
Envisaging the future

Our changing technological society: demands and links between numeracy performance and life outcomes for employment, education and training

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Acknowledgement

This paper was commissioned by the Australian Association of Mathematics Teachers Inc. and funded by the Commonwealth Department of Education, Training and Youth Affairs.

Note

This paper was prepared in 1998/1999 and some information in the paper may not reflect more recent developments.

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For I dipp'd into the future, far as human eye could see,
Saw the vision of the world, and all the wonder that would be.
(Locksley Hall, 1842)

Abstract

This paper was written to inform the development of material for a Commonwealth monograph on numeracy and to address the topic indicated in the title. While the focus of the discussion is on information and communication aspects of technology, implications for life outcomes for personal contexts have also been included. The readings undertaken to ensure the topic was addressed appropriately led to many formulations of the title and conclusions including:

- the nature of the technological society
- the interface between technology and education
- the nature of demands of working numerately and need for flexibility
- the demand from society for stronger inclusion in the education process
- the need for mathematics education to improve students' sense of contexts of application of mathematical learning
- the need for mathematics education to embrace the new approaches to assessment.

Introduction

Plato is credited with, or held responsible for, advocating the superiority of knowledge retained in written form over the oral traditions of previous time. Knowledge preserved in oral forms must meet linguistic conditions to enable memorisation, and may be:

stated in language that is regularly patterned, that is visually rich and imagic, that personifies impersonal phenomena... (Scribner, 1968, p. 175).

On the other hand, the use of written language enabled storage and retrieval of knowledge of a different form, allowing classificatory and philosophical expositions that do not necessarily lead to easy memorisation and which allow enquiry of knowledge to proceed in different ways. Of course, the nature of language itself had to change to allow this development of written literacy as the forum of education, with the development of 'syntax and lexicon suitable for the expression of conceptual thought' (Scribner, 1968, p. 175).

In the two and a half thousand years since Plato's time, in Western society in particular, we have progressively fostered the perception that knowledge is stored in, and is to be stored in, written paper and expository text format. The capacity to print volumes of texts relatively cheaply, and to write and to read such texts, have been seen as critical to our knowledge-driven society. Preservation of these perceptions has been an outcome of current education and bureaucratic systems which themselves have focused on knowledge storage and transmission in similar ways. Hence, while in law we may have brilliant barristers who declaim and win a case by their persuasive oral powers and logic, we have written transcripts of these cases and written judgements which become the source of study and reference in the continuation of legal practice, and assessment on these the gate-keeping to a career in this area.

Around this written world has existed an oral world of work practices and of community practices, a world that has been patronised as lesser — in demand, sophistication, even purpose — than the written world. However, in a very short period of time relative to the time since Plato directed our conceptions of knowledge and education, the world has changed dramatically. Computer technology and communication technology have brought about this change. While a primary role of computers in business may originally have been the storage of written text in cyberspace rather than paper, emulating the written world of the past, programs such as spreadsheets, specific purpose stock registers and also communication technology have led to changes in these purposes. The world of work in Western society, and increasingly throughout all of the world, has become a visual/oral world where reading requires recognition of individual words on menus and active selection of screen options. These screens allow information to be displayed, usually in tabular form with information in vertical columns, or for information to be input using similar forms. Professional communication in many settings now involves oral conversation about such tabular information. The new business triad for the clinching of a deal consists of two human interlocutors and a cybernetic intermediary. In recent times we have even seen startling consequences for the share market through deals made between cybernetic interlocutors without the intervention of a human intermediary.

This visual and oral world is the personal and work world for which education is preparing students. Cumming, Wyatt-Smith, Ryan and Doig (1998) have shown that the classroom environment for student learning partially replicates this world, with an emphasis on multimodal ways of communicating, that is, oral and visual interaction. However, education itself and the bureaucracies that surround education are in danger of becoming anachronisms, preciously preserving their world of paper and perceptions of knowledge as written expository or narrative text. This paper itself is part of such a world. However this paper has been prepared on a computer

screen surrounded by icons and list menus to facilitate its preparation, with time and battery symbols to be interpreted to manage my work, judgements to be made on the spot about font size, spacing between lines, margins, tabs and so on. The paper will be transmitted electronically where it can be read on screen or printed, or selected pages can be printed or scanned electronically for key words or phrases. Large chunks can be extracted and 'pasted' into another document enabling electronic writing to occur rather than new creativity.

More to the point, the information and argument to be developed in this paper could just as easily be presented in a table form or dot points, a more succinct form for the reader. However, the specifications require at least 5,000 words (already nearly 2,000 words using the Word Count menu option) and the implicit expectation is that the traditional expository argument will be used. Perhaps the resultant policy document will have more iconic form.

Just as the worlds of work and personal lives have changed over recent decades, so too has our understanding of the nature of learning and the role of teaching in facilitating learning. The need for education to develop flexible learners who will continue to generate meaning and understanding from what they have already learnt, while seeking to continue to learn, is seen as critical to a modern technological society. Currently-held learning theories emphasise socio-cultural and situated effects on learning (Brown, Collins & Duguid, 1989; CTGV, 1992; Vygotsky, 1978). The world of school learning cannot be divorced from the other worlds.

