Characterising the Perceived Value of Mathematics Educational Apps in Preservice Teachers

Boris Handal
boris.handal@nd.edu.au
The University of Notre Dame Australia
140 Broadway Rd 2007 NSW

Chris Campbell
chris.campbell@uq.edu.au
The University of Queensland
Brisbane QLD 4072

Michael Cavanagh
michael.cavanagh@mq.edu.au
Macquarie University
Sydney NSW 2109

Peter Petocz
peter.petocz@mq.edu.au
Macquarie University
Sydney NSW 2109

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This study validated the semantic items of three related scales aimed at characterising the perceived worth of mathematics-education-related mobile applications (apps). The technological pedagogical content knowledge (TPACK) model was used as the conceptual framework for the analysis. Three hundred and seventy three preservice students studying primary school education from two public and one private Australian universities participated in the study. The respondents examined three different apps using a purposive designed instrument in regard to either their explorative, productive or instructive instructional role. While construct validity could not be established due to a broad range of variability in responses implying a high degree of subjectivity in respondents' judgments, the qualitative analysis was effective in establishing content validity.

Key words: Mathematics education, mobile apps, pedagogy, technology, TPACK.

Introduction

Various instruments, mostly available via the Internet, have been developed to measure the perceived value of educational apps; however, they do not provide evidence of being grounded in educational theory and do not discuss their conceptual constructs (Watlington, 2011). Most of the Internet checklists represent a uni-dimensional structure and are not discipline specific (Kearney, Schuck, Burden & Aubusson, 2012). There is also research showing that such a lack of mathematics specific checklists adds to the burden of school teachers looking for good quality mobile resources (Attard, 2013; Attard & Curry, 2012; Calder, 2015; Larkin, 2015; Spencer, 2013).

Such literature deficit has implications for teachers' professional development and new pedagogical knowledge. For example, Jonas-Dwyer, Clark, Celenza and Siddiqui (2012) have indicated "There are often no formal guidelines to assist with evaluating apps" (p. 57). In making connections with so much needed relevant research on evaluating educational apps Kiekel and Kirk (2013) have remarked

Due to the fact that tablet devices have exploded into popularity in the past two years as well as the explosion of application development, very little research has been done to evaluate the instructional design and readability of the apps (p. 2171).

There are an impressive number of educational mobile learning apps that teachers need to appraise before integrating them into the classroom. To make this assimilation process more effective in classrooms, teachers need to learn the dynamics of assessing maths apps before these are introduced to students (MacCallum & Jeffrey, 2009).

The growing presence of ICTs in secondary maths education implies that pedagogies based only on print based resources need to concede space for teaching and learning with electronic materials. Moreover, the advent of new learning technologies brings new challenges and possibilities to the traditional way some teachers have been delivering the curriculum (Churchill & Churchill, 2008). Specifically, mobile learning, the latest addendum to the continuous process of innovation in educational technology, brings a technology that is ubiquitous in nature, highly portable and endowed with multimedia capabilities offering a new dimension to curriculum making learning accessible “anywhere, anytime”. Examples of successful projects delivering mathematics curricular objectives through mobile devices in the middle years of schooling have been documented by Baya'a and Daher (2009) and Franklin and Peng (2008) working with year 8 students, and Sollarvall, Otero, Milrad, Johansson and Vogel (2011, 2012) with year 6 students.

For the purpose of this study “Content validity refers to whether or not the content of the manifest variables (e.g. items of a test or questions of a questionnaire) is right to measure the latent concept (self-esteem, achievement, attitudes, etc) that we are trying to measure”(Muijs, 2011, p. 57) while construct validity relates “to the internal structure of an instrument and the concept it is measuring” (Muijs, 2011, p. 59).

Theoretical Framework

The TPACK theoretical rationale of the instrument used in this study has been discussed in more detail in the paper “Appraising maths apps using the TPACK model” (Handal, Campbell, Cavanagh & Dave, 2014). The TPACK model constitutes a conceptual framework that is valuable because it integrates three dimensions for using ICTs in teaching and learning; namely, pedagogical knowledge, technological knowledge and disciplinary content (Figure 1).

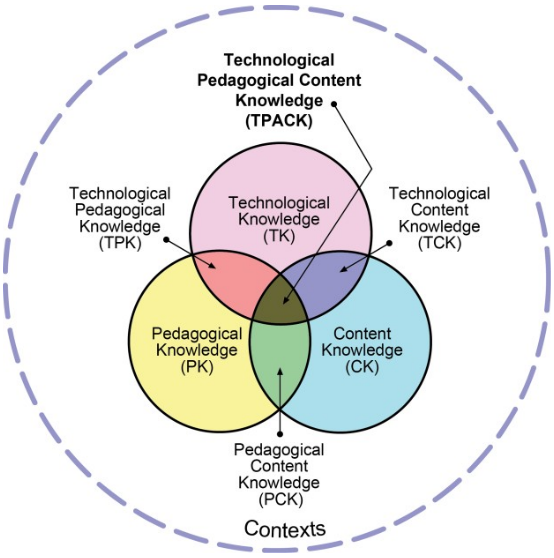

***Fig 1**: The TPACK model*
Source: http://tpack.org

Pedagogical knowledge (PK) represents teachers’ understanding of evidence-based quality teaching as well as expertise aimed at enhancing students’ experiences and therefore learning. In turn, technological knowledge (TK) represents those operational capabilities that teachers need to deploy technology. Content knowledge (CK) stands for teachers’ acquaintance with the subject matter, more specifically, expertise in a particular branch of learning that qualifies them as professionals in the field (Handal, Campbell, Cavanagh, Petocz & Kelly, 2012; 2013).

Furthermore, the zones resulting from the intersection of PK, TK and CK forms even larger constructs; namely, technological content knowledge (TCK), technological pedagogical knowledge (TPK) and pedagogical content knowledge (PCK). TCK relates to teachers' knowledge of digital technologies related specifically to the discipline of mathematics. TPK represents the skill set to use digital learning technologies to deliver educational outcomes throughout the curriculum. In turn, PCK is described by Shulman as the knowledge of pedagogies that are applicable to teaching a specific content (Shulman, 1986, 1987; Thomas & Ye Yoon Hong, 2013).

