On designing overlapping group mode-dependent H_∞ controllers of discrete-time Markovian jump linear systems with incomplete mode transition probabilities[†]

Xiaohua Ge and Qing-Long Han*

Centre for Intelligent and Networked Systems, Central Queensland University, Rockhampton, Qld 4702, Australia

SUMMARY

This paper is concerned with overlapping group mode-dependent H_∞ control for a discrete-time Markovian jump linear system, where global modes of the system are not completely available for controller design. Firstly, a randomly overlapping decomposition method is developed to reformulate the system by a set of locally overlapping switched groups with accessible group modes. The reformulated system switches among different group modes in an overlapping manner. Secondly, an overlapping group mode-dependent state feedback controller is delicately constructed. Compared with some existing mode-dependent controllers in the literature, the proposed controller has the following three features: (i) it does not require all exact knowledge of global modes; (ii) it takes full advantage of group mode information of the reformulated system; and (iii) it allows overlapping local modes to exist in the formed groups. Thirdly, sufficient conditions on the existence of a desired overlapping group mode-dependent state feedback controller are derived such that the resultant closed-loop system is stochastically stable with prescribed H_∞ performance. Furthermore, the proposed method is extended to design overlapping group mode-dependent state feedback controllers subject to incomplete mode transition probabilities. The proposed overlapping group mode-dependent framework is shown to be more general and covers traditional Markovian jump linear systems with completely accessible global modes as its special case. In the case of only one group in the reformulated system, it is shown that some existing result in the literature can be retrieved. Finally, two illustrative examples are given to show the effectiveness of the developed theoretical results.

KEY WORDS: Markovian jump linear systems, overlapping operation modes, randomly overlapping decomposition, H_∞ state feedback control, incomplete mode transition probabilities

1. INTRODUCTION

Markovian jump linear systems (MJLSs), which are known as a special class of stochastic hybrid systems including both time-evolving and event-driven mechanisms, are commonly deemed as suitable mathematical models to represent a class of dynamic systems subject to random abrupt variations in system structure and parameters caused by sudden environment changes, system component fault or failures, and subsystem interconnections changes. A number of practical applications can be modeled by MJLSs such as communications networks, economic systems, manufacturing systems, power systems, and aerospace systems [1]. Over the past several years, considerable research efforts on the control problem of MJLSs have been made with respect to stabilization [2–5], H_2 control [6], H_∞ state feedback control [7–10], H_∞ output feedback control [11, 12], observer-based control [13], and decentralized control [14–16]. However, an underlying assumption made in most of these works is that information of global operation modes of MJLSs should be completely accessible or available to a controller at every instant of time. Practically, such an assumption may sometimes be impossible to satisfy since it is usually difficult and costly to obtain complete knowledge of global operation modes. For example, there is no

[†]Correspondence to: Qing-Long Han, Tel. +61 7 4930 9270; Fax. +61 7 4930 6886; E-mail: q.han@cqu.edu.au.

time stamp information for transmitted mode signal in a networked control system [18]; there are limited computation and communication resources in the transmission network for broadcasting all information of system operation modes; and there are some changes in the system structure and parameter difficult to measure or associated to failures of components. To eliminate the need for broadcasting complete global mode information to a controller, three main approaches have been developed in the literature. The first approach, called the local mode-dependent control approach [12, 15], is to consider Markovian jump large-scale systems which consists of a set of interconnected lower dimensional subsystems in a decentralized fashion. Local decentralized controllers are designed based only on state and mode information locally available within the subsystems. Recently, the local mode-dependent control approach is refined in [16] by taking more local mode information of neighboring subsystems including the subsystem it controls into consideration. This approach is referred to as the neighboring mode-dependent control approach in [16]. The last approach is based on the cluster availability of global operation modes [6, 10, 11]. Complete operation modes are firstly split into several non-overlapping clusters. Then the controller is designed based on partial modes, forming clusters of observations.

In practice, there may exist “overlapping operation modes” within the framework of MJLSs. For example, we consider the speed control of an automotive transmission system through a gear-box [19], in which a lower gear, the neutral gear, and a higher gear are deemed as different operation modes of the automotive transmission system. The gear-box is in fact a switching device. To achieve a higher speed, we need to switch from a lower gear to a high gear through the neutral gear, which indicates that the switching order is described as “a lower gear mode — the neutral gear mode — a higher gear mode”. In this case, the switching law can be viewed as two locally overlapping stages “a lower gear mode — the neutral gear mode” and “the neutral gear mode — a higher gear mode”. The neutral gear mode can be viewed as an “overlapping operation mode” of the automotive transmission system in the process of achieving a higher speed. Take the workshop production process [19] as another example, where the predefined order of activated subsystems is supposed as “subsystem 1 — subsystem 2 — subsystem 3”, which can be regarded as two locally overlapping parts “subsystem 1 — subsystem 2” and then “subsystem 2 — subsystem 3”. In this case, subsystem 2 is regarded as an “overlapping operation mode” of the system during the activation. Another motivation example is the aircraft control system [20, 21] in which the system can be described by several decoupled linear models corresponding to finite operating points within the flight envelope and the switching between different operating points can be seen as some locally overlapping regions. There are other motivation examples of “overlapping operation modes” in numerous practical applications, e.g., electric power systems, mechanical systems, sensor networks, and multi-agent systems. However, none of the existing approaches is able to provide a systematic and general way of addressing the “overlapping mode-dependent control” problem for MJLSs subject to “overlapping operation modes”, which remains challenging and deserves much research effort from the community. It is, therefore, the purpose of this paper to make one of the first few attempts to deal with the overlapping mode-dependent control of MJLSs. We propose to address the following interesting questions. When global operation modes of an MJLS are not completely available to a controller and overlapping local operation modes exist in the MJLS, **i)** *how to identify those overlapping local modes?* **ii)** *how does the MJLS switch among different operation modes in an overlapping manner?* and **iii)** *how to design an overlapping mode-dependent controller to stabilize the MJLS by using incomplete and overlapping operation mode information?*

In this paper, a *novel overlapping group mode-dependent H_∞ control scheme* for a discrete-time MJLS with incomplete mode transition probabilities is presented, in which global operation modes of the system are not completely available for controller design and overlapping operation modes locally exist in the MJLS. Firstly, to identify overlapping local modes, a *randomly overlapping decomposition method* is developed. Applying this method, the MJLS under consideration is reformulated by a set of *locally overlapping switched groups* governed by some locally overlapping Markov chains and a high-level Markov chain. The reformulated system does not depend on complete knowledge of global modes, but only requires information of group operation modes and local operation modes within this group. Moreover, the jumps among different groups are governed

by the high-level Markov chain. Therefore, the reformulated system is allowed to either switch within a local group or to switch to an adjacent group, which means that the system switches among different group modes in an overlapping manner. Secondly, a new *overlapping group mode-dependent state feedback controller* is delicately proposed. The controller not only uses full state information and local operation mode information of the system, but also takes full advantage of group mode information of the reformulated system. Furthermore, overlapping local operation modes are allowed to exist in the formed groups. Thirdly, *sufficient conditions* on the existence of a desired overlapping group mode-dependent state feedback controller are derived such that the closed-loop system is stochastically stable with prescribed H_∞ performance. It is shown that the proposed framework is more general, which covers traditional MJLSs with completely accessible global modes as its special case. Furthermore, some existing result on global mode-dependent control can be recovered by our obtained result if one considers only one local group in the proposed framework. Finally, simulation examples are given to show the effectiveness of the proposed control design method.

Notation: For symmetric matrices X and Y , the notation $X \leq Y$ (respectively, $X < Y$) means that $X - Y$ is negative semidefinite (respectively, negative definite). The superscript “ T ” stands for vector/matrix transposition. For an invertible matrix Z , its inverse matrix is denoted by Z^{-1} . $\Pr\{\cdot\}$ denotes the occurrence probability of the event “ $\cdot$ ”. $\mathbb{E}\{\cdot\}$ represents for the expectation of a stochastic variable “ $\cdot$ ”. I represents an identity matrix of appropriate dimensions. $\mathbb{R}^n$ denotes the n -dimensional Euclidean space. $\mathbb{R}^{m \times n}$ denotes the set of all the real $m \times n$ matrices. $\emptyset$ stands for an empty set. $\|\cdot\|$ stands for the Euclidean vector norm or the induced matrix two-norm as appropriate. The symmetric term in a symmetric matrix is denoted as “ $\star$ ”. The space of square-summable vector functions over $[0, \infty)$ are denoted by $l_2[0, \infty)$, and for $w(k) \in l_2[0, \infty)$, its norm is given by $\|w(k)\|_2 = \sqrt{\sum_{k=0}^{\infty} w^T(k)w(k)}$. I refers to an identity matrix of appropriate dimensions. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

2. PROBLEM STATEMENT AND A RANDOMLY OVERLAPPING DECOMPOSITION METHOD

Consider the following discrete-time MJLS $\mathcal{G}_o$ whose dynamics is governed by a discrete-time Markov chain with a finite set $\mathcal{M} = \{1, 2, \dots, M\}$

$$\mathcal{G}_o : \begin{cases} x(k+1) = A(r(k))x(k) + B(r(k))u(k) + E(r(k))w(k) \\ z(k) = C(r(k))x(k) + D(r(k))u(k) + F(r(k))w(k), \end{cases} \quad (1)$$

where $x(k) \in \mathbb{R}^{n_x}$ is the state; $u(k) \in \mathbb{R}^{n_u}$ is the control input; $w(k) \in \mathbb{R}^{n_w}$ is the exogenous disturbance input belonging to $l_2[0, \infty)$; $z(k) \in \mathbb{R}^{n_z}$ is the objective output to be attenuated; $\{r(k), k \geq 0\}$ is a discrete-time Markov chain on the probability space with the transition probability matrix (TPM) $\Pi = [\pi_{ij}]_{M \times M}$ given by

$$\Pr\{r(k+1) = j | r(k) = i\} = \pi_{ij}, \quad \forall i, j \in \mathcal{M}, \quad (2)$$

where $\pi_{ij} \geq 0$ is the transition probability (TP) from mode i at time k to mode j at time $k+1$, and $\sum_{j=1}^M \pi_{ij} = 1$ for $\forall i \in \mathcal{M}$. In the following, we consider that global modes of the discrete-time Markov chain $\{r(k), k \geq 0\}$ are not completely available for controller design. In this case, most of the existing global mode-dependent control schemes [1–4, 7–9, 13] in the literature are invalid. We now propose to develop a new overlapping decomposition method for the MJLS $\mathcal{G}_o$. The discrete-time MJLS is then randomly decomposed into several locally overlapping switched groups. In this paper, by “overlapping” we mean that some local operation modes are commonly shared by adjacent groups.