At this point, mathematics educators may be asserting that mathematics is a symbolic language and that mathematics education focuses heavily on the visual and iconic forms, consistent with the modern society just depicted. However, in this paper it is stated that ironically, mathematics education, referred to by Landman (1997) as one of the new technologies, may be the most preciously protected area. Many teachers and others are reluctant to move from the textbook and written responses, and paper and pencil assessment of 'learning'. Debate continues about whether technological aids such as graphics and programmable calculators should be used in teaching or assessment. They are still not allowed to be used in many university examinations, for fear that if students are not calculating (with ordinary calculator assistance) solutions to equations themselves, they are not demonstrating understanding. Some view appropriate use of a programmable calculator somehow as cheating. Or are there other underlying reasons why practice is reluctant to change?

In this paper it is argued that, as we look at our changing technological society, and the demands and links that need to be made between numeracy performance and life outcomes in employment, education, training and personal contexts, the future of numeracy education for schools is to address the issues of education for the visual

and oral world that we occupy. In literacy, Freire (Freire & Macedo, 1987) advocated world literacy programs for empowerment and political reasons, to allow people to not only 'read the word' but also to 'read the world'. It is timely for numeracy education to go beyond the written word to the visual. Through all of our educational activities we must assist students to 'envisage the world'. This position will be explored through elaboration on a range of related topics and frameworks.

Relationships between mathematics and numeracy

While the following discussion does not explore in depth past and current conceptualisations of mathematics education and numeracy, as these will be addressed in other papers, sufficient discussion of this relationship is needed to provide a definitional or conceptual framework for this paper.

The first premise on which this paper is based relates to the basic definitions of the nature of mathematics. Mathematics is defined as:

The logical study of shape, arrangement, quantity, and many related concepts.
(James & James, *Mathematics Dictionary* (5th edition), 1992)

or, in educational terms, as:

A body of ideas structured by logical reasoning.
(*Encyclopedia of Educational Research*, 5th edition, 1982)

The importance of these basic definitions is their emphasis on relationships and representation. While the role of logic is also stressed, later discussion will show there is a need to question 'whose logic'.

An important understanding of the nature of mathematics is that it is about modelling, about providing explanations for the world in which we live (Landman, 1997). Hence at an elemental level mathematics is about providing a visual and symbolic representation of the world and its own spatial environment and the relationships between elements. While the direction of mathematics has been to develop abstracted representations, the origins of these should not be forgotten. Western mathematics is derived from various sources, including the long-established Chinese mathematics. Leung (1998) provides an overview of the traditional Chinese view of mathematics with the suggestion that this view needs to be revisited in current times. He cites Wang who summarised five major characteristics of ancient Chinese mathematics of which the first was *Pragmatics*.

...use of mathematical models to solve everyday life problems, marks the two most important characteristics of traditional Chinese mathematics: its algorithmic nature and its emphasis on application, (Leung, 1998, p. 71).

Leung contrasts the Greek philosophical system of mathematics, seen as an elitist system, with the Chinese emphasis on pragmatics and problem-solving, perhaps more suitable for the whole population.

It is timely to reconsider these core conceptualisations of mathematics if we examine notions of numeracy and also the needs of learners. Firstly, definitions of numeracy generally differ from traditional considerations of mathematics education in the area of application and practicality. Definitions go beyond knowledge of mathematics to include ability to choose the mathematics to use — perhaps not in the ways defined by school curriculum as standard — to make appropriate use of this mathematics, to obtain reasonable solutions and to check on this reasonableness, to determine appropriate degrees of accuracy, and to be critically aware of how mathematical information and arguments can be used to manipulate, empower or disempower individuals. The early numeracy definitions of Crowther (1959) and Cockcroft (1982) emphasised these components in the contexts of everyday lives. Critical awareness has become more valued as a result of similar work in literacy (see, for example, Freebody and Luke (1990), and parallel numeracy discussions in Johnston (1994)).

Work in literacy research has defined the difference between the type of reading activity that is undertaken in schooling and the reading activities that are used at work. In literacy, this has been defined most simply as the 'reading to learn' and 'reading to do'. Literacy work in this area by researchers such as Diehl, Mikulecky, Sticht, Kirsch, Guthrie, and Moe has been summarised in Freebody, Cumming & Falk (1993) as:

- workplace literacy differs from school literacy in that workplaces call for a variety of materials while schools do not;
- workplace writing is targeted toward a specific audience with specific intent such as to convey information or to persuade;
- workplace literacy is a social phenomenon which includes asking questions and gathering information from other workers (hence often orally based);
- workers tend to read job material with higher levels of proficiency than they do general material (through availability and use of cues that help activate appropriate schemata);
- workplace literacy is multi-modal in nature (rarely are reading, writing and speaking found in isolation);
- job performance is more closely related to metacognitive aspects of literacy than to the basic literacy abilities of achieving simple comprehension or communicating simple measures;
- two primary and distinctive types of reading exist, 'reading to do', the majority of work reading, and 'reading to learn', reading to learn information to be remembered for later use (Freebody et al., 1993, p. 55).

Parallels obviously exist for numeracy. In looking at the needs for numeracy education it is important to be considering issues in the learning of mathematics as a discipline, the numeracy needs for learning other discipline areas, the numeracy needs for the workplace and also for personal lives.

Elsewhere, (Cumming, 1996) I have argued that it is important to have separate notions of numeracy education and mathematics education, mostly in order to enable new thinking on the directions education in either should take. However, it has always been clear that the most encompassing definitions of good mathematics education practice, such as in the 1995 statement of the National Council of Teachers in Mathematics (NCTM) of the United States of America, incorporate problem-solving, real life applications, cultural and critical awareness.

