Improved Design Rules for Fixed Ended Cold-formed Steel Columns Subject to Flexural-torsional Buckling

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Abstract: This paper has presented the details of an investigation into the flexural and flexural-torsional buckling behaviour of cold-formed structural steel columns with pinned and fixed ends. Current design rules for the member capacities of cold-formed steel columns are based on the same non-dimensional strength curve for both fixed and pinned-ended columns. This research has reviewed the accuracy of the current design rules in AS/NZS 4600 and the North American Specification in determining the member capacities of cold-formed steel columns using the results from detailed finite element analyses and an experimental study of lipped channel columns. It was found that the current Australian and American design rules accurately predicted the member capacities of pin ended lipped channel columns undergoing flexural and flexural torsional buckling. However, for fixed ended columns with warping fixity undergoing flexural-torsional buckling, it was found that the current design rules significantly underestimated the column capacities as they disregard the beneficial effect of warping fixity. This paper has therefore proposed improved design rules and verified their accuracy using finite element analysis and test results of cold-formed lipped channel columns made of three cross-sections and five different steel grades and thicknesses.

Keywords: Cold-formed steel columns, Flexural and flexural-torsional buckling, Fixed ends, Warping fixity, Numerical studies, Design rules.

1. Introduction

Cold-formed steel members are becoming increasingly popular within the construction industry due to their superior strength to weight ratio and ease of fabrication as opposed to hot-rolled steel members. They are often subject to axial compression loads in a range of applications (see Figure 1). These thin-walled members can be subject to various types of buckling modes, namely local buckling, distortional buckling, flexural buckling and flexural-torsional buckling. Hence extensive research efforts have gone into the many investigations addressing the buckling behaviour of cold-formed steel columns.

Popovic et al. [1] performed tests on fixed and pinned ended angle columns. Their test specimens failed in flexural and flexural-torsional modes. They showed that the design capacity curve proposed in AS/NZS 4600 [2] is conservative for shorter specimens. Popovic et al. [1] reported that the reason for this is most likely the post-buckling reserve of the section in the torsional mode. Young [3,4] conducted experimental studies of angle columns and concluded that AS/NZS 4600 [2] design rule is conservative. Hence he proposed a new design rule for concentrically loaded compression members of fixed ended cold-formed steel plain angle sections. This was adopted in Silvestre et al. [5] to estimate the global column strength, which gave more accurate ultimate strengths for fixed-ended and pin-ended columns. Shifferaw and Schafer [6] found that plain and lipped angles with fixed end conditions exhibited post-buckling strength with respect to global torsional or flexural-torsional buckling modes due to the presence of warping fixity, which is ignored in all the current design rules.

Rasmussen and Hancock's [7] research into the flexural of cold-formed steel channels showed that AS/NZS 4600 [2] conservatively predicted the member strengths of fixed ended columns at intermediate and long lengths. Recently, Bandula Heva and Mahendran [8] carried out a series of compression tests of cold-formed steel channel members subjected to flexural-torsional buckling, and showed that AS/NZS 4600 [2] design rules conservatively predicted the strength of tested specimens. These findings therefore warrant further investigations into the behaviour of fixed ended lipped channel steel columns subject to flexural and flexural-torsional buckling.

The overall aim of this research is to investigate the accuracy of current design rules in determining the strengths of concentrically loaded cold-formed steel columns with fixed ends subject to flexural and flexural-torsional buckling. Experimental results for this study were

obtained from Bandula Heva and Mahendran [8]. In their study suitable test specimens were selected based on the standard sections, thicknesses and grades that are commonly used in structural and architectural applications and available literature. The dimensions and lengths of lipped channel section columns were selected based on a number of preliminary analyses using a finite strip analysis program CUFSM [9] so that flexural-torsional buckling governed the member behaviour. In this study, finite element models were developed using a finite element analysis program ABAQUS [10] to simulate the behaviour of long lipped channel section columns using suitable loading and boundary conditions. The developed finite element models were validated by comparing their results with Bandula Heva and Mahendran's [8] test results. The validated model was then used in a detailed numerical study into the axial compression strengths of lipped channel columns. Five different steel grades and thicknesses were considered in this numerical study to investigate the effect of using low and high grade steels. Three different section dimensions were also considered in this study with varying column lengths. The results obtained from this study were compared with the predicted ultimate loads from the current cold-formed Australian and American steel standards, based on which the accuracy of current design rules for pinned and fixed ended cold-formed steel compression members undergoing flexural and flexural-torsional buckling was investigated. This paper presents the details of this research study and the results.

2. Experimental Study

Bandula Heva and Mahendran [8] investigated the behaviour and strength of cold-formed steel lipped channel columns at both ambient and elevated temperatures. As part of their experimental study, they conducted six tests at ambient temperature to investigate the flexural-torsional buckling behaviour of fixed ended columns.

2.1. Test Specimens

The most common cold-formed steel column section of lipped channel was chosen for their tests. Test section dimensions and specimen lengths were selected based on preliminary numerical analyses and AS/NZS 4600 design rules so that flexural-torsional buckling governed the member behaviour. These analyses showed that the member lengths of 1600 mm or higher gave flexural-torsional buckling for the two selected lipped channel sections (55x35x9 and 75x50x15) with fixed ends. Therefore two specimen lengths of about 1800 mm and 2800 mm were selected in their tests. Three grades and thicknesses, G550-0.95, G450-1.90 and G250-1.95, were selected to represent the cold-formed steel domain, each with

nominal lengths of 1800 and 2800 mm. Table 1 gives the measured cross-sectional dimensions and lengths of six ambient temperature test specimens and the mechanical properties of steels.

Imperfections of all the specimens were measured along the specimens on all the surfaces except lips in the study of Bandula Heva and Mahendran [8]. Since the expected buckling mode was global buckling, only the global imperfection was measured. In most cases, the maximum imperfections were observed to be on the web. Table 1 shows the measured imperfections. The measured imperfections were significantly less than the tolerance value of $L/1000$ recommended by AS 4100 [11].

2.2. Test Setup and Procedure

A special test set-up was designed and built to test long columns of different heights inside a furnace, so that both ambient and elevated temperature tests can be done as required [8]. Hence the ambient temperature tests reported here were conducted inside the furnace while keeping the doors open as shown in Figure 2. The loading arrangement consists of two loading shafts at the top and bottom and a hydraulic loading system as shown in Figures 2 and 3. By using the nuts on each of the leveling bars in the special loading arrangement at the bottom, the base plate was leveled so that the guidance tube was vertical to allow the required application of load. The hydraulic jack was connected to a hydraulic pump through a pressure transducer. The pressure transducer was used to determine the applied axial load.

All the tests were carried out using fixed-end conditions. To achieve a fixed-end support, special end plates were made to fit the specimen ends (see Figure 3). A groove of 12 mm deep and 10 mm width in the shape of specimen cross-section was made on a 15 mm thick circular steel plate. A Rectangular Hollow Section (RHS) of 2 mm thick and 15 mm height was then welded to the plate. Geometric centre of the cross-section of the specimen was made to coincide with the centre of the plate. The specimen was placed within the groove, and 165 procreate coil grout mixed with water was then used to fill the groove and the end space up to the top of RHS. The specimen with these end plates was then placed between the two loading shafts and bolted to form fixed ends. Out of plane deflections of the specimens were measured at mid-height in both directions using stainless steel cables and LVDTs. Axial shortening was also measured using a LVDT attached to the bottom loading shaft. Using the data from the pressure transducer and LVDTs the load-deflection curves were obtained.

2.3. Results and Discussions

As expected, all the specimens failed in flexural-torsional buckling as shown in Figure 4. The ultimate loads obtained for the tests are given in Table 2. Comparing the results obtained from tests to code predictions, Bandula Heva and Mahendran [8] found that for concentrically loaded columns with fixed ends subject to flexural-torsional buckling, AS/NZS 4600 [2] design rules considerably underestimated the column capacities, ie. very conservative. Their test data showed that columns made of higher grade steels had greater inaccurate predictions from the design codes than those made of lower grade steels. In one case their experimental study found that a test specimen to have 77% higher member strength than what was predicted by the design code. Test results from [8] will be used in this study to validate the finite element models developed in this research and to compare with the column capacities predicted by the current design rules.

3. Finite Element Modelling

In this research ABAQUS [10] was used in the finite element analyses (FEA) of cold-formed steel columns subject to axial compressive loads. Finite element models were developed first to simulate the tested cold-formed steel lipped channel column specimens. The measured dimensions of lipped channel specimens including their base metal thicknesses (Table 1) were used in these analyses. Thereafter the developed finite element model was used with nominal dimensions to investigate the axial compression capacities of cold-formed steel columns with different section sizes, steel grades, thicknesses and lengths.

3.1. Element Type and Size

S4R shell element type with a 4 mm x 4 mm mesh size was selected based on detailed convergence studies considering accuracy, processing time and memory. The doubly curved general-purpose shell element S4R gives robust and accurate solutions in most applications and allows transverse shear deformations. It provided closer results as S4 elements but with less memory space and time.