The TPACK model was chosen as the conceptual framework because of its potential to integrate technological pedagogical knowledge (TCK), technological pedagogical knowledge (TPK) and pedagogical content knowledge (PCK). The intersection of these three TPACK aspects renders the highest level of instructional skill creating an environment where the mathematics curriculum is efficiently and holistically delivered to students.

The TPACK model provides a sound framework for integrating technology into the mathematics curriculum (Urban-Woldron, 2013). Originally conceptualised by Koehler and Mishra (2008), the model is renowned for its theoretical scope to amalgamate the subject-area and specific mathematical concepts and process. Various research instruments have been developed using this framework to explore important educational variables in the field of mathematics education such as teacher education (Lee & Hollebrands (2008), professional development (Niess, van Zee & Gillow-Wiles, 2011), students' academic performance (Lyublinskaya & Tournaki, 2011), curriculum development (Niess et al., 2009), assessment (Schmidt et al., 2009), and teachers' technological and pedagogical skills (Handal et al., 2012), among others.

Due to its multiple dimensions TPACK stands as a well articulated model to understand the instructional design of an app from various technical pedagogical perspectives (Handal et al., 2014).

Research Objectives

The research objectives for this project were:

- To establish the construct analysis of an instrument designed to characterise the perceived value of educational apps in mathematics education (quantitative analysis)
- To establish the content analysis of an instrument designed to characterise the perceived value of educational apps in mathematics education (qualitative analysis)

Methodology

Instrument Design

An instrument was developed to characterise the perceived value of mobile apps in mathematics education. The instrument was based on three aspects of the TPACK model; namely, content, technology and pedagogy, focusing on their intersection.

Four sub-scales were created to characterise the association between the three TPACK aspects and each sub-scale is shown in Table 1.

Table 1: Relationship between TPACK components and sub-scales

Sub-scale	*Code*	*TPACK component*
Task Structure	E (explorative), P (productive), I (instructive)	Technological Pedagogical Content Knowledge (TPACK)
Cognitive Involvement	CI	Pedagogical Content Knowledge (PCK)
General Pedagogical Issues	GPI	Technological Pedagogical Knowledge (TPK)
Operational Issues	OP	Technological Content Knowledge (TCK)

The Task Structure sub-scale examines the app's role according to the type of task promoted: explorative, productivity and/or instructive. In brief, explorative apps are useful for exploring and demonstrating mathematical models or concepts through manipulating objects that mimic or simulate complex physical situations. In turn, productivity apps are intended for measuring and graphically representing objects or concepts in 2D/3D, collecting data, making calculations, or creating multimedia materials. Finally, instructive apps are conducive to practising content through drill exercises, acquiring new skills through questions and answers (tutorials), or retrieving factual information (Handal, El-Khoury, Cavanagh & Campbell, 2013).

Hence, the Explorative items focus on how the apps encourages discovery learning or to which extent the app engages students on simulation or emulation investigative tasks. The Productive Task items focus on the app allowing the students to creatively work with the app, as well as representing mathematics content by creating and linking symbolic, numerical and graphical data and using a variety of different tools within the app. In turn, the Instructive Task items focus on whether the app was viewed as offering a variety of activities that were engaging and provided well-graded exercises with worthwhile feedback on children's progress.

The second sub-scale characterises the degree of cognitive involvement (CI) when a learner interacts with the app. It incorporates Bloom's Taxonomy of learning objectives allowing students to evaluate how well the app supports the development of children's mathematical learning. The Bloom Taxonomy was used because of its capacity to yield differential assessment categories as well as its usefulness for conceptualizing various levels of mathematics understanding (Webb, 2013). Hence, the level of cognitive interactivity between the apps and the user could be framed in the context of Bloom's taxonomy to depict a sequential order of skills complexity leading to higher order thinking. Anderson and Krathwohl (2001) revised the original Bloom's taxonomy (see Figure 2).

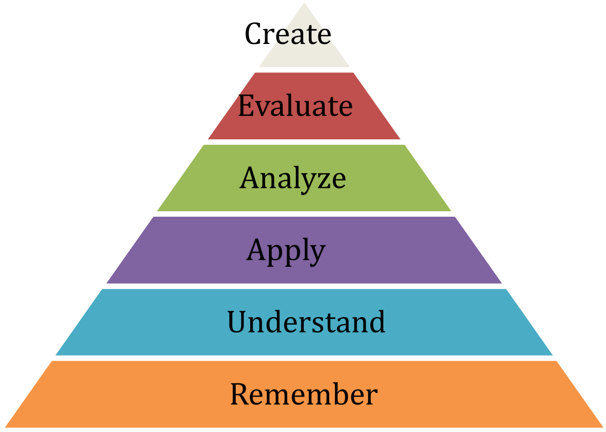

***Fig 2**: Revised Bloom's taxonomy*

Similarly, the General Pedagogic Issues (GPI) sub-scale included topics such as permits students to solve their own problems, allows for differentiation, delivers content in an appealing and motivating way, allows students to collect and record their own data, saves and keeps students work, and provides opportunities for collaboration. The fourth sub-scale deals with the app's operational and technical affordances.

The instrument framework was piloted with school teachers and mathematics educators to ensure content validity (Table 2) that is, to ensure that all important aspects of the apps are addressed in the set of questions presented to respondents and that they do not ask about irrelevant aspects of the apps as described in Handal et al. (2014).