A shift in the vision of learning mathematics towards investigating, formulating, representing, reasoning, and applying a variety of strategies to the solution of problems — then reflecting on these uses of mathematics — and away from being shown or told, memorizing, and repeating. This represents a shift from mechanical to cognitive work and also assumes the acquisition of a healthy disposition toward mathematics. Furthermore, cognitive work for all students is culturally dependent because students bring to each lesson their past experiences and the diverse facets of their cultural identities. Thus, instruction that capitalizes, and builds, on what students bring to a problem situation can motivate them to struggle with, and make sense of, the problem and share their thinking with classmates (NCTM, 1995, p. 2).

Resnick (1988) explored the notion of mathematics as an ill-structured discipline, rather than the concise discipline as it is sometimes viewed. She noted

mathematics is useful. It helps us describe and manipulate real objects and real events in the real world...

We encounter an explosion of interpretations when we include as potential referents for mathematical statements the actual things in the world to which abstract mathematical entities can be reliably mapped — what we might term ‘situations that we can mathematize’ (Resnick, 1988, p. 33–34).

Resnick believes that teaching that emphasises explanations and justifications

would aim to develop both capability and disposition for finding relationships among mathematical entities and between mathematical statements and situations involving quantities, relationships, and patterns (Resnick, 1988, p. 33).

While such statements from a noted mathematics educator and researcher serve to set the direction for the future of mathematics education, they do reveal a thinking that works from the mathematical content or discipline to the situation. As will be shown in later discussion, a move towards mathematics that derives from the modelling of situations may be the new direction for numeracy education.

Several states in Australia have been considering undertaking considerable professional development work in shifting teachers', children's and the community's perceptions of the purpose of mathematics education to be closer to these definitions of numeracy. The report of the Numeracy Education Strategy Development Conference (AAMT, 1997) similarly defined numeracy by saying:

This project identifies the following elements as central to any description of numeracy.

Numeracy involves

... using

... some mathematics

... to achieve some purpose

... in a particular context (AAMT, 1997, p. 13).

These recent conceptualisations of the goals of mathematics education and numeracy from an informed perspective have clearly merged. They represent a development even in a short time from the publication of *Mathematical Knowledge and Understanding for Effective Participation in Australian Society* (AAMT, 1996) which has a much more traditional focus on content, traditional areas of mathematics (Number, Space and so on), and separate headings for 'Representation of mathematical ideas' and 'Applying mathematics and solving problems'. In a conceptualisation of mathematics education based on numeracy, these latter two headings become the major frame. The content areas become relevant knowledge to be developed and called upon, again in a variety of ways, to support numerate performance.

For the remainder of this paper, the term numeracy will be considered to be synonymous with the goals of good mathematics education and the directions in which our future mathematical education practices should be heading. It will not be treated as having a separate role to play from mathematics education in the education of Australian students and adults. In making this move to consideration of a single term, all of the previous demands for mathematics education — 'to learn' mathematics, 'to learn' in other disciplines, 'to do' work and for personal lives — must be included.

Moving to the use of the terms numeracy education and performance in this manner will alleviate some of the problems encountered with popular perceptions of numeracy. Conversations in the general community on this topic usually revert to a focus on number and the needs of those with very limited numeracy skills, paralleling public attention to spelling or the skills of decoding in literacy. Just as literacy has been reconceptualised as a continuum, rather than the dichotomy literate/illiterate, recognising that we all have different degrees of literacy in different contexts, so numeracy can similarly be seen as a continuum of performance.

Just as literacy in the singular has been reconceptualised as literacies, even multiliteracies, identifying that in different contexts we call upon different types of literacy performance, often even within a single context (Cumming, Wyatt-Smith, Ryan & Doig, 1998), so too numeracy can be reconceptualised as complex and multiple. As later discussion will show, adults and children can demonstrate that they possess much numeracy knowledge and performance that has not been acquired in schooling, but which has been developed in various ways. This recognition of the nature of numerate performance is very important for future directions for mathematics education, once again going beyond such recent statements as:

It is in the national interest that schools promote high levels of achievement within rigorous mathematics programs, and that as many individuals as possible achieve those levels. All Australian citizens need a broad understanding of mathematics if they are to participate in decision-making about their social and physical environment (AAMT, 1996, p. 5).

As will be shown later, it is reasonable to assume that most Australians do have a broad background in mathematics, as they do already participate in numerate decision-making about their social and physical environment. As we build links between modern technological society and the numeracy education curriculum of schools, it will be important to remember the mathematical knowledges that all learners bring and the range of cultures and contexts in which they have been developed. A key factor in developments for the future will be for school mathematics education to embrace society and to look from out in, rather than from within the narrow traditions that have developed over time in school out to the much more complex but invigorating world.

Different numeracy education needs for different learners

A major consideration in looking out is to consider the different numeracy needs of learners for their life outside of and after schooling, and to consider whether one approach or philosophy of numeracy education can serve all of these needs. Stepping away from mathematics, a parallel discussion can be found to be occurring in science education. Scientific literacy is a term which is used as freely today as numeracy, and not surprisingly used with many different conceptualisations. In some ways, the appendage of the term 'literacy' to any term such as scientific or technological, is used as synonymous with a type of fluency, similar to some uses of the term numeracy. The terms are used to imply sufficient scientific knowledge and skills to be able to use these appropriately and with appropriate understanding and critical skill. Wilson (1998) has provided the following framework for considering scientific literacy or literacies:

...literacy in the context of school science refers to the ability to use practices associated with the formulation, representation and communication of meanings about natural phenomena, objects, events and their inter-relationships.

