Stochastic feedback control of quantum transport to realize a dynamical ensemble of two nonorthogonal pure states

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A Markovian open quantum system which relaxes to a unique steady state $\rho_{\rm ss}$ of finite rank can be decomposed into a finite physically realizable ensemble (PRE) of pure states. That is, as shown by Karasik and Wiseman [Phys. Rev. Lett. **106**, 020406 (2011)], in principle there is a way to monitor the environment so that in the long time limit the conditional state jumps between a finite number of possible pure states. In this paper we show how to apply this idea to the dynamics of a double quantum dot arising from the feedback control of quantum transport, as previously considered by one of us and co-workers [Phys. Rev. B **84**, 085302 (2011)]. Specifically, we consider the limit where the system can be described as a qubit, and show that while the control scheme can always realize a two-state PRE, in the incoherent tunneling regime there are infinitely many PREs compatible with the dynamics that cannot be so realized. For the two-state PREs that are realized, we calculate the counting statistics and see a clear distinction between the coherent and incoherent regimes.

I. INTRODUCTION

[illegible] [1,2]. [illegible] ρ_c [illegible] ρ [illegible] [2]. [illegible] [3], [illegible] [4–6], [illegible] [3,7,8]. [illegible] [3] [illegible] ρ.

[illegible] $\rho_{\rm ss}$ [illegible] $\rho_{\rm ss}$, [illegible] *physically realizable*[9] [illegible] *physically realizable ensembles* [illegible]

[illegible] *how* [illegible] [10] [illegible]

[illegible] [2]. [illegible] [2,11–16]. [illegible] [17,18] [illegible] [19,20].

[illegible] [10] [illegible] [14]. [illegible]

II. TRACKING THE STATE OF A QUBIT USING A BIT

[illegible]

$$\dot{\rho} = -i[\hat{H}, \rho] + \sum_l \mathcal{D}[\hat{c}_l]\rho \equiv \mathcal{L}\rho, \quad (1)$$

[illegible] $\mathcal{D}[\hat{O}]\bullet = \hat{O}\bullet\hat{O}^\dagger - 1/2\{\hat{O}^\dagger\hat{O}, \bullet\}$ [illegible] [2]. [illegible] $\hat{H}$ [illegible] $\{\hat{c}_l\}$ [illegible] $\mathcal{L}$ [illegible]

$$\mathcal{L} = \mathcal{S}[\hat{H}'] + \sum_{l=1}^{L} \mathcal{J}[\hat{c}_l] \quad (2)$$

[illegible] $\mathcal{S}[\hat{H}']\bullet = (-i\hat{H}')\bullet + \bullet(i\hat{H}'^\dagger)$ [illegible] $\mathcal{J}[\hat{c}]\bullet = \hat{c}\bullet\hat{c}^\dagger$ [illegible]

$$\hat{H}' = \hat{H} - \frac{i}{2}\sum_{l=1}^{L} \hat{c}_l^\dagger \hat{c}_l \quad (3)$$

[illegible] $\mathcal{L}$ [illegible] (??) [illegible] $\mathcal{M}$, [illegible]

$$\rho(t) = \mathcal{M}(t)\rho(0) = e^{\mathcal{L}t}\rho(0). \quad (4)$$

[illegible] $\hat{c}$ [illegible] $\mathcal{M}(t) = \sum_{n=0}^{\infty} \mathcal{M}^{(n)}(t)$ [illegible]

$$\mathcal{M}^{(n)}(t) = \int_0^t dt_n \int_0^{t_n} dt_{n-1} \cdots \int_0^{t_2} dt_1 \mathcal{Q}(t-t_n)\mathcal{J} \times \mathcal{Q}(t_n - t_{n-1})\mathcal{J} \cdots \mathcal{Q}(t_2 - t_1)\mathcal{J}\mathcal{Q}(t_1), \quad (5)$$

[illegible] $\mathcal{Q}(t) = e^{t\mathcal{S}[\hat{H}']}$ [illegible] $\mathcal{S}$. [illegible] $\mathcal{S}$, [illegible] $\mathcal{J} \equiv \mathcal{J}[\hat{c}]$.

[illegible] (??) [illegible]

$$\hat{c}_l \to \hat{c}_m = \sum_{l=1}^{L} \widetilde{U}_{ml}\hat{c}_l + \beta_m, \quad (6)$$

$$\hat{H} \to \hat{H} - \frac{i}{2}\sum_{m=1}^{M} (\beta_m^* \hat{c}_m - \beta_m \hat{c}_m^\dagger), \quad (7)$$

[illegible] β_m [illegible] $\widetilde{U}_{ml} \in \mathbb{C}^{M\times L}$ [illegible] $\widetilde{U}$) [illegible] β_l [illegible] l [illegible] $\{\beta_l\}, \{M_{lk}\}$ [illegible]

[illegible] $\mathcal{L}\rho_{\rm ss} = 0$ [illegible] $\rho_{\rm ss}$ [illegible] $\{(p_k, |\psi_k\rangle)\}_{k=1}^{K}$ [illegible] p_k [illegible] k [illegible] *preferred ensemble fact*[9]. [illegible] $\{\beta_m\}, \{\widetilde{U}_{ml}\}$ [illegible]

[illegible] [10], [illegible] [10] [illegible] $\mathcal{J}[\hat{c}_l]$ [illegible] $|\psi_k\rangle$.

