Prediction of scour at abutments using piecewise regression

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Scour is one of the main causes of bridge failure and scour depth prediction is thus a very important issue. Although many studies have been carried out to develop formulas for the prediction of scour depth at abutments, the existing formulas do not yield accurate results, mainly due to the complexity of the scour phenomena around abutments. This paper reports on a study of abutment scour using piecewise multiple linear regression analysis. First, scour depth data were collected from the literature and the existing formulas were evaluated and compared. To develop new formulas, the conventional governing parameters were selected as the independent variables. The piecewise multiple linear regression technique was applied to investigate the relationship between dimensionless scour depth and the governing dimensionless parameters in several subdomains. Errors of the proposed formula and previous models were also compared. Compared with existing formulas, the error measures in the proposed method show a significant improvement in prediction accuracy.

Notation

d_{50}	median size of sediment
F_c	$V_c/(\Delta Ig)^{1/2}$
Fr	$V/(yg)^{1/2}$
g	gravitational acceleration
I_a	index of agreement
K_d	sediment size factor
K_G	channel geometry factor
K_I	flow intensity factor
K_s	shape factor
K_{yl}	abutment length or flow depth factor
K_θ	angle of attack factor
K_σ	sediment gradation factor
k	number of independent variables
l	abutment length
n	number of measurements
V	flow velocity
V_c	critical flow velocity
$\bar{x}$	mean of measured values
x_i	measured value
y	flow depth
$\bar{y}$	mean of the predicted values
y_i	predicted value
y_s	equilibrium scour depth
θ	angle of abutment alignment
σ_g	geometrical standard deviation

1. Introduction

A bridge is arched over a river, valley or crossing to enable transport. Scour is one of the main causes of bridge failure

around the world. When an abutment is placed on an erodible bed, the flow pattern around the abutment will change. A change in flow field creates vortices and these vortices may finally result in scouring around the abutment. Scour is a complex phenomenon because it is the result of interaction between the fluid, the structure and sediment (Zhao and Fernando, 2007). The prediction of scour depth at the equilibrium stage is vital in the design of bridge abutments.

Many researchers have studied abutment scour, including Ahmad (1953); Dey and Barbhuiya (2004b); Froehlich (1989); Garde *et al.* (1961); Gill (1972); Kwan (1984); Laursen (1963); Liu *et al.* (1961); Melville (1992, 1997); Muzzammil (2010); and Richardson and Davis (1995). Most of these researchers used dimensionless parameters and laboratory experimental data to develop formulas for scour depth prediction.

Froehlich (1989) showed that scour depth is a function of the abutment length, flow depth, Froude number, abutment shape and abutment alignment. The US Federal Highway Administration (FHWA) suggests Froehlich's formula for short abutments and Hire's formula for long abutments (Richardson and Davis, 1995, 2001). Hire's formula implies that scour depth is a function of Froude number, abutment shape and abutment alignment. Melville (1992) developed a method to estimate the scour depth at abutments utilising different coefficients to consider the effects of abutment length, sediment size, sediment gradation and channel geometry on scour depth. Dey and Barbhuiya (2004b) investigated the scour depth formula for different abutments and showed that the scour depth also depends on the critical Froude number.

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Muzzammil (2010) used artificial neural networks and adaptive neuro fuzzy system models (Haykin, 1999; Kazeminezhad *et al.*, 2005) to increase the accuracy of the scour depth prediction. However, these soft computing methods are not transparent and are not easy for engineers either to understand and/or use.

The main purposes of this study are to compare the performances of different existing empirical methods and to develop a new set of formulas for predicting scour depth in various conditions. The presented formulas were developed using piecewise multiple linear regression by SPSS software (Levesque, 2007).

2. Scour at abutments

There are two conditions for local scour: clear-water and live-bed. Clear-water scour occurs when there is no movement of the bed material in the upstream. Live-bed scour occurs when bed material is transported from the upstream into the scour hole (Richardson and Davis, 2001). Recent studies of abutment scour (e.g. Kwan, 1988; Kwan and Melville, 1994) have shown that the scour mechanism at abutments is very complex. The down flow and the principal vortex at the upstream corner of the abutment, together with the secondary and wake vortices at the middle part and the downstream corner of the abutment, cause complex interactions between the fluid and the bed material leading to the scour at abutments.

