found for each specimen by comparing the simulated punching shear displacement Δ_p (i.e. Δ_c when punching shear occurs) with the experimental one. This calibration leads to three major findings:

(1) The maximum effective strain criterion is suitable for punching shear problems as both the punching shear surface and cone, and the associated sudden load drop can be well simulated.

(2) Development of the flexural behaviour is independent of punching shear, yet is terminated by punching failure. This is confirmed by the trial and error of $\bar{\varepsilon}_{c,max}$ which only altered the magnitude of Δ_p , but made no impact to the load-displacement curve during the flexural stage. The alteration of Δ_p can potentially lead to three scenarios depending on the deformation capacity of the specimen:

- (a) if the deformation capacity is low, the punching shear occurs at a small displacement well before the slab flexural capacity is fully developed, then the punching shear strength is less than the slab flexural capacity;
- (b) if the deformation capacity is of a critical value making the punching shear failure occurring just when the slab flexural capacity is fully developed, then the slab is under a “balanced” condition, of which its punching shear strength equals its flexural capacity;
- (c) if the deformation capacity is larger than the critical value in scenario (b), the slab flexural stage is terminated by a secondary punching shear failure (Park and Gamble, 2000) at a large displacement. In this case, the punching shear strength is in general also lower than the flexural capacity given the material softening behaviour.

(3) To quantify the deformation capacity of a SCC specimen corresponding to its punching shear failure, a holistic “flexibility index” λ symbolising the deformation capacity is proposed below, including the following key factors shown in Eq. (3).

$$\lambda = \frac{1}{\sqrt{f'_c} \sqrt[3]{\rho_{ct} + \rho_{st} + \frac{1}{2}(\rho_{cc} + \rho_{sc})}} \sqrt[3]{\frac{\mu L}{d}} \quad (3)$$

where, f'_c is the concrete compressive strength and the term $\sqrt{f'_c}$ represents the material stiffness; ρ_{ct} and ρ_{cc} are, respectively, the mean tensile and compressive slab reinforcement ratios of a specified slab section at the column position, whereas ρ_{st} and ρ_{sc} are, respectively, the mean tensile and compressive slab reinforcement ratios of a similar slab section at the supports. These specified slab sections take the width of the column size plus six times the slab effective depth d . The term $\sqrt[3]{\rho_{ct} + \rho_{st} + \frac{1}{2}(\rho_{cc} + \rho_{sc})}$, following that given in Eurocode (BS EN 1992-1-1, 2004; Regan, 1981), acknowledges that an increased amount of reinforcement can increase the stiffness and decrease the deformation capacity of the specimen (Kinnunen and Nylander, 1960; Muttoni, 2008). L is the distance from the column centre to the supports. The slenderness $\frac{\mu L}{d}$ reflects the overall structural flexibility (Fang et al., 1994; Lovrovich and McLean, 1990), where the effective length coefficient μ is taken as 0.5 and 1 for fully clamped and simply supported slabs, respectively.

Upon simulating all the eight specimens, a relationship between the calibrated $\bar{\varepsilon}_{c, \max}$ and λ is obtained in Fig. 9, leading to the following expression:

$$\bar{\varepsilon}_{c,\max} = 0.096\lambda^2 - 0.359\lambda + 0.354 \quad (4)$$

The failure criterion for reinforcing steel is satisfied when the effective plastic strain $\bar{\varepsilon}_s$ reaches the maximum, characteristic value $\bar{\varepsilon}_{s,\max}$. This criterion, considers both axial and shear actions developed in the reinforcement elements, enables the simulation of rebar fracture at a large deformation stage. A $\bar{\varepsilon}_{s,\max}$ of 0.083 was obtained by calibrating the simulated Δ_c of DPS-1 at the onset of rebar rupture against the corresponding experimental Δ_c . This value was also used for other models as all reinforcing bars came from the same material batch.

4.4 Model Validation

The characteristic effective strain for concrete $\bar{\varepsilon}_{c,\max}$ proposed in Eq. (4) and the calibrated characteristic effective plastic strain $\bar{\varepsilon}_{s,\max}$ for steel were used to simulate all the eight DPS/UPS specimens (Yang et al., 2018; Diao et al., 2018). In addition, another four SCC specimens (PM9-12) with integrity bars (Ruiz et al., 2013) were also simulated to further confirm the reliability of the modelling techniques developed herein.