2.1. Locally Overlapping Markov Chains

Definition 1

The discrete-time Markov chain $\{r(k), k \geq 0\}$ taking values in a finite set $\mathcal{M} = \{1, 2, \dots, M\}$ is

said to be *randomly high-level overlapping decomposable* (RHLOD), if there exist a high-level homogeneous Markov chain $\{\delta(k), k \geq 0\}$ taking values in a finite set $\mathcal{N} = \{1, 2, \dots, N\}$, and some other Markov chains $\{r^{\delta(k)}(k), k \geq 0\}$ with index $\delta(k) = m \in \mathcal{N}$, taking values in some finite sets $\mathcal{M}^{(m)} = \{l_1^{(m)}, l_2^{(m)}, \dots, l_{p_m}^{(m)}\}$, where $l_1^{(m)}, l_2^{(m)}, \dots, l_{p_m}^{(m)} \in \mathcal{M}$, $l_1^{(m)} < l_2^{(m)} < \dots < l_{p_m}^{(m)}$, and $p_m \leq M$, such that

- (i) $\bigcup_{m \in \mathcal{N}} \mathcal{M}^{(m)} = \mathcal{M}$;
- (ii) $\forall n \in \mathcal{N}, \mathcal{M}^{(n)} \cap (\bigcup_{m \in \mathcal{N}, m \neq n} \mathcal{M}^{(m)}) \neq \emptyset$;
- (iii) $\forall m, n \in \mathcal{N}, m \neq n, \mathcal{M}^{(m)} \neq \mathcal{M}^{(n)}$.

In this sense, the discrete-time Markov chains $\{r^{(m)}(k), k \geq 0, m \in \mathcal{N}\}$ are referred to as the *locally overlapping Markov chains* (LOMCs). Furthermore, the TPs and the TPMs of the LOMCs $\{r^{(m)}(k), k \geq 0, m \in \mathcal{N}\}$ are given by

$$Pr\{r^{(m)}(k+1) = j^{(m)} | r^{(m)}(k) = i^{(m)}\} = \pi_{i^{(m)}j^{(m)}}, \forall i^{(m)}, j^{(m)} \in \mathcal{M}^{(m)},$$

where $\Pi^{(m)} = [\pi_{i^{(m)}j^{(m)}}]_{p_m \times p_m}$ and $\pi_{i^{(m)}j^{(m)}} \geq 0$ is the TP from mode $i^{(m)}$ at time k to mode $j^{(m)}$ at time $k+1$, and $\sum_{j^{(m)} \in \mathcal{M}^{(m)}} \pi_{i^{(m)}j^{(m)}} = 1$ for $\forall i^{(m)} \in \mathcal{M}^{(m)}$. The TPM $\Omega = [\omega_{mn}]_{N \times N}$ of the high-level homogeneous Markov chain $\{\delta(k), k \geq 0\}$ is given by

$$Pr\{\delta(k+1) = n | \delta(k) = m\} = \omega_{mn}, \forall m, n \in \mathcal{N},$$

where $\omega_{mn} \geq 0$ is the TP from $\Pi^{(m)}$ at time k to $\Pi^{(n)}$ at time $k+1$, and $\sum_{n=1}^N \omega_{mn} = 1$ for $\forall m \in \mathcal{N}$.

Assumption 1

$\{\delta(k), k \geq 0\}$ is statistically independent of $\{r^{(m)}(k), k \geq 0, m \in \mathcal{N}\}$, i.e., $\{\delta(k), k \geq 0\}$ is statistically independent on $F(k-1) = \sigma\{r^{(m)}(1), r^{(m)}(2), \dots, r^{(m)}(k-1)\}$, where $F(k-1)$ is a σ -algebra generated by $\{r^{(m)}(1), r^{(m)}(2), \dots, r^{(m)}(k-1)\}$.

Remark 1

By Definition 1, one can see that the finite set $\mathcal{M}$ is randomly decomposed into several local subsets, and any two of these local subsets are overlapping. Therefore, Definition 1 provides a way to determine the overlapping local modes, which answers the first question raised in the Introduction. More specifically, property (i) implies that the random decomposition covers the whole initial set $\mathcal{M}$; whereas property (ii) renders that the overlapping decomposition is allowed, which guarantees the existence of some commonly shared local modes; while property (iii) ensures that there is no duplicated subset.

2.2. Locally Overlapping Switched Groups

Definition 2

If a discrete-time Markov chain $\{r(k), k \geq 0\}$ evolving upon the system (1) is RHLOD, then the MJLS $\mathcal{G}_o$ can be reformulated as

$$\mathcal{G}_l : \begin{cases} x(k+1) = A(r^{(m)}(k), \delta(k))x(k) + B(r^{(m)}(k), \delta(k))u(k) + E(r^{(m)}(k), \delta(k))w(k) \\ z(k) = C(r^{(m)}(k), \delta(k))x(k) + D(r^{(m)}(k), \delta(k))u(k) + F(r^{(m)}(k), \delta(k))w(k), \end{cases} \quad (3)$$

where $\{r^{(m)}(k), k \geq 0, m \in \mathcal{N}\}$ are the LOMCs. In this sense, the system (3) is referred to as the *locally overlapping switched groups* (LOSGs). $\forall r^{(m)}(k) = i^{(m)} \in \mathcal{M}^{(m)}; \delta(k) = m \in \mathcal{N}$, the system matrices are denoted by $A_{i^{(m)},m}, B_{i^{(m)},m}, E_{i^{(m)},m}, C_{i^{(m)},m}, D_{i^{(m)},m}$, and $F_{i^{(m)},m}$, which are known with appropriate dimensions.

Remark 2

By Definition 2, it is shown that the MJLS $\mathcal{G}_o$ with not available global mode information can be represented by a set of LOSGs governed by $\{r^{(m)}(k), k \geq 0, m \in \mathcal{N}\}$ and $\{\delta(k), k \geq 0\}$. In this sense, the reformulated system (3) is allowed to either switch within a local subset $\mathcal{M}^{(m)}$ or

to switch to an adjacent subset $\mathcal{M}^{(n)}$, $\forall m, n \in \mathcal{N}$. As a consequence, the reformulated system $\mathcal{G}_l$ switches among different group modes in an overlapping manner, which answers the second question raised in the Introduction. It should be pointed out that the localized formulation (3) is not deterministic or unique as the decomposition of the Markov chain $\{r(k), k \geq 0\}$ evolves in a random way. In this sense, we refer to the formulation (3) as a randomly overlapping decomposition provided that properties (i)-(iii) in Definition 1 hold.

Figure 1. The cooperative strategy for the robot players

2.3. An Illustrative Example for LOSGs

We use the LOSGs to demonstrate robot soccer systems. For simplicity of demonstration, we consider a robot soccer team with 7 robot players. In a typical robot soccer game, a cooperative strategy is required to coordinate all robot players to win the game in a robot soccer competition. Robot players usually start by breaking the strategy up into different parts and are assigned different roles including forwards (or attackers), midfielders (or assistants), and guards (or defenders), respectively, as human soccer teams do [22]. In this example, the robots 1, 2, and 3 are defined as forward players; the robots 4 and 5 are defined as midfielder players; and the robots 6 and 7 are defined as guards before the game starts. Their positions on the court are shown in Figure 1. We propose to model each robot soccer player as one local subsystem. Moreover, the cooperative strategy, which enables robots to cooperate with each other, is built by using an overlapping architecture. More specifically, we decompose the cooperative strategy into three different parts. In the first part, robot players are assigned to attack; in the second part, robot players are assigned to assist; and in the third part, robot players are assigned to defend. We model these three parts as the LOSGs with $\mathcal{N} = \{1, 2, 3\}$. The whole cooperative strategy for the robot soccer system is also shown in Figure 1. The cooperative strategy in each individual part for different robot players is governed by LOMCs $\{r^{(m)}(k), k \geq 0, m \in \mathcal{N}\}$ with $\mathcal{M}^{(1)} = \{1, 2, 3\}$, $\mathcal{M}^{(2)} = \{3, 4, 5\}$, $\mathcal{M}^{(3)} = \{3, 5, 6, 7\}$, and the strategy switching among different parts is assumed to be governed by a high-level homogeneous Markov chain $\{\delta(k), k \geq 0\}$ due to the fact that offense and defense are usually randomly switching in practical games. When the game starts, the first step of the strategy is role selection based on environment and strategy. Compared with dynamic role switching, permanent role fixing results in undesirable behaviors such as a forward robot not going for the ball even though the ball is near but outside an offense zone. Therefore, we consider a dynamic role switching mechanism for robots to accomplish the soccer competition, which means that same robot players may be assigned different roles in the common region of different LOSGs to illustrate the concept of “overlapping”. They can dynamically switch their roles among different LOSGs based on the team strategy. For example, from Figure 1, it is shown that when the whole team possesses the ball, the robots 1, 2 and 3 are defined as forward players who are responsible for attacking and scoring. When there is not any good chance to score, the robot 3 may switch its role to midfielder and together with the robots 4 and 5 accomplish the assistance for the forward players or the defenders. When the whole team losses its ball control at a given time, however, the robots 3 and 5 may switch their roles to defenders

and together with the robots 6 and 7 stand in their defence zone to fight against their opponents. In this case, the robots 3 and 5 are assigned “overlapping” roles in the whole team strategy. As a matter of fact, by defining these overlapping switching roles for robot players, the performance of the entire team for the competition can be maximized for a scenario [22].