[illegible] [10] [illegible] (??) [illegible]

$$\dot{\vec{r}} = A\vec{r} + \vec{b}, \quad (8)$$

[illegible] A [illegible] 3×3 [illegible] b [illegible] $\vec{r} = \langle(\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z)^\top\rangle$ [illegible] $A\vec{r}_{\rm ss} = -\vec{b}$. [illegible] A [illegible] $\{(p_k, \vec{r}_k)\}_{k=\pm}$

[illegible] [10]

$$\vec{r}_k = \vec{r}_{\rm ss} - k\alpha_k \vec{e}_v,$$

$$p_k = \frac{1}{2}\left[1 - \frac{k\vec{r}_{\rm ss}\cdot\vec{e}_v}{\sqrt{1-\|\vec{r}_{\rm ss}\|^2 + (\vec{r}_{\rm ss}\cdot\vec{e}_v)^2}}\right].$$

[illegible] $\vec{e}_v$ [illegible] A, [illegible] $\alpha_k = k\vec{r}_{\rm ss}\cdot\vec{e}_v + \sqrt{1-\|\vec{r}_{\rm ss}\|^2 + (\vec{r}_{\rm ss}\cdot\vec{e}_v)^2}$. [illegible] $\vec{r}_+$ [illegible] $\vec{r}_-$ [illegible] A [illegible] A [illegible]

III. STATE STABILIZATION OF A QUBIT USING FEEDBACK CONTROL

[illegible] (??) [illegible] [8]. [illegible] [21–29]. [illegible] [14]. [illegible] [34]. [illegible] [14], [illegible] *at most* [illegible] ($\mu_L \to \infty$, $\mu_R \to -\infty$)[35,36], [illegible] ϵ_L [illegible] ϵ_R [illegible] $|\emptyset\rangle \equiv |N_L, N_R\rangle$ [illegible] N_d [illegible] $d \in \{R, L\}$. [illegible] $|L\rangle \equiv |N_L + 1, N_R\rangle$, [illegible] $|R\rangle \equiv |N_L, N_R + 1\rangle$, [illegible] $|\emptyset\rangle \to |L\rangle$ [illegible] $|R\rangle \to |\emptyset\rangle$ [illegible] γ_L [illegible] γ_R, [illegible]

[illegible] (??) [illegible] [2,34].

$$\hat{H} = \frac{1}{2}\Delta\hat{\sigma}_z + T_c\hat{\sigma}_x,$$

[illegible] $\Delta \equiv \epsilon_L - \epsilon_R$ [illegible] T_c [illegible] $\hat{\sigma}_z = |L\rangle\langle L| - |R\rangle\langle R|$,

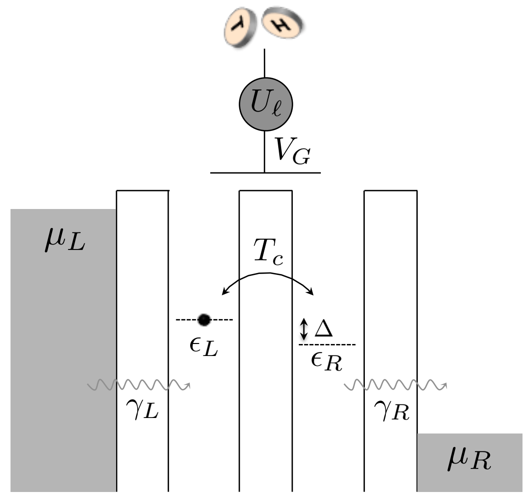

FIG. 1. Stochastic feedback control of a double quantum dot. Flow of one excess electron is monitored when it tunnels into the left dot at rate γ_L (e.g. by a QPC, not shown). A unitary control operation (which can be implemented for example with gate voltages V_G) then rotates the state of the system into one of two (randomly chosen) desired states. Here T_c is the coupling strength between the dots, Δ is the detuning, γ_R is the tunneling rate out of the right dot, and $\mu_{L(R)}$ are chemical potentials of the leads.