3.2. Mechanical Properties

The measured mechanical properties were used to enable the comparison of FEA and test results of cold-formed steel columns. The measured yield strength and elastic modulus of cold-formed steels used in this study are given in Table 1. Poisson's ratio of steel was assumed as 0.3. The elastic-perfect-plastic material model was used to simulate the behaviour

of cold-formed steel columns since the use of this simplified model did not affect the behaviour and ultimate load of cold-formed steel columns at ambient temperature.

3.3. Loading and Boundary Conditions

Both ends of the test columns were fixed against rotations and translations except that the bottom end was allowed to move axially. Hence in the numerical study also fixed support conditions were simulated and only axial translation at the bottom end was allowed as shown in Figure 5. These end boundary conditions allowed the specimens to fail symmetrically about the plane perpendicular to the axis of the column at mid-height. Due to the symmetry conditions of test specimen and loading, it is economical to simulate one half of the test columns in the analyses. Therefore a half-length model was used with appropriate boundary conditions as shown Figure 5. Rigid plate made of R3D4 elements was attached to the bottom end of the column, and all three rotations were restrained. This end of the column was restrained in the two major directions (UX and UY) while the axial compressive load was applied at the section centroid. At the other end of the model (ie. column mid-height), out of plane displacements were allowed but the axial displacement was restrained. In addition, twist rotation was allowed but other two rotations about X and Y axes were restrained.

3.4. Initial Geometric Imperfections and Residual Stresses

Cold-formed steel sections are likely to have initial geometric imperfections. Therefore a suitable initial geometric imperfection was included in the nonlinear analyses by introducing it to the appropriate buckling mode obtained from the bifurcation buckling analyses of cold-formed steel columns. Flexural-torsional buckling was observed as the first eigen mode in the elastic buckling analysis. Therefore this eigen mode was used to introduce the initial geometric imperfection with an appropriate amplitude. Since the expected buckling mode was flexural-torsional buckling, only global imperfections were measured in Bandula Heva and Mahendran [8]. These values are reported in Table 1. They were used in the validation of finite element models developed here. The effect of initial geometric imperfection amplitude on the ultimate capacity of cold-formed steel columns was further investigated using FEA. Table 3 shows the ultimate compression capacities obtained using different amplitudes of initial geometric imperfection. The use of $L/2000$ gave the best agreement with test results (mean of 0.99). However, $L/1000$ will be used as the initial global imperfection in the parametric study FEA as it will provide conservative results.

The residual stress is an important parameter influencing the axial compressive strength of cold-formed steel columns as this can cause premature yielding. A new set of residual stresses for lipped channel sections without the rounded corners was proposed in Ranawaka and Mahendran [12]. These residual stress values expressed as a percentage of the material yield stress (f_y) were used in the finite element models (see Figure 6).

3.5. Analyses

Two types of analyses, namely, bifurcation buckling and nonlinear analyses, were conducted using ABAQUS. The bifurcation buckling analyses were used to determine the elastic buckling loads and modes. The relevant buckling modes were then used to include the initial geometric imperfections in the nonlinear analyses. Finally the nonlinear analyses using the Riks On method were used to determine the ultimate loads of cold-formed steel columns.

3.6. Validation

It is important to validate the developed finite element model to determine whether it can simulate the desired buckling and ultimate behaviour and strength of cold-formed steel columns. In this research, the ultimate compression load, load-deflection curves and deflected shape from finite element analyses were compared with corresponding test results reported in [8].

Three deflection measurements were recorded in the tests. They were the out-of-plane deflections of web and flange elements and the axial shortening of test specimens. These deflections could also be obtained from finite element simulations. Figure 7 compares the load-shortening curves from finite element analyses and tests for some selected cases. Axial shortening in the tests was measured at the bottom end of the bottom loading shaft (Figure 2). Therefore the measured axial shortening in test curves included the shortening of loading shafts. Therefore the axial shortening in test curves is slightly higher than that in finite element analysis results. Having considered these facts, it can be concluded that the load-deflection curves from FEA and tests showed a reasonably good agreement.

Figures 8 and 9 compare the ultimate failure modes obtained from test and FEA. Test results [8] showed that columns failed by flexural-torsional buckling. Nonlinear finite element analyses also showed the same flexural-torsional buckling behaviour at failure. Figures 8 and 9 also show the stress components (von Mises) of tested cold-formed steel columns near

failure. The highest von Mises stress was equal to the material yield stress. This demonstrates that the ultimate loads of test columns have been achieved during the nonlinear FEA.

The developed finite element model was finally validated by comparing the ultimate loads obtained from tests and analyses. It was validated for different steel grades, thicknesses and lengths (Table 3). The mean value of Test/FEA ultimate load ratios is close to one while the associated coefficients of variation are also small as shown in Table 3. These comparisons show that the developed finite element model accurately predicts the ultimate capacities, load-deflection curves and failure modes of cold-formed steel long columns subjected to axial compression.

The validated finite element models were used thereafter in a parametric study to investigate the current design rules to predict the axial compressive strengths of cold-formed steel columns. This study included the effects of various parameters such as steel grade, steel thickness, section size, column lengths and end boundary conditions.

4. Current Design Rules

The unified approach developed by Pekoz [13] is used in the American Iron and Steel Institute Specification [14] and Australian/New Zealand Standard [2] for the design of cold-formed steel structural members.

Fixed-end supports provide restraints against major and minor axis rotations as well as twist rotations and warping. Hence fixed-ended columns failing by overall buckling were designed as concentrically loaded compression members using their effective length as equal to half the column length. In this case, the effective lengths for major and minor axis flexural buckling as well as torsional buckling are assumed to be equal to one half of the column length.

4.1. AISI [14] and AS/NZS 4600 [2] Design Rules

In this study, the design rules proposed by AISI [14] and AS/NZS 4600 [2] were considered in the design of concentrically loaded compression members. However, the design rules for compression members in AS/NZS 4600 [2] are identical to those in AISI [14], and hence only AS/NZS 4600 [2] design rules are mentioned in this paper.

According to AISI [14] and AS/NZS 4600 [2], the nominal axial strengths N_s (section capacity) and N_c (member capacity) of concentrically loaded compression members are calculated using Equations 1 (a) and (b), respectively.

$$N_s = A_{e1} f_y \quad (1a)$$

$$N_c = A_{e2} f_n \quad (1b)$$

where A_{e1} and A_{e2} are the effective areas calculated at the yield and critical stresses, respectively; f_y is the yield stress which is taken as the 0.2% proof stress; f_n is the critical stress, which accounts for overall instability, and is determined as,

$$f_n = (0.658^{\lambda_c^2}) f_y \text{ for } \lambda_c \leq 1.5 \quad (2a)$$

$$f_n = \left(\frac{0.877}{\lambda_c^2} \right) f_y \text{ for } \lambda_c > 1.5 \quad (2b)$$

where λ_c is the non-dimensional slenderness, given by,

$$\lambda_c = \sqrt{\frac{f_y}{f_{cre}}} \quad (3)$$

where f_{cre} is the minimum of elastic buckling stresses f_{ox} , f_{oy} , f_{oz} and f_{oxz} ; f_{ox} and f_{oy} are the elastic flexural buckling stresses about major and minor axes, respectively; f_{oz} and f_{oxz} are the elastic torsional and flexural-torsional buckling stresses, respectively. Further details about the current design rules can be found in AS/NZS 4600 [2] and Yu [15].

Popovic et al. [1] showed that the above procedure led to overly conservative P_n values due to the fact that the effect of torsional buckling stress comes into play twice (through f_n and A_e). In order to achieve more accurate ultimate strength predictions, they proposed to find f_n based on the flexural (minor-axis) buckling stress alone, and to find A_e based on the local (torsional) buckling stress. Popovic et al. [1] stated that ignoring the flexural-torsional buckling stress in computing the column strength does not imply that torsion is ignored in the design procedure. This is due to the fact that local buckling is used in determining the effective area and that the local mode is identical to the torsional mode at reducing lengths.

4.2. Young's [3] Design Rules

Young [3] tested fixed-ended angle columns and showed that the design method proposed by Popovic et al. [1] was conservative for short column lengths, but not conservative for intermediate and long column lengths. According to Young [3], the design method considering only the flexural buckling mode is reliable for non-slender sections, but not for slender sections. In order to obtain more accurate estimates, he proposed the use of a modified global strength curve, given by Equations 4(a) and 4(b). The design equations proposed by Young [3] require only small modifications to the current critical inelastic and elastic stress equations. In Equations (2a) and (2b), the values of 0.658 and 0.877 have been replaced with 0.5, and the non-dimensional slenderness λ_c has been adjusted to 1.4 for a smooth transition of elastic and inelastic buckling stresses.

$$f_n = \left(0.5^{\lambda_c^2}\right) f_y \text{ for } \lambda_c \leq 1.4 \quad (4a)$$

$$f_n = \left(\frac{0.5}{\lambda_c^2}\right) f_y \text{ for } \lambda_c > 1.4 \quad (4b)$$

where the non-dimensional slenderness λ_c is identical to that in Equation (3), except that the elastic buckling stress f_{cre} is calculated for minor-axis flexural buckling only.