Table 2: Questionnaire Items

SECTION 1: Explorative apps (Task Structure)
E1: App closely mirrors a model or real-life situation
E2: Students can enter their own data and observe changes in the model
E3: Exploration is guided within a predetermined learning discovery framework
E4: Tasks are goal oriented driving student interest and curiosity
E5: There are elements of ambiguity and uncertainty fostering personal investigation
SECTION 2: Productivity Apps (Task Structure)
P1: App lets students to creatively come up with their own design and/or concept
P2: App allows representing maths content by linking symbolic, numerical and graphical data
P3: Students are guided in creating their own content/understandings
P4: Students can represent or present maths content using a variety of different tools (e.g., audio/video recording, measuring devices, etc.)
P5: App tools are intuitive and easy to use
SECTION 3: *Instructive Apps (Task Structure)*
I1: App contains a variety of different activities/exercises
I2: Appropriate feedback is provided to students
I3: Activities/exercises cater for a range of student ability levels
I4: Content is meaningful, fostering engagement and rich problem solving
I5: App contains activities/exercises that are graded and summary data is provided
Cognitive Involvement - The app encourages students to …
CI1: retrieve and review maths concepts/skills/procedures (Remembering)
CI2: demonstrate understanding of maths concepts/skills/procedures (Understanding)
CI3: apply their knowledge of maths concepts/skills/procedures in practical contexts (Applyin
CI4: critically analyse maths content in text, graphs and/or animations (Analysing)
CI5: appraise and justify maths ideas or products (Evaluating)
CI6: construct new and meaningful maths ideas or products (Creating)
General Pedagogical Issues - The app …
GP1: permits students to pose their own problems
GP2: allows for differentiation through sequentially designed degrees of difficulty
GP3: gives students control over their learning
GP4: delivers content in an appealing and motivating way according to the age group
GP5: provides meaningful teaching and learning guidelines
GP6: integrates maths with content from other Key Learning Areas
GP7: allows students to collect and record their own data
GP8: shows a reading level appropriate to the student's level
GP9: saves and keeps students' work
GP10: provides opportunities for collaboration
Operational Issues - The app …
OP1: has an intuitive and user friendly navigation
OP2: contains helpful technical instructions to the user and/or a Help function
OP3: lets students alter its settings to customise the app to their needs
OP4: allows file sharing, streaming of content and/or online communications
OP5: is flexible permitting students to move in different directions
OP6: has a supporting Web page providing additional useful information
OP7: easily works with a range of media (audio, video, image, text, animations)
OP8: can interface with social media tools (e.g., Facebook, wikis, blogs, Twitter, YouTube)
OP9: presents an uncluttered display which is visually stimulating
OP10: permits a student leave at any time and begin where he or she left

Responses were made on a four-point scale ranging from ("Always", "To some extent", "Never" and "Not applicable").

Procedure

The instrument was administered to 373 primary teacher education students from The University of Notre Dame Australia (UNDA), The University of Queensland (UQ) and Macquarie University, all in Australia. Participants were requested to examine a specific app representative of each of the three categories: Explorative (e.g., Fraction Fiddle), Productivity (e.g., ShowMe Interactive Whiteboard) and Informative (e.g. Long Division Touch). These apps were selected on the basis of (a) meeting as much as possible the pedagogical/technical characteristics described by Handal et al. (2014); and (b) available for free downloaded free from the iTunes store.

Participants were asked to examine the apps using the TPACK-based app instrument and were provided iPad devices in class. In each university cohort students were randomly assigned to one of the three groups with one of the three apps allocated to each. Students were given approximately 30 minutes to complete the task. The breakdown of participants in shown in Table 3:

Table 3: Participants by University

University	Participating Students
Macquarie	160
(43%) UNDA	169
(45%) UQ	44
(12%)	
Total	373 (100%)

Quantitative Data Analysis

The quantitative analysis focused on determining construct validity of the instrument, in other words, that the three theoretical sub-scales were indeed reflected in the numerical results obtained from participants. The statistical analysis aimed to show through principal components analysis (PCA) how the questionnaire items fit with each of the three sub-scale scales by the way respondents discriminated items across the three scales. These three scales were Cognitive Involvement (CI), General Pedagogical Issues (GPI) and Operational Issues (OP). The Task Structure sub-scales were not considered in the PCA because they represent specific apps attributes; namely, explorative, productive and instructive roles. Handal et al. (2012) successfully used PCA to establish construct validity of TPACK based scales in the field of mathematics education.

PCA factors that turned up as ambiguous or cross-factored across the three scales, because of their loadings, were deemed to be problematic based on items loadings accepted with the range of -0.4 and 0.4 (Muijs, 2010). The PCA analysis revealed that the three groups of questions (CI, GPI and OP) do not correspond with the three dimensions identified via factor analysis, irrespective of the rotation method. So although the questions are listed under these three patterns, they are not perceived that way by the respondents. Analysing all the questions together (the CI, GPI and OP questions) leads to an unclear pattern revealing that there is no obvious scale(s) to look at for the purposes of statistical validation. In other words, the resulting groupings did not seem to be useful as there is no obvious descriptor or common feature, particularly as this analysis was for all three apps combined.

If the questions are analysed separately by app groups, the results are even less clear. Constructing average scores for the three groups of questions (CI, GPI and OP) and running a multivariate ANOVA indicates that the *university* and *app* variables are both significant and the interaction *university***app* is also significant (Table 4). This indicates that each university has a different pattern of results for each app, a difficult result to interpret.

Table 4: Multivariate Tests

Effect	F	*df*	*Sig*
Uni	9.200	6	.000
App	26.283	6	.000
Uni*app	2.475	12	.003

Hence, the apps are seen differently in the three groups of questions, and the respondents from the universities see them differently, but they also show a different pattern of responses to each app at the different universities. A confirmatory factor analysis does not seem to be needed as the results are quite clear that the sub-scales are not supported

For all the above reasons, construct validity of the questionnaire could not be established, and we must conclude that our postulated dimensions of Cognitive Involvement, General Pedagogical Issues and Operational Issues do not seem to be distinguished as such by our respondents.