[illegible] $\hat{\sigma}_x = |L\rangle\langle R| + |R\rangle\langle L|$. [illegible] $\Delta = 0$ [illegible]

$$\hat{c}_1 = \sqrt{\gamma_L}|L\rangle\langle\emptyset|, \qquad \hat{c}_2 = \sqrt{\gamma_R}|\emptyset\rangle\langle R|.$$

[illegible] (??) [illegible] (??) [illegible] (??) [illegible]

$$\hat{H}' = -i\frac{\gamma_L}{2}|\emptyset\rangle\langle\emptyset| + \sum_{\ell=\pm}\varepsilon_\ell|\phi_\ell\rangle\langle\widetilde{\phi_\ell}|,$$

[illegible]

$$|\phi_\ell\rangle = \frac{(i\gamma_R + \ell\kappa)|L\rangle + 4T_c|R\rangle}{\sqrt{16T_c^2 + |i\gamma_R + \ell\kappa|^2}},$$

$$\varepsilon_\ell = -\frac{1}{4}(i\gamma_R + \ell\kappa), \quad \kappa = \sqrt{16T_c^2 - \gamma_R^2}.$$

[illegible] $|\phi_\ell\rangle$ [illegible] $\langle\widetilde{\phi_\ell}|$ [illegible] $\hat{H}'$ [illegible] $\langle\widetilde{\phi_\ell}| \neq \langle\phi_\ell|$. [illegible] $\langle\emptyset|\phi_\ell\rangle = 0$ [illegible] $\langle\phi_\ell|\phi_{-\ell}\rangle \neq 0$

[illegible] [14], [illegible] [30,31], in [illegible] [32,33] [illegible]

[illegible] $|L\rangle$, [illegible]

$$\mathcal{J}[c_1]|\emptyset\rangle\langle\emptyset| = \gamma_L |L\rangle\langle L|. \tag{6}$$

[illegible] $\hat{H}'$, [illegible] $|\phi_\ell\rangle$[14]. [illegible] V_G, [illegible]

$$\hat{U}_\ell |L\rangle = |\phi_\ell\rangle. \tag{7}$$

[illegible] $\varrho_\ell = |\phi_\ell\rangle\langle\phi_\ell|$ [illegible]

$$\mathcal{S}\big[\hat{H}'\big]\varrho_\ell = 2\Im(\varepsilon_\ell)\varrho_\ell. \tag{8}$$

[illegible]

$$\mathcal{J}[c_2]\varrho_\ell = \gamma_R^\ell |\emptyset\rangle\langle\emptyset|, \tag{9}$$

[illegible] $\gamma_R^\ell \equiv -2\Im(\varepsilon_\ell)$, [illegible]

[illegible] $\mathscr{H}^3 \equiv$ [illegible] $|\emptyset\rangle, |\phi_-\rangle, |\phi_+\rangle$). [illegible] ℓ [illegible] $-$) [illegible] γ_L [illegible] γ_R^ℓ [illegible] (??) [illegible] (??), [illegible]

$$\rho_{\rm ss} = \frac{1}{2\gamma_L + \gamma_R^\ell}\big(\gamma_R^\ell |\emptyset\rangle\langle\emptyset| + 2\gamma_L \varrho_\ell\big). \tag{10}$$

[illegible] $\gamma_L \gg \gamma_R, T_c$, [illegible] [14]. [illegible] $|\emptyset\rangle\langle\emptyset|$ [illegible] ϱ_ℓ [illegible] $|L\rangle\langle L|$ [illegible] ϱ_ℓ. [illegible] U_ℓ) [illegible] $\varrho_\ell = |\phi_\ell\rangle\langle\phi_\ell|$, [illegible] U_ℓ [illegible] $|\phi_\ell\rangle$ [illegible]

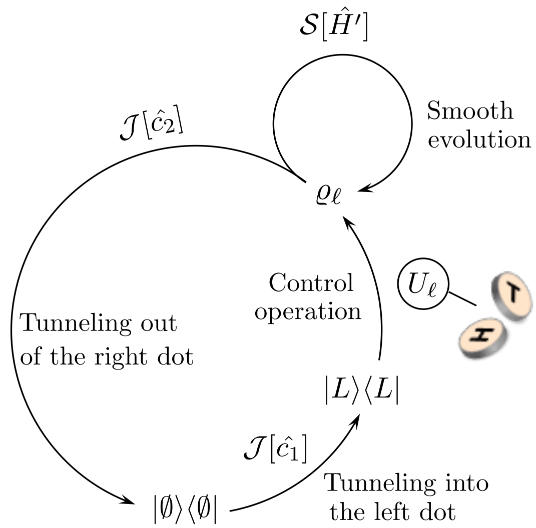

FIG. 2. A schematic representation of the state evolution of the DQD.