In this study the existing design rules given in AS/NZS 4600 [2] and the equations proposed by Young [3] were checked for their accuracy in determining the member compression capacities of cold-formed steel columns. It should be noted that the plots obtained using Young's [3] design rules considers only the flexural buckling mode as suggested by Popovic et al. [1].

5. Effect of Warping Restraint on Elastic Buckling Loads

Table 4 presents the elastic buckling loads obtained from finite strip analyses (CUFSM) and finite element analyses (ABAQUS) and compares them with theoretical predictions. These buckling loads were obtained for 2800 mm long 55x35x9 lipped channel columns made of 0.95 mm thick steel. Pinned and fixed end boundary conditions with warping free and fixed cases were considered. Nodal compression loads were applied at one end of the column to simulate warping free, pinned and fixed end boundary conditions as shown in Figure 10. On the other hand, an axial compression load was applied to the reference node of the rigid body

(attached to one end) to simulate warping fixed, pinned and fixed end boundary conditions as shown in Figure 5.

Elastic buckling loads obtained from CUFSM, ABAQUS and theoretical formulae agreed well (4.08 and 4.06 kN) for pin-pin warping free boundary conditions when effective length factors were taken as 1.0 about all three axes in the theoretical formula, ie. $kL_1=kL_2=kL_3=L$. However, pin-pin warping fixed boundary condition had a significant impact on the effective length in torsion (kL_3). Elastic buckling load obtained from the theoretical formula (6.52 kN) assuming $kL_3=0.5L$ (fixed-fixed effective length in torsion) is very close to the elastic buckling load obtained from ABAQUS (6.48 kN) assuming pin-pin warping fixed boundary conditions. The results are slightly unconservative compared to FEA. These effective length observations are possibly due to the fact that the effective length for St. Venant torsion is not $0.5L$ while for warping torsion it is $0.5L$. The theoretical buckling load assuming $kL_3=0.654L$ matched well with ABAQUS prediction (6.48 kN) for this particular column.

Elastic buckling loads obtained from the selected methods (CUFSM, ABAQUS and theoretical formula) agreed well (11.68 and 11.81 kN) for fixed-fixed warping fixed boundary conditions when effective length factors were taken as 0.5 about all three axes in the theoretical formula, ie. $kL_1=kL_2=kL_3=0.5L$ (Table 4). Elastic buckling load obtained from the theoretical formula, assuming $kL_1=kL_2=0.5L$ (fixed-fixed effective length in flexure) and $kL_3=L$ (pin-pin effective length in torsion) is more conservative compared to that obtained from ABAQUS assuming fixed-fixed warping free boundary conditions (5.32 and 4.53 kN). However, the theoretical buckling load assuming $kL_3=0.875L$ matched well with ABAQUS for the selected column. Effective length observations here are possibly due to the fact that the effective length for St. Venant torsion is not L while the warping torsion is at a kL of L . It should be noted that the elastic buckling load obtained for fixed-fixed warping free boundary condition (5.32 kN) is lower than the elastic buckling load obtained for pin-pin warping fixed boundary conditions (6.48 kN), indicating the influence of warping fixity on cold-formed steel column stability.

In summary the following effective length factors are recommended for 0.95x55x35x9 lipped channel columns with four different end support conditions: (1) Pinned and Warping Free $kL_1=kL_2=kL_3=1.0L$ (2) Fixed and Warping Fixed $kL_1=kL_2=kL_3=0.5L$ (3) Pinned and Warping Fixed $kL_1=kL_2=L$; $kL_3=0.5L$ (4) Fixed and Warping Free $kL_1=kL_2=0.5L$; $kL_3=$

0.875L. For other lipped channel sections and lengths they will be identical for Cases (1) and (2), but for the other two cases similar effective length factors are expected.

For the flat-ended wall studs used in unlined cold-formed steel wall frame systems shown in Figure 1, their effective length factors about all three axes are recommended as 0.65 by Miller and Pekoz [16]. Telue and Mahendran [17,18] investigated this further using both tests and numerical analyses, and recommended that the effective length factor is a function of track to stud flexural rigidity ratio for unlined studs that are screw fastened to the top and bottom tracks. They showed that it varied from 0.62 to 1.0 with increasing track to stud flexural rigidity ratio. However, if the studs are welded to the top and bottom tracks instead of screw fastening, the effective length will reduce and approach the case of a fully fixed boundary condition (effective length of 0.5L about all three axes). When the effects of plasterboard lining were included in their tests and numerical analyses, Telue and Mahendran [19] found that the effective length factor about the major axis can still be taken as that of unlined studs, but recommended that the factors for the other two axes to be equal to the ratio of screw spacing to stud length. These findings by other researchers for columns with realistic end conditions agree reasonably well with those recommended in this paper.

6. Investigation of Pinned and Fixed Ended Columns

In order to investigate the behaviour of cold-formed steel columns, the developed finite element model described in Section 3 was used in a detailed parametric study. The rigid end plate (hence warping fixed) shown in Figure 5 was used to simulate the fixed end support conditions while nodal loads (hence warping free) were used in pinned ended columns to apply the axial compressive load as shown in Figure 10.

Instead of the measured thicknesses and initial imperfections, nominal thicknesses and standard imperfection values ($b/150$ for local buckling and $L/1000$ for global buckling) were used here. The mechanical properties used in this investigation are given in Table 5, which were obtained from Dolamune Kankanamge and Mahendran [20] and Ranawaka and Mahendran [21]. Other essential geometric parameters such as section sizes and specimen length were selected based on the range of parameters investigated.

6.1. Pinned Ended Columns (Warping Free)

Table 6 shows the parameters considered in the parametric study of pinned ended (warping free) columns. Pinned end boundary condition was investigated with five different steel grades and thicknesses for the lipped channel section 55x35x9. The specimen lengths were based on the preliminary analyses carried out using finite strip analyses (see Figure 11). Multiples of half wave lengths for local buckling were considered for shorter columns (138, 276, 552 and 1104 mm) while three additional lengths (1800, 2400 and 2800 mm) were considered for longer columns. Table 7 shows the calculated elastic buckling stresses and effective areas of pinned ended columns based on AS/NZS 4600 [2] design rules. Effective length l_e was taken as equal to the column length about all three axes in these calculations. According to AS/NZS 4600 [2], the effective areas at the yield and critical stresses were used to find the section and member capacities of a column, respectively. Hence the effective area at the critical stress ($A_e @ f_n$) increases with member length and is equal to the gross area (A_g) at longer lengths indicating pure global buckling, ie. no local and global buckling interactions.

Table 8 compares the predicted elastic buckling (P_{cr}) and ultimate loads (N_c and P_{ult}) from AS/NZS 4600 [2] and FEA. Elastic buckling loads from AS/NZS 4600 [2] as given in Table 8 were calculated by considering both flexural and flexural torsional buckling modes. However, the buckling load obtained from FEA considered both local and global buckling modes. In FEA, the local buckling load was predicted as 38 kN (compared to the squash load P_{sq} of 84 kN) and remained the same for up to a length of 552 mm for G550 0.95x55x35x9 steel column (see Figure 12 (a)). It should be noted that the calculated effective area at the critical stress was also not equal to the gross area as shown in Table 7. Hence a local buckling mode was observed in FEA as shown in Table 8. In this case, an amplitude of $b/150$ was used as the initial geometric imperfection with local buckling mode to obtain the ultimate load. The ultimate loads were higher than the elastic buckling loads indicating the presence of post-buckling strength for shorter columns. The flexural torsional buckling mode was observed in FEA for longer columns (1104 mm or more) as shown in Figure 12 (b) and the elastic buckling loads obtained from FEA and AS/NZS 4600 formulae agreed well (see Table 8). In this case, an amplitude of $L/1000$ was used as the initial geometric imperfection with the critical global buckling mode to obtain the ultimate load. The ultimate loads were either less or equal to the elastic buckling load for longer columns. Similar behaviour was observed for G250 0.95x55x35x9 and G500 1.15x55x35x9 steel columns. Table 8 compares the ultimate

loads from AS/NZS 4600 (N_c) and FEA (FEA/AS 4600 column), but also presents f_n/f_y and λ_c values for the purpose of comparing them directly with the column curve of f_n/f_y versus λ_c .

Elastic buckling loads obtained from FEA for short (up to 276 mm) G250 1.95 mm and G450 1.90 mm steel columns were much higher than the squash load, which indicates that these short columns will fail by yielding. Hence the ultimate loads obtained for these short columns from FEA were close to the squash load. The flexural torsional and flexural buckling modes as shown in Figures 12 (b) and (c) were observed for longer columns as expected (see Table 8). As seen in Table 8, none of the columns selected in this research failed by distortional buckling or local and global buckling interactions.