Qualitative Data Analysis

The main purpose of the qualitative analysis was to establish the content validity of the instrument. There were 303 comments in the open-ended questionnaire item for almost each app scale item signaling that respondents did think about the items. The open-ended question in the instrument requested respondents to: “Write here any other comments you might have about the quality of your maths app” It is noteworthy that the instrument, due to its originality, did not have a data baseline. Similarly, in order to capture respondents’ subjectivity the instrument did not assume right or wrong answers as part of the items design.

Because the quantitative analysis found a significant variability in responses by apps, within each university cohort and between university cohorts, the qualitative analysis focused on (a) each sub-scales’ content validity and (b) possible variabilities in students’ responses.

In order to establish content validity the analysis focused on four criteria:

(1) The analysis was carried out at each level within the app.
(2) The analysis focused on matching at least one student’s comment on each scale semantic item, regardless of whether the comment was positive (like) or negative (dislike). The matchings across scales were later reviewed by the team of researchers to maximise inter-reliability judgement.
(3) If possible, examples from both positive and negative comments for each item were drawn to characterise the degree of variability in the responses.
(4) If no comments were given on a particular item then the analysis centred on whether the omission was due either to item irrelevance, the app itself, or the item wording.

The discussion of findings that follows confirmed that students did think about most of the sub-scale items, which supports our conclusion that the survey has content validity. It also showed the great variability among students responses identified previously by the quantitative analysis. The following three sub-sections discuss respondents' comments on the basis of their favourable or unfavourable statements about the three apps (i.e., explorative, productive and instructive apps).

The Explorative Scale

"Fraction fiddle: Reach the target" is an app developed by Education Services Australia (ESA)(2014) targeting Year 5 Australian Curriculum mathematics content. The app can be downloaded from the iTunes Store to use with iPads. The key learning objectives described by ESA are as follows:

- Students explore the effect of changing the numerator and denominator on the type and size of a fraction.
- Students identify pairs of fractions that add up to a target number.
- Students find equivalent fractions.

According to ESA, the "Fraction fiddle: Reach the target" app is designed with a focus on experiential learning, independent learning, problem solving and visual learning with student activity focused on interactivity, analysis and modelling.

Explorative Task Structure Sub-Scale (E)

Responses show that most preservice students thought about the authenticity context of the app (E1) with acknowledgment of the animation showing a plane hitting a bulls-eye following a correct answer. However, the majority of students felt that there could be better examples of fractions in real life. For example, filling a jar, slicing a pizza, or partitioning a pie or cake. Some students were able to recognise the issue of dynamic modelling through the multimedia interface (E2). For example, a respondent remarked: "It's visual and really helpful to see, side by side, the denominator changing and seeing the partitions change accordingly." However, a number of students did not recognise the digital modelling affordance such as the respondent who wrote: "The questions are just basically fiddling with numbers."

The learning discovery capacity of the app was apparent in some of the responses (E3) such as: "It is a non-threatening app where students are given a problem and can manipulate the model to learn how fractions work." However, negative comments considered that the app "allows the child to keep repeating the exercise and not actually think about the problem." Similarly, opinions were divided on whether the app causes interest and curiosity (E4). Positive comments were "I would recommend this app to younger high school or Yr5/6 as it encourages them to solve the problems without using a specified method" while negative comments aligned to thoughts such as "I would not use this app in the classroom as it is not very engaging and does not give specific instructions on where students go wrong."

On the issue of ambiguity and uncertainty for problem solving (E5) favourable remarks include: "It provides mathematical thinking and extends the user to think outside the box devising new ways of approaching fraction." Negative comments mentioned that the app "is not cognitively stimulating" and "quite predictable & could become boring for students."

Cognitive Involvement Sub-Scale (CI)

Following the pattern of like/dislike responses, the items CI1 to CI6 received a broad range of opinions. For example for the item CI1 (Remembering) some students wrote about the app: "This is a good app for solidifying knowledge. It is easy to use, involves easy instructions and turns math learning into a game." However, negative comments related the item to “rote learning ... and not much else.” In regard to the item CI2 (Understanding) some respondents considered that the app “allows students to further explore fractions and gain understanding of them”, while others thought that “there are better ways to teach the concepts and processes which involve a deeper and more meaningful understanding.” In turn, the item CI3 (Applying) received comments ranging from: “The app is a great extension for students to apply their understanding of fractions through manipulating the app and interacting with its features” to those asserting that the app does not assist students “to make meaning or to develop mathematical thinking.”

Moreover, the item CI4 (Analysing) received favourable remarks including “It provides mathematical thinking and extends the user to think outside the box devising new ways of approaching fraction” and unfavourable remarks such as: “The design is too easy just to flick the numbers until you see the meter fill up without having a solid understanding of why that's the case.”

Some students thought positively that, in terms of the item CI5 (Evaluating), the app was effective as: “It is useful for providing feedback as it says where they are on the number line and where the desired answer is so they can edit their work” while others thought that it focuses on “repetition more than actually extending the child’s knowledge.” In regard to the CI6 (Creating) item a respondent thought that “the app requires students to add fractions and gives opportunities to consider other fractions that could add to the same product” while another wrote that “students are exposed to different portions/fractions but their background knowledge is not developed further.

General Pedagogic Issues Sub-scale (GPI)

Similar to the Task Structure and Cognitive Involvement sub-scales, respondents expressed a variety of opinions corroborating no discernible response pattern in the response patterns. Some exceptions are represented by the item GP1 “permits students to pose their own problems”, where students considered that “the fact you can't create your own questions negatively impacts the app” and that the app “doesn't have flexibility to choose which fraction to make.”

Responses matching the items GP8 and GP9 leaned mostly in the negative direction. In regard to whether the apps shows a reading level appropriate to the student’s level (GP8) students wrote: “You can't adjust the settings in terms of the child's grade so the formalisation of the language may only work for younger students” and “I would not recommend because you can't differentiate or contextualise for student ability (reading and math ability);. Also, regarding the app saving and keeping students’ work (GP9) students remarked that “the downside is that it does not save results so teachers can see nor does it have a timer” and “and can't record results to see progress.”