Fig. 10 shows the results comparison for DPS-1 to facilitate detailed discussions in Section 5, whereas the comparisons for the other seven DPS/UPS specimens are presented in Fig. 11. The results of a typical specimen (PM9) of Ruiz et al. (2013) is also shown in Fig. 11(h). Fig. 10 highlights the punching (first peak) and post-punching shear (second peak) capacities, denoted as V_p and V_{pp} respectively. Note that, V_{pp} is of critical importance as being the secondary defence to resist progressive collapse triggered by punching shear failure (Habibi et al., 2014; Ruiz et al., 2013; Melo and Regan, 1998).

Figs. 10 and 11 reveal that the numerical models are able to capture the flexural, punching, and post-punching shear behaviours of various interior SCCs. Particularly, the slab initial stiffness, the flexural capacity development, the sudden load drop induced by punching shear failure and the re-ascending post-punching behaviour were all simulated in good agreements with the experimental results. Tables 2 and 3 summarise, respectively, V_p and V_{pp} and their corresponding displacements. Accurate numerical predictions were achieved with average experiment-to-simulation ratios of 0.97 and 0.92, with coefficients of variation of 9% and 11%, for V_p and V_{pp} , respectively.

The experimentally observed crack patterns and damage modes of DPS-1 in the post-punching failure stage, are compared in Fig. 12 to the simulated damage index (refer to as “effective plastic strain” in LS-DYNA for the CSCM concrete model) contours. Both the top and bottom cracks were well reproduced. Column perforation and bottom rebars ripping out of the slab soffit were also satisfactorily replicated numerically. To better display the well-simulated punching shear cone wrapped around by the critical, inclined punching shear surface, the slab-column joint of PM9 specimen, having a relatively larger slab thickness, is shown in Fig. 13 at the onset of punching shear failure.

5. Detailed Discussions on Load-Carrying Mechanism of SCC

5.1 Overall Behaviour

Detailed structural behaviours in each load-carrying mechanism stage are discussed using a representative load-displacement comparison for DPS-1 (Fig. 10). The entire load-carrying behaviour exhibits two stages, viz. the flexural stage and suspension stage distinguished by the punching shear failure as the first peak load (Point A). Especially, the first suspension sub-stage, terminated by fracture of the first reinforcing bar (Point C), is most critical to progressive

collapse resistance of an RC flat plate structure as mentioned before.

During the early loading stage of the load-displacement curves shown in Fig. 10, flexural cracks were developed on the slab soffit at the slab-column interface of DPS-1 (see Fig. 14 (a)). As the load increased, more flexural cracks propagate to a larger area and further extend upward, leading to an elevated neutral axis of the slab cross-section within this area. A gradually decreased stiffness (Fig. 10) in the later flexural stage is noticed due to the reduced effective cross-section and softening of the local concrete materials.

Fig. 14(b) shows the damage status of DPS-1 just before punching shear occurs (point A in Fig. 10). Noted that some concrete elements surrounding the slab-column interface are deleted, leaving only a concrete compression zone connecting the column and the slab. This observation substantiates the proposition given by other researchers that punching shear occurs concurrently with brittle rupture of the concrete compression zone (Taylor and Hayes, 1965; Regan, 1981; Kinnunen and Nylander, 1960; Broms, 2016). During the brittle punching failure (Fig. 14(c)), an inclined punching shear surface (symbolised by red dashed lines), on which the elements are deleted almost simultaneously, is abruptly formed. Moreover, the punching shear failure is also reflected as a sudden and considerable load drop in reference to point B in Fig. 10.

Subsequent to punching shear, all the concrete elements at the interface connecting the slab and the column fail, with only the reinforcing bars serving as the tie. Therefore, in the suspension stage, the applied load is solely carried by the reinforcing bars between the separated slab and column owing to their dowel action. Specifically, as the column stub being continuously pushed down, the bottom layer reinforcing bars rip out from the slab soffit (see Fig. 14(d)). Combined

with the already bent rebars in the top layer, a suspension system forms, by which further increased applied load can be resisted. This re-ascending load-carrying feature is attributable to the increased inclination of the reinforcing bars as well as the strain-hardening behaviour of the steel. At the end of the suspension sub-stage 1, the V_{pp} is reached in reference to point C in Fig. 10 followed by the fracture of a top reinforcing bar going through the column. Afterwards, the entire system still exhibits a level of residual load-carrying capacity, which is terminated by another reinforcing bar fracture marked by point E. Nonetheless, this residual capacity is deemed less useful in the design consideration as the system in sub-stage 2 becomes unstable which is susceptible to subsequent failures.

