

Experimental work on Reinforced and Prestressed Concrete Deep Beams with Various Web Openings

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ABSTRACT: This paper presents the outcome of recent research undertaken on concrete deep beams with and without web openings. The paper also highlights the current design code methods and attempts to extend a previously published design formula for reinforced (RC) and prestressed concrete (PC) deep beams. An experimental program has been undertaken to compare with the proposed design method. The emphasis of the experimental work is based on deep beams with varying web opening size, location and concrete strength. Apart from highlighting the experimental set-up, typical crack patterns, failure modes, and load-deflection behaviour are also reported. The main purpose of this paper is to summarise research previously done to highlight the inadequacy in existing design methods based on the current test results.

1 INTRODUCTION

Currently, deep beam design has been actively investigated as their necessity as a construction option has become more apparent. Although there has been extensive research into developing a system of design equations specifically for reinforced and prestressed deep beams with web openings, it has fallen short of a definitive means for accurate design. Although there have been suggestions as to the correct methods for designing deep beams, they are not conclusive enough to substantiate their use as a design model in national design codes. This is particularly true when complexities exist like prestressing or web openings. Especially, the presence of opening cause geometric discontinuity which directly effect the strut strength which in turn leads to a reduced ultimate strength.

The addition of the prestressing force in a deep beam increases the shear capacity of the beam counteracting the effect of the web opening. The Australian Standard (AS3600-01) and the associated suggestions have margins for improvement. Hence this paper focuses on laboratory tests undertaken on RC and PC deep beams with various web opening size and location, and varying concrete strengths. These tests cover an area where previous research is limited. Apart from highlighting the experimental set-up, failure loads, load-deflection behaviour and typical crack patterns of the test specimens are also reported. Experimental results are then compared with those predicted by existing design methods.

2 CURRENT CODE METHOD (AS3600-01)

In the Australian standard (AS3600-01), Section 12 specifies deep beam design method based on the strut-and-tie model. The code suggests that a cantilever deep beam has the ratio of clear span or projec-

tion to overall depth less than 1.5. A simply supported and a continuous deep beam has the ratio of clear span to depth less than 3 and 4, respectively.

The current AS3600-01 suggests the ultimate strength of strut be less than $b_c d_c f_{c,cal}$, where b_c and d_c are the width and depth of the compress strut, respectively, and $f_{c,cal}$ is equal to $(0.80 - f' / 200) \times f'_c$. The code mainly focuses on anchorage and bearing failure in the design and is limited in application when either high strength concrete ($f'_c > 65\text{MPa}$) or web opening is introduced. Hence, the current AS3600-01 method is inadequate and a reliable design formula is much needed.

3 DESIGN EQUATIONS PROPOSED BY OTHER RESEARCHERS

3.1 RC deep beams with and without web openings

Ray (1966) developed design formulae to calculate the ultimate strength of a deep beam using modified Mohr-Coulomb's criterion and the shear friction theory. An experimental work was undertaken to investigate the effect of web reinforcement and the test results were compared to the design formulae. The study showed that the experimental ultimate strengths of the beams were very much higher than the predicted values.

The design formula for the concrete beams without any web reinforcement is,

$$P_{ult} = \frac{2 \times b \times h \times c}{\sin \beta \cos \beta (\tan \beta + \tan \phi_0)} + 2F_t \left[\tan \beta - \frac{1}{\cos^2 \beta (\tan \beta + \tan \phi_0)} \right] \quad (1)$$

$$\text{where } c = \frac{\sqrt{(\sigma_{uc} \times \sigma_{ut})}}{2} \quad \text{and} \quad \tan \phi_0 = \frac{\sigma_{uc} - \sigma_{ut}}{2\sqrt{(\sigma_{uc} \sigma_{ut})}}$$

in which b = width of rectangular beam, h = overall depth of deep beams, c = internal cohesion of concrete, β = inclination of the rupture plane with hori-

zonal, ϕ_0 = coefficient of internal friction of concrete, σ_{uc} = compressive strength of concrete, and σ_{ut} = tensile strength of concrete.