There was recognition that the app does not allow much differentiation (GP2). There were comments as “great for advanced learners but not so great for strugglers”, or “it needs some sort of differentiation for different levels” or it “does not provide opportunity for differentiation. You cannot make it easier or harder depending on skill level.”

A similar dichotomous pattern appears for the item GP3 in regard to whether the app gives students control over their learning. Some believed that “students can move at their own pace” while another student wrote that the app “does not let students move at their own pace … because students have to go in order of the game even if they are not up to fractions of that level.” As per the item GP4 “delivers content in an appealing and motivating way according to the age group”, a number of opinions reveal that the app is “more suitable for older primary students”, “for younger Yr 3/4 students” and “ideal for upper primary.” However, there were negative remarks such as “the app is boring and isn't engaging enough.”

For the item GP6 a student thought that s/he did not find applications of the app in other areas of curriculum but another was of the opinion that the app “can be used across all subjects.” A variety of responses were also found for the item characterising the app for its capacity to allow students to collect and record their own data (GP7). An example of a favourable comment was: “Can use recording function and other functions to engage students and allow them to participate’ whereas a negative comment was “I would not use this app as it does not necessarily test/measure learning.”

Some students believed that the app “does not offer an opportunity for differentiation or group involvement” and “there is no choice for group collaboration” (GP10) while another thought that, “the teacher can use it as a whole class activity as well but I think it is more suited for individual or group work” or “only applicable to a small group.” Similarly, for item GP5 remarks varied from the app “does not provide enough explanation/meaning behind why certain answers are correct or how to achieve the correct answer” to “there was as a scaffold for visual learners which was good.”

Operational Issues Sub-Scale (OP)

Like the previous sub-scales OP items were endorsed as valid through students’ comments. Items OP2, OP3, OP6, OP8 and OP10 were responded to with only unfavourable responses. In contrast, item OP7 was validated through only favourable comments because of the app’s capacity in the “use of different media” such as “smartboards”, videos and “interactive technology.” As a respondent wrote: “I can imagine it would be a good, short multimedia activity to play every so often.” The OP4 item could not be validated because the app products cannot be streamed online like other apps.

Finally, items OP1, OP5 and OP9 received both positive and negative responses. In regard to having an intuitive and user friendly navigation, typical positive comments included: “It is user friendly and students can progress at their own pace” and unfavourable comments referred to the buttons and arrows which are “hard to access and too small” and the fact that the “icons stopped working after a few turns.”

At the task level, respondents appeared to endorse the app as “flexible, permitting students to move in different directions” (OP5) because “students can just fiddle around with the number arrows until the numbers selected align with the target.” However, at the sequence level, thoughts revolved around the idea that “students have to go in order of the game even if they are not up to fractions of that level.”

Similarly, opinions were divided on whether the app presents an uncluttered display which is visually stimulating (OP9). A positive stance was “I would recommend this app as it has easy navigation features as well as an attractive layout” while unfavourable comments conveyed opinions like “screen is too cluttered - student would have to look in too many places.”

As stated above, the exploratory scale allow teachers to teachers the nature and dimensions of the investigative and discovery learning affordances of an app. Participants’ responses offered both positive and negative opinions for almost each of the scale items. The implications of the students’ responses will be discussed later in the article.

The Productive Scale

ShowMe Interactive Whiteboard is an app developed by Learnbat Inc (2015) and allows students to use a device to record voice-over tutorials and share them online or with each other on their devices. Although this app suits a range of topics, only mathematics has been used as an example in this project. There are a number of features of the app which makes it not only interesting to use, but also quite interactive for content creation . Features include recording, text, colour, erase, importing of images and documents from various locations, creating video from other documents, share services, and unlimited length of creations. Whilst there is a paid version of the app which includes additional features, only the free version was evaluated for this project.

Productive Task Structure Sub-Scale (P)

Comments were generally positive with students writing about being able to "create lessons to show to students at another time" and that it "provides students with an opportunity to draw or add pictures, record their voice and present information in their own way." Positive comments include the app being the same as pen and paper, but being better for "saving working out or information." However, one student felt that it was boring and that "students would become disinterested pretty quickly" and that it is "basically just drawing, and you can use 'paint' on the computer," which is actually an interesting comment as the app also allows the recording of information and for this to be played back.

Students were very positive about the app being easy to use. Comments included, "It is a great app to visually show working whilst recording thoughts spoken out loud", while another in-depth comment indicated that:

> The app could be used by a teacher to create 'flip the classroom' style homework lessons. It could also be used to record live lessons for absent student or for students to use for revision. The app could also be used by students to peer teach. Also, presentations for the class on various topics/revision of topics created by the students. The 'Explore' feature would also be used by the teacher to find other teachers' perspectives on various concepts.

Cognitive Involvement Sub-Scale (CI)

This sub-scale incorporates Bloom's Taxonomy. One comment states, "This app gives flexibility of choice of learning for a student. In the classroom it can be incorporated as a reinforcement tool to review a procedure; or step by step, prove, and repeat for students", while another suggests:

> I would recommend this app for use in a classroom. It is user friendly and visually stimulating so would engage students more easily in a given topic. It would be a perfect tool for maths assessments or for homework. It could be used as a reference for students when completing individual tasks or as an additional source of information for students struggling with a topic.

This suggests that the app allows students to retrieve information readily. It also allows for demonstrating understanding of maths concepts, skills and procedures (Understanding in Bloom's Taxonomy) as one comment includes "I would recommend this as it allows them to narrate their learning! This allows teachers to recognise if the student understands the concept."

Students are able to appraise and justify maths ideas or products (Evaluating). One comment suggested “I would use this app to ask students to create their own tutorials to explain their own process therefore as they can draw and explain at the same time”, while another suggestion is that it was “great for students creating assessable presentations.” Another comment suggested that they:

> Would use this app in the classroom in the following ways: (a) explicitly modelling a maths concept to students; (b) as an assessment tool whereby students create their own tutorial to show their understanding of maths concepts, (c) students explore to find and evaluate good explanations/tutorials for teaching different maths concepts.