5.2 Punching Shear Failure Behaviour

The stress states of the elements on and adjacent to the punching shear surface just before the punching shear failure are also explored. Typically, the maximum principal stress σ_1 vectors of these elements are plotted in Fig. 15(a), with a particular emphasis on those elements on the punching shear surface (surface elements). Whilst the directions of σ_1 of the adjacent elements are scattered, those of the surface elements on the punching shear surface are normal to the punching shear surface. Similar inspections are conducted for the intermediate (σ_2) and minimum (σ_3) principal stresses, showing that these surface elements, except for the top one, are in the state of bi-axial compression (σ_2 and σ_3) and tension (σ_1) in the other axis (see Fig. 15(b)). This tensile stress σ_1 , with the presence of the other two compressive stresses (σ_2 and σ_3), results in a deduction of the shear strength (i.e. the internal friction within the punching shear surface) of the concrete, which probably explains the formation of the critical punching shear surface. Note that the top element in the compression zone is in a tri-axial compression state (Fig. 15(a)), failing after the other surface elements. This element fails differently due to a subsequent overloading and excessive distortion after the deletion of other surface elements.

Given a tri-axial compression state, this top element is under a ductile-type damage accumulation exhibiting a high residual strength, if the built-in concrete strain-softening algorithm in the CSCM model (Murray, 2007) is solely used. This implies that the CSCM model cannot independently solve the punching shear problems as also stated in Section 4.2.2. This further confirms the necessity of incorporating an additional element failure criterion, i.e. the maximum effective strain, to simulate the punching shear failure as described in Section 4.3.

The punching shear capacities of SCCs are attributable to different sources. Referring to Fig. 10 again, one can see that the simulated load drop due to punching shear failure exceeds the experiment one in an amount of v_a . This is because the failed elements in the punching area are numerically deleted making no further contribution whereas in reality, concrete aggregate interlocking and friction (Muttoni, 2008) on the punching shear surface also contribute to the residual strength, thereby causing this discrepancy, v_a . Section 5.1 indicates that the predicted residual load-carrying capacity v_s after the punching shear failure is solely attributed to the reinforcement. This implies that the experimental load drop v_c is sourced from the strength of un-crushed concrete slab cross-sections. These different contributing sources v_c , v_a and v_s as well as the ratio of individual source to the total punching shear capacity (V_p) are summarised in Table 4 for all the twelve models. Compared to v_c and v_a , v_s is relatively stable owing to the similar amount of reinforcing bars going through the column in the DPS/UPS specimens. Figs. 16(a) and (b) depict the axial stress status of the reinforcing bars in the vicinity of the slab-column joint before and at the onset of punching shear failure. At the slab-column interface, kinking of two reinforcing bars (R_{tx0} and R_{bx0}) going through the column is also observed in Fig. 16(b) at the punching shear failure. In addition, R_{tx0} experiences a local bending at the intersection with R_{ty1} outside the slab-column interface (Fig. 16(b)), suggesting that R_{ty1} serves

as a support to R_{tx0} .

Figs. 16(c) and (d) show the axial stress status as well as the deformed shape of the reinforcing bars during the post-punching suspension sub-stage 1 up until its end. Along with the incremental downward displacement of the column stub, the inclination of through-column rebars becomes steeper, starting from the slab-column interface and spreading outwards. In consequence, the top bar R_{ty1} , serving as the first support (Fig. 16(c)) to R_{tx0} , is pulled down until the inclination of R_{tx0} reaches R_{ty2} . The suspension sub-stage 1 is terminated by fracture of R_{tx0} shown in Fig. 16(d). A side view displayed in Fig. 17 gives a more clear deformed configuration of the suspension system produced by the top and bottom reinforcing nets, suggesting that the downward displacement of the slab after punching failure is localised in the adjacent area surrounding the column. This phenomenon matches with the observation from an RC flat plate substructure test underwent a punching failure at its interior column area (Xue et al., 2018).

6. Conclusions

To investigate the punching and post-punching shear behaviours of slab-column connections without shear reinforcement, a set of numerical modelling techniques is proposed. Validation of the numerical against published experimental tests supports the reliability and accuracy of the proposed methods. Major conclusions are highlighted:

1. The calibrated concrete material properties and element failure criteria can satisfactorily simulate the punching shear failure behaviour featuring a critical punching shear surface and a sudden drop in the applied load. The post-punching shear response in the form of re-ascending load resistance being ceased by rebar fracture in the suspension stage is also well

captured.

2. The development of the flexural behaviour of slab-column connections is independent of the punching shear failure yet is terminated by such a failure, which is governed by its deformation capacity.
3. The tensile maximum principal stress perpendicular to the punching shear surface, with the presence of a bi-axial compression status in the other two principal directions, reduces the concrete shear strength thereby leading to punching shear failure.
4. The contributions of various sources to the punching shear capacity are distinguished, viz. the un-crushed concrete slab cross-section, aggregate interlocking and friction, as well as through-column reinforcing bars.