2.4. An Overlapping Group Mode-Dependent H_∞ Controller

In the sequel, the state of the system (3) is assumed to be measurable. We consider an overlapping group mode-dependent state feedback controller of the following structure

$$u(k) = K(r^{(m)}(k), \delta(k))x(k), \quad (4)$$

where $K_{i^{(m)}, m}, \forall r^{(m)}(k) = i^{(m)} \in \mathcal{M}^{(m)}; \delta(k) = m \in \mathcal{N}$, are the controller gains to be determined.

Remark 3

It is noteworthy that the proposed overlapping group mode-dependent state feedback controller (4) is different from the mode-dependent state feedback controllers in [1–4, 7–9, 12, 13, 15–17]. More specifically, the controllers in [1–4, 7–9, 13, 17] rely on complete global operation mode information of the considered MJLSs. This type of controllers are known as the *global mode-dependent controllers*. The controller in [12, 15], however, only uses mode information locally available within the subsystem and the controller is designed based only on local mode information of this subsystem it actuates. In this sense, this type of controllers are named as the *local mode-dependent controllers*. Recently, a *neighboring mode-dependent control* approach was proposed in [16] to address the state feedback controller design for Markovian jump large-scale systems. The key idea of this approach is that each local controller makes use of not only local mode information of the subsystem it controls but also mode information of its neighboring subsystems. This type of controllers are denoted as the *neighboring mode-dependent controllers*. The main differences between the controllers in [12, 15, 16] and the one proposed in this paper lie in the following aspects: (i) the controllers in [12, 15, 16] are considered in continuous-time case, while we consider the controller (4) in discrete-time case; (ii) the control design problem addressed in [12, 15, 16] is within the framework of Markovian jump large-scale systems which consists of a number of interconnected subsystems, whereas we consider the control issue for MJLSs; (iii) the local mode processes of the controllers in [12, 15, 16] are non-Markov and may statistically depend on each other, which means that the standard framework of MJLSs is not directly applicable to solve the proposed control problem. However, the local mode processes $\{r^{(m)}(k), k \geq 0, m \in \mathcal{N}\}$ of the controller (4) are still governed by Markov chains and statistically independent on each other. Therefore, one may solve the control problem tractably since the standard framework of MJLSs can be applied, which will make clear later in the paper; (iv) the local controller in [12, 15, 16] is dependent on only local mode information or local mode information of its neighboring subsystems including the subsystem it controls, whereas the proposed controller (4) depends on both group operation mode information of the reformulated system and local mode information within this group; and (v) for the first time, we allow overlapping local modes to exist in the formed groups of the controller (4), while the overlapping structure of local mode processes for the controllers in [12, 15, 16] is not considered, which makes a distinct difference between [12, 15, 16] and this paper. Even though at a certain time instant, the knowledge of current global operation mode is not available, the proposed controller can be fully autonomous provided that one knows that in which group the MJLS is operating and among which local modes within this group the LOSG is switching. This requirement is much loose than the knowledge of complete global operation modes and much rigorous than the knowledge of only local operation modes. It should be noted that such group mode dependence of the proposed controller (4) provides an unique feature of the proposed control scheme. To emphasize this feature, we refer to this type of controllers as the *overlapping group mode-dependent controllers*.

2.5. The Overlapping Group Mode-Dependent H_∞ Control Problem

Combining (3) and (4), we have the closed-loop system $\mathcal{G}_s$ given by

$$\mathcal{G}_s : \begin{cases} x(k+1) = \tilde{A}(r^{(m)}(k), \delta(k))x(k) + \tilde{E}(r^{(m)}(k), \delta(k))w(k) \\ z(k) = \tilde{C}(r^{(m)}(k), \delta(k))x(k) + \tilde{F}(r^{(m)}(k), \delta(k))w(k), \end{cases} \quad (5)$$

where

$$\begin{aligned} \tilde{A}_{i^{(m)},m} &= A_{i^{(m)},m} + B_{i^{(m)},m}K_{i^{(m)},m}, & \tilde{E}_{i^{(m)},m} &= E_{i^{(m)},m} \\ \tilde{C}_{i^{(m)},m} &= C_{i^{(m)},m} + D_{i^{(m)},m}K_{i^{(m)},m}, & \tilde{F}_{i^{(m)},m} &= F_{i^{(m)},m} \end{aligned}$$

for all $r^{(m)}(k) = i^{(m)} \in \mathcal{M}^{(m)}; \delta(k) = m \in \mathcal{N}$.

The *overlapping group mode-dependent H_∞ control problem* to be addressed is stated as follows: For a prescribed level of disturbance attenuation $\gamma > 0$, design an overlapping group mode-dependent H_∞ controller in the form of (4) such that

- (i) The closed-loop system $\mathcal{G}_s$ is stochastically stable, i.e., the state trajectory $x(k)$ of system $\mathcal{G}_s$ with $w(k) \equiv 0$ satisfies

$$\mathbb{E} \left\{ \sum_{k=0}^{\infty} \|x(k)\|^2 \mid x(0), r^{(m)}(0), \delta(0) \right\} < \infty;$$

- (ii) For all nonzero $w(k) \in l_2[0, \infty)$, the H_∞ disturbance attenuation performance $\frac{\|z(k)\|_{E_2}}{\|w(k)\|_2} < \gamma$ is guaranteed under zero initial condition, where $\|z(k)\|_{E_2} = \sqrt{\mathbb{E}\{\sum_{k=0}^{\infty} z^T(k)z(k)\}}$.

3. H_∞ PERFORMANCE ANALYSIS

In this section, suppose that mode TPs of the Markov chains $\{r^{(m)}(k), k \geq 0, m \in \mathcal{N}\}$ and $\{\delta(k), k \geq 0\}$ are completely known. An H_∞ performance criterion is presented such that the closed-loop system $\mathcal{G}_s$ is stochastically stable with the prescribed H_∞ performance.

Theorem 1

For a given scalar $\gamma > 0$, the closed-loop system $\mathcal{G}_s$ with complete TPs is stochastically stable and the H_∞ norm of the system (5) can be calculated by: $\|\mathcal{G}_s\|_\infty = \sup_{0 \neq w(k) \in l_2[0, \infty), r^{(m)}(0) \in \mathcal{M}^{(m)}, \delta(0) \in \mathcal{N}} \frac{\|z(k)\|_{E_2}}{\|w(k)\|_2} < \gamma$ if there exist symmetric positive definite matrices $P_{j^{(m)},n}$ of appropriate dimensions such that

$$\begin{bmatrix} P_{i^{(m)},m} & \star & \star & \star \\ 0 & \gamma^2 I & \star & \star \\ \tilde{A}_{i^{(m)},m} & \tilde{E}_{i^{(m)},m} & \tilde{P}_{i^{(m)},m}^{-1} & \star \\ \tilde{C}_{i^{(m)},m} & \tilde{F}_{i^{(m)},m} & 0 & I \end{bmatrix} > 0 \quad (6)$$

for $\forall i^{(m)}, j^{(m)} \in \mathcal{M}^{(m)}; m, n \in \mathcal{N}$, where $\tilde{P}_{i^{(m)},m} = \sum_{n \in \mathcal{N}} \omega_{mn} \sum_{j^{(m)} \in \mathcal{M}^{(m)}} \pi_{i^{(m)}j^{(m)}} P_{j^{(m)},n}$.

Proof: Choose the stochastic Lyapunov functional candidate as

$$V(x(k), r^{(m)}(k), \delta(k)) = x^T(k) P_{\varphi(r^{(m)}(k), \delta(k))} x(k), \quad (7)$$

where $\varphi(r^{(m)}(k), \delta(k))$ takes values in the pairs $(i^{(m)}, m)$, $\forall i^{(m)} \times m \in \mathcal{M}^{(m)} \times \mathcal{N}$, and $P_{j^{(m)},n}$, $\forall j^{(m)} \in \mathcal{M}^{(m)}; n \in \mathcal{N}$, are positive definite constant matrices of appropriate dimensions. Define $\Delta V(x(k), r^{(m)}(k), \delta(k)) = \mathbb{E}\{V(x(k+1), r^{(m)}(k+1), \delta(k+1))\} - V(x(k), r^{(m)}(k), \delta(k))$ and $\varphi(r^{(m)}(k+1), \delta(k+1)) = (j^{(m)}, n)$, $\forall j^{(m)} \times n \in \mathcal{M}^{(m)} \times \mathcal{N}$. Then by Assumption 1, we have

$$\Delta V(x(k), r^{(m)}(k), \delta(k)) = x^T(k+1) \tilde{P}_{i^{(m)},m} x(k+1) - x^T(k) P_{i^{(m)},m} x(k). \quad (8)$$

Firstly, we consider the stochastic stability of system (5) with $w(k) = 0$. Clearly, (6) implies that $P_{i(m),m} - \tilde{A}_{i(m),m}^T \tilde{P}_{i(m),m} \tilde{A}_{i(m),m} > 0$ by using Schur complement. Then one has $\Delta V(x(k), r^{(m)}(k), \delta(k)) < 0$ from (8). Following a similar proof line of Theorem 1 in [7], one can see that $\mathbb{E}\{\sum_{k=0}^{\infty} \|x(k)\|^2\} < \infty$ under any initial condition. Therefore, the system (5) with $w(k) = 0$ is stochastically stable.