The ultimate load results obtained from finite element analyses for 55x35x9 lipped channel columns with pinned ends and warping free boundary conditions were compared with the predicted column curve based on Equations 2 and 3 of AS/NZS 4600 [2] in Figure 13 using the format of non-dimensionalised ultimate stress (f_n/f_y) versus slenderness (λ_c). The results for shorter lipped channel columns subjected to elastic local buckling failures are not plotted in this figure. Figure 13 shows that FEA results closely agreed with AS/NZS 4600 predictions for slender ($\lambda_c > 1.5$) columns subject to flexural and flexural torsional buckling modes.

6.2. Fixed Ended Columns (Warping Fixed)

Effects of fixed end (warping fixed) boundary conditions were investigated initially with the same lipped channel section (55x35x9) used for pinned ended case (see Table 6). The maximum column length of 6000 mm was considered to obtain slenderness ratios up to 5. Tables 9 and 10 show the calculated parameters including effective areas, elastic buckling stresses and section and member capacities based on AS/NZS 4600 design rules, and comparison with FEA results for fixed ended columns with a section size of 55x35x9. Elastic local buckling mode was observed up to 1000 mm lengths for thinner (G500 0.95, G250 0.95 and G500 1.15) steel columns and thereafter flexural torsional buckling mode was observed up to a length of 6000 mm. However, elastic buckling mode changed from flexural-torsional to flexural at 5000 mm for thicker steel (G250 1.95 and G450 1.90) columns. As seen in Tables 9 and 10, none of the columns selected in this research failed by distortional buckling or local and global buckling interactions. Tables 7 to 10 are presented in the same format as Tables 7 and 8, and hence the explanations provided for Tables 7 and 8 apply to all of them.

Figure 14 shows the comparison of ultimate loads of 55x35x9 lipped channel columns with fixed ends and warping fixity with the column curve based on Equations 2 and 3 of AS/NZS 4600 [2] in the non-dimensionalised ultimate stress (f_n/f_y) versus slenderness (λ_c) format. Experimental results from Bandula Heva and Mahendran [8] are also plotted in this figure. However, the results for shorter lipped channel columns subjected to elastic local buckling failures are not plotted in this figure. Figure 14 shows that the curve given in AS/NZS 4600 [2] is very conservative for fixed ended columns compared to pinned ended columns (see Figure 13). This is due to the fact that the column curve given in AS/NZS 4600 [2] does not take into account the warping fixity of fixed ended columns.

The design rules proposed by Popovic et al. [1] and Young [3] were also used to predict the ultimate loads of these fixed ended cold-formed steel columns. Based on their research, Popovic et al. [1] recommended that flexural-torsional buckling mode should be excluded from the design procedure and to consider only the minor axis flexural buckling. Young [3] used Popovic et al.'s [1] recommendation and proposed new column curves as defined by Equations 4(a) and (b). This design procedure was used to plot the ultimate load results from our research in Figure 15 for comparison with Young's column curve. Figure 15 shows that the design rules proposed by Popovic et al. [1] and Young [3] do not accurately predict the axial compression capacities of cold-formed steel lipped channel section columns. Young's [3] design rules are still conservative for long columns. Hence there is a need to develop a new set of equations for fixed ended columns with warping fixity.

6.3. Pinned and Fixed Ended Columns with Varying Warping Restraint

Effects of warping fixity were further investigated for columns with pinned and fixed end conditions. Figure 16 compares the ultimate loads of pinned and fixed ended columns with warping free and fixed cases (total of four cases) with the column curve in AS/NZS 4600 in the non-dimensionalised ultimate stress (f_n/f_y) versus slenderness (λ_c) format. This comparison shows that the ultimate loads of pinned end columns follow the AS/NZS 4600 curve irrespective of their warping boundary conditions. The ultimate loads of fixed ended columns with warping free boundary conditions also follow the AS/NZS 4600 curve quite well. However, fixed ended columns with warping fixed boundary conditions showed the presence of post buckling strength due to warping fixity and hence their ultimate load results do not agree with the AS/NZS 4600 column curve.

7. Proposed Design Rules

There are mainly two regions in the column curve of AS/NZS 4600. The first region is for shorter and intermediate columns (where λ_c is less than or equal to 1.5) which is defined by Equation 2 (a) in AS/NZS 4600 [2]. Last section has shown that the current AS/NZS 4600 column curve is too conservative for slender columns with fixed ends and warping fixity. Therefore this paper investigates the second region for slender columns (where λ_c is greater than 1.5) defined by Equation 2 (b), and proposes a new set of equations.

Figure 17 shows the ultimate loads of fixed ended (warping fixed) long columns in the form of non-dimensionalised ultimate stress versus slenderness curves. The ultimate loads of these columns plotted higher compared to the curve given in AS/NZS 4600 [2]. A closer look at the results indicated that the ultimate loads of columns subjected to flexural buckling agreed well with the column curve given in AS/NZS 4600 [2] as shown in Figure 17. On the other hand the ultimate capacities of fixed ended columns subjected to flexural-torsional buckling were much higher (up to 88% more) compared to the predictions of AS/NZS 4600 [2]. The reason for this is that the warping rigidity of fixed ended columns will be useful only when the columns are subjected to flexural-torsional buckling and not when they are subjected to flexural buckling. Hence a new set of equations was proposed (Equations 5 (a) and (b)) for fixed ended columns with warping fixity when they are subjected to flexural-torsional buckling. It should be noted that the aim of this paper is to modify the curve for longer columns ($\lambda_c > 1.5$). However the equation for shorter columns ($\lambda_c \leq 1.5$) is also modified in Equation 5 (a) to provide continuity for the full design capacity curve. Figure 17 shows a close agreement of test and FEA results with the proposed equations.

$$f_n = (0.69^{\lambda_c^2}) f_y \text{ for } \lambda_c \leq 1.5 \quad (5a)$$

$$f_n = \left(\frac{0.8}{\lambda_c^{1.5}} \right) f_y \text{ for } \lambda_c > 1.5 \quad (5b)$$

The behaviour and strength of fixed ended (warping fixed) columns subjected to flexural and flexural-torsional buckling was further investigated using two other lipped channel section sizes, 75x50x15 and 90x40x15, to confirm the findings obtained for Section 55x35x9. In order to avoid any local and global buckling interactions, only the thicker steel sections and longer lengths were considered for Sections 75x50x15 and 90x40x15. Tables 11 and 12 show

the calculated parameters and comparisons with FEA results for the fixed ended columns with these section sizes. Figure 18 uses the ultimate load results in Tables 11 and 12 for these two lipped channel sections and compares them with the predictions based on AS/NZS 4600 and the proposed design rules. As expected, the ultimate loads of columns with fixed ends and warping fixity agreed well with AS/NZS 4600 [2] column curve if the columns were subject to flexural buckling. On the other hand, if they were subjected to flexural torsional buckling they achieved higher ultimate capacities, which agreed well with the proposed equations (Eq. 5(b)) in this paper. In the case of lipped channel studs used in cold-formed steel wall frame systems (Figure 1), their compression strengths in the case of flexural torsional buckling should be calculated using Equations 5a and 5b if the studs are rigidly connected to the top and bottom tracks.

Capacity Reduction Factor

The American cold-formed steel structures code [14] recommends a statistical model to determine the capacity reduction factors. This model accounts for the variations in material, fabrication and the loading effects. The capacity reduction factor Φ is given by Equation 6 (a).

$$\Phi = 1.52 M_m F_m P_m e^{-\beta_0 \sqrt{V_m^2 + V_f^2 + C_p V_p^2 + V_q^2}} \quad (6a)$$

where, M_m , V_m = Mean and coefficient of variation of the material factor = 1.1, 0.1

F_m , V_f = Mean and coefficient of variation of the fabrication factor = 1, 0.05

V_q = coefficient of variation of load effect = 0.21

β_0 = Target reliability index = 2.5

$$C_p = \text{Correction factor depending on the number of tests} = \left(1 + \frac{1}{n}\right) \left(\frac{m}{m-2}\right)$$

P_m = mean value of the tested to predicted load ratio

V_p = Coefficient of variation of the tested to predicted load ratio

n = Number of tests

m = Degree of freedom = $n-1$

V_p and P_m values have to be determined from experiments or analyses. In this investigation ultimate loads obtained from FEA were considered. Hence V_p and P_m are the mean and coefficient of variation of the ratio of ultimate loads from FEA and design standards.