IV. STOCHASTIC FEEDBACK CONTROL OF THE DQD

[illegible]

$$\hat{c}_1 \to \hat{c}_1' = \hat{U}_\ell\, \hat{c}_1 = \sqrt{\gamma_L}\, |\phi_\ell\rangle\langle\emptyset| \tag{11}$$

[illegible]

$$\dot{\rho} = -i[\hat{H}, \rho] + \big(\mathcal{D}[c_1'] + \mathcal{D}[c_2]\big)\rho. \tag{12}$$

[illegible] $D = 2$ [illegible] ϱ [illegible] $\{|L\rangle, |R\rangle\}$, [illegible] (??) [illegible]

$$\dot{\varrho} = -i[\hat{H}, \varrho] + \mathcal{D}[c_\ell]\varrho, \tag{13}$$

[illegible]

$$\hat{c}_\ell = \sqrt{\gamma_R}\, \hat{U}_\ell |L\rangle\langle R| = \sqrt{\gamma_R}\, |\phi_\ell\rangle\langle R|. \tag{14}$$

[illegible] $|\phi_\ell\rangle$ [illegible]

$$\hat{H}' = \hat{H} - \frac{i}{2}\hat{c}_\ell^\dagger \hat{c}_\ell = \varepsilon_\ell |\phi_\ell\rangle\langle\widetilde{\phi_\ell}|. \tag{15}$$

[illegible] ϱ_ℓ.

[illegible] $|\phi_\pm\rangle$ [illegible]

[illegible] $\hat{U}_\pm$ [illegible] $0 \le \wp_\ell \le 1$ [illegible] $\hat{U}_\ell$ [illegible] $\wp_- + \wp_+ = 1$ [illegible]

$$\dot{\varrho} = -i[\hat{H}, \varrho] + \sum_{\ell=\pm} \wp_\ell \, \mathcal{D}[c_\ell]\varrho,$$

[illegible]

$$\varrho_{\rm ss} = \sum_{\ell=\pm} p_\ell \varrho_\ell.$$

[illegible] p_ℓ [illegible] $\wp_\ell$ [illegible] p_ℓ [illegible]

[illegible] (??) [illegible] A [illegible] b [illegible] A [illegible] $\dot{\vec{r}}_{\rm ss} = 0$ [illegible]

A. Coherent tunneling ($\gamma_R < 4T_c$)

[illegible] $|L\rangle$ [illegible] $|R\rangle$ [illegible] $\wp_- = 0$ [illegible] $\wp_+ = 1$ [illegible] *vice versa,* [illegible] $\vec{r}_+$ [illegible] $\vec{r}_-$, [illegible] H' [illegible]

$$\vec{r}_\ell = \left(-\ell \frac{\kappa}{4T_c}, -\frac{\gamma_R}{4T_c}, 0 \right), \quad \gamma_R < 4T_c.$$

[illegible] A [illegible]

[illegible] p_ℓ [illegible] ρ_ℓ [illegible] $\wp_\ell$. [illegible] $\sum_\ell \wp_\ell = 1$ [illegible] p_ℓ. [illegible]

$$\tau_\ell = (\gamma_R \, \varrho_\ell^{RR})^{-1},$$

[illegible]

$$\varrho_\ell^{RR} = \langle R|\varrho|R\rangle_\ell = \frac{1 - \langle \hat{\sigma}_z \rangle_\ell}{2}.$$

[illegible] (??) [illegible] $\langle \hat{\sigma}_z \rangle_\pm = 0$ [illegible] $\tau_\ell = 2/\gamma_R$ [illegible] $\wp_\ell$ [illegible] ϱ_ℓ [illegible] p_l [illegible]

$$p_\ell = \wp_\ell.$$

B. Incoherent tunneling ($\gamma_R > 4T_c$)

[illegible] $\tilde{\kappa}$ [illegible] $\kappa = i\kappa$ [illegible] κ [illegible]

$$\vec{r}_\ell = \left(0, -\frac{4T_c}{\gamma_R}, -\ell \frac{\tilde{\kappa}}{\gamma_R} \right), \quad \gamma_R > 4T_c.$$

[illegible] (??) [illegible] A [illegible] (??) [illegible] (??) [illegible] A. [illegible] A. [illegible]