Next, we establish the H_{∞} performance index $J_w = \mathbb{E}\{\sum_{k=0}^{\infty} [z^T(k)z(k) - \gamma^2 w^T(k)w(k)]\}$ for all nonzero $w(k) \in l_2[0, \infty)$ and a prescribed $\gamma > 0$. Assuming that $w(k) \neq 0$ and zero initial conditions, one has

$$\begin{aligned} J_w &\leq \mathbb{E} \left\{ \sum_{k=0}^{\infty} [z^T(k)z(k) - \gamma^2 w^T(k)w(k) + \Delta V(\xi(k), r^{(m)}(k), \delta(k))] \right\} \\ &= \begin{bmatrix} x(k) \\ w(k) \end{bmatrix}^T \begin{bmatrix} \tilde{A}_{i(m),m} & \tilde{E}_{i(m),m} \\ \tilde{C}_{i(m),m} & \tilde{F}_{i(m),m} \end{bmatrix}^T \begin{bmatrix} \tilde{P}_{i(m),m} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \tilde{A}_{i(m),m} & \tilde{E}_{i(m),m} \\ \tilde{C}_{i(m),m} & \tilde{F}_{i(m),m} \end{bmatrix} \begin{bmatrix} x(k) \\ w(k) \end{bmatrix} \\ &\quad - \begin{bmatrix} x(k) \\ w(k) \end{bmatrix}^T \begin{bmatrix} P_{i(m),m} & 0 \\ 0 & \gamma^2 I \end{bmatrix} \begin{bmatrix} x(k) \\ w(k) \end{bmatrix}. \end{aligned} \quad (9)$$

Applying Schur complement to (6), it can be seen from (9) that $J_w < 0$ holds, which means that $\|z(k)\|_{E_2} < \gamma \|w(k)\|_2$. This completes the proof. $\square$

4. OVERLAPPING GROUP MODE-DEPENDENT STATE FEEDBACK CONTROL WITH COMPLETE MODE TRANSITION PROBABILITIES

Based on Theorem 1, we are in a position to present the following theorem which provides a sufficient condition on the existence of an overlapping group mode-dependent controller to stabilize the resultant closed-loop system $\mathcal{G}_s$ with complete TPs.

Theorem 2

For a given scalar $\gamma > 0$, there exists an overlapping group mode-dependent H_{∞} state feedback controller in the form of (4) such that the closed-loop system $\mathcal{G}_s$ with complete TPs is stochastically stable with the prescribed H_{∞} disturbance attenuation performance level γ if there exist symmetric positive definite matrices $X_{j(m),n}$, matrices $H_{i(m),m}$, $G_{i(m),m}$, $Y_{i(m),m}$, and $Z_{i(m),j(m),m,n}$, $\forall i^{(m)}, j^{(m)} \in \mathcal{M}^{(m)}$; $m, n \in \mathcal{N}$, of appropriate dimensions such that

$$\begin{bmatrix} G_{i(m),m} + G_{i(m),m}^T - X_{i(m),m} & \star & \star & \star \\ 0 & \gamma^2 I & \star & \star \\ A_{i(m),m} G_{i(m),m} + B_{i(m),m} Y_{i(m),m} & E_{i(m),m} & H_{i(m),m} + H_{i(m),m}^T - \Theta_{i(m),m} & \star \\ C_{i(m),m} G_{i(m),m} + D_{i(m),m} Y_{i(m),m} & F_{i(m),m} & 0 & I \end{bmatrix} > 0 \quad (10)$$

$$\begin{bmatrix} Z_{i(m),j(m),m,n} & \star \\ H_{i(m),m} & X_{j(m),n} \end{bmatrix} > 0, \quad \forall i^{(m)}, j^{(m)} \in \mathcal{M}^{(m)}; m, n \in \mathcal{N}, \quad (11)$$

where $\Theta_{i(m),m} = \sum_{n \in \mathcal{N}} \omega_{mn} \sum_{j^{(m)} \in \mathcal{M}^{(m)}} \pi_{i(m),j^{(m)}} Z_{i(m),j^{(m)},m,n}$. Moreover, the controller gains are given by $K_{i(m),m} = Y_{i(m),m} G_{i(m),m}^{-1}$.

Proof: Assume that (10) and (11) hold. By virtue of Schur complement, we have $Z_{i(m),j^{(m)},m,n} > H_{i(m),m}^T X_{j^{(m)},n}^{-1} H_{i(m),m}$ from (11). Meanwhile, it can be verified that

$$\begin{aligned} &H_{i(m),m} + H_{i(m),m}^T - \Theta_{i(m),m} \\ &= H_{i(m),m} + H_{i(m),m}^T - \sum_{n \in \mathcal{N}} \omega_{mn} \sum_{j^{(m)} \in \mathcal{M}^{(m)}} \pi_{i(m),j^{(m)}} Z_{i(m),j^{(m)},m,n} \\ &< H_{i(m),m} + H_{i(m),m}^T - H_{i(m),m}^T \tilde{\Theta}_{i(m),m}^{-1} H_{i(m),m} \\ &= \tilde{\Theta}_{i(m),m} - (H_{i(m),m} - \tilde{\Theta}_{i(m),m})^T \tilde{\Theta}_{i(m),m}^{-1} (H_{i(m),m} - \tilde{\Theta}_{i(m),m}) \\ &\leq \tilde{\Theta}_{i(m),m}, \end{aligned} \quad (12)$$

where $\tilde{\Theta}_{i^{(m)},m} = (\sum_{n \in \mathcal{N}} \omega_{mn} \sum_{j^{(m)} \in \mathcal{M}^{(m)}} \pi_{i^{(m)}j^{(m)}} X_{j^{(m)},n}^{-1})^{-1}$. On the other hand, we have $G_{i^{(m)},m}^T X_{i^{(m)},m}^{-1} G_{i^{(m)},m} \geq G_{i^{(m)},m} + G_{i^{(m)},m}^T - X_{i^{(m)},m}$ since $(G_{i^{(m)},m} - X_{i^{(m)},m})^T X_{i^{(m)},m}^{-1} (G_{i^{(m)},m} - X_{i^{(m)},m}) \geq 0$. Therefore, inequality (10) still holds if its (1, 1)-th and (3, 3)-th diagonal blocks are replaced by $G_{i^{(m)},m}^T X_{i^{(m)},m}^{-1} G_{i^{(m)},m}$ and $\tilde{\Theta}_{i^{(m)},m}$, and we denote this new inequality as $\widehat{(10)}$. Multiplying both sides of $\widehat{(10)}$ with $\text{diag}\{G_{i^{(m)},m}^{-T}, I, I, I\}$ to the left and its transpose to the right, we obtain

$$\begin{bmatrix} X_{i^{(m)},m}^{-1} & \star & \star & \star \\ 0 & \gamma^2 I & \star & \star \\ \tilde{A}_{i^{(m)},m} & \tilde{E}_{i^{(m)},m} & \tilde{\Theta}_{i^{(m)},m} & \star \\ \tilde{C}_{i^{(m)},m} & \tilde{F}_{i^{(m)},m} & 0 & I \end{bmatrix} > 0. \quad (13)$$

Let $X_{j^{(m)},n} = P_{j^{(m)},n}^{-1}$. It is found that (13) is equivalent to (6) for the closed-loop matrices. The rest of the proof is straightforward by using Theorem 1. This completes the proof. $\square$

Remark 4

Theorem 2 is based on complete knowledge of the mode TPs. Without loss of generality, one may set $G_{i^{(m)},m} = X_{i^{(m)},m}$. However, these slack variables may play an important role in reducing conservatism for controller design with parameter uncertainties in TPs or in system matrices. Inspired by [10], the inequality constraint (11) is proposed to linearize the nonlinear term $\tilde{\Theta}_{i^{(m)},m}$ during the design procedure, e.g., in (12). Another approach to linearize the nonlinear term is to use some augmented blocks, see, e.g., [2, 9].

5. OVERLAPPING GROUP MODE-DEPENDENT STATE FEEDBACK CONTROL WITH INCOMPLETE MODE TRANSITION PROBABILITIES

In this section, we assume that the mode TPs of the Markov chains $\{r^{(m)}(k), k \geq 0, m \in \mathcal{N}\}$ and $\{\delta(k), k \geq 0\}$ are incomplete or partially accessible. More specifically, two different cases, i.e., completely known TPs and completely unknown TPs, are considered. For example,

$$\Pi^{(1)} = \begin{bmatrix} \pi_{11} & ? & \pi_{13} & ? \\ \pi_{21} & ? & ? & \pi_{24} \\ ? & ? & ? & ? \\ ? & ? & \pi_{43} & \pi_{44} \end{bmatrix}, \quad \Omega = \begin{bmatrix} ? & \omega_{12} & \omega_{13} & ? \\ ? & ? & ? & ? \\ ? & ? & \omega_{33} & ? \\ ? & ? & ? & \omega_{44} \end{bmatrix},$$

where “?” represents the completely unknown TP. For clarity of notation, we denote $\mathcal{M}^{(m)} = \tilde{\mathcal{M}}_{kn}^{i^{(m)}} + \tilde{\mathcal{M}}_{ukn}^{i^{(m)}}$, and $\mathcal{N} = \tilde{\mathcal{N}}_{kn}^m + \tilde{\mathcal{N}}_{ukn}^m$ with

$$\begin{aligned} \tilde{\mathcal{M}}_{kn}^{i^{(m)}} &\triangleq \{j^{(m)} : \pi_{i^{(m)}j^{(m)}} \text{ is completely known for } j^{(m)} \in \mathcal{M}^{(m)}\} \\ \tilde{\mathcal{M}}_{ukn}^{i^{(m)}} &\triangleq \{j^{(m)} : \pi_{i^{(m)}j^{(m)}} \text{ is completely unknown for } j^{(m)} \in \mathcal{M}^{(m)}\} \end{aligned} \quad (14)$$

$$\begin{aligned} \tilde{\mathcal{N}}_{kn}^m &\triangleq \{n : \omega_{mn} \text{ is completely known for } n \in \mathcal{N}\} \\ \tilde{\mathcal{N}}_{ukn}^m &\triangleq \{n : \omega_{mn} \text{ is completely unknown for } n \in \mathcal{N}\} \end{aligned} \quad (15)$$

for every $i^{(m)} \in \mathcal{M}^{(m)}; m \in \mathcal{N}$.