Based on experimental studies, Kong et al. (1970, 1973, 1998) and Kong & Sharp (1977) derived design equations for normal and lightweight concrete deep beams with web openings. The ultimate shear strength equations for reinforced concrete deep beams are:

$$Q = C_1 \left[1 - 0.35 \frac{x}{D} \right] f_t b D + C_2 \sum a \frac{y}{D} \sin^2 \alpha_1 \quad (2)$$

for solid deep beam, and

$$Q = C_1 \left[1 - 0.35 \frac{k_1 x}{k_2 D} \right] k_2 f_t b D + C_2 \sum \lambda C_2 a \frac{y}{D} \sin^2 \alpha_1 \quad (3)$$

for deep beam with web opening

where, C_1 = empirical coefficient (1.40 for normal strength concrete, 1.35 for light weight concrete), b = breadth (thickness) of beam, D = overall depth, f_t = cylinder-splitting tensile strength of concrete, x = clear-shear-span distance, C_2 = empirical coefficient (300N/mm² for deformed steel bar, 130N/mm² for plain steel bar), y = depth at which a typical bar intersects the potential critical diagonal crack in solid deep beam, which is approximately at the line joining the loading and reaction points, and λ = an empirical coefficient, equal to 1.5 for web bars and 1.0 for main bars. Other geometric notations are described in Figure 1.

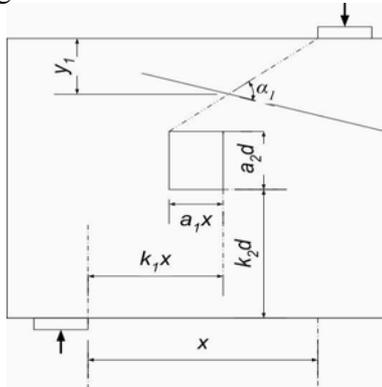


Figure 1. Notation for size and location of opening (Kong and Shrap, 1977)

Kong et al. (1970, 1973, 1998) made significant contributions to the development of the British Standard. The first term on the right side of Equation 2 expresses the load capacity of strut. When an opening is in the natural loading path, the first term considers the lower load path. The second term on the right side of the equation articulates the contribution of reinforcement in deep beams. However, these equations are only applicable for the concrete strength less than 46 MPa.

Smith and Vantsiotis (1982) conducted extensive laboratory tests on deep beams to identify the con-

tributions of web reinforcement upon changing the a/d ratio. The study concluded that the minimum reinforcement ratio should be greater than 0.18 and 0.23 for vertical and horizontal reinforcement, respectively and the contribution of web reinforcement can not exceed $4(\sqrt{f_c})bd$. However, the clear span to depth ratio of the tested beams was between 0.77 and 1.34 which were much less than the ratios suggested by AS3600-01.

Tan et al. (1995, 1997 & 2003) and Leong & Tan (2003) investigated the effects of high strength, shear span to depth ratios and web reinforcement ratios of the beams using both experimental program and numerical analysis. The design formula for high strength concrete deep beams is

$$V_n = \frac{1}{\frac{\sin 2\theta_s}{f_t A_c} + \frac{1}{f'_c A_{str} \sin \theta_s}} \quad (4)$$

$$\text{where } f_t = \frac{2A_s f_y \sin \theta_s}{A_c / \sin \theta_s} + \frac{2A_w f_{yw} \sin(\theta_s + \theta_w)}{A_c / \sin \theta_s} \cdot \frac{d_w}{d} + f_{ct} \quad (5)$$

$$\text{and } \tan \theta_s = \frac{h - \frac{l_a}{2} - \frac{l_c}{2}}{a} \quad (6)$$

in which, θ_s = angle between the longitudinal tension reinforcement and the diagonal strut, f_t = combined tensile strength of reinforcement and concrete, A_c = area of concrete section, A_{str} = cross-sectional area of diagonal strut, f_y = yield strength of longitudinal steel reinforcement, A_w = area of web reinforcement, f_{yw} = yield strength of web reinforcement, θ_w = angle between the web reinforcement and the axis of beams at the intersection of the reinforcement and diagonal strut, d_w = distance from the beam top to the intersection of the web reinforcement with the line connecting the support centre and the load centre, d = effective depth, f_{ct} = tensile strength of concrete, h = overall height of deep beam, l_a = height of bottom node, l_b = width of support bearing plate, and a = shear span measured between concentrated load and support point.

Maxwell (2000) investigated the strut-and-tie model using experimental tests and the test specimens have a large opening near the support. The study concluded that the specimens performed very well following the prediction of the strut-and-tie model theory. However, those tested beams were of normal strength concrete and relatively thin deep beams with large web openings.

