Robust Ring Detection In Phase Correlation Surfaces

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Abstract— This paper presents a novel hybrid technique for robust detection of rings in phase correlation surfaces. Phase correlation of affine transformed images has been shown to result in rings on the correlation surface. The reliability of the classical approach to ring detection via the Hough Circle Transform is sensitive to correlation noise. Alternative methods that are more resilient are very computationally intensive. A hybrid method that preserves the relative efficiency of the HCT that is robust in the presence of correlation noise is proposed.

Keywords— Circle Detection, Hough Transform,

I. INTRODUCTION

Recently phase correlation based image registration has been extended to permit recovery of affine transformation parameters [1]. Rather than forming a peak on the correlation surface, affine transformations result in a ring formation. In contrast to the correlation surfaces resulting from similarity transforms, the noise level is significantly higher when affine transforms are involved. The resulting rings typically have poor support. In this scenario, even the classical Hough Circle Transform (HCT) [2] struggles. Various means have been proposed for coping with noise in Hough Transforms.

One such technique is the probabilistic Hough Transform [3]. While its main objective is to minimize computation time it claims to provide benefits in noisy data. It operates by carefully selecting which points are considered by the Hough Transform, the limiting factor being the amount of noise. For features with the amount of support close to counts due to noise, a full transform must be performed [4]. The standard Hough transform has been shown more accurate than a range of probabilistic and non-probabilistic Hough Transforms [5].

Surround suppression [6] has also been proposed to improve the Hough Transform's detection accuracy. This improves the accuracy where noise is clustered around otherwise well defined data points. This is not typical of correlation surfaces and hence is not effective at mitigating the effect of noise in this case.

Other variants of the HCT that exploit gradient direction, or pairs of connected points may be robust but they can not be applied to correlation surfaces as the underlying data formation process does not generate data that permits the application of these Hough Transform variants.

This paper evaluates the accuracy of different methods for ring detection and proposes a hybrid technique that is robust to correlation noise. It further demonstrates that the method is

applicable in the case of general Gaussian noise. Section two presents relevant theory; section three discusses methodology and section four presents some experimental results.

II. THEORY

A. Hough Circle Transform

Detecting a ring on an surface $f(i,j)$ where $[i,j] \in \mathbb{Z}^2$ via the Hough Circle Transform requires a 3D accumulator space $A(x,y,r)$. Each value on the correlation surface at (i, j) represents a set of circles corresponding to each possible radius r centered at (i, j) :

$$C(x,y,r) = (x-i)^2 + (y-j)^2 - r^2 \quad \forall r: 0 \leq r < R \quad (1)$$

This set of circles forms an inverted cone in the accumulator space as depicted in Fig. 1.

Fig. 1. Inverted cone in the accumulator space.

The Hough Circle Transform over the entire correlation surface $f(i, j)$ becomes the sum of all circles resulting from the all of the points on the 2D correlation surface.

$$A(x,y,r) = \sum_i \sum_j f(i,j) \quad \forall x,y,r : C(x,y,r) \quad (2)$$

All the circles of a given radius formed from points on the perimeter of a ring of the same radius will intersect at the center of that ring as shown in Fig. 2. Rings can accordingly be detected from maxima in the accumulator.

Fig. 2. Ring centre marked by intersecting circles on it's perimeter.

B. Noise Reduction

When the correlation surface has a low signal to noise ratio the noise readily inundates the classical HCT process creating false positives. A possible solution is to first truncate noise from the correlation surface below a given threshold t such that the process becomes:

$$A(x, y, r) = \sum_i \sum_j \begin{cases} f(i, j) & f(i, j) > t \\ 0 & \text{otherwise} \end{cases} \quad \forall x, y, r : C(x, y, r) \quad (3)$$

Additionally the influence of the noise can be reduced through appropriate filtering. As the noise tends to be spatially uncorrelated following a Gaussian distribution, local noise averaging can significantly improve the signal to noise ratio. Each point in the correlation surface is renormalized according to the mean local noise:

$$m(x, y) = \frac{1}{4r^2} \sum_{i=-r}^{+r} \sum_{j=-r}^{+r} f(x+i, y+j) \quad f'(x, y) = \left[\frac{f(x, y)}{m(x, y)} \right] \quad (4)$$