Remark 5

In practice, it is the TPs of jumping modes that determine the performance and behavior of MJLSs. However, in most previous works, it is commonly assumed that the TPs are accessible and given *a priori*. In practice, this assumption is not applicable since the likelihood of measuring all ideal knowledge of the TPs is usually difficult and costly [9]. Currently, there have been two representative cases of the study of unknown TPs: *uncertain TPs* and *incomplete TPs*. The first one allows the TPMs to be uncertain and subject to norm-bounded uncertainties or to vary in a convex polytope; while the second one is assumed that the TPMs are incomplete or partially accessible with

some elements in the TPMs completely known and others completely unknown. We refer readers to [4, 8, 10, 11, 23] and [3, 5, 9, 24] for more details of these two cases. It is worth mentioning that, in the case of uncertain TPMs studied in the literature, it is often assumed that all knowledge of the bounds of norm-bounded uncertainties or the vertices of the polytopic-type uncertainties needs to be known *a priori*, which can still be viewed as accessible TPMs knowledge. Therefore, our assumption on the TPMs $\Pi^{(m)}$ and Ω is more natural. Also note that the incomplete model (14)-(15) covers the completely known TPMs ($\mathcal{M}^{(m)} = \tilde{\mathcal{M}}_{kn}^{i(m)}, \tilde{\mathcal{M}}_{ukn}^{i(m)} = \emptyset; \mathcal{N} = \tilde{\mathcal{N}}_{kn}^m, \tilde{\mathcal{N}}_{ukn}^m = \emptyset$) and completely unknown TPMs ($\mathcal{M}^{(m)} = \tilde{\mathcal{M}}_{ukn}^{i(m)}, \tilde{\mathcal{M}}_{kn}^{i(m)} = \emptyset; \mathcal{N} = \tilde{\mathcal{N}}_{ukn}^m, \tilde{\mathcal{N}}_{kn}^m = \emptyset$) as two special cases.

The following theorem provides another sufficient condition on the existence of an overlapping group mode-dependent H_∞ state feedback controller in the case of incomplete TPMs.

Theorem 3

For a given scalar $\gamma > 0$, there exists an overlapping group mode-dependent H_∞ state feedback controller in the form of (4) such that the closed-loop system $\mathcal{G}_s$ with incomplete TPMs described by (14)-(15) is stochastically stable with the prescribed H_∞ disturbance attenuation performance level γ if there exist symmetric matrices $X_{j(m),n}$, and matrices $H_{i(m),m}, G_{i(m),m}, Y_{i(m),m}, W_{i(m),m}$, and $Z_{i(m),j(m),m,n}, \forall i(m), j(m) \in \mathcal{M}^{(m)}; m, n \in \mathcal{N}$, of appropriate dimensions such that (11) and

$$\begin{bmatrix} G_{i(m),m} + G_{i(m),m}^T - X_{i(m),m} & \star & \star & \star \\ 0 & \gamma^2 I & \star & \star \\ A_{i(m),m} G_{i(m),m} + B_{i(m),m} Y_{i(m),m} & E_{i(m),m} & \tilde{\Xi}_{i(m),m} & \star \\ C_{i(m),m} G_{i(m),m} + D_{i(m),m} Y_{i(m),m} & F_{i(m),m} & 0 & I \end{bmatrix} > 0 \quad (16)$$

$$\sum_{n \in \tilde{\mathcal{N}}_{kn}^m} \omega_{mn} \Xi_{i(m),j(m),m,n} < 0, \quad \forall j(m) \in \tilde{\mathcal{M}}_{ukn}^{i(m)} \quad (17)$$

$$\sum_{j(m) \in \tilde{\mathcal{M}}_{kn}^{i(m)}} \pi_{i(m)j(m)} \Xi_{i(m),j(m),m,n} < 0, \quad \forall n \in \tilde{\mathcal{N}}_{ukn}^m \quad (18)$$

$$\Xi_{i(m),j(m),m,n} < 0, \quad \forall j(m) \in \tilde{\mathcal{M}}_{ukn}^{i(m)}; n \in \tilde{\mathcal{N}}_{ukn}^m, \quad (19)$$

where

$$\begin{aligned} \tilde{\Xi}_{i(m),m} &= H_{i(m),m} + H_{i(m),m}^T + W_{i(m),m} - \sum_{n \in \tilde{\mathcal{N}}_{kn}^m} \omega_{mn} \sum_{j(m) \in \tilde{\mathcal{M}}_{kn}^{i(m)}} \pi_{i(m)j(m)} \Xi_{i(m),j(m),m,n} \\ \Xi_{i(m),j(m),m,n} &= W_{i(m),m} + Z_{i(m),j(m),m,n}. \end{aligned}$$

Moreover, the controller gains are given by $K_{i(m),m} = Y_{i(m),m} G_{i(m),m}^{-1}$.

Proof: Recalling that $\pi_{i(m)j(m)} \geq 0, \omega_{mn} \geq 0, \forall i(m), j(m) \in \mathcal{M}^{(m)}; m, n \in \mathcal{N}$, we have

$$\begin{aligned} \Phi_{i(m),m} - \sum_{n \in \tilde{\mathcal{N}}_{kn}^m} \omega_{mn} \sum_{j(m) \in \tilde{\mathcal{M}}_{ukn}^{i(m)}} \pi_{i(m)j(m)} \Psi_{i(m),j(m),m,n} - \sum_{n \in \tilde{\mathcal{N}}_{ukn}^m} \omega_{mn} \sum_{j(m) \in \tilde{\mathcal{M}}_{kn}^{i(m)}} \pi_{i(m)j(m)} \Psi_{i(m),j(m),m,n} \\ - \sum_{n \in \tilde{\mathcal{N}}_{ukn}^m} \omega_{mn} \sum_{j(m) \in \tilde{\mathcal{M}}_{ukn}^{i(m)}} \pi_{i(m)j(m)} \Psi_{i(m),j(m),m,n} > 0, \quad (20) \end{aligned}$$

where $\Phi_{i(m),m}$ denotes the left-hand side of the inequality defined in (16), $\Psi_{i(m),j(m),m,n}$ denotes a 4 by 4 block matrix with the (3,3)-th block $\Xi_{i(m),j(m),m,n}$ and others zero. Clearly, (20) is equivalent to (10) due to the fact that $(1 - \sum_{n \in \mathcal{N}} \omega_{mn} \sum_{j(m) \in \mathcal{M}^{(m)}} \pi_{i(m)j(m)}) W_{i(m),m} = 0$, where $W_{i(m),m}, \forall i(m) \in \mathcal{M}^{(m)}; m \in \mathcal{N}$ are arbitrary matrices, $\sum_{n \in \mathcal{N}} \omega_{mn} = \sum_{n \in \tilde{\mathcal{N}}_{kn}^m} \omega_{mn} + \sum_{n \in \tilde{\mathcal{N}}_{ukn}^m} \omega_{mn}$, and $\sum_{j(m) \in \mathcal{M}^{(m)}} \pi_{i(m)j(m)} = \sum_{j(m) \in \tilde{\mathcal{M}}_{kn}^{i(m)}} \pi_{i(m)j(m)} + \sum_{j(m) \in \tilde{\mathcal{M}}_{ukn}^{i(m)}} \pi_{i(m)j(m)}$. Then following Theorem 2, we complete the proof. $\square$

Remark 6

Notice that when information of the mode TPs in the TPMs is completely known, Theorem 3 reduces to Theorem 2. However, when the mode TPs of the system under consideration are partially accessible, one may use Theorem 3 to design an overlapping group mode-dependent controller to account for the case of incomplete TPs described by (14)-(15).

With Theorem 2 and Theorem 3, the problem to be addressed can be transformed into the following optimization problem

$$\min_F \lambda \quad \text{subject to} \quad \lambda = \gamma^2,$$

where F is the set of all feasible solutions from LMIs (10)-(11) for Theorem 2, and (11), (16)-(19) for Theorem 3. The optimal H_∞ performance level $\gamma = \sqrt{\lambda}$ and desired controller gains $K_{i(m),m}$, $\forall i(m) \in \mathcal{M}^{(m)}; m \in \mathcal{N}$, can be obtained by solving the minimization problem mentioned above. In this sense, we say that the third question raised in the Introduction is solved.