This parallels closely the previous considerations of numeracy. Further, Wilson identifies that to participate in the discourse involves: learning to understand science; learning to act in a domain of science (current notions of scientific or mathematical knowledge); and learning through engagement in problem tasks. Wilson identifies that these are all essential but to be developed to different degrees for four types of users following science education: the knowledgeable citizen; the technical/trades person; the professional such as engineer or doctor; and the professional scientist.

Clearly, these same four types of users should be the outcomes of mathematics education. However, mathematics or numeracy education in the past has had trouble identifying similar outcomes. Courses in secondary schooling which focus on life skills mathematics have usually been taken by those who have been identified through schooling as 'not good at mathematics', and courses with 'traditional' mathematics content taken by other students regardless of their expected future use of such mathematics. Taking a numerate approach to mathematics education, paralleling a scientific understanding and process approach to science education, means focusing on the context of application and problem-solving strategies, rather than primarily on the domain or content knowledge of the discipline. This does not underplay the importance of either domains of the mathematical or scientific discipline knowledges, but reinforces that these become learned as necessary support for numerate or scientific behaviour. Definitions of numeracy or good mathematics education have included, for a long time phrases, such as 'ability to select and apply mathematics in appropriate contexts and ways'. Hence it would seem important to focus on the development of this aspect of mathematics — application, appropriateness, problem-solving — with supporting domain knowledge, rather than on the learning of domain content in ways that make it meaningless for students. This can be achieved.

Pacing

Cumming et al. (1998) identified that a major literacy demand of post-compulsory schooling related to focus on curriculum content and the pacing of teaching to ensure coverage of this content. Students who have difficulties at any phase of delivery, who have not acquired the modes of operating within a subject, or the prerequisite domain knowledge, or who do not have easy access to the vocabulary of the subject to participate in the discourse of learning, will fail. The following transcript from 55

seconds of a mathematics lesson to Year 12 students demonstrates this point. The lesson continued in this way for 70 minutes.

T: ...Now where am I up to. On to twenty-six six now. Probably where we have little bit more important things to go through here. OK.

This one here is a parabola (*reference to equation in textbook problem*). You have to recognise it as a parabola. The way to do that of course is to look at the x squared and the y squared. In this particular case only the y is squared and the x is not. With the parabola you also have to realise which way they are oriented. If it's y squared equals four a x, the parabola is oriented this way or this way (*making visual direction sign with hand over the OHT*). If it's x squared equals four a y it's going to be concave upwards or concave downwards and you have to recognise that and apply the appropriate formula (Cumming, 1998, p. 269).

The pace in this lesson was fast, the demands in terms of prior knowledge were high, the presentation to develop meaning was visual and oral, through teacher talk and reference to an overhead with a prepared diagram and some lines of equation solutions.

While pacing emerged as an issue in the Cumming et al. (1998) study across all subjects, it was most apparent in mathematics classes. An overall recommendation of the study was 'that authorities examine more flexible structures for curriculum delivery in schooling in the post-compulsory years'. It was expected that similar issues would arise in earlier years. A predominant concern in mathematics education has been the focus on addressing content, regardless of the degree to which students acquire such content, rather than addressing student learning. Mathematics education has been a process of selecting out, rather than bringing in, mathematics learners. Mathematics teachers can identify stages at which students start to fail in mathematics, such as:

- Grade 4, when the curriculum moves from simple computational and spatial frameworks to encompass tasks in word form,
- Grade 7 or 8, when students leave the integrated education setting of primary school and engage in subjects called mathematics, with higher and lower levels,
- Grade 10, when certification and pacing issues can impact on student learning,
- Grade 11, when again students choose between levels of mathematics and can be labelled as taking 'vegie' maths and failure is attributed to the introduction of more 'difficult' content or levels of abstraction.

The mathematics content is seen as void of context, with problems in learning attributed to the learner, not the mathematics. Yet as Bruner (for example, 1960) has long stated, '...the foundations of any subject may be taught to anybody at any age in some form', while notions of difficulty are often curriculum and teaching induced.

The effects of pacing and perceived needs to cover curriculum content may be more problematic in secondary schooling than primary schooling. Syllabus documents are directed by State Boards of Studies and must be met for certification. Teachers in secondary schools have subject-specific roles. They define themselves as teachers of 'history' or 'mathematics', not as teachers of learners. These current curriculum and pacing demands are bound to result in selecting out. We select out on the basis of handling of quadratic equations, trigonometry, regardless of the applicability of such content to the eventual numeracy needs of the learner. An approach that focuses on application and problem-solving, with the introduction of domain knowledge, not only as needed but also when the learner has an appropriate background, will enable much more 'selecting in' of learners.

Such an approach would enable schooling to meet the needs of the four types of numerate person previously considered: the knowledgeable citizen; the technical/trades person; the professional such as engineer or doctor; and the professional mathematician. At present mathematics curriculum focuses on building the repertoire of domains of knowledge to the most abstract forms and expects the development of problem solving abilities to develop within these contexts. This is the view reinforced in the statement of *Mathematical Knowledge and Understanding for Effective Participation in Australian Society* (AAMT, 1996). The development of problem-solving abilities and the ability to identify that a new mathematical knowledge is needed is a more appropriate framework of development. In this way, the repertoire of mathematical domain knowledge needed by an engineer¹ can be developed. In this way, also, students need not fail and be filtered out at an early age, but may over time develop sufficient mathematical interest and expertise to wish to undertake further studies.