The substitution of all the above values leads to the following equation.

$$\Phi = 1.672P_m e^{-2.5\sqrt{0.0566+C_p V_p^2}} \quad (6b)$$

Equation 6 (b) was used to determine the capacity reduction factors for the values obtained from AS/NZS 4600 [2] and the proposed design rules. AS/NZS 4600 [2] recommends a capacity reduction factor of 0.85 for compression members. Table 13 compares the FEA results with existing and proposed equations for fixed ended columns subjected to flexural-torsional buckling. The mean value and the capacity reduction factor according to AS/NZS 4600 [2] Equations 2 (a) and (b) is 1.39 and 1.10, respectively, which emphasize the need for improved design rules. The mean value and the capacity reduction factor according to the proposed equations (5a and 5b) are 0.99 and 0.89, respectively. In order to achieve a capacity reduction factor of 0.85, a second set of equations is also proposed in this paper (Equations 7(a) and 7(b)). However, it should be noted that the mean values in this case are considerably low for very slender columns as seen in Table 13. Hence Equations 5(a) and (b) are recommended.

$$f_n = (0.7^{\lambda_c^2}) f_y \text{ for } \lambda_c \leq 1.5 \quad (7a)$$

$$f_n = \left(\frac{0.8}{\lambda_c^{1.45}} \right) f_y \text{ for } \lambda_c > 1.5 \quad (7b)$$

8. Conclusions

This research has presented the details of an investigation on the behaviour and design of cold-formed steel columns subject to flexural and flexural-torsional buckling. Elastic buckling and ultimate capacities of pinned and fixed ended columns subject to flexural and flexural-torsional buckling were evaluated in detail based on experimental and numerical studies and by comparison with the current design rules given in Australian and American design standards for cold-formed steel structures. Current design rules in AS/NZS 4600 and North American Specification were able to predict the member capacities of pinned and fixed ended columns subjected to flexural or flexural torsional buckling, but not in the case of columns with fixed ends and warping fixity subjected to flexural-torsional buckling. Hence improved design rules were proposed for this case. The accuracy of the proposed design rules was verified using the available test and FEA results of cold-formed lipped channel columns made of five different steel grades and thicknesses. Three different cross-sections were also

considered in this study with varying column lengths. The agreement between the ultimate loads from tests, FEA and proposed design rules was very good. This paper has provided good improvements to the current member capacity design rules for cold-formed steel columns with fixed ends.

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Table 1: Measured Dimensions and Mechanical Properties of Test Specimens

Lipped Channels	f_y (MPa)	E (MPa)	Measured Dimensions (mm)					Imperf.
			Web	Flange	Lip	Thickness	Length	
G550x0.95x55x1800	615	205000	54.94	34.88	8.00	0.95	1740	L/2485
G550x0.95x55x2800	615	205000	54.82	34.78	7.82	0.95	2820	L/1446
G250x1.95x75x1800	271	188000	74.82	50.06	14.87	1.95	1740	L/2558
G250x1.95x75x2800	271	188000	74.97	49.88	14.69	1.95	2820	L/4086
G450x1.90x75x1800	515	206000	74.67	49.94	14.51	1.88	1740	L/2949
G450x1.90x75x2800	515	206000	74.27	49.78	14.92	1.88	2820	L/2073

Table 2: Comparison of Ultimate Loads from Tests and AS/NZS 4600 [2]

Lipped Channels	Test (kN)	AS/NZS 4600 (kN)	<u>Test</u> AS/NZS 4600
G550x0.95x55x1800	24.7	20.1	1.23
G550x0.95x55x2800	15.9	9.0	1.77
G250x1.95x75x1800	87.9	76.2	1.15
G250x1.95x75x2800	54.1	49.4	1.10
G450x1.90x75x1800	120.4	107.4	1.12
G450x1.90x75x2800	61.3	53.5	1.15

Table 3: Comparison of Ultimate Loads from Tests and FEA

Lipped Channels	Test (kN)	FEA EBL (kN)	FEA Ultimate Load (kN)			Test/FEA		
			Measured Imperf.	L/2000	L/1000	Measured Imperf.	L/2000	L/1000
G550x0.95x55x1800	24.7	24.2	25.6	25.5	25.3	0.96	0.97	0.98
G550x0.95x55x2800	15.9	10.2	15.4	15.4	15.3	1.03	1.03	1.04
G250x1.95x75x1800	87.9	134.4	90.6	89.3	84.1	0.97	0.98	1.05
G250x1.95x75x2800	54.1	57.1	55.6	53.9	51.4	0.97	1.00	1.05
G450x1.90x75x1800	120.4	139.0	128.0	125.0	117.0	0.94	0.96	1.03
G450x1.90x75x2800	61.3	59.4	63.8	63.7	62.8	0.96	0.96	0.98
Mean						0.97	0.99	1.02
COV						0.03	0.03	0.03

Note: EBL – Elastic Buckling Load

Table 4: Elastic Buckling Loads of 0.95x55x2800 Cold-formed Steel Columns

Method	Pin-Pin		Fixed-Fixed	
	Warping Free	Warping Fixed	Warping Free	Warping Fixed
CUFSM	4.07 kN	-	-	11.83 kN
ABAQUS	4.08 kN	6.48 kN	5.32 kN	11.68 kN
THEORY	4.06 kN $kL_1 = kL_2 = L$; $kL_3 = L$	6.48 kN $kL_1 = kL_2 = L$; $kL_3 = 0.654L$	5.32 kN $kL_1 = kL_2 = 0.5L$; $kL_3 = 0.875L$	11.81 kN $kL_1 = kL_2 = 0.5L$; $kL_3 = 0.5L$
		6.52 kN $kL_1 = kL_2 = L$; $kL_3 = 0.5L$	4.53 kN $kL_1 = kL_2 = 0.5L$; $kL_3 = L$	

Note:

0.95x55x2800 Column cross-section dimensions are 55x35x9 mm

kL_1 : Effective length for flexural buckling about the major (x-x) axis

kL_2 : Effective length for flexural buckling about the minor (y-y) axis

kL_3 : Effective length for torsional buckling (z-z) axis

Table 5: Mechanical Properties of Cold-formed Steel Columns

Grade x Thickness (mm)	f_y (MPa)	E (MPa)
G550x0.95	615	205000
G250x0.95	320	200000
G500x1.15	569	213520
G250x1.95	271	188000
G450x1.90	515	206000

Table 6: Parameters Used in the Investigation

Boundary Condition	Section Web x Flange x Lip (mm x mm x mm)	Grade (MPa) x Thickness (mm)	Lengths (mm)
Pinned	55x35x9	G550x0.95	138, 276, 552, 1104, 1800, 2400, 2800
		G250x0.95	138, 276, 552, 1104, 1800, 2400, 2800
		G500x1.15	138, 276, 552, 1104, 1800, 2400, 2800
		G250x1.95	138, 276, 552, 1104, 1800, 2400, 2800
		G450x1.90	138, 276, 552, 1104, 1800, 2400, 2800
Fixed	55x35x9	G550x0.95	300, 1000, 1800, 2400, 2800, 4000, 5000, 6000
		G250x0.95	300, 1000, 1800, 2400, 2800, 4000, 5000, 6000
		G500x1.15	300, 1000, 1800, 2400, 2800, 4000, 5000, 6000
		G250x1.95	300, 1000, 1800, 2400, 2800, 4000, 5000, 6000
		G450x1.90	300, 1000, 1800, 2400, 2800, 4000, 5000, 6000
	75x50x15	G250x1.95	1800, 2400, 2800, 4000, 5000, 6000
		G450x1.90	1800, 2400, 2800, 4000, 5000, 6000
	90x40x15	G250x1.95	2800, 4000, 5000, 6000
		G450x1.90	2800, 4000, 5000, 6000

**Table 7: Calculated Parameters for Pinned Ended 55x35x9 Lipped Channel Columns
based on AS/NZS 4600 [2]**

Lipped Channels	A_g (mm ²)	l_e (mm)	Min. (f_{ox} & f_{oy}) (MPa)	f_{oxz} (MPa)	f_{oc} (MPa)	λ_c	f_n (MPa)	A_e @ f_n (mm ²)	A_e @ f_y (mm ²)
G550x0.95x55x138	136	138	19744	7771	7771	0.28	595	79	77
G550x0.95x55x276	136	276	4936	1952	1952	0.56	539	82	77
G550x0.95x55x552	136	552	1234	497	497	1.11	366	97	77
G550x0.95x55x1104	136	1104	309	133	133	2.15	117	134	77
G550x0.95x55x1800	136	1800	116	57	57	3.28	50	136	77
G550x0.95x55x2400	136	2400	65	37	37	4.08	32	136	77
G550x0.95x55x2800	136	2800	48	30	30	4.54	26	136	77
G250x0.95x55x138	136	138	19263	7581	7581	0.21	314	102	102
G250x0.95x55x276	136	276	4816	1904	1904	0.41	298	104	102
G250x0.95x55x552	136	552	1204	485	485	0.81	243	113	102
G250x0.95x55x1104	136	1104	301	130	130	1.57	114	134	102
G250x0.95x55x1800	136	1800	113	56	56	2.40	49	136	102
G250x0.95x55x2400	136	2400	64	36	36	2.98	32	136	102
G250x0.95x55x2800	136	2800	47	29	29	3.31	26	136	102
G500x1.15x55x138	164	138	20567	8100	8100	0.27	553	110	109
G500x1.15x55x276	164	276	5142	2038	2038	0.53	506	114	109
G500x1.15x55x552	164	552	1285	523	523	1.04	361	131	109
G500x1.15x55x1104	164	1104	321	144	144	1.99	126	164	109
G500x1.15x55x1800	164	1800	121	65	65	2.96	57	164	109
G500x1.15x55x2400	164	2400	68	43	43	3.62	38	164	109
G500x1.15x55x2800	164	2800	50	36	36	3.98	32	164	109
G250x1.95x55x138	279	138	18119	7163	7163	0.19	267	279	279
G250x1.95x55x276	279	276	4530	1825	1825	0.39	255	279	279
G250x1.95x55x552	279	552	1132	490	490	0.74	215	279	279
G250x1.95x55x1104	279	1104	283	155	155	1.32	130	279	279
G250x1.95x55x1800	279	1800	107	83	83	1.81	72	279	279
G250x1.95x55x2400	279	2400	60	61	60	2.13	53	279	279
G250x1.95x55x2800	279	2800	44	52	44	2.48	39	279	279
G450x1.90x55x138	272	138	19853	7846	7846	0.26	501	245	243
G450x1.90x55x276	272	276	4963	1997	1997	0.51	462	251	243
G450x1.90x55x552	272	552	1241	534	534	0.98	344	272	243
G450x1.90x55x1104	272	1104	310	167	167	1.75	147	272	243
G450x1.90x55x1800	272	1800	117	88	88	2.41	78	272	243
G450x1.90x55x2400	272	2400	66	65	65	2.81	57	272	243
G450x1.90x55x2800	272	2800	48	56	48	3.27	42	272	243