The research on continuous deep beams with and without openings was conducted by Ashour (1996, 2000). Based on the upper-bound theory, a design equation for continuous beams was developed.

Zararis (2003) proposed an equation based on the upper-bound theory of failed reinforced concrete

members by using free body diagram from crack patterns. The equation is

$$V_u = \frac{bd}{a/d} \left[\frac{1 - 0.5c_s/d + 0.5(\rho_h/\rho)(1 - c_s/d)c_s}{1 + (\rho_h/\rho)(1 - c_s/d)} \frac{c_s}{d} f'_c + 0.5\rho_v f_{yv} \left(1 + \frac{c_s}{d} \right)^2 \left(\frac{a}{d} \right)^2 \right] \quad (7)$$

where, c_s = depth of compression zone above critical diagonal crack, ρ = ratio of main tension reinforcement, ρ_h = ratio of horizontal web reinforcement, ρ_v = ratio of vertical web reinforcement, and f_{yv} = yield strength of vertical web reinforcement.

Recent research on continuous deep beams with web opening was carried out by Ashour & Rishi (2000). The experimental test results were compared with the strut-and-tie model predictions.. The study concluded that the support reactions were influenced by the size and location of web openings. However, this research only considered the effect of normal strength concrete only.

3.2 PC deep beams with and without web openings

Alshegeir (1992) tested a series of I-shape prestressed deep beams and test specimens were fabricated by AASHTO standard type I and II. The test results were analyzed and compared with the strut-and-tie model predictions. The study concluded that the presence of prestressing in deep beam delays the occurrence of the inclined web-cracking, but it is not necessarily the critical factor in the inclination of the failure diagonal crack.

Tan & Mansur (1992) and Teng et al. (1998) tested prestressed deep beams and predicted the ultimate shear strength by using the strut-and-tie method. Tan & Mansur (1992) pointed out that the national standard is unsafe or too much conservative when comparing the test results with the national code.

4 LIMITATION OF EXISTING DESIGN EQUATIONS

Table 1 gives an overview of work done on RC and PC deep beams by the abovementioned researchers. Most studies and experimental work focused on the normal strength concrete without an opening and the findings of such work has contributed to the development of the national code equations. Therefore, the current practices for calculating the failure loads do not consider the contributions from web openings with high strength concrete. As a consequence research work on high strength concrete deep beams with web openings is yet to be incorporated in the design codes.

Table 1. Summary of existing deep beam tests

Name	Number of beam	f'c (MPa)	a/d ratio	Opening
Kong et al. (1970)	35	19~28	0.35~	No
Kong, Sharp(1977)	32	32~44	0.2~0.3	Yes
Smith, Vantisiltis(1982)	52	16~23	1~2.1	No
Ashour, Morley(1982)		Theoretical analysis		No
Alshegeir (1992)	3	61~62	4.32~4.7	No(p)
Tan et al. (1992)	8	31~67	1.5,2.0	No(p)
Kong & Sharp(1993)	24	29~37	0.25~0.4	Yes
Kong et al.(1995)	17	39~46	0.3	Yes
Tan et al.(1995)	19	41~58	0.3~2.7	No
Maxwell, Breen(1997)	4	28	1	Yes
Tan et al.(1997)	22	51~94	0.1~2.5	No
Teng et al.(1998)	30	16~82	~2.5	No(p)
Tan et a.l(1999)	12	34~47	0.5~1.0	No(p)
Ashour, Rishi(2000)	16	20~28	0.8	Yes
Leong, Tan(2003)				
Tan, et al.(2003)		Theoretical research		
Zararis(2003)				

*P = prestressed deep beam.

5 EXPERIMENTAL PROGRAM

5.1 Test panel and set-up

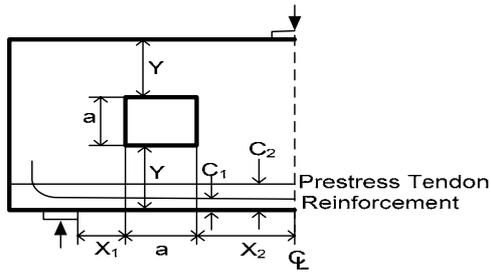
In an attempt to evaluate the performance of the existing design equations, 16 beams were tested to failure and they are divided into two stages.