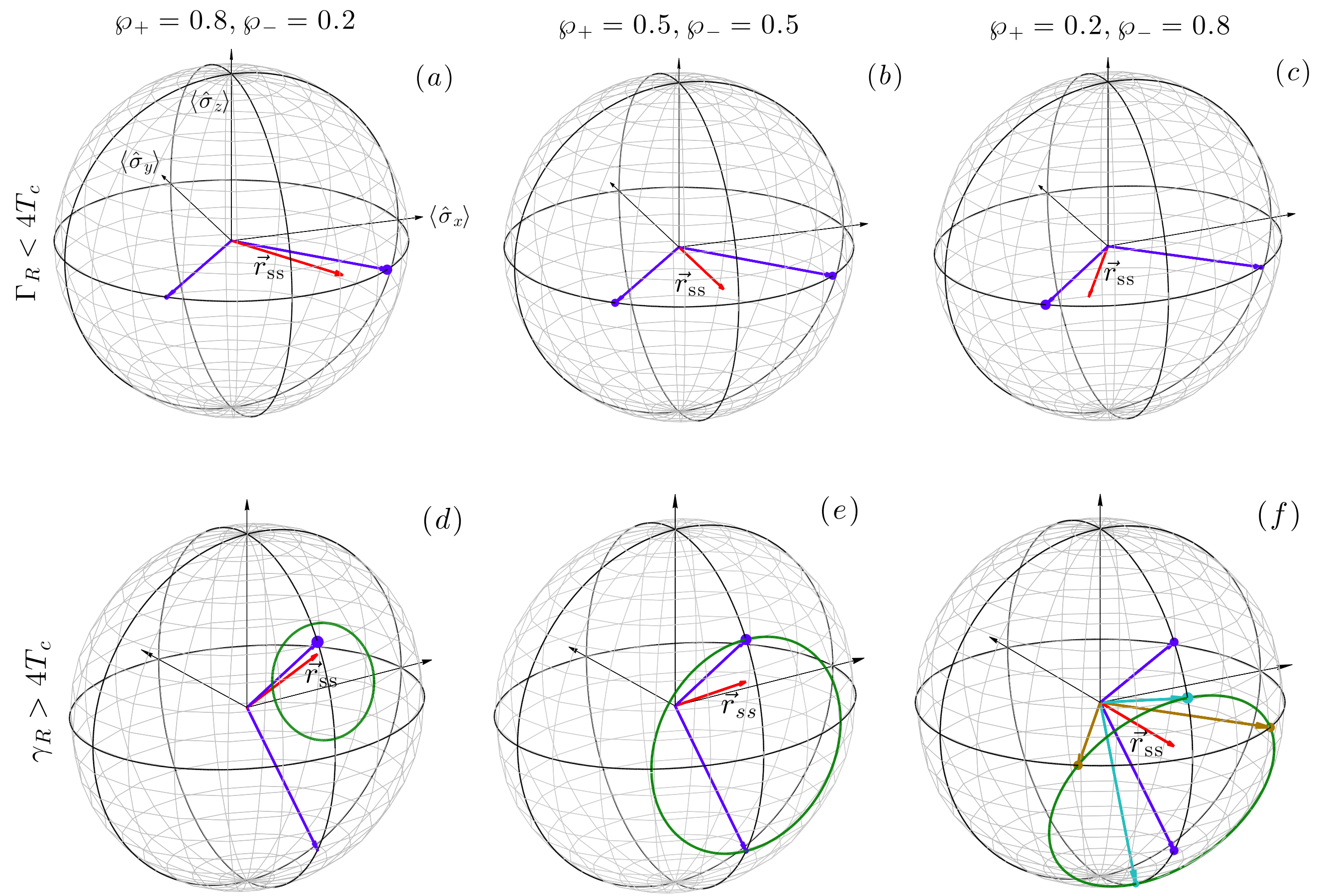

FIG. 3. (Color online). Bloch representation of TSEs. The top row shows, for $\gamma_R = 3, T_c = 1$, Bloch vectors of two-state hopping for three different probabilities defining the stochastic feedback: (a) $\wp_+ = 0.8, \wp_- = 0.2$, (b) $\wp_+ = 0.5, \wp_- = 0.5$, and (c) $\wp_+ = 0.2, \wp_- = 0.8$. Blue arrows (dark arrows in the azimuthal plane) represent the state vectors of the TSE with associated probabilities p_ℓ Eq. (10), indicated by the volume of the sphere at the tip of each vector. The steady state $\vec{r}_{\rm ss}$ is depicted by a red arrow. In this regime, the probabilities $\wp_\ell$ are equal to the probabilities p_ℓ that the system is found in a particular state of the TSE. The bottom row shows TSEs for $\gamma_R = 5, T_c = 1$ for the same control probabilities as in the top row. Here, in contrast, in addition to the eigenvectors of the effective Hamiltonian $\hat{H}'$, shown by blue arrows (dark arrows in the polar plane), an infinite number of other two-state TSEs exist. The solid green circle depicts the locus of them. By comparing this circle in (d), (e), and (f) it is clear that increasing $\wp_+$ shrinks its radius such that at $\wp_+ = 1$ it eventually collapses to a single point. In (f) we plot the states of two other TSEs each corresponding to a real eigenvector of A shown in light brown (in the azimuthal plane) and cyan (dark arrows in the polar plane). In the bottom row, $\wp_\ell$ and p_ℓ are entirely different because the latter is a function of the lifetimes of states in the right dot, Eq. (**??**).

[illegible] $\wp_\ell = \wp_{-\ell} = 0.5$ [illegible] (**??**) [illegible] (**??**) [illegible] $\wp$, [illegible] $\wp$.