Previous investigations have shown that the abutment scour depth is related to parameters such as the approach flow velocity and critical flow velocity. The ratio between these two velocities can be used as an indicator for clear-water or live-bed scour conditions. If the mean velocity is equal to or less than the critical velocity of the median diameter of the bed material, then scour will be clear-water scour. If the mean velocity is greater than the critical velocity, live-bed scour will occur. The equation to determine the critical velocity for a given flow depth and size of bed material is given by Richardson and Davis (2001). The abutment length and flow depth are two other important parameters that also influence scour depth.

To consider the effects of sediment size on scour, it is important to distinguish between clear-water and live-bed conditions. Under live-bed conditions, Laursen (1960) argued that there is no significant effect of sediment size on the local scour. For clear-water conditions, studies such as those of Laursen (1960) have shown that sediment size affects local scour. Wong (1982) showed that, under clear-water conditions, scour depth increases with an increase in sediment size. However, Ahmad (1953), Izzard and Bradley (1958) and Laursen and Toch (1956) stated that scour depth is independent of sediment size.

A river's bed materials are generally non-uniform. A measure of the non-uniformity of the sediment is the geometrical standard deviation. Sediment gradation reduces scour depth with the formation of armour layers in the scour holes. It has been noticed that, when a protective layer (known as the armour layer)

develops on the bed surface, sediments of all sizes are not in motion. This means, once an armour layer has been formed on the river bed, further erosion of the river bed becomes very difficult. However, finer sediment particles from tributaries may be delivered to the downstream over the armour layer. Dey and Barbhuiya (2004c) conducted experiments to study the effects of a thin armour layer on the scour depth at abutments. They concluded that the scour depth at abutments with an armour layer in clear-water scour conditions is always greater than that of the case of no armour layer for the same bed sediments (Dey and Barbhuiya, 2004a).

The local scour depth also depends on the shape of the abutment. Streamlined bodies, such as semi-circular (SC), spill-through and wing-wall (WW) abutments, produce vortices of feeble strength while blunt obstructions, such as vertical-wall (VW) abutments, are capable of producing stronger turbulent vortices. Consequently, a relatively larger scour depth is observed at blunt obstructions (Dey and Barbhuiya, 2004a). The effect of shape is expressed by the shape factor (Table 1). In addition, the abutment alignment or angle of attack, defined as the angle of the abutment axis with respect to the approaching flow direction, significantly influences the scour depth. For an abutment angled downstream, the scour depth decreases, whereas the scour depth increases for an abutment angled upstream. Richardson and Davis (2001) expressed the effect of angle by K_θ

$$1. \quad K_\theta = (\theta/90)^{0.13}$$

where θ is the angle of attack: $\theta < 90^\circ$ if the abutment points downstream and $\theta > 90^\circ$ if the abutment points upstream.

3. Description of datasets

Several researchers have carried out experiments on abutment scour. This study used the experimental datasets of Abou Seida *et al.* (2009), Coleman *et al.* (2003), Cunha (1975), Dey and Barbhuiya (2004b), Dongol (unpublished abutment scour data, 1990), Dongol (1994), Garde *et al.* (1963), Gill (1972), Kandasamy (1989), Kwan (1984), Kwan (1988), Ladage (1998), Lim (1997), Liu *et al.* (1961), Rajaratnam and Nwachukwu (1983) and Tey (1984). In total, 827 data points were collected and the dataset is a collection of various experimental conditions; it was assumed that an equilibrium state was achieved in all the experiments. Table 2

Abutment shape	K_s
Vertical wall	1.00
Vertical-wall abutment with semi-circular end	0.75
45° wing-wall	0.75
Spill-through ($H:V = 0.5:1$)	0.60
Spill-through ($H:V = 1:1$)	0.50
Spill-through ($H:V = 1.5:1$)	0.45

Table 1. Shape factors for different abutments (Melville, 1992)