The first stage (8 deep beams) was focused on varying web opening size of reinforced and prestressed deep beams. The opening sizes were of 150×150mm, 180×180mm, 210×210mm, and 240×240mm. For prestressing beams, 5 mm diameter tendons were used. The concrete strength was 39.31 MPa. The dimensions of the deep beams are detailed in Table 2 where the symbols R and P indicate reinforced and prestressed beams, respectively and the subsequent symbol O or S denotes opening or solid specimen, respectively. The first two digits following the symbols represent the concrete strength. The last digit of the beam reference denotes the loading condition with 1 for single point loading and 2 for two point loading conditions. Other geometric symbols are illustrated in Figures 2 (a) and (b).

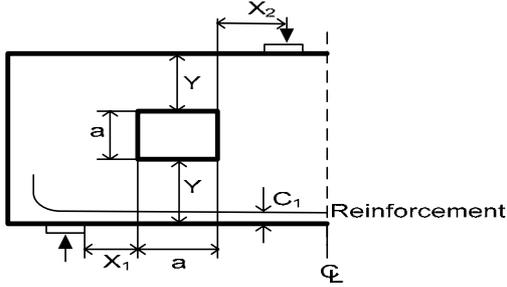
Further testing of deep beams (second stage) was conducted using high strength concrete (= 77.6 MPa) with steel reinforcement only. Table 2 also shows dimensions of the beams with one and two point loading. Also, the loading position was varied as indicated in Figure 2 (b). In this second stage, two solid beams (RS80-1 and RS80-2) were also tested to provide a control data.

The test frame was designed to support a jack of 80 tonne capacity. Dial gauges were used to measure the deflections of the beams during testing. The beams were loaded in about 10 kN increments with a reduced load close to failure. At each load incre-

ment, crack patterns and deflections were recorded. The latter allowed the load-deflection history to be accurately traced. The ultimate loads of the beams are presented in Table 3.



(a) Deep beam under single point load



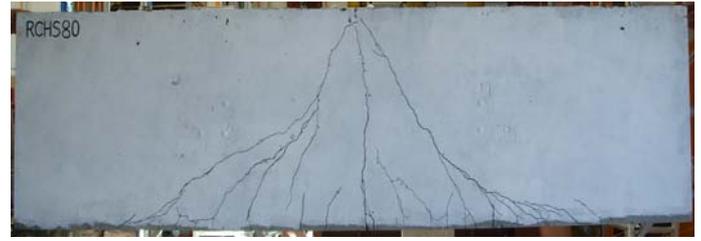
(b) Deep beam under two point load
Figure 2 Loading conditions

Table 2 Dimensions of deep beams

	Beam refer- ence	a mm	X ₁ mm	X ₂ mm	Y mm	C ₁ mm	C ₂ mm
Stage 1	RO40-150/1	150	322.5	427.5	225	20	N/A
	RO40-180/1	180	307.5	412.5	210	20	N/A
	RO40-210/1	210	292.5	397.5	195	20	N/A
	RO40-240/1	240	277.5	382.5	180	20	N/A
	PO40-150/1	150	322.5	427.5	225	20	100
	PO40-180/1	180	307.5	412.5	210	20	100
	PO40-210/1	210	292.5	397.5	195	20	100
	PO40-240/1	240	277.5	382.5	180	20	100
Stage 2	RS80-1	0	850	N/A	N/A	20	N/A
	RS80-150/1	150	322.5	427.5	225	20	N/A
	RS80-180/1	180	307.5	412.5	210	20	N/A
	RS80-210/1	210	292.5	397.5	195	20	N/A
	RS80-2	0	850	N/A	N/A	20	N/A
	RS80-150/2	150	322.5	427.5	225	20	N/A
	RS80-180/2	180	307.5	412.5	210	20	N/A
	RS80-210/2	210	292.5	397.5	195	20	N/A

Table 3 Experimental test results

Beam reference	Ultimate Load (kN)
RO40-150/1	88.09
RO40-180/1	86.52
RO40-210/1	79.46
RO40-240/1	72.40
PO40-150/1	107.70
PO40-180/1	103.20
PO40-210/1	88.29
PO40-240/1	82.99
RS80-1	472.65
RO80-150/1	125.57
RO80-180/1	93.20
RO80-210/1	82.40
RS80-2	657.56
RO80-150/2	583.11
RO80-180/2	334.52
RO80-210/2	174.42