**Table 8: Comparison of Results from FEA and AS/NZS 4600
for Pinned Ended 55x35x9 Lipped Channel Columns**

Lipped Channels	AS/NZS 4600				FEA				FEA/ AS4600	λ_c	f_n/f_y
	P_{cr} (kN)	N_s (kN)	N_c (kN)	P_{sq} (kN)	P_{cr} (kN)	EB Mode	Imp. (mm)	P_{ult} (kN)			
G550x0.95x55x138	1056	47.6	46.8	83.5	38.0	L	0.367	46.4	0.99	1.48	0.56
G550x0.95x55x276	265	47.6	44.3	83.5	39.0	L	0.367	45.7	1.03	1.46	0.55
G550x0.95x55x552	67.5	47.6	35.6	83.5	38.5	L	0.367	46.1	1.29	1.47	0.55
G550x0.95x55x1104	18.1	47.6	15.6	83.5	18.0	FT	1.104	16.1	1.03	2.15	0.19
G550x0.95x55x1800	7.75	47.6	6.80	83.5	7.78	FT	1.80	7.52	1.11	3.28	0.09
G550x0.95x55x2400	5.01	47.6	4.39	83.5	5.03	FT	2.40	5.00	1.14	4.07	0.06
G550x0.95x55x2800	4.06	47.6	3.56	83.5	4.08	FT	2.80	4.00	1.12	4.52	0.05
G250x0.95x55x138	1030	32.5	32.2	43.5	37.0	L	0.367	30.3	0.94	1.08	0.70
G250x0.95x55x276	259	32.5	31.1	43.5	38.0	L	0.367	29.9	0.96	1.07	0.69
G250x0.95x55x552	65.83	32.5	27.3	43.5	37.6	L	0.367	29.8	1.09	1.08	0.68
G250x0.95x55x1104	17.61	32.5	15.2	43.5	17.6	FT	1.104	15.5	1.02	1.57	0.36
G250x0.95x55x1800	7.56	32.5	6.63	43.5	7.59	FT	1.80	7.16	1.08	2.39	0.16
G250x0.95x55x2400	4.89	32.5	4.29	43.5	4.91	FT	2.40	4.75	1.11	2.97	0.11
G250x0.95x55x2800	3.96	32.5	3.47	43.5	3.98	FT	2.80	3.82	1.10	3.30	0.09
G500x1.15x55x138	1332	61.9	60.9	93.6	69.8	L	0.367	63.4	1.04	1.16	0.68
G500x1.15x55x276	335	61.9	57.8	93.6	71.7	L	0.367	63.0	1.09	1.14	0.67
G500x1.15x55x552	86.0	61.9	47.1	93.6	70.9	L	0.367	63.0	1.34	1.15	0.67
G500x1.15x55x1104	23.7	61.9	20.8	93.6	23.7	FT	1.104	21.5	1.04	1.99	0.23
G500x1.15x55x1800	10.7	61.9	9.34	93.6	10.7	FT	1.80	10.4	1.11	2.96	0.11
G500x1.15x55x2400	7.15	61.9	6.27	93.6	7.18	FT	2.40	6.98	1.11	3.61	0.07
G500x1.15x55x2800	5.92	61.9	5.19	93.6	5.94	FT	2.80	5.69	1.10	3.97	0.06
G250x1.95x55x138	1997	75.6	74.4	75.6	251	Y	0.367	73.1	0.98	0.55	0.97
G250x1.95x55x276	509	75.6	71.0	75.6	227	Y	0.367	74.5	1.05	0.58	0.99
G250x1.95x55x552	137	75.6	59.9	75.6	133	FT	0.552	65.9	1.10	0.75	0.87
G250x1.95x55x1104	43.2	75.6	36.3	75.6	43.1	FT	1.104	37.8	1.04	1.32	0.50
G250x1.95x55x1800	23.0	75.6	20.2	75.6	23.0	FT	1.80	21.4	1.06	1.81	0.28
G250x1.95x55x2400	16.7	75.6	14.7	75.6	16.6	F	2.40	15.9	1.09	2.13	0.21
G250x1.95x55x2800	12.3	75.6	10.8	75.6	12.2	F	2.80	11.8	1.10	2.49	0.16
G450x1.90x55x138	2132	125	123	140	262	Y	0.367	129	1.05	0.73	0.92
G450x1.90x55x276	543	125	116	140	231	Y	0.367	135	1.16	0.78	0.96
G450x1.90x55x552	145	125	93.5	140	141	FT	0.552	105	1.12	1.00	0.75
G450x1.90x55x1104	45.5	125	39.9	140	45.4	FT	1.104	42.5	1.07	1.76	0.30
G450x1.90x55x1800	24.0	125	21.1	140	24.0	FT	1.80	23.0	1.09	2.42	0.16
G450x1.90x55x2400	17.7	125	15.5	140	17.6	FT	2.40	16.1	1.04	2.82	0.11
G450x1.90x55x2800	13.1	125	11.5	140	13.0	F	2.80	13.2	1.15	3.28	0.09

Note : L – Local buckling; F, FT – Flexural, Flexural-torsional buckling; Y – Yielding

**Table 9: Calculated Parameters for Fixed Ended 55x35x9 Lipped Channel Columns
based on AS/NZS 4600 [2]**

Lipped Channels	A_g (mm ²)	I_e (mm)	Min. (f_{ox} & f_{oy}) (MPa)	f_{oxz} (MPa)	f_{oc} (MPa)	λ_c	f_n (MPa)	A_e @ f_n (mm ²)	A_e @ f_y (mm ²)
G550x0.95x55x300	136	300	16712	6579	6579	0.31	591	79	77
G550x0.95x55x1000	136	1000	1504	603	603	1.01	401	94	77
G550x0.95x55x1800	136	1800	464	194	194	1.78	170	128	77
G550x0.95x55x2400	136	2400	261	114	114	2.32	100	136	77
G550x0.95x55x2800	136	2800	192	87	87	2.66	76	136	77
G550x0.95x55x4000	136	4000	94	48	48	3.57	42	136	77
G550x0.95x55x5000	136	5000	60	35	35	4.20	31	136	77
G550x0.95x55x6000	136	6000	42	27	27	4.74	24	136	77
G250x0.95x55x300	136	300	16304	6419	6419	0.22	313	102	102
G250x0.95x55x1000	136	1000	1467	588	588	0.74	255	111	102
G250x0.95x55x1800	136	1800	453	189	189	1.30	158	129	102
G250x0.95x55x2400	136	2400	255	111	111	1.69	98	136	102
G250x0.95x55x2800	136	2800	187	85	85	1.94	74	136	102
G250x0.95x55x4000	136	4000	92	47	47	2.61	41	136	102
G250x0.95x55x5000	136	5000	59	34	34	3.07	30	136	102
G250x0.95x55x6000	136	6000	41	27	27	3.46	23	136	102
G500x1.15x55x300	164	300	17408	6858	6858	0.29	550	110	109
G500x1.15x55x1000	164	1000	1567	634	634	0.95	391	126	109
G500x1.15x55x1800	164	1800	484	208	208	1.65	182	162	109
G500x1.15x55x2400	164	2400	272	125	125	2.14	109	164	109
G500x1.15x55x2800	164	2800	200	96	96	2.43	84	164	109
G500x1.15x55x4000	164	4000	98	56	56	3.20	49	164	109
G500x1.15x55x5000	164	5000	63	41	41	3.71	36	164	109
G500x1.15x55x6000	164	6000	44	33	33	4.14	29	164	109
G250x1.95x55x300	279	300	15336	6070	6070	0.21	266	279	279
G250x1.95x55x1000	279	1000	1380	587	587	0.68	223	279	279
G250x1.95x55x1800	279	1800	426	212	212	1.13	159	279	279
G250x1.95x55x2400	279	2400	240	137	137	1.40	119	279	279
G250x1.95x55x2800	279	2800	176	112	112	1.56	98	279	279
G250x1.95x55x4000	279	4000	86	74	74	1.92	64	279	279
G250x1.95x55x5000	279	5000	55	58	55	2.22	48	279	279
G250x1.95x55x6000	279	6000	38	49	38	2.66	34	279	279
G450x1.90x55x300	272	300	16804	6648	6648	0.28	499	245	243
G450x1.90x55x1000	272	1000	1512	641	641	0.90	368	272	243
G450x1.90x55x1800	272	1800	467	229	229	1.50	201	272	243
G450x1.90x55x2400	272	2400	263	148	148	1.86	130	272	243
G450x1.90x55x2800	272	2800	193	120	120	2.07	105	272	243
G450x1.90x55x4000	272	4000	95	79	79	2.56	69	272	243
G450x1.90x55x5000	272	5000	60	62	60	2.92	53	272	243
G450x1.90x55x6000	272	6000	42	52	42	3.50	37	272	243