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Appendix

All relevant parameters given below are reproduced from (Murray, 2007) unless otherwise specified.

Elastic		$E_c = 21.5 \times 10^3 \times \left(\frac{f_c}{10} \right)^{\frac{1}{3}}$ $\nu = 0.2$ $K_c = \frac{E_c}{3(1-2\nu_c)}$ $G_c = \frac{E_c}{2(1+\nu_c)}$
Shear	Tri-axial	$\alpha = -3.0000 \times 10^{-3} (f_c)^2 + 3.1697 \times 10^{-1} f_c + 7.7047$

surface	compression	$\theta = 1.3216 \times 10^{-5} (f_c)^2 + 2.3548 \times 10^{-3} f_c + 2.1401 \times 10^{-1}$ $\lambda = 1.0500 \times 10^1$ $\beta = 1.9290 \times 10^{-2}$
	Torsion	$\alpha_1 = 7.4735 \times 10^{-1}$ $\theta_1 = -3.8859 \times 10^{-7} (f_c)^2 - 3.9317 \times 10^{-6} f_c + 1.5820 \times 10^{-3}$ $\lambda_1 = 1.7000 \times 10^{-1}$ $\beta_1 = -1.9972 \times 10^{-5} (f_c)^2 + 2.2655 \times 10^{-4} f_c + 8.1748 \times 10^{-2}$
	Tri-axial extension	$\alpha_2 = 6.6000 \times 10^{-1}$ $\theta_2 = -4.8697 \times 10^{-7} (f_c)^2 - 1.8883 \times 10^{-6} f_c + 1.8822 \times 10^{-3}$ $\lambda_2 = 1.6000 \times 10^{-1}$ $\beta_2 = -1.9972 \times 10^{-5} (f_c)^2 + 2.2655 \times 10^{-4} f_c + 8.1748 \times 10^{-2}$
Strain-hardening		$N_H = 1.0$ $C_H = 0$
Cap surface		$R = 5$ $X_0 = 8.7692 \times 10^{-3} (f_c)^2 - 7.3302 \times 10^{-2} f_c + 8.4850 \times 10^1$ $D_1 = 2.5000 \times 10^{-4}$ $D_2 = 3.4920 \times 10^{-7}$
Strain-softening		$B = 10$ $D = 0.1$ $G_{F0} = 5.2083 \times 10^{-8} d_{\max} + 7.5000 \times 10^{-4} d_{\max} + 1.9344 \times 10^{-2}$ $C_{ms} = 4.2120 \times 10^{-2} l_e + 2.6380 \times 10^{-2}$ (proposed in this study) $GFS = C_{ms} G_{F0} \left(\frac{f_c}{10} \right)^{0.7}$ $GFT = GFS$ $GTC = 100GFS$ $PWRC = 5.0$ $PWRT = 1.0$ $PMOD = 0.0$

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Fig. 1 Test Setup.

Fig. 2 Typical DPS specimen details.

Fig. 3 Stress-strain relationship of slab reinforcement.

Fig. 4 Numerical model of quarter DPS-1 (with steel elements mirrored).

Fig. 5 Effect of mesh size on load-displacement responses (numbers in the legend donate the element size).

Fig. 6 Mesh size effect on concrete internal energy accumulation.

Fig. 7 Relationship between slope ratio and mesh size.

Fig. 8 Load-displacement responses of different mesh-sized models with regulated fracture energy.

Fig. 9 Relationship between the characteristic effective strain and the “flexibility index”.

Fig. 10 Load-displacement responses of DPS-1.

(a)

(b)

(c)

(d)

(e)

(f)

(g)

(h)

Fig. 11 Load-displacement response comparison.

(a)

(b)

(c)

(d)

Fig. 12 Failure mode comparison: (a) top view in the test; (b) top view in the simulation; (c) bottom view in the test; (d) bottom view in the simulation (shown for DPS-1 with model mirrored twice).

Fig. 15 Principal stress states of (a) all elements in punching shear region; (b) a typical element on punching shear surface (shown for DPS-1).

Fig. 16 Reinforcement stress (MPa) nearby the slab-column joint (a) before punching shear; (b) at punching shear; (c) in suspension sub-stage 1; (d) rebar fracture (shown for DPS-1 with models mirrored).

Fig. 17 Side view of the suspension system, shown for DPS-1 with model mirrored.