C. Exhaustive Search Metrics

An alternate approach to ring detection is to exhaustively search for rings on the correlation surface. A number of different metrics based on integrating the values around each potential circumference can be used. Using polar coordinates (r, θ) the circumference of a circle centered at (ρ, ϕ) with radius r is given by:

$$K(\theta) = \rho \cos(\theta - \phi) + \sqrt{r^2 - \rho^2 \sin^2(\theta - \phi)} \quad (5)$$

The signal to noise ratio (SNR) of the ring can now be calculated as the inverse coefficient of variation (where N is the number of points in the circumference):

$$SNR = \frac{\mu}{\sqrt{\frac{1}{N-1} \oint_K (f(\theta) - \mu)^2 \cdot d\theta}} \quad (6)$$

$$\text{Where} \quad \mu = \frac{1}{N} \oint_K f(\theta) \cdot d\theta \quad (7)$$

It is also possible to obtain a similar measure to that of the classical HCT by considering the total ring energy:

$$E = N \cdot \mu \quad (8)$$

One of the problems with this metric (8) is that it is biased towards larger rings. For a given mean value, larger rings will always give higher total energy than smaller ones. Rings with higher means but lower values of N will be outvoted. Using the mean as a metric on its own would bias detection to small noise clusters. A solution to overcome this problem is to give the mean greater weight as follows:

$$H = N \cdot \mu^2 \quad (9)$$

Another alternative is to consider only the actual number of nonzero points η on the ring perimeter, rather than its actual circumference N , to give the total support energy:

$$\bar{E} = \eta \cdot \mu \quad (10)$$

A hybrid of metrics (9) & (10) can also be used:

$$\bar{H} = \eta \cdot \mu^2 \quad (11)$$

Metrics (8-11) utilize N in calculation of the mean. Instead, utilising η in the calculation of the mean in equation (9) results in another metric; the square of the total ring energy divided by the number of supporting points:

$$J = \frac{E^2}{\eta} = \frac{(N \cdot \mu)^2}{\eta} \quad (12)$$

III. METHOD DESCRIPTION

To investigate the effectiveness of the various ring detection measures, a number of instances of correlation surface images with ground truth data were obtained from [1]. These were 512 x 512 in size containing of rings of radii from 5 to 20 pixels at a fixed location on the correlation surface. In addition a second set of 6080 correlation surfaces pertaining to rings of radii from 2 to 16 pixels incremented by 2 with located at 760 discrete locations on the correlation surface. A typical correlation surface image is shown in Figure 3.

Fig. 3. Inverse image of correlation surface with ring of radius 11.

The classical HCT was used as the benchmark and its accuracy, defined as the ratio of correct detections to the sample size over this data set was noted.

Exhaustive searches were then performed using metrics (8)-(11) and the accuracy of each noted. To assess the merit noise reduction, each data value below a given threshold, across a range of predetermined thresholds was truncated to zero and each of the measures recalculated.

As exhaustively searching over the entire correlation surface is computationally intensive, a hybrid scheme was devised. This aimed to provide the computational benefits of the Hough transform with improved detection accuracy. This involved creating a short list of the twenty-five most likely candidates generated by the Hough transform and applying metrics (8)-(11) to each candidate.

Synthetic images were then generated with Gaussian noise similar to that of produced by phase registration of affine transformed images ($\mu = 0$ $\sigma = 0.25$). Results were generated to evaluate effects of noise truncation thresholds against ring support. A fourth set of results was obtained to assess different noise distributions varying from $\sigma = 0.0 - 1.0$ in steps of 0.1 against ring support. Rings were inserted in the images using a Bernoulli distribution with probabilities varying from 0 to 1 in

steps of 0.05 and constant intensity of 0.5. A sample image generated in this manner is shown in Figure 4. All of the preceding experiments were performed on this synthetic dataset to determine how well the various approached performed with general Gaussian noise.