Society's acceptance of poor mathematics education: 'I'm no good at mathematics'

The affective outcomes of current approaches to mathematics education, albeit being addressed in the early years of schooling through advocacy of different approaches, have been highly negative for many generations. It is not a public embarrassment to say 'I'm no good at mathematics', as it is for literacy (Cumming, 1996; Kerka, 1995). And yet,

¹ For example, a final year mechanical engineering student about to undertake an examination in robotics explained that the questions would involve some simple mathematics questions to do with going forwards and backwards, which relate to matrix and vector algebra, and draw on the trigonometry knowledge such as cos and sin functions developed in high school. For this student all of the domain knowledge developed as a result of schooling is important. However, for the majority of students this would not be so relevant and work in these areas may have impeded numerate development.

...mathematics is used in many everyday situations — cooking, shopping, crafts, financial transactions, traveling, using VCRs and microwave ovens, interpreting information in the media, taking medications, different people need different sets of math skills, and then numeracy needs change in response to changes in life circumstances, such as buying a car or house or learning a new hobby. (Kerka, 1995)

This elaboration of the different numeracy knowledges that adults demonstrate, perhaps in spite of schooling, shows that it should be a public embarrassment to mathematics curriculum and policy developers and educators that expression of failure in mathematics is such a common statement. Schools in the past have convinced many people that they cannot do mathematics, despite their success in adult lives in many numerate behaviours, and such adults will always identify mathematics with specific types of ‘sums’ or ‘word problems’ that they could not do in school. While efforts are being made in primary schooling to address such concerns, practices in secondary schooling have changed little. Many high school graduates, despite high enrolments in and participation in mathematics studies, are still essentially maths-phobic, an outcome that is attributable to the nature of curriculum content and pacing, not the aspirations of teachers.

Assessment issues, effects of context and future directions

One issue in mathematics assessment that will have impact in the future on mathematics education relates to current initiatives in the national numeracy benchmarks (see, as an example, the approved literacy benchmarks for Years 3 and 5, Curriculum Corporation, 1998) and state-wide assessment procedures. The draft numeracy benchmarks represent statements of minimum domains of knowledge and processes that students should achieve at various stages of schooling. When finalised and implemented, the benchmarks should provide positive guidance for all students. After the difficult work of defining the benchmarks, reporting mechanisms based on the benchmarks need to emphasise best assessment practice.

Lokan and Ainley (1998), in a report on the Third International Mathematics and Science Study (TIMSS), noted differences in student performance on the closed TIMSS items and more open performance assessment tasks which were more demanding to administer but more clearly related to current conceptions of good mathematics education.

The relationships (between success on performance assessment tasks with achievement on the written tests) are positive, as would be expected on measures of achievement, but generally lower than one would find between measures of

achievement assessed on related written tests. The correlations are low enough² to indicate that the performance assessment tasks are likely to be measuring somewhat different skills from those measured in the written tests. (Lokan & Ainley, 1998).

Past assessment practices in mathematics have clearly been on the traditional lines of the written tasks, while progress is being made towards more problem-solving, holistic and open tasks. Australian students overall performed well on the traditional tasks as well as on the performance tasks in the TIMSS, although these were not undertaken by many countries due to implementation constraints. Lokan and Ainley's finding confirms that how something is assessed influences what it is that is assessed, and hence the need for assessment practices to reflect new assessment paradigms. Overemphasis on outcomes of the more limited tasks from the TIMSS, that do allow full international comparison, could lead to a narrow focus for mathematical outcomes. It should be noted that countries that outperformed Australia on the written tasks, mainly the Asian countries of Singapore and Hong Kong, are seeking to enhance performance in problem-solving even at the risk of a drop in other areas of performance. Plenary speakers from these countries at the TIMSS Conference held by ACER in Melbourne in 1997, discussed the changes being made to meet these goals. Zhang, Leung and Wong (1998) found that while Chinese students do very well in the traditional and timed written tests, they show the same lack of understanding on nonsense problems, and in calculating reasonable answers, as in other countries. Zhang et al. concluded that the Chinese curriculum, with too much emphasis on rote learning and the practising of identified word problem types, was not doing sufficient work to develop good problem-solving skills.

Cumming and Maxwell (1999) provide a detailed consideration of ways in which the search for the development of authentic achievement in general has been redefined into pseudo-authentic assessment approaches. Mathematics education is one of the worst users of these, where both assessment tasks and teaching tasks have been embedded in pseudo-contexts which do not encourage the 'mathematising of context' (Boaler, 1993, cited in Zevenbergen, 1997). Cumming and Maxwell note that in many circumstances what eventuates is a poor level of camouflage which still involves meaningless activity. Worse still, research by Black (1991), (derived from the work by Schofield and others, 1988), has shown that translating scientific tasks into everyday contexts (both presented in written form) can reduce student success.

Contexts of assessment have been shown to be important not only because they influence what is measured, but also because context can affect performance negatively or positively. Research work has shown that performance on decontextualised tasks can be poor relative to performance on contextualised

² .25, .28, .35, .36, .43, .45

activities. For example, work by Lave, Murtagh and de la Rocha (1984) showed that adults who scored an average of 59 per cent on arithmetic tasks presented in a school-test manner, had accuracy rates of 98 per cent on similar tasks when shopping in the supermarket. Nunes, Schliemann and Carraher (1993), however, have shown that such differences in performance may not necessarily be due to the influence of school or 'real life' contexts, but due to differences in performance between oral and written mathematical knowledges and skills. What is perhaps more important is the finding by Jenkins and Kirsch (1994), in the context of an adult literacy survey, that adults who had used in their jobs or lives the types of quantitative literacy tested were more proficient on those tasks than those who had not. Familiarity with a task in context has long been known to have a positive effect on performance (Cumming & Maxwell, 1999).