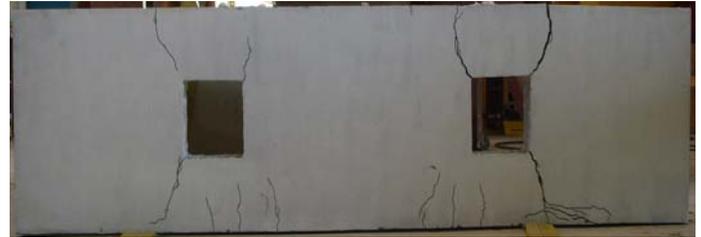
(a) Single point loading solid beam



(b) Two points loading case of solid RC beam



(c) Single point loading case of RC beam with web openings



(d) Two point loading case of RC beams with web openings



(e) Single point loading case of PC beam with web openings
Figure 3 Typical crack patterns of deep beams

5.2 Test results and discussion

The typical crack patterns for three beams are illustrated in Figure 3. Figures 3 (a) & (b) show that the inclined cracks gave the appearance of a tied-arch system to the beam and this arch-type behavior was apparent in all the test specimens. For the deep beam with web openings (see Figures 3 (c), (d) & Figure 4), the first visible diagonal cracks generally appeared at the top and bottom corners of web openings towards the load points and supports at different load levels. Also for Figures 3 (d), the mode of fail-

ure was similar to the solid deep beams, with minor cracks occurring below and above the outer edge of the openings. These minor cracks were also presented in the prestressed deep beams due to the prestressing force increasing the yield strength of the beam (see Figure 3(e)).

As can be seen from Table 3, the ultimate loads for the single-point loaded beams and thereby the shear strengths of the beams were much smaller compared to their two-point loaded counterparts. Also the ultimate strength decreases as the opening size increases in both RC and PC deep beams.

Further, Table 3 indicates that for normal strength concrete deep beams, an increase in the opening size from 150mm to 240mm has led to a strength reduction of 17.8% and 22.9% for RC and PC beam respectively. Moreover, the size of the opening was a more important factor in high strength concrete deep beams and it was observed that an increase in the opening size from 150mm to 240mm has led to a strength reduction of 34.4% and 70.0% for single- and double-point loaded beams, respectively.

The higher strength reduction reflects the size of openings and the strength of concrete. This can be the reason that the larger the opening size in double-point loaded beams, the more the shear path being disturbed and hence the greater the reduction in ultimate load capacity was observed.

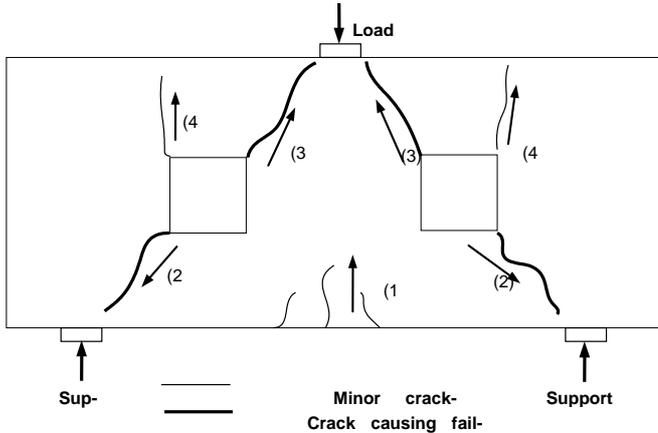


Figure 4 Crack propagation of deep beams

Figure 5 illustrates the load-deflection response of normal strength RC and PC deep beams under single point loading (Stage 1). It is obvious that the failure loads for the PC beams are higher than the corresponding RC specimens. Also the ultimate strength decreases as the opening size increases in both RC and PC deep beams.

Figure 6 illustrates load-deflection response of high strength concrete deep beams for one and two point loading cases with and without web openings. The curves for the solid beams exhibited non-flexural behaviour. However, the single point load case of high strength concrete beams with openings exhibited a ductile failure type.