Fig. 4. Inverse ring image with 20% support in noise $\mu = 0.2$ $\sigma = 0.15$

IV. EXPERIMENTAL RESULTS

A. Correlation Surface Data

The results from the first set of correlation surfaces using exhaustive search and local noise averaging are shown in table 1 and figure 5. The columns correspond to the metrics given by equations (5)-(11). The rows in the table are for different truncating thresholds. All values in the correlation surfaces below the threshold were set to zero. Table 1 shows that the J metric performed the best on average with 79% accuracy for all thresholds. While it did not provide the highest accuracy at any single threshold level it consistently placed near the top, especially at higher noise levels resulting from lower thresholds. The poor performance of the SNR metric is due to the high level of variance in the ring's support. The accuracy of all metrics generally reduces as the truncation threshold begins reduces the support of the rings.

TABLE I. ACCURACY OF EXHAUSTIVE SEARCH METRICS

Threshold	SNR	E	H	$\bar{E}$	$\bar{H}$	J
0.00	0.00	0.50	1.00	0.75	1.00	0.92
0.14	0.08	0.83	1.00	0.83	0.92	0.92
0.28	0.17	1.00	1.00	0.92	0.67	0.92
0.42	0.17	0.92	0.67	0.83	0.42	0.75
0.56	0.25	0.83	0.42	0.67	0.17	0.67
0.70	0.17	0.58	0.33	0.42	0.17	0.58
Average	0.14	0.77	0.58	0.74	0.56	0.79

Fig. 5. Accuracy of metrics using exhaustive search for different thresholds

The results for the first set of correlation noise images using the HCT and its hybrids are shown in table 2 and figure 6. The first column lists the truncation thresholds. The column labeled HT shows the accuracy for the standard HCT at each threshold. The remaining columns are the HCT hybrid schemes. From this one can see that the performance of the classical HCT is identical to that of an exhaustive search using the total ring energy $\bar{E}$ metric. With the exception of the J metric the hybrid methods were unable to better the classical HCT. The hybrid J metric performed the best overall, correctly identifying rings in 83% of all test cases, but achieved 100% accuracy across the broadest range of thresholds. The hybrid J metric provided better performance than the exhaustive search version.

TABLE II. ACCURACY FOR THE HOUGH AND HYBRID METHODS

Threshold	HT	SNR	E	H	$\bar{E}$	$\bar{H}$	J
0.00	0.75	0.42	0.50	0.92	0.75	0.92	1.00
0.19	0.83	0.58	0.83	1.00	0.83	1.00	1.00
0.38	0.92	0.83	1.00	1.00	0.92	1.00	1.00
0.56	0.83	0.33	0.92	0.67	0.83	0.50	0.75
0.75	0.67	0.17	0.83	0.42	0.67	0.25	0.67
0.75	0.42	0.17	0.50	0.33	0.42	0.17	0.58
Average	0.74	0.42	0.76	0.72	0.74	0.64	0.83

Fig. 6. Accuracy of hybrid Hough based methods for different thresholds

Using the larger second set of correlation surface images with the hybrid methods produces similar results, albeit with lower overall accuracy as shown in table III. The reduced accuracy in this data set is predominantly due to increased noise introduced by the different locations of the ring. For the purpose of these experiments a ring was deemed to be reliably detected if its coordinate values was within 25% of its true location. The value of 25% was chosen rather arbitrarily since error distribution tends to be a step function: where the rings are correctly identified the error is typically below 15% and when not the average error is above 40%.

TABLE III. ACCURACY FOR THE HYBRID METHODS USING 2ND DATASET

Threshold	HT	E	H	$\bar{E}$	$\bar{H}$	J
0.00	0.67	0.65	0.70	0.70	0.77	0.79
0.20	0.76	0.74	0.76	0.77	0.80	0.82
0.40	0.85	0.82	0.82	0.84	0.85	0.85
0.60	0.79	0.72	0.72	0.73	0.73	0.73
0.80	0.50	0.45	0.45	0.45	0.56	0.56
Average	0.67	0.59	0.66	0.69	0.65	0.83

Fig. 7. Accuracy of metrics using hybrid methods for second dataset

Similar to the results in table II, the J metric provided the best average accuracy across all truncation thresholds, and came equal first at a truncation threshold of 0.4. The J metric appears to be the least affected by the noise level, although it, along with the other metrics appeared to benefit from some degree of noise truncation. In all of these experiments the classical HCT proved inferior to both the exhaustive and hybrid form of the J metric. Without noise reduction via truncation, the J metric significantly provided between 24% to 33% better accuracy than the classical HCT. When noise truncation is utilized to boost the HCT performance, it still under performed in some cases compared to the J metric. Without the use of preliminary local noise averaging all the methods evaluated performed very poorly and were generally incapable of detecting rings of radii greater than two pixels.