**Table 10: Comparison of Results from FEA and AS/NZS 4600
for Fixed Ended 55x35x9 Lipped Channel Columns**

Index	AS/NZS 4600				FEA						
	P _{cr} (kN)	N _s (kN)	N _c (kN)	P _{sq} (kN)	P _{cr} (kN)	EB Mode	Imp. (mm)	P _{ult} (kN)	FEA/ AS4600	λ_c	f _n /f _y
G550x0.95x55x300	894	47.6	46.6	83.5	39.3	L	0.367	55.9	1.20	1.46	0.67
G550x0.95x55x1000	81.9	47.6	37.6	83.5	39.4	L	0.367	45.9	1.22	1.46	0.55
G550x0.95x55x1800	26.4	47.6	21.8	83.5	25.8	FT	1.80	26.4	1.21	1.80	0.32
G550x0.95x55x2400	15.5	47.6	13.6	83.5	15.3	FT	2.40	19.4	1.43	2.34	0.23
G550x0.95x55x2800	11.8	47.6	10.4	83.5	11.7	FT	2.80	16.6	1.60	2.67	0.20
G550x0.95x55x4000	6.56	47.6	5.76	83.5	6.51	FT	4.00	10.8	1.88	3.58	0.13
G550x0.95x55x5000	4.73	47.6	4.15	83.5	4.69	FT	5.00	7.61	1.83	4.22	0.09
G550x0.95x55x6000	3.71	47.6	3.26	83.5	3.69	FT	6.00	5.53	1.70	4.76	0.07
G250x0.95x55x300	872	32.5	32.1	43.5	38.4	L	0.367	35.6	1.11	1.06	0.82
G250x0.95x55x1000	79.9	32.5	28.2	43.5	38.4	L	0.367	34.5	1.22	1.06	0.79
G250x0.95x55x1800	25.7	32.5	20.3	43.5	25.1	FT	1.80	22.0	1.08	1.32	0.51
G250x0.95x55x2400	15.1	32.5	13.3	43.5	14.9	FT	2.40	15.3	1.15	1.71	0.35
G250x0.95x55x2800	11.5	32.5	10.1	43.5	11.4	FT	2.80	12.9	1.28	1.95	0.30
G250x0.95x55x4000	6.40	32.5	5.62	43.5	6.35	FT	4.00	9.11	1.62	2.62	0.21
G250x0.95x55x5000	4.61	32.5	4.05	43.5	4.58	FT	5.00	6.93	1.71	3.08	0.16
G250x0.95x55x6000	3.62	32.5	3.18	43.5	3.60	FT	6.00	5.25	1.65	3.48	0.12
G500x1.15x55x300	1128	61.9	60.7	93.6	72.3	L	0.367	73.3	1.21	1.14	0.78
G500x1.15x55x1000	104	61.9	49.2	93.6	72.4	L	0.367	68.5	1.39	1.14	0.73
G500x1.15x55x1800	34.2	61.9	29.4	93.6	33.5	FT	1.80	33.6	1.14	1.67	0.36
G500x1.15x55x2400	20.5	61.9	18.0	93.6	20.2	FT	2.40	24.8	1.38	2.15	0.27
G500x1.15x55x2800	15.8	61.9	13.9	93.6	15.6	FT	2.80	21.4	1.54	2.45	0.23
G500x1.15x55x4000	9.14	61.9	8.02	93.6	9.06	FT	4.00	14.0	1.75	3.21	0.15
G500x1.15x55x5000	6.79	61.9	5.95	93.6	6.73	FT	5.00	9.87	1.66	3.73	0.11
G500x1.15x55x6000	5.46	61.9	4.79	93.6	5.42	FT	6.00	7.11	1.48	4.16	0.08
G250x1.95x55x300	1693	75.6	74.2	75.6	289	Y	0.001	74.6	1.01	0.51	0.99
G250x1.95x55x1000	164	75.6	62.3	75.6	156	FT	0.367	68.1	1.09	0.70	0.90
G250x1.95x55x1800	59.0	75.6	44.2	75.6	58.1	FT	1.80	47.9	1.08	1.14	0.63
G250x1.95x55x2400	38.3	75.6	33.1	75.6	37.8	FT	2.40	35.1	1.06	1.41	0.46
G250x1.95x55x2800	31.1	75.6	27.3	75.6	30.7	FT	2.80	29.9	1.10	1.57	0.40
G250x1.95x55x4000	20.5	75.6	18.0	75.6	20.2	FT	4.00	20.7	1.15	1.93	0.27
G250x1.95x55x5000	15.4	75.6	13.5	75.6	15.3	F	5.00	14.3	1.06	2.22	0.19
G250x1.95x55x6000	10.7	75.6	9.38	75.6	10.6	F	6.00	10.1	1.08	2.66	0.13
G450x1.90x55x300	1806	125	122	140	299	Y	0.001	134	1.10	0.68	0.96
G450x1.90x55x1000	174	125	100	140	166	FT	0.367	113	1.13	0.92	0.81
G450x1.90x55x1800	62.3	125	54.7	140	61.4	FT	1.80	59.6	1.09	1.51	0.43
G450x1.90x55x2400	40.3	125	35.3	140	39.7	FT	2.40	43.4	1.23	1.88	0.31
G450x1.90x55x2800	32.6	125	28.6	140	32.2	FT	2.80	37.3	1.30	2.08	0.27
G450x1.90x55x4000	21.4	125	18.7	140	21.1	FT	4.00	24.2	1.29	2.58	0.17
G450x1.90x55x5000	16.4	125	14.4	140	16.4	F	5.00	15.8	1.10	2.92	0.11
G450x1.90x55x6000	11.4	125	10.0	140	11.4	F	6.00	11.1	1.11	3.51	0.08

Note : L – Local buckling; F, FT – Flexural, Flexural-torsional buckling; Y – Yielding

Table 11: Calculated Parameters for Fixed Ended 75x50x15 and 90x40x15 Lipped Channel Columns based on AS/NZS 4600 [2]

Lipped Channels	A_g (mm ²)	l_e (mm)	Min. (f_{ox} & f_{oy}) (MPa)	f_{oxz} (MPa)	f_{oc} (MPa)	λ_c	f_n (MPa)	A_e @ f_n (mm ²)	A_e @ f_y (mm ²)
G250x1.95x75x1800	400	1800	897	364	364	0.86	198	400	389
G250x1.95x75x2400	400	2400	505	214	214	1.13	159	400	389
G250x1.95x75x2800	400	2800	371	162	162	1.29	135	400	389
G250x1.95x75x4000	400	4000	182	90	90	1.74	79	400	389
G250x1.95x75x5000	400	5000	116	64	64	2.06	56	400	389
G250x1.95x75x6000	400	6000	81	50	50	2.33	44	400	389
G450x1.90x75x1800	390	1800	983	397	397	1.14	299	377	294
G450x1.90x75x2400	390	2400	553	233	233	1.49	204	390	294
G450x1.90x75x2800	390	2800	406	177	177	1.71	155	390	294
G450x1.90x75x4000	390	4000	199	97	97	2.30	85	390	294
G450x1.90x75x5000	390	5000	127	69	69	2.73	61	390	294
G450x1.90x75x6000	390	6000	89	54	54	3.09	47	390	294
G250x1.95x90x2800	390	2800	244	190	190	1.19	149	389	359
G250x1.95x90x4000	390	4000	119	107	107	1.59	94	390	359
G250x1.95x90x5000	390	5000	76	78	76	1.88	67	390	359
G250x1.95x90x6000	390	6000	53	62	53	2.26	47	390	359
G450x1.90x90x2800	380	2800	267	207	207	1.58	181	371	314
G450x1.90x90x4000	380	4000	131	116	116	2.11	102	380	314
G450x1.90x90x5000	380	5000	84	84	84	2.48	73	380	314
G450x1.90x90x6000	380	6000	58	67	58	2.98	51	380	314

**Table 12: Comparison of Results from FEA and AS/NZS 4600
for Fixed Ended 75x50x15 and 90x40x15 Lipped Channel Columns**