[illegible] $\{p_\ell\}$ [illegible] $\{\wp_\ell\}$. [illegible] (**??**) [illegible] $\tau_\ell = 2/[\gamma_R + \ell\tilde{\kappa}]$. [illegible] p_ℓ [illegible] τ_ℓ [illegible]

$$p_\ell = \frac{\wp_\ell \tau_\ell}{\sum_{j=\pm} \wp_j \tau_j},$$

[illegible] ℓ [illegible] $\wp_\ell$ [illegible] $\tau_1 = \tau_2$, [illegible] (**??**) [illegible] $\gamma_R = 4T_c$, [illegible] $\vec{r}_{\rm ss} = \vec{r}_\pm = (0, -1, 0)$ [illegible]

[illegible] $\wp_\ell$ [illegible]

Physical meaning of the other TSEs

[illegible] (??) [illegible]

[illegible] $|\phi_\ell\rangle$ [illegible] $|\xi\rangle$, [illegible] [28,29]. [illegible]

$$|\Psi\rangle \propto \sum_r |\phi_{\ell|r}\rangle|\xi_r\rangle.$$

[illegible] r [illegible] $|\phi_\pm\rangle$ [illegible] (??) [illegible] [37,38] [illegible] [39].

V. COUNTING STATISTICS OF QUANTUM TRANSPORT

[illegible]

[illegible] $P(n,t) = \mathrm{Tr}\,\varrho^{(n)}(t)$ [illegible] $P(n,t)$ [illegible] n [illegible] t [illegible] $\varrho^{(n)}(t)$ [illegible] t [illegible] n [illegible] *e.g.* [illegible] n [illegible] n [illegible] (??) [illegible] $\varrho(t) = \sum_{n=0}^{\infty}\varrho^{(n)}(t)$ [illegible] [32] [illegible]

$$\dot{\varrho}^{(n)} = \mathcal{S}\varrho^{(n)} + \sum_{\ell=\pm}\wp_\ell\mathcal{J}[c_\ell]\varrho^{(n-1)}.$$

[illegible] n [illegible] n [illegible] $\varrho(\chi) = \sum_n e^{in\chi}\varrho^{(n)}$ [illegible] *counting field* χ [illegible] n. [illegible] (??) in χ [illegible]

$$\dot{\varrho}(\chi) = \Big(\mathcal{S} + \sum_{\ell=\pm}\wp_\ell\mathcal{J}[c_\ell]e^{i\chi}\Big)\varrho(\chi) \equiv \mathcal{L}(\chi)\varrho(\chi).$$

[illegible] $\mathcal{F}(\chi,t)$ [illegible] [32]

$$e^{\mathcal{F}(\chi,t)} = \sum_{N=0}^{\infty} P(n,t)e^{in\chi},$$

[illegible]

$$\mathcal{F}(\chi,t) = \ln\left(\mathrm{Tr}\left[e^{\mathcal{L}(\chi)(t-t_0)}\varrho(t_0)\right]\right).$$

[illegible] $t\to\infty$ [illegible] $\mathcal{L}(\chi)$ [illegible] (??) [illegible]

$$\mathcal{F}(\chi,t) = \lambda_0(\chi)t + [illegible],$$

[illegible] λ_0 [illegible] $\mathcal{L}(\chi)$ [illegible] n [illegible]

$$\left\langle S^{(n)}\right\rangle = \frac{d}{dt}\frac{\partial^n}{\partial(i\chi)^n}\mathcal{F}(\chi,t)|_{\chi=0,t\to\infty} = \frac{\partial^n}{\partial(i\chi)^n}\lambda_0(\chi)|_{\chi=0}.$$

[illegible] $\left\langle S^{(1)}\right\rangle$ [illegible]

[illegible] [32]

$$F^{(n)} = \frac{\langle S^{(n)} \rangle}{\langle S^{(1)} \rangle}. \tag{4}$$

[illegible] $n = 4$ [illegible] γ_R/T_c [illegible] (θ, ϑ_c) [illegible] $\wp_\pm$ [illegible] Eq. (??) [illegible] $\gamma_R \leq 4T_c$. [illegible] $F^{(n)} = 1$ [illegible] γ_R [illegible] $4T_c$, [illegible] Eq. (??) [illegible] $\wp_+ = 1$ [illegible] $\wp_- = 0$ [illegible] *vice versa*, [illegible] [14] [illegible]

VI. CONCLUSION

[illegible] [10,40] [illegible] [41,42].

[illegible] [43].