6. A SPECIAL CASE: ONLY ONE GROUP IN THE LOSGS

If the system (3) contains only one local group, e.g., letting it be governed by $r^{(1)}(k)$ and denoting $r^{(1)}(k)$ by $\theta(k)$, the reformulated $\mathcal{G}_l$ reduces to the following traditional MJLS with completely accessible global operation modes

$$\mathcal{G}_t : \begin{cases} x(k+1) = A(\theta(k))x(k) + B(\theta(k))u(k) + E(\theta(k))w(k) \\ z(k) = C(\theta(k))\xi(k) + D(\theta(k))u(k) + F(\theta(k))w(k), \end{cases} \quad (21)$$

where $\{\theta(k), k \geq 0\}$ is a discrete-time Markov chain with the associated TPM $\Theta = [\vartheta_{sl}]_{S \times S}$, $\forall s, l \in \mathcal{S} = \{1, 2, \dots, S\}$. Moreover, we assume that the mode TPs of $\{\theta(k), k \geq 0\}$ are partially known and the finite set $\mathcal{S}$ is denoted as $\mathcal{S} = V_{kn}^s + V_{ukn}^s$ with

$$\begin{aligned} V_{kn}^s &\triangleq \{l : \vartheta_{sl} \text{ is completely known for } l \in \mathcal{S}\} \\ V_{ukn}^s &\triangleq \{l : \vartheta_{sl} \text{ is completely unknown for } l \in \mathcal{S}\}. \end{aligned} \quad (22)$$

The objective is to design a global mode-dependent H_∞ state feedback controller in the form of $u(k) = K(\theta(k))x(k)$ such that the closed-loop system

$$\mathcal{G}_c : \begin{cases} x(k+1) = \tilde{A}(\theta(k))x(k) + \tilde{E}(\theta(k))w(k) \\ z(k) = \tilde{C}(\theta(k))x(k) + \tilde{F}(\theta(k))w(k), \end{cases} \quad (23)$$

where $\tilde{A}_s = A_s + B_s K_s$, $\tilde{E}_s = E_s$, $\tilde{C}_s = C_s + D_s K_s$, and $\tilde{F}_s = F_s$, $\forall \theta(k) = s \in \mathcal{S}$, is stochastically stable with the prescribed H_∞ disturbance attenuation performance level γ . In this case, the proposed framework of LOSGs covers the traditional MJLSs framework as its special case. Furthermore, it should be stressed that some existing result on global mode-dependent control in the literature can be regarded as a special case of our proposed result. Before making corresponding conclusion, the following lemma from [25] is recalled.

Lemma 1

For a given scalar $\gamma > 0$, given controller gains K_s , the closed-loop system $\mathcal{G}_c$ with complete TPs is stochastically stable and the H_∞ norm of system (23) can be calculated by: $\|\mathcal{G}_c\|_\infty = \sup_{0 \neq w(k) \in l_2[0, \infty), \theta(0) \in \mathcal{S}} \frac{\|z(k)\|_{E_2}}{\|w(k)\|_2} < \gamma$ if and only if there exist symmetric positive definite matrices P_s , $\forall s \in \mathcal{S}$, of appropriate dimensions such that

$$\begin{bmatrix} P_s & \star & \star & \star \\ 0 & \gamma^2 I & \star & \star \\ \tilde{A}_s & \tilde{E}_s & \tilde{P}_s^{-1} & \star \\ \tilde{C}_s & \tilde{F}_s & 0 & I \end{bmatrix} > 0 \quad (24)$$

where $\tilde{P}_s = \sum_{l \in \mathcal{S}} \vartheta_{sl} P_l$, $\forall s \in \mathcal{S}$.

Next we are in a position to present the following corollary which provides a sufficient and necessary condition on the existence of a global mode-dependent H_∞ state feedback controller to stabilize the closed-loop system (23).

Corollary 1

For a given scalar $\gamma > 0$, there exists a global mode-dependent H_∞ state feedback controller in the form of $u(k) = K(\theta(k))x(k)$ such that the closed-loop system $\mathcal{G}_c$ with complete TPs is stochastically stable with the prescribed H_∞ disturbance attenuation performance level γ if and only if there exist symmetric matrices X_s and matrices H_s, G_s, Y_s , and $Z_{s,l}$, $\forall s, l \in \mathcal{S}$, of appropriate dimensions such that

$$\begin{bmatrix} G_s + G_s^T - X_s & \star & \star & \star \\ 0 & \gamma^2 I & \star & \star \\ A_s G_s + B_s Y_s & E_s & H_s^T + H_s - \sum_{l \in S_{kn}^s} \vartheta_{sl} Z_{s,l} & \star \\ C_s G_s + D_s Y_s & F_s & 0 & I \end{bmatrix} > 0 \quad (25)$$

$$\begin{bmatrix} Z_{s,l} & \star \\ H_s & X_l \end{bmatrix} > 0, \quad \forall s, l \in \mathcal{S}. \quad (26)$$

Moreover, the controller gains are given by $K_s = Y_s G_s^{-1}$.

Proof: The proof of sufficiency is straightforward from Theorem 2. To prove the necessity, we assume that (24) holds. Multiplying (24) by $\text{diag}\{G_s^T, I, I, I\}$ to the left and its transpose to the right, and defining two new variables $X_s = P_s^{-1}$ and $Y_s = K_s G_s$, we obtain

$$\begin{bmatrix} G_s^T X_s^{-1} G_s & \star & \star & \star \\ 0 & \gamma^2 I & \star & \star \\ A_s G_s + B_s Y_s & E_s & \tilde{X}_s^{-1} & \star \\ C_s G_s + D_s Y_s & F_s & 0 & I \end{bmatrix} > 0, \quad (27)$$

where $\tilde{X}_s = \sum_{l \in \mathcal{S}} \vartheta_{sl} X_l^{-1}$. Let $H_s = \tilde{X}_s^{-1}$ and $Z_{s,l} = \tilde{X}_s^{-1} X_l^{-1} \tilde{X}_s^{-1} + \epsilon I$ with $\epsilon > 0$. Applying Schur complement, the inequality $Z_{s,l} - \tilde{X}_s^{-1} X_l^{-1} \tilde{X}_s^{-1} > 0$ implies (26). On the other hand, we have

$$H_s^T + H_s - \sum_{l \in S_{kn}^s} \vartheta_{sl} Z_{s,l} = \tilde{X}_s^{-T} + \tilde{X}_s^{-1} - \tilde{X}_s^{-T} \tilde{X}_s \tilde{X}_s^{-1} - \epsilon \sum_{l \in S_{kn}^s} \vartheta_{sl} I = \tilde{X}_s^{-1} - \epsilon I. \quad (28)$$

Therefore, if one chooses a sufficiently small $\epsilon > 0$ and denotes $G_s = X_s$, the inequality (25) is verified by (27) and (28). This completes the proof. $\square$

Remark 7

Considering the particular case with only one local group in the LOSGs (3), Theorem 1 of [10] can be retrieved and expressed in an equivalent form as Corollary 1, which verifies that the proposed framework of overlapping group mode-dependent H_∞ control is more general than that of global mode-dependent H_∞ control in the literature.

Similar conclusion can be drawn from Theorem 3 to account for the case of incomplete TPs. The proof is straightforward from Theorem 3 and is omitted.

Corollary 2

For a given scalar $\gamma > 0$, there exists a global mode-dependent H_∞ state feedback controller in the form of $u(k) = K(\theta(k))x(k)$ such that the closed-loop system $\mathcal{G}_c$ with incomplete TPs described by (22) is stochastically stable with the prescribed H_∞ disturbance attenuation performance level γ if there exist symmetric matrices X_s and matrices H_s, G_s, W_s, Y_s , and $Z_{s,l}$, $\forall s, l \in \mathcal{S}$, of appropriate dimensions such that (26) and

$$\begin{bmatrix} G_s + G_s^T - X_s & \star & \star & \star \\ 0 & \gamma^2 I & \star & \star \\ A_s G_s + B_s Y_s & E_s & H_s^T + H_s + W_s - \sum_{l \in V_{kn}^s} \vartheta_{sl} (Z_{sl} + W_s) & \star \\ C_s G_s + D_s Y_s & F_s & 0 & I \end{bmatrix} > 0 \quad (29)$$

$$Z_{sl} + W_s < 0, \quad \forall l \in V_{ukn}^s. \quad (30)$$

Moreover, the controller gains are given by $K_s = Y_s G_s^{-1}$.

7. ILLUSTRATIVE EXAMPLES

In this section, two illustrative examples will be given to show the effectiveness of the proposed theoretical results.

7.1. Example 1

We consider the traditional MJLS in the form of (21) with four operation modes and the following system matrices borrowed from [9]

$$\begin{aligned} A_1 &= \begin{bmatrix} 1.00 & -1.25 \\ 2.50 & 2.50 \end{bmatrix}, A_2 = \begin{bmatrix} 0.50 & -0.83 \\ 2.50 & 3.50 \end{bmatrix}, A_3 = \begin{bmatrix} 0.25 & -0.25 \\ 2.50 & 3.00 \end{bmatrix}, A_4 = \begin{bmatrix} 0.75 & -0.57 \\ 2.50 & 2.75 \end{bmatrix} \\ B_1 = B_2 = B_3 = B_4 &= \begin{bmatrix} 0.50 & 0.10 \end{bmatrix}^T, E_1 = E_2 = E_3 = E_4 = \begin{bmatrix} 0.08 & 0.10 \end{bmatrix}^T \\ C_1 = C_2 = C_3 = C_4 &= \begin{bmatrix} 1 & 0 \end{bmatrix}, D_1 = D_2 = D_3 = D_4 = 0.80, F_1 = F_2 = F_3 = F_4 = 0.60. \end{aligned}$$

Firstly, we assume that four global operation modes are completely accessible for controller design. The objective is to design a global mode-dependent H_∞ state feedback controller for the system above with incomplete TPs. More specifically, we consider two different cases of incomplete TPM Θ for the Markov chain $\{\theta(k), k \geq 0\}$ studied in [9]

$$\text{Case 1: } \Theta = \begin{bmatrix} 0.3 & 0.2 & 0.1 & 0.4 \\ ? & ? & 0.3 & 0.2 \\ 0.1 & 0.1 & 0.5 & 0.3 \\ 0.2 & ? & ? & ? \end{bmatrix}; \quad \text{Case 2: } \Theta = \begin{bmatrix} 0.3 & ? & 0.1 & ? \\ ? & ? & 0.3 & 0.2 \\ ? & 0.1 & ? & 0.3 \\ 0.2 & ? & ? & ? \end{bmatrix}.$$

Applying Corollary 2, we find that the corresponding closed-loop system is stochastically stable with the minimal H_∞ performance level $\gamma_{min} = 1.3166$ in Case 1 and $\gamma_{min} = 1.3283$ in Case 2, respectively. Moreover, the corresponding controller gains are given by

$$\begin{aligned} K_1 &= [-5.6000 \quad -2.2045], K_2 = [-5.0776 \quad -4.6962] \\ K_3 &= [-4.4126 \quad -4.3290], K_4 = [-5.5967 \quad -4.0000] \end{aligned}$$

in Case 1, and

$$\begin{aligned} K_1 &= [-5.8643 \quad -2.4655], K_2 = [-5.0639 \quad -4.6724] \\ K_3 &= [-4.4034 \quad -4.3184], K_4 = [-5.5979 \quad -3.9981] \end{aligned}$$

in Case 2, respectively. However, in these two cases, the minimal H_∞ performance levels are obtained as $\gamma_{min} = 1.8782$ in Case 1, and $\gamma_{min} = 1.9388$ in Case 2, respectively, in [9], which means that Corollary 2 in this paper is less conservative than that of [9] for this example.