Scribner (1975) has presented different issues that relate to the influences of cultural context on equity of assessment outcomes. In her work, she showed that assumptions that all cultural groups had the same logic and the same taxonomic organisation (the way of organising categories of information mentally) of knowledge, were not supportable. In work with the Kpelle and Vai people of Liberia she was finally able to identify the taxonomic procedures that they used, by altering tasks until she found one that induced the organisation of recall by taxonomic categories. Straight Westernised tasks were not successful. Similar issues are of concern in Australia where systems of numeracy assessment, which assume common logics and taxonomic organisations, may result in cultural bias rather than adequate representation of what the student does know. Scribner questioned the use of inappropriate schema that would inevitably demonstrate 'the inherent stupidity of millions of children' (p. 81) rather than demonstration of capability. This is of particular concern with indigenous Australians who in some contexts may be working on different organisational structures of information than urban white children. Yet frequently we still implement such questionable assessment programs.

Scribner (1976) certainly raised the overall question of comparability in international assessment regimes by noting that:

... achieving equal familiarity of problem content in two cultures or in two populate groups within a culture does not ensure comparable task difficulty; the dimension of familiarity may be an irrelevant dimension for one group, a facilitating dimension for a second, and a disruptive dimension for a third (Scribner, 1976, p. 100).

While few questions appear to be raised about the TIMSS outcomes, questions have certainly been raised about the cultural equivalence of items on the International Adult Literacy Survey (IALS), in which Australia participated through the ABS survey (ABS, 1997). For Australia, cultural distance from the initiators of the IALS, the USA and Canada, is not great. However, a study currently being undertaken by

France and other European and non-English speaking countries, examining the outcomes for the adult literacy survey, is demonstrating concern with item comparability with non-English speaking cultures.

Contexts of time may also be relevant to considerations of assessment issues. Many research claims about numeracy performance have been made on the basis of fairly narrow conceptualisations of mathematics and narrow paper-and-pencil testing, albeit at a time when advances in conceptualising the nature of mathematics were not greatly advanced. For example, McGaw, Long, Morgan and Rosier (1988/1989) in a study of 'numeracy' in Victorian schools, used paper and pencil testing to examine student performance on isolated items such as reading scales in grams, comparing digital and analogue clocks to calculate difference in the time depicted, and picking of a correct option to complete a symmetry diagram.

Data released recently (Marks & Ainley, 1997) provided a comparison of performance on the same or similar items by Year 9 students from cohorts twenty years apart. Overall, they noted that there had been little change in performance on these types of tasks, although there was little consideration of the relative relevance of such tasks in 1975 and 1995. In 1975 Macintosh computers had not been invented; a Commodore 64 was the highest technology available for homes and even professional computer users had restricted computer memory available for programs often submitted on punched cards; autotellers were not around; microwaves and VCRs did not exist. Word processing on computers was an extremely arduous task with no relation to the written word and hand calculators did the most basic tasks. Computer spreadsheets and graphical calculators were virtually unknown. The virtual world of today did not exist. Is it defensible to be comparing 'numeracy' performance over such a twenty years of Australian society using items that may even have been questioned in 1975? At what point will we start developing assessment instruments that will demonstrate what students can do? Many numerate behaviours that we expect Year 9 students to exhibit today were not even able to be considered in 1975.

Therefore, it is important as we move towards new numeracy policies for the new millennium that we ensure that the numeracy assessment policy moves also. As Land (1997) has noted, in the Proceedings from the 1996 CRESST Conference, the changes to good performance assessment which are identified by the mathematics education and the assessment community as essential, will not only have to be responsive to the technological societal context but may also be making use of technologies as the following examples show.

Future generations of tests will need to tap nontraditional constructs, base test designs on cognitive principles, and increase the diversity of problem types.

Bennett³ predicted that large-scale assessments would soon include computer-based presentations of problem types not possible with paper-and-pencil tests. Bennett shared multimedia prototype items using historical speeches and newscasts to illustrate the potential of presenting and asking students to respond to 'dynamic stimuli' (Land, 1997, p. 20).

Land also reports that a colleague was able to demonstrate:

... the use of neural network technology to permit real-time assessment of complex problem solving. In one of Stevens' prototypes, medical students were presented with realistically sketchy information about a patient's symptoms, a set of diagnostic tests that they could order, and a 'library' of reference materials. As the students worked through the options presented by the computer program, their choices were recorded and could be compared with patterns of hypotheses generated by expert diagnosticians investigating the same problems (Land, 1997, p. 20).

Such directions in assessment, for both large-scale assessments and school-based assessments, accord with directions numeracy education should take, towards the holistic context and emphasis on problem solving and emulation of expert strategies of solution. There are clear implications of course, for resources, cost, equity of access and practice, telecommunication technologies and security of data for national assessment programs. However, the desirability of the type of information that might be able to be obtained about student understanding and reasoning processes is far removed from the narrow confines of previous paper-and-pencil tests in mathematics.

New directions in conceptualising mathematics education: Towards the visual, spatial and oral, modeling and representation, problem-solving

It is being argued in this paper that the modern technological world is increasingly visual and oral, returning to the world of the past but with much greater access to information and communication presented in tabular and visual forms. As Owston (1997, p. 29–30) notes:

...students in our public schools and in a good many colleges and universities do not know a world without the computer... It is an integral part of their world: they play with, are entertained by, and learn with the computer. They tend to be more visual learners than previous generations because their world is rich in visual stimuli... So it is fitting that we design learning materials and

³ Randy Bennett, Educational Testing Services, in presentation to 1996 CRESST Conference.

opportunities that capitalize on what we know about how our students prefer to learn.