Lipped Channels	AS/NZS 4600				FEA						
	P _{cr} (kN)	N _s (kN)	N _c (kN)	N _{sq} (kN)	P _{cr} (kN)	EB Mode	Imp. (mm)	P _{ult} (kN)	FEA/ AS4600	λ_c	f _n /f _y
G250x1.95x75x1800	145	106	79.3	108	141	FT	1.80	88	1.10	0.88	0.81
G250x1.95x75x2400	85.4	106	63.7	108	84.1	FT	2.40	68	1.07	1.13	0.63
G250x1.95x75x2800	64.9	106	53.8	108	64.2	FT	2.80	57	1.05	1.30	0.52
G250x1.95x75x4000	35.8	106	31.4	108	35.5	FT	4.00	37	1.17	1.75	0.34
G250x1.95x75x5000	25.6	106	22.5	108	25.4	FT	5.00	29	1.30	2.06	0.27
G250x1.95x75x6000	20.0	106	17.6	108	19.8	FT	6.00	24	1.37	2.34	0.22
G450x1.90x75x1800	155	152	113	201	150	FT	1.80	125	1.11	1.16	0.62
G450x1.90x75x2400	90.7	152	79.5	201	89.4	FT	2.40	85	1.07	1.50	0.42
G450x1.90x75x2800	68.8	152	60.3	201	68.1	FT	2.80	70	1.16	1.72	0.35
G450x1.90x75x4000	37.8	152	33.2	201	37.5	FT	4.00	48	1.45	2.31	0.24
G450x1.90x75x5000	27.0	152	23.7	201	26.8	FT	5.00	38	1.60	2.74	0.19
G450x1.90x75x6000	21.0	152	18.4	201	20.8	FT	6.00	30	1.62	3.10	0.15
G250x1.95x90x2800	74.2	97.4	58.0	106	74	FT	2.80	62	1.06	1.20	0.58
G250x1.95x90x4000	41.8	97.4	36.7	106	41	FT	4.00	40	1.09	1.60	0.38
G250x1.95x90x5000	29.8	97.4	26.1	106	30	F	5.00	27	1.03	1.89	0.26
G250x1.95x90x6000	20.7	97.4	18.2	106	21	F	6.00	19	1.06	2.26	0.18
G450x1.90x90x2800	78.6	161	67.4	196	77.9	FT	2.80	75	1.12	1.58	0.38
G450x1.90x90x4000	44.1	161	38.6	196	43.7	FT	4.00	48	1.23	2.12	0.24
G450x1.90x90x5000	31.8	161	27.9	196	31.7	F	5.00	30	1.08	2.49	0.15
G450x1.90x90x6000	22.1	161	19.4	196	22.0	F	6.00	21	1.09	2.98	0.11

Note : FT – Flexural-torsional buckling; F – Flexural buckling;

**Table 13: Comparison of Predictions with FEA Results
for Fixed Ended (Warping Fixed) Columns**

Lipped Channels	Section	FEA	Current		Equation 5		Equation 7	
		P _{ult} (kN)	N _c (kN)	FEA/ Pred.	N _c (kN)	FEA/ Pred.	N _c (kN)	FEA/ Pred.
G550x0.95x55x1800	55x35x9	26.4	21.8	1.21	26.6	0.99	27.3	0.97
G550x0.95x55x2400	55x35x9	19.4	13.6	1.43	18.9	1.03	19.7	0.98
G550x0.95x55x2800	55x35x9	16.6	10.4	1.60	15.4	1.08	16.2	1.03
G550x0.95x55x4000	55x35x9	10.8	5.76	1.88	9.92	1.09	10.6	1.02
G550x0.95x55x5000	55x35x9	7.61	4.15	1.83	7.76	0.98	8.33	0.91
G550x0.95x55x6000	55x35x9	5.53	3.26	1.70	6.47	0.85	7.00	0.79
G250x0.95x55x2400	55x35x9	15.3	13.3	1.15	15.8	0.97	16.2	0.95
G250x0.95x55x2800	55x35x9	12.9	10.1	1.28	12.8	1.00	13.3	0.97
G250x0.95x55x4000	55x35x9	9.11	5.62	1.62	8.27	1.10	8.68	1.05
G250x0.95x55x5000	55x35x9	6.93	4.05	1.71	6.47	1.07	6.84	1.01
G250x0.95x55x6000	55x35x9	5.25	3.18	1.65	5.40	0.97	5.74	0.91
G500x1.15x55x1800	55x35x9	33.6	29.4	1.14	34.5	0.97	35.4	0.95
G500x1.15x55x2400	55x35x9	24.8	18.0	1.38	24.0	1.04	24.9	1.00
G500x1.15x55x2800	55x35x9	21.4	13.9	1.54	19.7	1.09	20.6	1.04
G500x1.15x55x4000	55x35x9	14.0	8.02	1.75	13.1	1.07	13.9	1.01
G500x1.15x55x5000	55x35x9	9.87	5.95	1.66	10.5	0.94	11.2	0.88
G500x1.15x55x6000	55x35x9	7.11	4.79	1.48	8.89	0.80	9.55	0.74
G250x1.95x55x2800	55x35x9	29.9	27.3	1.10	31.1	0.96	31.8	0.94
G250x1.95x55x4000	55x35x9	20.7	18.0	1.15	22.7	0.91	23.5	0.88
G450x1.90x55x1800	55x35x9	59.6	54.7	1.09	60.8	0.98	62.8	0.95
G450x1.90x55x2400	55x35x9	43.4	35.3	1.23	44.0	0.99	45.4	0.96
G450x1.90x55x2800	55x35x9	37.3	28.6	1.30	37.6	0.99	39.0	0.96
G450x1.90x55x4000	55x35x9	24.2	18.7	1.29	27.3	0.88	28.7	0.84
G250x1.95x75x4000	75x50x15	36.7	31.4	1.17	37.8	0.97	38.8	0.95
G250x1.95x75x5000	75x50x15	29.2	22.5	1.30	29.4	0.99	30.5	0.96
G250x1.95x75x6000	75x50x15	24.1	17.6	1.37	24.4	0.99	25.5	0.95
G450x1.90x75x2400	75x50x15	85.0	79.5	1.07	88.3	0.96	91.1	0.93
G450x1.90x75x2800	75x50x15	69.8	60.3	1.16	71.9	0.97	73.9	0.94
G450x1.90x75x4000	75x50x15	48.0	33.2	1.45	45.9	1.05	47.9	1.00
G450x1.90x75x5000	75x50x15	37.8	23.7	1.60	35.6	1.06	37.5	1.01
G450x1.90x75x6000	75x50x15	29.9	18.4	1.62	29.5	1.01	31.2	0.96
G250x1.95x90x4000	90x40x15	39.8	36.7	1.09	42.2	0.94	43.2	0.92
G450x1.90x90x2800	90x40x15	75.2	67.4	1.12	77.2	0.97	79.0	0.95
G450x1.90x90x4000	90x40x15	47.5	38.6	1.23	51.2	0.93	53.1	0.89
Mean			1.39		0.99		0.95	
COV			0.18		0.07		0.07	
Φ			1.10		0.89		0.85

Figure 1: Cold-formed Steel Columns used in Steel Frames

Figure 2: Loading Arrangement at the Bottom End of the Furnace [8]

Figure 3: Loading Arrangement at the Upper End of the Furnace [8]

(a) Failure modes of shorter columns

(b) Failure modes of longer columns

Figure 4: Flexural-torsional Buckling of Test Specimens [8]

Figure 5: Loading and Warping Fixed End Boundary Conditions

Figure 6: Residual Stress Distributions Assumed in FEA [12]

(a) G450 1.95

(b) G550 0.95

Figure 7: Comparison of Load-Axial Shortening Curves from Tests and FEA

Figure 8: Comparison of Failure Modes from Tests and FEA for Shorter Columns

Figure 9: Comparison of Failure Modes from Tests and FEA for Longer Columns

Figure 10: Loading and Warping Free End Boundary Conditions

Figure 11: Results from Finite Strip Analyses of Pinned Ended Columns (G550x0.95x55x35x9)

(a) G550x0.95x55x552 - Local

(b) G550x0.95x55x1104 –
Flexural Torsional

(c) G250x1.95x55x2400 - Flexural

Figure 12: Elastic Buckling Modes

Figure 13: Comparison of Results for Pinned Ended (Warping Free) Columns with AS/NZS 4600 [2] Design Rules

Figure 14: Comparison of Results for Fixed Ended (Warping Fixed) Columns with AS/NZS 4600 [2] Design Rules

Figure 15: Comparison of Results for Fixed Ended (Warping Fixed) Columns with AS/NZS 4600 [2] and Young's [3] Design Rules

Figure 16: Comparison of Results with AS/NZS 4600 Design Rules for Columns with Different End Boundary Conditions

Figure 17: Comparison of Results with Proposed Design Rules for Fixed Ended (Warping Fixed) Columns Subjected to Flexural (F) Flexural-torsional (FT) Buckling

Figure 18: Comparison of Results with AS/NZS 4600 and Proposed Design Rules for Fixed Ended (Warping Fixed) Columns with Different Cross-sections