Figure 2. The overlapping structure of the LOSGs with four operation modes

Secondly, we assume that four global operation modes of the system above are not completely accessible and local overlapping operation modes exist. Let us consider the following scenario: the MJLS (21) with four operation modes is randomly decomposed into two LOSGs in the form of

(3), i.e., $\mathcal{N} = \{1, 2\}$. Suppose that LOSG 1 contains subsystems 1, 2 and 3, and LOSG 2 contains subsystems 2, 3 and 4. The overlapping structure of four operation modes is shown in Figure 2. We assume that the overlapping subsystem shares the same system parameter between different LOSGs, i.e., the system matrices of the reformulated LOSGs are given by $A_{i_m, m} = A_{i_m}$, $B_{i_m, m} = B_{i_m}$, $C_{i_m, m} = C_{i_m}$, $D_{i_m, m} = D_{i_m}$, and $E_{i_m, m} = E_{i_m}$, $i_m \in \mathcal{M}^{(m)}$; $m \in \mathcal{N}$, where $\mathcal{M}^{(1)} = \{1, 2, 3\}$, and $\mathcal{M}^{(2)} = \{2, 3, 4\}$. The objective is to design an overlapping group mode-dependent H_∞ state feedback controller in the form of (4) for the system above such that the corresponding closed-loop system is stochastically stable with the prescribed H_∞ disturbance attenuation performance. Theorem 2 and Theorem 3 are used to verify the proposed controller design method by resorting to the Matlab LMI Toolbox.

The following four different cases of the TPMs Ω and $\Pi^{(m)}$, $m \in \mathcal{N}$ are considered.

$$\begin{aligned}
 \text{Case I: } \Omega &= \begin{bmatrix} 0.2 & 0.8 \\ 0.8 & 0.2 \end{bmatrix}, \Pi^{(1)} = \begin{bmatrix} 0.4 & 0.4 & 0.2 \\ 0.2 & 0.5 & 0.3 \\ 0.2 & 0.7 & 0.1 \end{bmatrix}, \Pi^{(2)} = \begin{bmatrix} 0.2 & 0.2 & 0.6 \\ 0.1 & 0.6 & 0.3 \\ 0.1 & 0.5 & 0.4 \end{bmatrix}; \\
 \text{Case II: } \Omega &= \begin{bmatrix} 0.2 & 0.8 \\ 0.8 & 0.2 \end{bmatrix}, \Pi^{(1)} = \begin{bmatrix} 0.4 & ? & ? \\ ? & ? & 0.3 \\ ? & 0.7 & ? \end{bmatrix}, \Pi^{(2)} = \begin{bmatrix} 0.2 & ? & ? \\ ? & ? & ? \\ ? & 0.5 & ? \end{bmatrix}; \\
 \text{Case III: } \Omega &= \begin{bmatrix} 0.2 & 0.8 \\ 0.8 & 0.2 \end{bmatrix}, \Pi^{(1)} = \begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix}, \Pi^{(2)} = \begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix}; \\
 \text{Case IV: } \Omega &= \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}, \Pi^{(1)} = \begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix}, \Pi^{(2)} = \begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix}.
 \end{aligned}$$

Applying Theorem 2 and Theorem 3 in Case I, and Theorem 3 in Cases II-IV, we have the minimal value of the H_∞ disturbance attenuation performance level γ in different cases shown in Table I. It is clearly seen from Table I that the more information of mode TPs in the TPMs one has, the smaller the optimal H_∞ disturbance attenuation performance can be achieved, which provides the tradeoff between the cost of acquiring exact knowledge of the mode TPs and the system performance. For the optimal H_∞ performance in Case II, the corresponding controller parameter matrices in (4) under different operation modes are given as

$$\begin{aligned}
 K_{1,1} &= [-5.3037 \quad -2.2246], K_{2,1} = [-4.9045 \quad -4.5329], K_{3,1} = [-4.6344 \quad -4.5687] \\
 K_{2,2} &= [-5.0287 \quad -4.7260], K_{2,3} = [-4.7823 \quad -4.7515], K_{2,4} = [-5.4048 \quad -3.8930].
 \end{aligned}$$

Finally, we assume that the disturbance signal is chosen as $w(k) = \sin^2(k)/(0.5 + (k-1)^2)$ and the initial condition is taken as $x(0) = x_0 = [-\frac{\pi}{3} \quad \frac{\pi}{6}]^T$. It can be easily checked that the open-loop LOSGs ($u(k) = 0$) in the form of (3) with the system parameters given above is unstable. Connecting the overlapping group mode-dependent controller (4) with parameter matrices obtained in Case II and the traditional global mode-dependent controller with parameter matrices obtained in Case I to the system above, the state responses of the closed-loop system are shown in Figure 4 under possible system mode evolutions, as illustrated in Figure 3. From Figure 3, it should be noted that at time instant $k = 1s$, the system (3) switches from group 1 to group 2 and starts to operate at local mode 3. Correspondingly, at time instant $k = 2s$, the system (3) switches from group 2 to group 1. However, the system remains at local mode 3 due to that local operation mode 3 is defined to be “overlapping”. Similar situations can be considered, e.g., on time intervals $[3s, 6s]$ and $[32s, 25s]$ for overlapping local mode 2. In this case, both local modes 2 and 3 perform their “overlapping roles” in different groups during the entire system modes evolution. On the other hand, it can be seen from Figure 4 that applying the proposed controller design method, the obtained overlapping group mode-dependent H_∞ state feedback controller can not only stabilize the system, but also impose an H_∞ level for the closed-loop system with not available global modes and incomplete TPs, which verifies the effectiveness of the obtained theoretical results.

Table I. The minimal value of the H_∞ disturbance attenuation performance level γ in different cases

	Case I	Case II	Case III	Case IV
Theorem 2	0.8990	-	-	-
Theorem 3	0.8990	1.0365	1.0847	1.9466

Figure 3. Modes evolution of system (21) and LOSGs

Figure 4. State responses of the closed-loop system under traditional global mode-dependent H_∞ control and the proposed overlapping group mode-dependent H_∞ control

Figure 5. The inverted pendulum system on a cart

Table II. System parameter values of the discretized pendulum system

Parameters	a_1^s	a_2^s	a_3^s	a_4^s	b_1^s	b_2^s
$\theta(k) = s = 1$	1.0083	0.0322	0.5245	1.0095	-0.0008	-0.0068
$\theta(k) = s = 2$	1.0103	0.0288	0.5169	1.0086	-0.0011	-0.0032
$\theta(k) = s = 3$	1.0099	0.0324	0.5186	1.0101	-0.0023	-0.0098
$\theta(k) = s = 4$	1.0065	0.0376	0.5254	1.0066	-0.0002	-0.0047

7.2. Example 2

In this example, the effectiveness of the proposed overlapping group mode-dependent control scheme is illustrated by means of an inverted pendulum system [26]. The pendulum is attached to the side of a cart through a pivot and is allowed to swing in the xy -plane, as shown in Figure 5. Then a force u is given to the cart in the x -direction to keep the pendulum balanced upright.

We now consider the inverted pendulum system with the following continuous-time state-space model [26]:

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ \frac{3(M+m)g}{l(4M+m)} & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ -\frac{3}{l(4M+m)} \end{bmatrix} u(t) \quad (31)$$

where the parameters $M = 8.0\text{kg}$, $m = 2.0\text{kg}$, $l = 0.5\text{m}$, and $g = 9.8\text{m/s}^2$. The system is discretized with a zero-order holder and sampling time $T_s = 30\text{ms}$. Moreover, we assume that abrupt changes are imposed in the system dynamics, and these variations are assumed here to be driven by a homogeneous Markov chain $\{\theta(k), k \geq 0\}$. Then we have the discretized model for the above pendulum system in (31) with an external disturbance signal $w(t)$ given by

$$x(k+1) = \underbrace{\begin{bmatrix} a_1^{\theta(k)} & a_2^{\theta(k)} \\ a_3^{\theta(k)} & a_4^{\theta(k)} \end{bmatrix}}_{A(\theta(k))} x(k) + \underbrace{\begin{bmatrix} b_1^{\theta(k)} \\ b_2^{\theta(k)} \end{bmatrix}}_{B(\theta(k))} u(k) + E(\theta(k))w(k) \quad (32)$$

with the system parameters given in Table II. Other system matrices are chosen as $E_1 = E_2 = E_3 = E_4 = [0 \ 1]^T$, $C_1 = C_2 = C_3 = C_4 = [1 \ 0]$, $D_1 = D_2 = D_3 = D_4 = 0.3$, and $F_1 = F_2 = F_3 = F_4 = 0.2$.