The visual world is already the world of the expert. Woolnough et al. (1997) found, in a cross-cultural comparison of scientists, that scientists believe that they

... communicate better in diagrams than in words. This latter characteristic is perhaps the most striking, and most significant of all. In all countries the scientists perceive themselves as being able to communicate best in diagrams rather than in words. This ability to be able to think pictorially, with good spatial awareness, is important for scientists. The ability to be able to communicate well in words is one which scientists need to develop if they are to be able to communicate with the 'non-scientific' world and each other (Woolnough et al., 1997, p. 117).

Students in all countries, who planned to be scientists, also responded more positively to working on extended practical projects as the focus of their work, preferring to plan their own experiments more than non-scientists, who preferred to be given clear instructions.

Good mathematicians are also able to relate to the mathematical world visually and spatially, and prefer to work on holistic problems. Hence, not only is it suggested that today's students participate in a visual world, but also spatial skills and modelling appear to be key needs of numeracy education. Nunes et al. (1993) found that differences in performance between 'street' arithmetic and 'school' arithmetic were due not to the context but to the modes of operation. They consider that 'street arithmetic is oral and school arithmetic is written' (p. 27) and show consistent results in school contexts where students performed better orally than in writing. I have often argued elsewhere (Cumming, 1993b) that adults deemed to have numeracy problems possess considerable mathematical knowledge and logical argument in oral contexts but lack the written code of formal mathematics.⁴ Nunes et al. (1993) found that when students are dealing with written mathematical tasks there is loss of meaning and hence poor performance. When tasks are undertaken orally, meaning is preserved. Nunes et al. suggest that oral practices demand semantic knowledge (p. 54), a finding consistent with the finding by Cumming et al. (1998) that when students and teachers focus on the development of knowledge and expertise around a task ('doing'), whether 'brains-on' or 'hands-on', then the development of meaning and transfer are more likely to occur. Scribner (1985b) found similar results and loss of meaning in work with drivers who regularly calculated bills for goods.

On a paper-and-pencil arithmetic test such as those administered in school, drivers, whose on-the-job accuracy rate was near perfect, made many errors on

⁴ It is through examining not just the needs but also the expertise of adult numeracy learners that I realised a focus on the building blocks of school mathematics curriculum was not only inappropriate for their instruction but also most likely what had been detrimental to their learning.

decimal multiplication problems similar in format to their pricing problem (Scribner, 1985b, p. 326).

Cumming and Morris (1991) and Cumming, (1993a) documented exemplary practice in adult numeracy instruction that focused on visual and spatial explanation. Current mathematics curriculum reinforces problems for children as it seeks to make as much work as possible into a written form, leaving out as much of the specifics of the situation in order to strive for generality. Goals for mathematics education include ability to communicate findings to others or a general audience. Generally, this communication skill is then tested in writing, again changing the nature of the expectation and the performance that could be demonstrated. In fact, Cooper (1994), raising a similar cultural context concern as Scribner (1975, 1976), has shown that the introduction of writing and communication into the area of mathematics assessment is seen by some to introduce equity issues and to be a shifting of the 'goal posts'. This was particularly felt by groups without English as a first language who felt that they had just mastered the expectations of the old curriculum, when the new requirements were introduced.

The attempt to create generality in mathematics education, and hence abstraction, is of course related to notions of transfer. If the essence of an activity is extracted and taught, it is assumed that that knowledge will then be applied in a range of contexts. For example, if we teach all children the addition tables, we assume they will use these tables in everyday contexts. The evidence on transfer and situation specificity of performance and knowledge development causes questioning of this basic assumption. There is considerable educational and psychological debate about the extent to which knowledge gained in generic or abstracted ways is used by people in other contexts. There is a clear need for an appropriate context to be established for any learning activity. Nunes et al. (1993) found that if tasks are appropriately designed

...preservation of meaning in setting up the problem-solving strategy illustrates how a mathematical schema of a situation can represent both situational aspects and mathematical relationships. A schema of this sort is naturally somewhat general but also somewhat specific to similar situations (Nunes et al., 1993, p. 61).

Nunes et al. address concerns about specificity, generalisation and transfer, pointing again to the role of mathematics education in developing modeling ability and problem-solving.

Modeling is not concerned with the quantification of one object but with the mathematization of situations. The relationships between quantities are the essential aspects of a model (Nunes et al., 1993, p. 137).

The role of specific domain knowledge is still important, with students and workers needing to 'understand the mathematical invariants as well as the particulars of the situations'(p. 139). Street kids, adults shopping in supermarkets, pool builders

(Zevenbergen, 1997), workers in dairy factories (Scribner, 1968) all used idiosyncratic approaches to their solutions in different contexts, but all use common mathematical elements such as number, sequencing and addition. Therefore these invariant components do emerge as essential to general and specific situations. As Landman (1997, p. 18–19) notes, it is important that when we are developing these mathematical knowledges we need to get across to students that the ‘mathematics that is taught is applicable, powerful, and ultimately will be necessary’ for their future work and life. Teachers will only be able to do this, of course, if first it is true, and second they believe it themselves.

Along with these elements of knowledge, however, it is also important to consider the aspects of modeling, interpretation and reasoning, and particularly the ‘ideas that bring to the fore the importance of forms of representation in thinking’ (Nunes et al., 1993, p. 144–5). Nunes et al. echoed the concerns of many others regarding the difficulty of finding suitable problems for classroom use that enable students to:

...work on understanding relationships between variables, to develop models, and to be able to proceed from their own informal mathematics constructions to what could be accepted as informal mathematics (Streefland, 1990, cited in Nunes et al., 1993, p. 148–9).