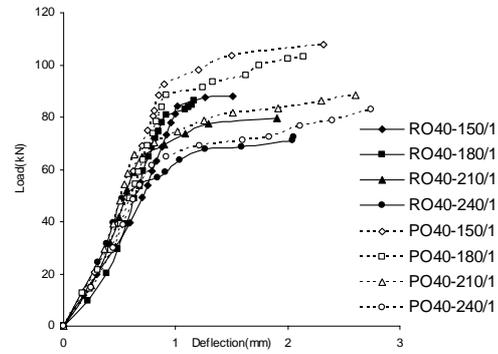


Figure 5 Load-deflection response of normal strength RC and PC deep beams

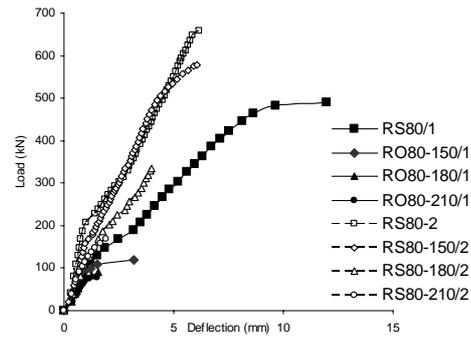


Figure 5 Load-deflection response of high strength deep beams with and without web openings (single and two point loading conditions).

6 COMPARISONS

The design equations proposed by previous researchers were compared with the experimental test results of the four normal and eight high strength concrete deep beams from Stages 1 and 2. The mean ratio (Calculated/Experimental) and its standard deviation were calculated and presented in Table 4.

Table 4 Comparisons with experiment results and design equations from literature

Beam Reference	Calculated Result (kN)		Eq.(3) Eq.(4)	
	Eq. (3)	Eq. (4)	Exp	Exp
RO40-150/1	146.76	93.53	1.67	1.06
RO40-180/1	105.64	92.78	1.22	1.07
RO40-210/1	68.12	93.17	0.86	1.17
RO40-240/1	23.68	91.76	0.33	1.27
RS80-1	331.94	544.74	0.70	1.15
RO80-150/1	68.71	176.46	0.55	1.41
RO80-180/1	52.66	163.31	0.57	1.75
RO80-210/1	35.28	145.10	0.43	1.76
RS80-2	803.60	482.15	1.22	0.73
RO80-150/2	236.76	389.20	0.41	0.67
RO80-180/2	186.20	342.50	0.56	1.02
RO80-210/2	164.64	297.80	0.94	1.71
Average			0.79	1.23
Standard deviation			0.41	0.37

As stated in Section 4, none of the current design formulae give a good prediction for prestressed deep beams with high strength concrete. Only Equations (2) & (3) and Equation (4) are included in Table 4. Also as stated in Section 2, Zararis (2003) equation

is not applicable to the beam with openings and is therefore not included in this comparison.

The application of Kong and Sharp's equation (1977) indicates that the calculated/experimental ratios vary from 0.41 to 1.67, with a mean ratio of 0.79 and a standard deviation of 0.41, showing that the equation generally underestimates the ultimate strength. This could be due to the fact that one of the terms in the design equation is $\left(1 - 0.35 \frac{k_1 x}{k_2 D}\right)$ which

is concerned with the position and size of the web opening. As the opening size increases and approaches the base of the beam, this term will produce a negative value. The value obtained from this term is then multiplied by the empirical coefficient for normal strength or lightweight concrete which in turn would result in a negative value for the concrete strength. Hence the failure load predictions obtained from this method produce greater discrepancies. On the other hand, Tan et al.'s (2003) method shows the calculated/experimental ratios consistently greater than 1, suggesting that the equation overestimates the failure load, which could be due to the fact that the equation does not consider the effect of concrete of such a high strength. Even though Tan et al. (2003) investigated and proposed design equations for normal and high strength concrete deep beams, modifications are still required to produce a satisfactory design formula for high strength concrete deep beams particularly when openings are present.

7 CONCLUSION

The Australian Standard (AS3600-2001) design method was found to be inadequate for deep beams with web openings, particularly with high strength concrete. An experimental program has been undertaken to investigate the performance of the existing design equations in predicting the ultimate load of concrete deep beams with and without openings. A comparative study indicates that available design methods proposed by previous researchers are inadequate in terms of the large discrepancies in their strength predictions. In view of these significant shortcomings, a more appropriate design formula is required for normal and high strength reinforced and prestressed concrete deep beams with web openings.

Despite the conduct of extensive tests on concrete deep beams, research on high strength concrete deep beams with various openings remains relatively unexplored and needs more focused research in the future.

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