It can be easily checked that the discretized system (32) without control is unstable. Applying

Corollary 1 and Corollary 2 with completely known TPs in the TPM $\Theta = \begin{bmatrix} 0.3 & 0.2 & 0.1 & 0.4 \\ 0.3 & 0.2 & 0.3 & 0.2 \\ 0.1 & 0.1 & 0.5 & 0.3 \\ 0.2 & 0.2 & 0.1 & 0.5 \end{bmatrix}$,

a stabilizing global mode-dependent H_∞ state feedback controller is obtained with the controller gains given by

$$K_1 = [400.9629 \ 101.8343], K_2 = [373.6459 \ 93.5232] \\ K_3 = [218.6136 \ 55.6297], K_4 = [654.4070 \ 169.3943]$$

and the minimal value of the H_∞ disturbance attenuation performance $\gamma_{\min} = 50.4116$. Next, by applying the proposed overlapping decomposition method, we randomly decompose the system (32) with four operation modes into two LOSGs governed by two LOMCs $\{r^{(1)}(k), k \geq 0\}$, $\{r^{(2)}(k), k \geq 0\}$, and a high-level homogeneous Markov chain $\{\delta(k), k \geq 0\}$. We assume that LOSG 1 contains local modes 1, 2 and 3; and LOSG 2 contains local modes 3 and 4. The system matrices of the newly constructed LOSGs are chosen as $(A, B, C, D, E)_{i_m, m} = (A, B, C, D, E)_{i_m}$, $i_m \in \mathcal{M}^{(m)}$; $m \in \mathcal{N}$, where $\mathcal{M}^{(1)} = \{1, 2, 3\}$, $\mathcal{M}^{(2)} = \{3, 4\}$, and $\mathcal{N} = \{1, 2\}$.

Let $\Omega = \begin{bmatrix} 0.1 & 0.9 \\ 0.3 & 0.7 \end{bmatrix}$, $\Pi^{(1)} = \begin{bmatrix} ? & ? & 0.2 \\ ? & 0.5 & ? \\ 0.3 & 0.3 & 0.4 \end{bmatrix}$ and $\Pi^{(2)} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$. Applying Theorem 3, it

is found that the proposed overlapping group mode-dependent H_∞ control problem is solvable with a minimum H_∞ norm $\gamma_{min} = 30.2593$. The controller gains are given by

$$K_{1,1} = [37.5343 \ 9.9415], \ K_{2,1} = [31.1204 \ 8.0325], \ K_{3,1} = [61.8286 \ 15.5056] \\ K_{3,2} = [69.3848 \ 17.4014], \ K_{4,2} = [21.5467 \ 7.4169].$$

Figure 6. Modes evolution of system (32) and LOSGs

Furthermore, connecting the controller above with the system (32), taking the disturbance signal as $w(k) = \sin^2(k)/(0.5 + (k - 1)^2)$, giving possible system modes evolution as illustrated in Figure 6, and choosing the initial condition $x(0) = x_0 = [-1.2 \ 0.6]^T$, the state responses of the closed-loop system under the traditional global mode-dependent controller design method and the proposed overlapping group mode-dependent controller design method are shown in Figure 7, respectively. From the curves in Figure 7, one can clearly see that the proposed controller design method with incomplete TPs is effective.

Figure 7. State response of the closed-loop system under traditional global mode-dependent H_∞ control and the proposed overlapping group mode-dependent H_∞ control

8. CONCLUSIONS

The problem of overlapping group mode-dependent H_∞ state feedback control for discrete-time Markovian jump linear systems whose global operation modes are not completely accessible

for controller design has been studied. A randomly overlapping decomposition method has been developed to reformulate the system by a set of locally overlapping switched groups with accessible group modes. Overlapping local operation modes have been allowed to exist in the formed groups. An overlapping group mode-dependent H_∞ state feedback controller has been delicately constructed. The proposed controller has not only adopted full state information and local operation mode information, but also taken full knowledge of group modes into consideration. Sufficient conditions on the existence of a desired overlapping group mode-dependent H_∞ state feedback controller have been derived based on the stochastic Lyapunov functional approach. The proposed framework has been shown to be more general and has covered the traditional Markovian jump linear systems with completely accessible global modes as its special case. In the case of only one local group, it has also been shown that some existing result in the literature can be retrieved. Finally, two illustrative examples have been given to show the effectiveness of the developed controller design method.

9. ACKNOWLEDGMENT

This work was supported in part by the Australian Research Council Discovery Projects under Grant DP1096780, the Research Advancement Awards Scheme Program (January 2010–December 2012) at Central Queensland University, Australia.

REFERENCES

- Costa O, Fragoso M, Marques R, *Discrete-time Markovian jump linear systems*, Springer, 2005.
- de Souza C, Robust stability and stabilization of uncertain discrete-time Markovian jump linear systems, *IEEE Transactions on Automatic Control* 2006; **51**(5):836–841.
- Zhang L, Boukas E-K. Stability and stabilization of Markovian jump linear systems with partly unknown transition probabilities, *Automatica* 2009; **45**(2): 463–468.
- Xiong J, Lam J, Gao H, Ho D. On robust stabilization of Markovian jump systems with uncertain switching probabilities, *Automatica* 2005; **41**(5):897–903.
- Zhang L, Lam J. Necessary and sufficient conditions for analysis and synthesis of Markov jump linear systems with incomplete transition descriptions, *IEEE Transactions on Automatic Control* 2010; **55**(7):1695–1701.
- do Val J, Geromel J, Gonçalves A. The H_2 -control of jump linear systems: cluster observations of the Markov state, *Automatica* 2002; **38**(2):343–349.
- Boukas E, Liu Z. Robust H_∞ control of discrete-time Markovian jump linear systems with mode-dependent time-delay, *IEEE Transactions on Automatic Control* 2001; **46**(12):1918–1924.
- Boukas E. H_∞ control of discrete-time Markovian jump linear systems with bounded transition probabilities, *Optimal Control Applications and Methods* 2009; **30**(5):477–494.
- Zhang L, Boukas E. H_∞ control for discrete-time Markovian jump linear systems with partly unknown transition probabilities, *International Journal of Robust and Nonlinear Control* 2009; **19**(8):868–883.
- Gonçalves A, Fioravanti A, Geromel J. H_∞ robust and networked control of discrete-time MJLS through LMIs, *Journal of the Franklin Institute* 2012; **349**(6):2171–2181.
- Fioravanti A, Gonçalves A, Geromel J. Discrete-time H_∞ output feedback for Markov jump systems with uncertain transition probabilities, *International Journal of Robust and Nonlinear Control* 2013; **23**(8):894–902.
- Xiong J, Ugrinovskii V, Petersen I. Local mode dependent output feedback control of uncertain Markovian jump large-scale systems, *Proceedings of 2009 American Control Conference*, St. Louis, MO, USA 2009;4254–4259.
- Zhang Y, Xu S, Chu Y. Sliding mode observer-controller design for uncertain Markovian jump systems with time delays, *International Journal of Robust and Nonlinear Control*, 2012; **22**(4):355–368.
- Li L, Ugrinovskii V, Orsi R. Decentralized robust control of uncertain Markov jump parameter systems via output feedback, *Automatica* 2007; **43**(11):1932–1944.
- Xiong J, Ugrinovskii V, Petersen I. Local mode dependent decentralized stabilization of uncertain Markovian jump large-scale systems, *IEEE Transactions on Automatic Control* 2009; **54**(11):2632–2637.
- Ma S, Xiong J, Ugrinovskii V, Petersen I. Robust decentralized stabilization of Markovian jump large-scale systems: A neighboring mode dependent control approach, *Automatica* 2013; **49**(10):3105–3111.
- Bolzern P, Colaneri P, Nicolao G. Markov Jump Linear Systems with switching transition rates: Mean square stability with dwell-time”, *Automatica*, 2010; **46**(6):1081–1088.
- Kim D, Park P, Ko J. Output-feedback control of systems over communication networks using a deterministic switching system approach, *Automatica* 2004; **40**(7):1205–1212.
- Ge S, Sun Z. Switched controllability via bumpless transfer input and dwell-time switching, *Proceedings of the 46th IEEE Conference on Decision and Control*, New Orleans, LA, USA, 2007;714–719.
- Hou Z, Dong C, Wang Q. Stability analysis of switched linear systems with locally overlapped switching law, *Journal of Guidance, Control, and Dynamics* 2010; **33**(2):396–403.
- Xu L, Wang Q, Li W, Hou Y. Stability analysis and stabilization of full-envelope networked flight control systems: switched system approach, *IET Control Theory & Applications* 2012; **6**(2):286–296.
- Martinson E, Arkin R. Learning to role-switch in multi-robot systems, *Proceedings of ICRA* 2003; 2727–2734.

23. Karan M, Shi P, and Kaya C. Transition probability bounds for the stochastic stability robustness of continuous and discrete-time Markovian jump linear systems, *Automatica* 2006; **42**(12):2159–2168.
24. Ge X, Han Q-L. Distributed fault detection over sensor networks with Markovian switching topologies, *International Journal of General Systems* 2014; **43**(3-4):305–318.
25. Seiler P, Sengupta R. A bounded real lemma for jump systems, *IEEE Transactions on Automatic Control* 2003; **48**(9):1651–1654.
26. Gao H, Chen T. New results on stability of discrete-time systems with time-varying delay, *IEEE Transactions on Automatic Control* 2007; **52**(2):328–334.