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	Number of data points	Abutment type	d_{50} : mm	σ_g	V/V_c	l/y
Abou Seida <i>et al.</i> (2009)	24	VW	—	—	0.33–0.95	0.62–1.27
Coleman <i>et al.</i> (2003)	68	VW	0.80–1.02	—	0.46–0.99	0.25–151.3
Cunha (1975)	7	VW	1.6–5.8	—	0.45–0.94	2.2
Garde <i>et al.</i> (1963)	7	VW	0.3–1	—	0.83–2.15	1.23–2.06
Gill (1972)	23	VW	0.9	1.2	0.65–1.49	1.96–8.82
Kwan (1984)	19	VW	0.85–0.9	1.3	0.9–1	1.64–28
Ladage (1998)	4	VW	0.8	—	0.49–0.7	0.75
Lim (1997)	11	VW	0.94	1.25	0.41–0.74	0.33–1
Liu <i>et al.</i> (1961)	4	VW	0.56–0.64	1.17	0.5–0.94	2–3.75
Rajaratnam and Nwachukwu (1983)	6	VW	1.4	1.3	0.4–0.67	0.99–1.43
Kandasamy (1989)	30	WW	0.9	1.3	0.95–2.25	1–69
Kwan (1988)	5	WW	0.9	1.3	1	2.4–9.5
Tey (1984)	8	VW-SC	0.8–0.9	1.26	0.78–1	1.65–6
Dongol (unpublished 1990)	46	VW-WW	0.9	1.3	1–6.4	0.25–1.5
Dongol (1994)	116	VW-WW	0.9–1.8	1.3	0.76–4	0.25–151.3
Dey and Barbhuiya (2004b)	449	VW-WW-SC	0.26–3.1	1.17–3.73	0.95	0.16–2.4
TOTAL	827		0.26–5.8	1.17–3.73	0.33–6.4	0.16–151.3

Table 2. Summary of test conditions in different experimental works

summarises the test conditions in different studies. The range of l/y shows that the dataset includes both short and long abutments. The range of V/V_c shows that the data points were collected in both clear-water and live-bed conditions. In addition, the range of σ_g shows that most of the experiments were conducted using uniform sediments and the d_{50} ranges indicate that both fine and coarse sediments were studied. In addition, 100 field measurements collected by Lombard and Hodgkins (2008) from different bridges were used in this study.

4. Analysis of existing formulas

The existing formulas of scour depth prediction and their limitations are as follows.

The FHWA method (Richardson and Davis, 2001) suggests

$$2. \quad y_s/y = 2.27K_sK_\theta(l/y)^{0.43}\text{Fr}^{0.61} + 1 \quad \text{for } l/y < 25$$

$$3. \quad y_s/y = (4/0.55)K_sK_\theta\text{Fr}^{0.33} \quad \text{for } l/y > 25$$

Melville (1992) proposed

$$4. \quad y_s = K_{yl}K_1K_dK_\sigma K_sK_\theta K_G$$

Dey and Barbhuiya (2004b) proposed the following (for $l/y < 1$)

■ vertical

$$5a. \quad y_s/l = 5.857F_c^{0.314}(y/l)^{0.126}(l/d_{50})^{-0.167}$$

■ 45° wing-wall

$$5b. \quad y_s/l = 6.4547F_c^{0.312}(y/l)^{0.101}(l/d_{50})^{-0.231}$$

■ semi-circular

$$5c. \quad y_s/l = 7.287F_c^{0.192}(y/l)^{0.103}(l/d_{50})^{-0.296}$$

Sensitivity analysis aims to describe how much model output values are affected by changes in the input values. The discrepancy ratio (DR), defined as the ratio of predicted and measured values, was used to quantify the sensitivity of different formulas to different variables. A DR value of 1 indicates a perfect agreement, while values greater (or smaller) than 1 show over (or under) estimation of the scour depth.