Students also found a relation of the app to the item CI6 “being able to construct new and meaningful maths ideas or products.” One comment is:

> This app would be good when creating and presenting data/information for an assignment. It could be utilised in the classroom to replace individual whiteboards (if there was enough for 1 per person or group) as work can be saved, retrieved and shared. It is a great way for teachers to assess and differentiate learning levels as there is video (visual recording) of how work was solved paired with sound recording.

Another comment is not as positive about the app, but does feel that students can create and explore.

> I would recommend this app for the classroom, however I think there needs to be improvements. The voice is rather boring and slow, along with the corresponding writing. Children could easily lose interest. Also the videos could be made fun without explanations for teachers to show the whole class and then give students a chance to explore/create for themselves. More animations rather than just what a teacher would usually write on the board anyway.

General Pedagogic Issues Sub-scale (GPI)

The ShowMe app appeared to address all of the items of the GPI sub-scale. One comment was “This app gives flexibility of choice of learning for a student. In the classroom it can be incorporated as a reinforcement tool to review a procedure; or step by step, prove, and repeat for students.”

Generally there were quite a few comments on collaborative learning and using the app in general, such as:

> I found the 'explore' tab really helpful in learning about the types of things you could do with 'show me'. I would recommend this app to teachers as it provides students with multiple ways to share and demonstrate knowledge in creative and meaningful ways. The app supports collaborative learning and would be a good tool to aid reflection. This app is limited because it requires internet connection to access the 'Explore' tab.

Another comment highlighted the group work potential: “I would recommend this app. It is interactive and opens up a large range of opportunities for teachers. It is easy to use and is stimulating. I would perhaps use it for group work, where each group had an iPad and created a presentation for a range of mathematics concepts. If there were enough iPads this could be done individually.”

One negative comment in this section was “I find it too complicated for primary school students. It is difficult to navigate. Does not have enough reliable mathematics resources.” Other comments suggest that some students may be too young to understand how to use the app and that teacher guidance may be required; for instance, “this app is great for older students to design their own questions and problems” and “it’s is great to record, but the teacher must give heavy instruction and really guide students about what they are to do.”

Operational Issues Sub-Scale (OP)

The operational issues sub-scale provides a variety of areas and comments that were generally positive, although some sections were mixed. Some felt that the app was very easy to use, as is evidenced above; however, as evidence above shows, others felt that it may be difficult to use with young children and also may need strong teacher guidance.

Finally two comments sum up the app nicely “it is a great app to visually show working whilst recording thoughts spoken out loud”, and:

> I would recommend this app for use in a classroom. It is user friendly and visually stimulating so would engage students more easily in a given topic. It would be a perfect tool for maths assessments or for homework. It could be used as a reference for students when completing individual tasks or as an additional source of information for students struggling with a topic.

As noted earlier, the productive scale facilitates the examination of an app in terms of its ability to function as a tool to create learning objects or to perform an specific tasks like measuring or drawing. From the student responses, it is clear from the analysis above that for most scale items responses appear oriented in both positive and negative directions. Later in this article, the implication of students’ responses will be discussed.

The Instructive Scale

“Long Division Touch” is an app developed by Regular Berry Software (2013) targeting children working to achieve the outcomes of Year 5 Mathematics in the Australian Curriculum (ACARA, 2014). The app can be downloaded from the iTunes Store to use with iPhone, iPad, or iPod Touch. The app is described on the iTunes website as follows:

> Learn the mechanics of long division with a touch interface. Drag digits down, slide the decimal into the correct position, and tap to identify repeating decimals. … Learning long division can be enjoyable when you’re not slowed down by pencil/paper or lost without instant feedback.

Lessons available via the app cover the basic procedures associated with long division for whole numbers, using remainders, division with decimals divisors and repeating decimals.

Instructive Task Structure Sub-Scale (I)

Many students felt that this app did not meet their expectations in regard to engagement and differentiation and would therefore not be appropriate for general classroom use. The main reasons students gave for rejecting the app were its lack of visual appeal and their belief that it would not engage children who would easily become disinterested and bored. Students also noted that the app provided minimal guidance and instruction on the long division algorithm.

> This app is not very engaging and if students did not know long division they would have difficulty using this app. It isn't very stimulating at all. This app could be used as a side activity in a lesson but I wouldn't use it in the classroom.

Some respondents thought, however, that the app “has good examples and provides practice questions for kids to practise their learnt skills at each stage” and “students can do practice problems and easily review the lessons and instructions on how to use the app to do long division.”

There was general consensus that if the app were to be used, it would be more appropriate as a revision tool after children had learned the long division algorithm. This was because students felt that the app “does not support student learning” since it offered no differentiation to cater for a range of ability levels “with limited feedback for students who don't know what they're doing.” On the positive side, a student remarked that the app is “good because it goes through a wide range of problems and outcomes.”

According to some respondents: “This is a good app that help explain to students on how to do long divisions”; “I would only use this app for rote practice of long division, It definitely helped understand the process after some examples”; “It is a good app for showing the steps and long division” and “I would include this in the classroom due to the visual and interactive nature the app provides - concepts and strategies are clearly presented and easy to understand.”

Cognitive Involvement Sub-Scale (CI)

For the item CI1 (retrieve and review maths concepts/skills/procedures (Remembering)), some students thought that the app could provide opportunities for drill and practice of the long division algorithm: “For individual work to consolidate progress and repetition for those students who need it, or for homework.” However, most comments related the item to “rote learning” and “just a practice app.” These comments must be taken in context as the app employs a tutorial design which basically presents drill-and-practice exercises augmented by problem- solving guidelines and feedback.