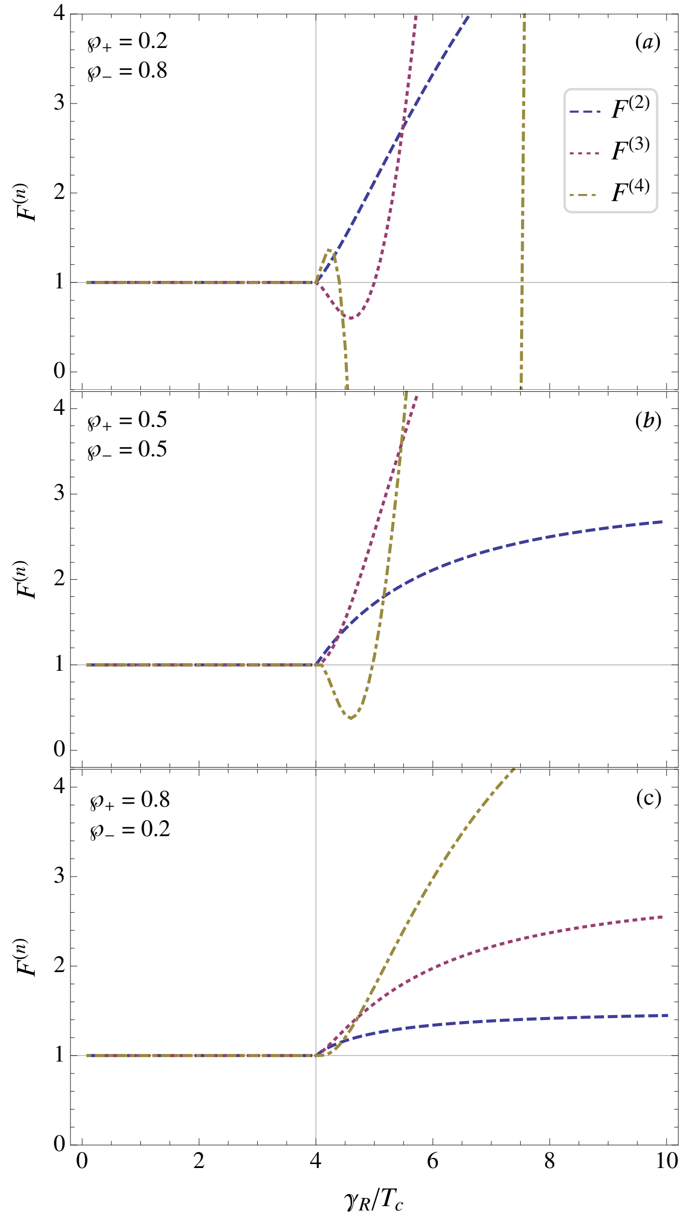

FIG. 4. (Color online). The value of Fano factors $F^{(n)}$, Eq. (??), for the DQD versus the ratio of tunneling rates γ_R/T_c: $n = 2$ (blue dashed), $n = 3$ (pink dotted), and $n = 4$ (light brown dashed-dotted). Three different sets of probabilities are shown (a) $\wp_+ = 0.2, \wp_- = 0.8$, (b) $\wp_+ = 0.5, \wp_- = 0.5$, and (c) $\wp_+ = 0.8, \wp_- = 0.2$. As long as $\gamma_R \leq 4T_c$ all Fano factors are equal to one no matter what value is assigned to $\wp_\pm$. This is correspond to the Poissonian statistics of the current which is independent of the order of the current correlation. In contrast, as γ_R becomes larger than $4T_c$ Fano factors depend on the probabilities $\wp_\pm$ to the extent that $F^{(n)} = 1$ only holds if and only if $\wp_+ = 1$ and $\wp_- = 0$, or *vice versa*.

$\hat{H}'$

p_ℓ $\wp_\ell$

ACKNOWLEDGEMENT

Appendix A: Supplemental Material

1. Adiabatic elimination of the null state

$$\rho = \varrho + \delta|\emptyset\rangle\langle\emptyset|$$

ϱ $\{|L\rangle, |R\rangle\}$ $\delta = O(\gamma_R/\gamma_L)$ δ

$$\begin{aligned}\dot{\delta} &= \langle\emptyset|\dot{\rho}|\emptyset\rangle \\ &= -\gamma_L\delta + \gamma_R\langle R|\varrho|R\rangle,\end{aligned}$$

(??) $\gamma_L \gg \gamma_R$

$$\delta \approx \frac{\gamma_R}{\gamma_L}\langle R|\varrho|R\rangle.$$

$$\dot{\varrho} = \hat{\Pi}_{\rm qb}\,\dot{\rho}\,\hat{\Pi}_{\rm qb}$$

$\hat{\Pi}_{\rm qb} = |L\rangle\langle L| + |R\rangle\langle R|$ (??) (??) (??)

$$\begin{aligned}\dot{\varrho} = -i[H,\varrho] + \gamma_R|\phi_\ell\rangle\langle\phi_\ell|\langle R|\varrho|R\rangle - \\ \frac{\gamma_R}{2}\big(|R\rangle\langle R|\varrho + \varrho|R\rangle\langle R|\big) \\ = -i[H,\varrho] + \mathcal{D}[c_\ell]\varrho.\end{aligned}$$