First, DR was plotted against the logarithm of l/y (or relative length of the abutment). The results of the FHWA method (Froehlich formula for short abutments and Hire formula for long abutments) are shown in Figure 1(a). The mean, maximum and minimum DR values of these formulas are 2.21, 7.28 and 0.32,

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Figure 1. Variation of DR with $\log(l/y)$ for (a) FHWA, (b) Melville (1997) and (c) Dey and Barbhuiya (2004b) formulas. The dashed lines show the 5–95% confidence interval

respectively. With an increase in l/y , the DR decreases and becomes less than 1. Generally, the predictions using these formulas are overestimated, especially at low values of l/y . This is because they are design formulas and must have an inherent safety factor. The results using the method of Melville (1997) are shown in Figure 1(b). The DR varies from 0.16 to 4.7 with a mean value of 1.14. With an increase of l/y , the accuracy of prediction reduces and the predictions become overestimated. However, this formula is accurate at low values of l/y . Figure 1(c)

depicts the results from the Dey and Barbhuiya (2004b) formula. The mean, maximum and minimum DR values of this formula are 1.38, 8.03 and 0.12, respectively; the accuracy of prediction is reduced for both low and high values of l/y . This formula is not accurate at high values of l/y because it is suggested for very short abutments ($l/y < 1$). Similar to the previous methods, the formula is conservative in most cases.

In the second step, DR was plotted against the logarithm of V/V_c (Figure 2). Figure 2(a) shows that, by increasing V/V_c (in live-bed

Figure 2. Variation of DR versus $\log(V/V_c)$ for (a) FHWA, (b) Melville (1997) and (c) Dey and Barbhuiya (2004b) formulas. The dashed lines show the 5–95% confidence interval

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conditions), the accuracy of prediction is reduced when using the FHWA method. The formula of Melville (1997) (Figure 2(b)) is more accurate in the clear-water condition than the other formulas and yields acceptable results for live-bed conditions. The results of the Dey and Barbhuiya (2004b) formula are shown in Figure 2(c). Interestingly, this formula performs well in live-bed conditions, even though it was developed for the clear-water condition. This is mainly due to the fact that it was derived from experiments with $V/V_c = 0.95$. In brief, the FHWA method and the Dey and Barbhuiya (2004b) formulas overestimate the scour depth for very low values of V/V_c because V/V_c is not considered in these formulas. In clear-water condition, flow velocity is much less than the critical velocity and the scour depth is minimal.

5. Development of new formulas

The most important task in developing new scour depth formulas is to find the most reliable parameters to be included in the formula. The influential parameters on equilibrium scour depth at an abutment are generally expressed in the following functional form, assuming a constant relative density of sediment and the absence of viscous effects

$$6. \quad y_s = f(y, l, g, V, V_c, \sigma_g, K_s, K_\theta)$$

In the empirical approaches, parameters involved in abutment scour are correlated through dimensional analysis, and different combinations of these parameters were used for developing the formulas. Here, the simplest and most accurate formulas are presented. As previous studies have shown, the most important dimensionless parameters are l/y , Fr , V/V_c , σ_g , K_s and K_θ , and these parameters were thus used in the new formulas. Equation 6 may be reduced, in terms of a set of non-dimensional parameters, to

$$7. \quad y_s/y = f(l/y, Fr, V/V_c, \sigma_g, K_s, K_\theta)$$

A power-law form of Equation 7 may be expressed as

$$8. \quad y_s/y = a(l/y)^b Fr^c (V/V_c)^d K_s$$

where a is a coefficient and b , c and d are exponents of the equation.

The effects of sediment non-uniformity will be discussed later because the sediments were uniform in this dataset and variations in σ_g were very small. In addition, the effect of sediment diameter was found to be negligible and was excluded from the formula. K_θ was also equal to 1 in all of the data points.

Piecewise regression was used to derive the formulas. Piecewise regression is similar to the decision-tree method and a multi-variable function is suggested at each leaf for output. Noting the

difference between live-bed and clear-water conditions, the dataset was first divided into two parts: $V/V_c > 1$ and $V/V_c < 1$. Then, a multiple linear regression model was developed for each condition. The multiple linear regression technique was applied to develop the relationship between the dependent variable and several independent variables in Equation 8. Datasets were divided into two parts for training and testing the formulas. Accurate results for training data do not necessarily guarantee the generalisation capability of the proposed formula. The performance of the developed formula should always be checked with the testing data (Haykin, 1999). To develop the formulas, 747 data points were used (663 for clear-water conditions and 84 for live-bed conditions); 70% were used for training and the remaining 30% for testing the formulas. Equations 9a and 9b (called PR1) were obtained using SPSS software