In regard to the item CI2 (demonstrate understanding of maths concepts/skills/procedures (Understanding)), most respondents considered that the app would not be appropriate since “the students who don't really understand long division will struggle with understanding why they are doing the drills and what it is about.” There were also a few favourable comments such as: “I would recommend this app for use in the classroom as it gives students a good understanding on how long division works in a simple and interactive way.”

Against the item CI3 (apply their knowledge of maths concepts/skills/procedures in practical contexts (Applying)), the following comment was typical of students’ responses: “[The app] lacks creativity and is very instrumental and instructional. It does not allow students to see maths as something helpful and worthwhile in our daily lives.” The app’s practicality was only perceived as useful “for students who prefer to use digital technology than just learning, reading, speaking and writing.”

Item CI4 (critically analyse maths content in text, graphs and/or animations (Analysing)) also received mostly unfavourable comments such as the following:

> Students may not understand the concepts being recognised in this app. They may just be following actions mentioned in the guidelines and more concerned with pressing the button until the answer is right … [I] suggest it be used with supervision and for short periods followed up by teacher modelling and then children also attempting to solve equations (sic) in writing.

Nevertheless, some positive comments were found for the item CI4 such as the following remark:

> I like it as a hands-on activity. I can see that students could clearly see the necessary steps required to undertake long division. Some explanation or tie back to 'why' may be good but overall a great concrete way for students of varying levels to practise long division … I could see it being useful in a learning support room where you are able to give students more one-on-one time prior to them having to do their own examples on paper.

Item CI5 (Evaluating) drew similarly negative responses because the app does not require children to provide any justification for their work. Some respondents did note that the app could be useful as a tool for remediation as per this statement: “students could practice [long division] without the app and if they find they keep getting it wrong go back to the app.” Another respondent thought that the app “would be a good tool to aid reflection.” With a few exceptions there was not much support to the item CI6 (Creating).

General Pedagogic Issues Sub-scale (GPI)

For the item GP1 (permits students to pose their own problems) students generally considered that “it is useful in a classroom to demonstrate how to do long division but it does not allow students to create their own problems to solve.”

There was a general consensus among students that the app does not allow for differentiation through sequentially designed degrees of difficulty (GP2). A typical comment was as follows: “[The app] does not allow students to view their progress. There are no instructions, no differentiation. ... No set age group or activities to challenge different needs.” For the item GP3 in regard to whether the apps gives students control over their learning, students agreed that this was not the case because there was “not enough student control - must follow directions.” For the item GP4 (delivers content in an appealing and motivating way according to the age group) a majority of opinions revealed that the app lacked visual appeal and that children, particularly young ones, would quickly lose interest in using it:

> [I] would use it for older years as practice for long divisions, however it's a little boring so I wouldn't use it for a very long period of time or very often.

Item GP5 (provides meaningful teaching and learning guidelines) encouraged a range of responses from generally positive comments such as “I would recommend this app. It's concise, clear in its instructions, and easy to use” through to less favourable responses like “I would not recommend this app for use in the classroom as it is not only disengaging for the students but does not explain why or how the division is done this way - no meaningful explanation.” There were no student responses categorised as pertaining to GP6 (integrates maths with content from other Key Learning Areas) or for GP7 (allow students to collect and record their own data).

Students’ responses classified for category GP8 (shows a reading level appropriate to the student's level) showed that students felt that the instructions for using the app could have been explained more simply since they "required [a] higher reading level." For GP9, students noted that the app does not save nor keep children's work and so "does not allow students to view their progress."

In regard to the item GP10, which focused on student collaboration, respondents believed that the app was essentially something that the teacher could demonstrate to the whole class through direct instruction or which children would use individually as there were no features of the app that made it suitable as a collaborative tool. However, a student thought the opposite: "The app supports collaborative learning" although there was not elaboration for his/her response.

Operational Issues Sub-Scale (OP)

This sub-scale relates to the user-friendliness of the app, how easy it is to navigate and use, and the extent to which it can be integrated with the full range of web platforms and multimedia forms. With respect to OP1 (has an intuitive and user friendly navigation), students' comments generally noted that the app "has an intuitive and user friendly navigation."

For OP2 (contains helpful technical instructions to the user and/or a Help function), students noted that the instructions were brief but generally easy to follow. In response to item OP3 (lets students alter its settings to customise the app to their needs), students noted that there was no such facility with this app. There were no student responses categorised as pertaining to OP4 (allows file sharing, streaming of content and/or online communications).

Comments related to item OP5 (is flexible permitting students to move in different directions) showed that students did not agree that the app offered this facility because it "does not allow you to get the wrong number" and "does not allow students the ability to manipulate the data freely" and "students also don't get the option to select their own problem." In response to sub-scale OP6 (has a supporting Web page providing additional useful information), one student noted the 'explore' tab which is accessed from the internet, while there were no student responses categorised as pertaining to OP7 (easily works with a range of media e.g., audio, video, image, text, animations) or OP8 (can interface with social media tools e.g., Facebook, wikis, blogs, Twitter, YouTube).

For OP9 (presents an uncluttered display which is visually stimulating), there was general agreement among students that this was not the case although some students praised "the visual and interactive nature the app provides … concepts and strategies are clearly presented and easy to understand." Furthermore, there were no comments with regard to OP10 (permits a student leave at any time and begin where he or she left). Moreover, all the Instructive scale items were validated with the exception of CI6 and GP6 since the app does not afford construction of new and meaningful mathematical ideas nor does it integrate mathematical content with that from other Key Learning Areas. Similarly, items OP4, OP7, OP8 and OP10 were not validated since the app has limited integration affordances. The remaining scales were validated, though predominantly through unfavourable commentary showing little appreciation for the drill-and-practice and tutorial structure of the app.

In general, the analysis of responses for each of the three scales showed no discernible patterns whether, as a whole, the participants approved or disapproved the instructional role, cognitive involvement, pedagogical and operational issues of the each app.