B. Synthetic Data

In the case of the synthetic dataset, results were generated for test cases replicating the characteristics of the correlation surface images. These used Gaussian noise model parameters

of $\mu = 0$ $\sigma = 0.25$ with ring support following a Bernoulli ring distribution with maximum intensity of 0.5. Results for the hybrid Hough methods for various degrees of noise truncation are shown in tables IV and figure 8. Each element in the tables represents the average accuracy of each method at the given threshold across 20 levels of ring support from 0-100%. No local averaging was applied. As in the correlation surfaces, metric J consistently performed well, equal best or second best at all truncation thresholds and 5% better than the HCT.

TABLE IV. RESULTS OF THE HYBRID METHODS FOR SYNTHETIC DATA

Threshold	HT	SNR	E	H	$\bar{E}$	$\bar{H}$	J
0.00	0.65	0.65	0.5	0.65	0.6	0.7	0.7
0.10	0.65	0.65	0.5	0.7	0.6	0.7	0.7
0.20	0.65	0.7	0.55	0.7	0.6	0.7	0.7
0.30	0.65	0.75	0.55	0.75	0.6	0.75	0.75
0.40	0.65	0.75	0.6	0.75	0.7	0.75	0.7
0.50	0.7	0.75	0.6	0.75	0.7	0.75	0.75
0.60	0.75	0.75	0.65	0.75	0.75	0.75	0.75
Average	0.67	0.71	0.56	0.72	0.65	0.73	0.72

Fig. 8. Accuracy of hybrid hough methods for synthetic data

Synthetic data was also generated for mean Gaussian noise of $\mu = 0$ and from $\sigma = 0$ to 1.0 without noise truncation (ie threshold = 0) or local averaging. The detection accuracy for the hybrid Hough methods is shown in tables V and figure 9. Each element in the table represents the average accuracy of each method at across 20 levels of ring support from 0-100%. In this experiment the J metric provided equal best accuracy, also 5% better than the standard HCT.

TABLE V. HYBRID METHODS FOR VARIOUS MEAN NOISE LEVELS

Std.Dev.	HT	SNR	E	H	$\bar{E}$	$\bar{H}$	J
0.0	0.95	0.8	0.95	1	0.95	1	1
0.10	0.85	0.7	0.85	0.95	0.85	0.95	0.95
0.20	0.8	0.55	0.8	0.85	0.8	0.85	0.85
0.30	0.8	0.65	0.75	0.8	0.8	0.8	0.8
0.40	0.75	0.7	0.65	0.75	0.75	0.75	0.75
0.50	0.65	0.65	0.5	0.65	0.6	0.7	0.7
0.60	0.45	0.55	0.3	0.55	0.4	0.55	0.55
Average	0.75	0.66	0.69	0.79	0.74	0.80	0.80

Fig. 9. Accuracy of hybrid hough methods for different noise levels

With or without noise reduction when used as a post-processing step to the HCT the J metric almost always provides improved ring detection in noisy data over the standard HCT. In the few tests where the J metric was not the outright best out of all the metrics tested, it was equal or second best.

V. CONCLUSIONS

The classical HCT provides good performance for many circle detection tasks. While it is computationally intensive it is nevertheless much faster than exhaustively searching images for possible circles. In challenging conditions, with noisy data and poor support such as with correlation surfaces produced by phase registration of affine transformed images its weaknesses become apparent. This paper has presented a hybrid algorithm that performs preliminary local noise averaging prior to the selecting of candidate rings via the HCT. A novel metric is then used to identify the most likely ring from this shortlist. In virtually all cases the accuracy of the classical HCT can be improved with this post-processing step. In the case of correlation surfaces the improvement is significant; 33% compared to using the HCT without any additional noise reduction and even when the HCT is assisted with noise reduction the proposed metric is still able to provide a 12% average improvement on accuracy.

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