$$y_s/y = 1.85(l/y)^{0.397} Fr^{0.134} (V/V_c)^{0.859} K_s$$

$$9a. \quad \text{for } V/V_c < 1$$

$$y_s/y = 1.1(l/y)^{0.422} Fr^{0.182} (V/V_c)^{0.136} K_s$$

$$9b. \quad \text{for } V/V_c > 1$$

The set of equations PR1 shows the dependency of scour depth on l/y , Fr , V/V_c and abutment shape. PR1 implies that scour depth increases significantly with an increase in l/y . The powers of l/y in both formulas are also close to that in the Froehlich (1989) formula. This finding is also supported by the previous findings of Liu *et al.* (1961). The scour depth moderately increases with an increase in Fr in PR1. The powers of Fr in both formulas are less than that of other formulas such as Froehlich (1989) and Hire (Richardson and Davis, 1995). This can be justified by noting that, in the previous formulas, the effect of velocity is considered in one parameter (Fr or V/V_c). However, in PR1, both Fr and V/V_c are considered, and the effect of approaching flow is included in both Fr and V/V_c . Hence, the power of Fr reduces. The main difference in clear-water and live-bed conditions is in the transport of the upstream sediments, which depends on V/V_c . The main splitting value in PR1 is $V/V_c = 1$. This shows that PR1 classifies the dataset into two parts: $V/V_c < 1$, which is the clear-water condition, and $V/V_c > 1$, which is the live-bed condition. The power of V/V_c is 0.859 in Equation 9a, showing that, in the clear-water condition, V/V_c is an important parameter. However, in the live-bed condition ($V/V_c > 1$), V/V_c is not as important, and its power decreases. This is in line with the findings of Melville (1992), where the power of V/V_c is one for clear-water and zero for live-bed conditions. K_s was also obtained from Table 1. It can thus be said that the PR1 formulas are physically sound and are successful in capturing the relationships among the dimensionless scour parameters.

Figure 3, which compares the measured scour depths and those

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Figure 3. Comparison of measured and predicted dimensionless scour depths

predicted by PR1 (all data points), shows that most of the data points are concentrated on the line of perfect agreement. At very high values of y_s/y , PR1 overestimates the dimensionless scour depth. However, the overestimation in these cases is marginal and negligible. For a quantitative comparison between the predicted and measured scour depth values, error measures such as index of agreement (I_a), scatter index (SI) and Bias were used

$$10. \quad I_a = 1 - \frac{\sum_{i=1}^n (y_i - x_i)^2}{\sum_{i=1}^n (|x_i - \bar{x}| + |y_i - \bar{y}|)^2}$$

$$11. \quad SI = 100 \frac{(1/n \sum_{i=1}^n (x_i - y_i)^2)^{1/2}}{\bar{x}}$$

$$12. \quad \text{Bias} = \bar{y} - \bar{x}$$

Table 3 shows the error measures of PR1 and other formulas. In total, 225 data points were used to calculate the error measures. However, only 143 data points of this dataset are in the range of applicability of the Dey and Barbhuiya (2004b) formula ($l/y < 1$), and hence this formula was evaluated using only these 143 data

Method	Number of data points	I_a	SI: %	Bias
Dey and Barbhuiya (2004b)	143	0.78	34	0.20
PR1	143	0.77	30	0.10
Melville (1992)	225	0.88	95	0.42
FHWA	225	0.81	49	0.81
PR1	225	0.98	26	0.01

Table 3. Error measures of different formulas

points. The SI of PR1 is reduced by 69% compared with the Melville (1992) formula and by 23% compared with the FHWA method. Moreover, the index of agreement (I_a) of PR1 is 0.17 greater than that of the FHWA method and 0.10 greater than the Melville formula. Bias shows the over- or underestimation of formulas. The Bias of the FHWA and Melville (1992) formulas shows that the FHWA formula is more conservative than the Melville formula. The error measures for 149 data points indicate that PR1 is also more accurate than the formula proposed by Dey and Barbhuiya (2004b). In brief, the higher values of I_a and lower values of Bias and SI indicate that PR1 is more accurate than the other formulas considered. It should be mentioned that using all data for developing new formulas did not improve the accuracy of predictions significantly.