Discussion and Conclusion

The study revealed that respondents could not differentiate the four TPACK constructs of TPACK, PCK, TPK and TCK. These four dimensions were represented by the Task Structure sub-scale, pedagogical content knowledge represented by the Cognitive Involvement sub-scale, the technological pedagogical knowledge represented by the General Pedagogical Issues sub-scale and technological content knowledge represented by the Operational Issues sub-scale.

Such a wide variability can be attributed to the fact that students viewed each app from their own personal/subjective perspective irrespective of the type of app or university. However, respondents' comments could be matched against most of the semantic items in the three scales validating the relevance of the items in characterising mathematics apps and therefore establishing content validity. Construct validity could not be established as factor analysis showed disparate patterns in students' responses. This leads to the conclusion that the TPACK constructs are not clearly perceived as such by these users of these apps.

The findings might imply that scales characterising educational apps provoke a great deal of subjectivity. A probable explanation for such variability could be that many/these preservice students were not formally taught principles of multimedia/mobile instructional design, which normally include types of app, by educational roles, degree of cognitive involvement, general pedagogical eLearning issues, and operational/ technical issues affecting apps. The qualitative analysis revealed that students need to understand clearly the difference in instructional roles among explorative, productive and instructive apps. For example, in regard to the instructive app group's responses to the item (CI6) "construct new and meaningful maths ideas or products", there were not many positive responses. Many students did not recognise that attribute in the app, perhaps because of their limited understanding that instructive apps take drill-and-practice or tutorial formats which, by their highly direct instructional nature, do not allow much space for users to generate new products or ideas. For instance, two students wrote: "The content is very limited and the students cannot create their own problems" and "It's not colourful, not relational, and very algorithmic." When positive comments appear, these seem an acknowledgement that this is a tutorial tool rather than an explorative application which, by nature, take a more learning discovery orientation.

The qualitative analysis on the productivity app (ShowMe) revealed the student appreciation for its hands-on nature, although divergent comments seem to refer to diverse potential uses by the teacher thus creating response variability. For example, some students thought the app could be worthwhile for a mathematics classroom while some other students suggested it would have a better use in curriculum areas other than mathematics. Students also commented on the app's capacity to generate student cognitive involvement, particularly in constructing new and meaningful mathematics ideas or products because the tool itself, because of its utilitarian nature, was content-neutral.

The third reason for response variability may rest on students' own articulation about what constitutes meaningful teaching guidelines as provided by the explorative app (Fraction Fiddle). These can be grouped under three categories: presence/absence of teaching instructions, comments on feedback, and opinions on prior knowledge and supplementary learning. In regard to teaching instructions, remarks varied from negative to extremely negative: "no guidelines or instructions", "no explanation of how to do fractions". Similar to the instructive apps case, the difficulty seems to arise because some respondents have made up their mind about what is the correct instructional role that an app should have. While in the instructive app analysis some students thought the app should be more exploratory oriented, in the explorative app some students believed that the "role model" app should be more tutorial driven. Hence, conceptions about feedback are more associated with drill-and-practice exercises than to a discovery learning approach. Furthermore, a number of comments recommended the need for students to have prior knowledge on fractions, or the app being used after fractions are taught explicitly by the teachers, or for complementary teaching like using a whole-class discussion on an interactive whiteboard.

The fact that respondents came from three different teacher education programs, with different curricula, might have exacerbated this variability in respondents' subjectivity. Nevertheless, it might be concluded that the scales can be very effective in providing respondents with a discourse/script to systematically characterise mobile apps as well as suggesting issues to look at before selecting an app for teaching and learning.

Although the present study focused on response variability, further research should consider analysing preservice teachers' responses based on experts' judgments. Such a study would make use of inter-judge reliability techniques to ensure that personal appraisals are normed particularly when, as this research proves, individual subjectivity is a significant distorting factor in analyzing the affordances and constraints of a particular app.

Various authors have already raised their voice for the need to assist teachers in guiding them through the technicalities involved in appraising the worth of an educational app (Attard, 2013Attard & Curry, 2012; Jonas-Dwyer, Clark, Celenza & Siddiqui, 2012; Kiekel & Kirk, 2013; Spencer, 2013). Such a professional development would reassure classroom teachers that technology does make a difference in mathematics teaching and learning. By scaffolding teachers in the design principles underpinning the construction of good educational apps, teachers can later apply these principles to the development of their own instructional resources.

Selecting an effective and appropriate app, once the instructional role of a particular task has been defined, is central to creating a rich learning experience. As stated previously, there are hundred of thousands of apps in the market which are labelled as educational apps in online stores; however, many of them are more appealing commercial games than pedagogical objects – hence the need to examine each educational app on the basis of systematic criteria (Handal, 2015; Larkin, 2015; Watlington, 2011).

Limitations of the Study

There are some limitations of the design of the study which may have influenced the results which we obtained. Firstly, a limitation of the study is that there may have been some subjectivity in the nature of one or two of the questionnaire items. For example item P5 operational item asked about the intuitiveness navigation of the app suggesting that such experiences are possibly being individually and culturally based (Handal, 2015). Typically, items in Likert-type scales are written to offer the participant a broad range of options, from strong agreement to strong disagreement to pre-established statements. However, those items, due to their own discrete nature, do not allow participants to express openly their own understanding or to question the meaning embedded in the semantic item (Cohen, Manion & Morrison, 2007). It is only when data triangulation occurs through qualitative and qualitative methods that additional meanings and caveats emerge helping to mitigate the subjectivity risk.

While the preservice students appeared to have been given sufficient time in their tutorial classes to use their allocated app, and to write up their questionnaire responses, perhaps the 30 minutes spent examining the apps may have been inadequate. The preservice teachers may have benefitted from more time to use the app so that their questionnaire responses were more deeply considered. Also, the primary teacher education curricula at each of the three universities which participated in the study are quite different in scope and sequence. Consequently, the preservice teachers are also likely to have different skills and different abilities in relation to TPACK. Future research would also benefit from increasing the time employed by students in getting familiar with an app before the appraisal process in order to explore the effect on response variability due to better acquaintance with its pedagogical and operational features.

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