2. Bloch equations and control parameters

A b

$$\begin{aligned}
A_{11} &= -2\gamma_R\sum_{\ell=\pm}\wp_\ell\big(|u_y^\ell|^2 + |u_z^\ell|^2\big),\\
A_{12} &= \gamma_R\sum_{\ell=\pm}\wp_\ell\Big[\big(u_x^\ell u_y^{\ell\,*} + u_x^{\ell\,*}u_y^\ell\big) - \\
&\quad i\big(u_z^\ell w^{\ell\,*} - u_z^{\ell\,*}w^\ell\big)\Big] - \Delta,\\
A_{13} &= \gamma_R\sum_{\ell=\pm}\wp_\ell\Big[\big(u_x^\ell u_z^{\ell\,*} + u_x^{\ell\,*}u_z^\ell\big) + \\
&\quad i\big(u_y^\ell w^{\ell\,*} - u_y^{\ell\,*}w^\ell\big)\Big],\\
A_{21} &= \gamma_R\sum_{\ell=\pm}\wp_\ell\Big[\big(u_x^\ell u_y^{\ell\,*} + u_x^{\ell\,*}u_y^\ell\big) + \\
&\quad i\big(u_z^\ell w^{\ell\,*} - u_z^{\ell\,*}w^\ell\big)\Big] + \Delta,\\
A_{22} &= -2\gamma_R\sum_{\ell=\pm}\wp_\ell\big(|u_x^\ell|^2 + |u_z^\ell|^2\big),\\
A_{23} &= \gamma_R\sum_{\ell=\pm}\wp_\ell\big(u_y^\ell u_z^{\ell\,*} + u_y^{\ell\,*}u_z^\ell\big) - \\
&\quad i\big(u_x^\ell w^{\ell\,*} - u_x^{\ell\,*}w^\ell\big)\Big] - 2T_c,
\end{aligned}$$

$$A_{31} = \gamma_R \sum_{\ell=\pm} \wp_\ell \Big[\big(u_x^\ell {u_z^\ell}^* + {u_x^\ell}^* u_z^\ell\big) - i\big(u_y^\ell {w^\ell}^* - {u_y^\ell}^* w^\ell\big)\Big],$$

$$A_{32} = \gamma_R \sum_{\ell=\pm} \wp_\ell \big(u_y^\ell {u_z^\ell}^* + {u_y^\ell}^* u_z^\ell\big) + i\big(u_x^\ell {w^\ell}^* - {u_x^\ell}^* w^\ell\big)\Big] - 2 T_c,$$

$$A_{33} = -2\gamma_R \sum_{\ell=\pm} \wp_\ell \big(|u_x^\ell|^2 + |u_y^\ell|^2\big).$$

and

$$b_1 = i\,2\gamma_R \sum_{\ell=\pm} \wp_\ell \big(u_y^\ell {u_z^\ell}^* - {u_y^\ell}^* u_z^\ell\big),$$

$$b_2 = i\,2\gamma_R \sum_{\ell=\pm} \wp_\ell \big(u_z^\ell {u_x^\ell}^* - {u_z^\ell}^* u_x^\ell\big),$$

$$b_3 = i\,2\gamma_R \sum_{\ell=\pm} \wp_\ell \big(u_x^\ell {u_y^\ell}^* - {u_x^\ell}^* u_y^\ell\big).$$

[illegible] $\underset{\sim}{u}^\ell = (u_x^\ell, u_y^\ell, u_z^\ell, w^\ell)$ [illegible]

$$u_x^\ell = \frac{1}{2}\Big[\cos(\vartheta_c^\ell/2) - i\sin(\vartheta_c^\ell/2)\cos(\theta^\ell)\Big],$$

$$u_y^\ell = i\,u_x^\ell,$$

$$u_z^\ell = \frac{i}{2}\sin(\vartheta_c^\ell/2)\sin(\theta^\ell),$$

$$w^\ell = -u_z^\ell.$$

[illegible] θ^ℓ and ϑ_c^ℓ [illegible]

$$\hat{U}_\ell = e^{-i\vartheta_c^\ell \vec{n}_\ell . \vec{\sigma}/2},$$

[illegible] $\vec{n}_\ell = (\sin\theta^\ell, 0, \cos\theta^\ell)$ [illegible] ϑ_c^ℓ. [illegible] ϑ_c [illegible] θ_C in Ref. [14].

[illegible] $\gamma_R < 4T_c$ [illegible]

$$\theta^\ell = [\text{illegible}]\Big(\frac{\ell\,\kappa}{\sqrt{[\text{illegible}]\,T_c^2 - \gamma_R^2}}\Big),$$

$$\vartheta_c^\ell = [\text{illegible}]\Big(\frac{\gamma_R}{\sqrt{[\text{illegible}]}\,T_c}\Big),$$

[illegible] $\gamma_R > 4T_c$ [illegible]

$$\theta^\ell = \pi/2,$$

$$\vartheta_c^\ell = [\text{illegible}]\Big(\frac{4T_c}{(\gamma_R + \ell\,\tilde{\kappa})}\Big).$$

[illegible] $\gamma_R = 4\,T_c$ [illegible] $\theta^\ell = \vartheta_c^\ell = \pi/2$

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