Equations 9a and 9b were further tested for their statistical reliability and significance. An analysis of variance (Anova) approach was used to test the statistical significance of the derived formulas. To test the hypothesis claiming that the amount of variation explained by the regression model is more than the variation explained by the average, the F ratio was used (Hair *et al.*, 1995). The null hypothesis is rejected if $F_0 > F_{\alpha, k, n-p}$. An example of the model significance test of Equations 9a and 9b is given in Table 4. In the F distribution table with $F_{0.05}$, $k = 4$ (number of independent variables), $p = k + 1$ and n is equal to 467 and 55 for Equations 9a and 9b, respectively. Therefore, $F_{0.05, 4, 462} = 2.37$ for Equation 9a and $F_{0.05, 4, 50} = 2.59$ for Equation 9b. However,

$$F_0 = MS_R / MS_E = 2499.9 > F_{0.05, 4, 462}$$

for Equation 9a where MS_R is mean square regression and MS_E is mean square error and

$$F_0 = MS_R / MS_E = 799.5 > F_{0.05, 4, 50}$$

for Equation 9b.

The null hypothesis is thus rejected for both, and the PR1 formulas are accepted as significant.

Figure 4 shows the results of PR1 (for all data). Figure 4(a) shows that, at low values of l/y , the predicted values are overestimated in some cases and PR1 is more accurate at high values of l/y . Figure 4(b) shows that at very low values of V/V_c , PR1 is conservative and PR1 is more accurate in live-bed conditions.

Non-uniformity of the sediments is another influential parameter on scour depth. In Figure 5, variations of the ratio between measured and predicted (by PR1) values are plotted against σ_g to investigate the effect of σ_g on the scour depth. In Figure 5, 80 data points from the experiments of Dey and Barbhuiya (2004b) are used. In this dataset, σ_g varies from 1.17 to 3.73 with a mean

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Source of variation	Sum of squares	Degree of freedom	Mean squares
Equation 9a			
Regression	$SS_R = 2099.944$	$k = 4$	$MS_R = SS_R/k = 524.98$
Error	$SS_E = 98.71$	$n - p = 462$	$MS_E = SS_E/(n - p) = 0.21$
Total	$SS_{YY} = 2198.65$	$n - 1 = 466$	
Equation 9b			
Regression	$SS_R = 63.95$	$k = 4$	$MS_R = SS_R/k = 15.99$
Error	$SS_E = 1.124$	$n - p = 50$	$MS_E = SS_E/(n - p) = 0.02$
Total	$SS_{YY} = 65.08$	$n - 1 = 54$	

Table 4. Anova table of Equations 9a and 9b. SS_E , error variance; SS_R , regression variance; SS_{YY} , total variance

Figure 4. DR versus (a) $\log(l/y)$ and (b) $\log(V/V_c)$ for PR1 (Equations 9a and 9b). The dashed lines show the 5–95% confidence interval

value of 2.17. The effect of non-uniformity can be considered by K_σ . This factor was obtained by best fitting as

$$13. \quad K_\sigma = 1.41\sigma_g^{-1.31} \quad 1.2 < \sigma_g < 3.7$$

It should be mentioned that this formula is only valid in the range of measured values and may not be extrapolated for other cases. Hence, the modified formulas (here called PR2) for non-uniform sediments will be

Figure 5. Variation of the ratio between measured and predicted (by PR1) scour depths against σ_g

$$y_s/y = 1.85(l/y)^{0.397} Fr^{0.134} (V/V_c)^{0.859} K_s K_\sigma$$

14a. for $V/V_c < 1$

$$y_s/y = 1.1(l/y)^{0.422} Fr^{0.182} (V/V_c)^{0.136} K_s K_\sigma$$

14b. for $V/V_c > 1$

Scour depth significantly decreases with increasing σ_g . The power of σ_g in these formulas is equal to -1.31 , which means that the effects of σ_g are very important. This is also in line with the physics of the phenomena because, by increasing σ_g , a stronger armour layer is formed, which prevents scouring. Dey and Barbhuiya (2004b) suggested that K_σ can be determined using a graph of K_σ versus σ_g . It can be seen that the presented K_σ is in agreement with the results obtained by Dey and Barbhuiya (2004b).

Table 5 shows the error measures of PR2 and the previous formulas for field data collected by Lombard and Hodgkins (2008). The l/y values in this dataset vary from 0 to 244.6 (i.e.

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	Number of data points	I_a	SI: %	Bias
Melville (1997)	100	−0.33	668	2.79
FHWA	100	−0.28	353	1.74
PR2	100	0.07	234	−0.31

Table 5. Error measures of different formulas (field measurements)

the dataset includes both short and very long abutments). The range of V/V_c is 0.05–2.37, meaning that the data points are in both clear-water and live-bed conditions. The range of σ_g is 2.4–44.8, showing that all sediments are non-uniform. To calculate the error measures of the different formulas, 100 data points were used. Only four data points from this dataset are in the range of applicability of the Dey and Barbhuiya (2004b) formula ($l/y < 1$), which is insufficient to calculate the error measures.

Generally, the predicted values are not in perfect agreement with the measurements, but the PR2 equations are more accurate than the other formulas. The SI of scour depths predicted by PR2 is significantly less than those of the FHWA and Melville (1997) formulas, and the I_a of PR2 is higher. The Bias of the FHWA and Melville results shows that these formulas are very conservative; the Bias of PR2 is less than that of the other two formulas and its predictions agree with the measurement. This is due to two reasons. Firstly, the other formulas do not include the effect of V/V_c on scour depth, whereas PR2, by considering V/V_c , is able to predict the scour depth in both clear and live conditions. The second reason is non-uniformity of sediments. In this dataset, all the measurements were taken in non-uniform sediment, and the FHWA and Melville formulas cannot consider the effect of this on scour depth. PR2 does consider K_σ , and can thus predict scour depth more accurately.

Commonly used models for estimating the scour depth at the abutment typically provide conservative values (Tables 3 and 5). However, the safety factors used in them are not known, and therefore the risk of bridge failure using these formulas is unknown. This limitation can be overcome by considering a safety factor related to the desired or allowable level of risk as follows

$$y_s/y = (1.85 + N\sigma_1)(l/y)^{0.397} Fr^{0.134} (V/V_c)^{0.859} K_s K_\sigma$$

15a. for $V/V_c < 1$

$$y_s/y = (1.1 + N\sigma_2)(l/y)^{0.422} Fr^{0.182} (V/V_c)^{0.136} K_s K_\sigma$$

15b. for $V/V_c > 1$

where N is the safety factor and σ_1 and σ_2 are the standard

deviations (0.6 and 0.3, respectively). The values of N can be obtained from the normal distribution curve. For example, for 5% acceptable risk, N is equal to 1.64.

6. Summary and conclusions

The scour depth at bridge abutments has been investigated. The data used for the development of new formulas were collected from previous experimental small-scale studies. The selection of input parameters has a large impact on a model's simplicity and accuracy, and hence different dimensionless parameters were used for formula development. Piecewise multiple linear regression was used to predict the dimensionless scour depth. Both live-bed and clear-water conditions were considered, and different dimensionless parameters were examined in developing the formulas. The utilised approach showed high performance for all cases, leading to reduced error measures.

PR1 (Equations 9a and 9b) was developed for uniform sediments using l/y , Fr , V/V_c and K_s , and the results were compared with those derived using previously proposed formulas. It was found that PR1 is physically sound and more accurate than the previous empirical formulas. PR1 was then modified to consider sediment non-uniformity, and σ_g was used in the modified formulas comprising set PR2 (Equations 14a and 14b). PR2 and other formulas were evaluated using field data. The results were not in perfect agreement with the measurements, but PR2 was more accurate than the other formulas considered. Finally, by considering the safety factor, design formulas were presented for the prediction of scour depth at an abutment.

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