Curriculum that presumes the appropriate mathematical models and seeks to teach students to learn and apply the same will clearly be ineffective.

As noted previously by Wilson (1998), the discourse of learning, or the oral discussion of the learning environment, is as important to the learning of mathematics as considerations of what is important to be learned. Hiebert and Wearne (1993) exemplify research that has shown that how we teach affects what is learned and how successful learners are. A traditional approach in Western mathematics education has been the completion of many examples of problems or algorithms in a single lesson. However, in an experimental study with students in a number of Second grade classrooms in the USA, Hiebert and Wearne found that:

...students (who) received fewer problems and spent more time with each problem, were asked more questions requesting them to describe and explain alternative strategies, talked more using longer responses... showed higher levels of performance or gained more by the end of the year on most types of items. The results suggest that relationships between teaching and learning are a function of the instructional environment, different relationships emerged in the alternative classrooms than those that have been reported for more traditional classrooms (Hiebert & Wearne, 1993, p. 393).

This reinforces the previous statement that students’ failing in schooling, and even eventual early school leaving, may well be in response to teaching, not an internal failing of the students themselves. Hiebert and Wearne’s results concur with recent findings of an investigation of the nature of learning activities in Japan, undertaken

as a result of their success in the recent TIMSS, and again with the findings on classroom events in Cumming et al. (1998). More talk, more conversations between students and teachers *about* the topic, more exploration of meaning and understanding through orality, less concern with curriculum content coverage, lead to better learning outcomes.

Developing specific skills for future numeracy: memory, pattern recognition, clustering/classification skills

Mathematics education in the past has incorporated a number of desirable goals which have become decontextualised from their purpose. Three such goals are estimation, memorisation and clustering or categorisation. Each of these areas is important for numerate behaviours. Unfortunately education practice has led to the teaching of these as 'topics' and the testing of these as 'topics' rather than in situation. For example, estimation or knowing the degree of precision appropriate for a context is very important. In mathematics text books, estimation becomes a series of items such as:

1240 ÷ 29 is about

A 20 B 30 C 40 D 50 E 60

(1989 YIT and AYS Maths/Numeracy Test, ACER)

Teachers and students could focus on the teaching of such items and remove notions of estimation from situations where they might occur, some of which are documented in following sections analysing the ways in which workers work.

Much mathematics education and public debate focuses on memorisation of tables or procedural applications of computations, formulae and so on. Certainly all of the analyses of workers demonstrate that number knowledge and computational skills are called upon by workers on a very regular basis, at all levels of expertise, and resorting to calculator support for every occurrence may not be very efficient. However, more important than the debate about whether instruction should focus on these memorisation skills, is a debate about whether through various areas of learning, including future mathematics education, students should be helped to develop strategies to develop memorisation skills. In a learning environment that expects that all learning occurs by osmosis, and that memory will develop, some students will not develop such skills. They will then be disadvantaged in the workplace, where memorised information and strings of number or letter sequences are frequently encountered. In a recent example, when buying new tyres, the tyre salesman went from my car to the computer screen and typed in a sequence of eight numbers and letters which corresponded to the details of the tyre needed. The

computer records information on some 2,000 tyre lines. By entering the code, for which there are some 180 odd distinctive codes which the salesman had memorised, the options were restricted to about 20 types from which a choice was then to be made. Through the modern workplace, the typing of similar strings into computers is common. Watch next time you make a purchase or request information. Most computer software requires some knowledge of memorised sequences on the part of the user to make progress. Tools such as pre-labelled cash registers in some restaurants are rare.

Good learners will of course quickly develop the necessary knowledge in context. They most likely will already have well-developed memorisation skills and strategies for chunking information. Poor learners will not, and will not demonstrate efficiencies in the workplace, particularly in sufficient time to be employed or maintain employment. Direct instruction could assist here, with the proviso that students understand the contexts for which they are learning such skills and practise them in appropriate contexts — perhaps the memorisation of library access codes, computer Personal Identification Numbers (PINs), personal IDs, subject codes and so on. And with the proviso that students are not assessed on whether they can remember sequences of numbers, and determined to be failures if they cannot.

As mentioned earlier, one component of mathematics education has been the repeated solution of problem types in order to develop pattern familiarity and build repertoire. This may still have a place to play, but its role is no longer so important. Firstly, those students who may become mathematicians or professionals using considerable mathematics, never did need to practise as much as the whole class was required. Research over time has shown that most teachers aim their teaching at mastery of content by students at about the 25 percentile of performance. These were the students who indicate how much practice of repeated tasks needed to occur. Students below the 25 percentile most likely went through the rituals but were still not gaining understanding of the processes they were undertaking, nor their applicability. Hence it will be more important for all students to undertake less repetition and more work with the visual and technological aids to learning in order to enhance understanding. The students who are most at ease with mathematics will still build repertoire and can be given additional work to extend their knowledge of the core components of traditional mathematics knowledge. It will be important to build the generic knowledge and flexibility indicated in later discussion as essential numeracies for future employment. It is interesting that Sowder, quoted in Fennema and Carpenter (1998, p. 12), has noted that:

Research since 1955 has shown that ability at non-standard computation that calls for idiosyncratic, on-the-spot methods for finding solutions is positively related to mathematical achievement... and confidence...

A focus on repeated practice of tasks, without reference to problem-solving contexts and appropriate selection and usage of mathematics, will therefore not disadvantage the better students, but will continue to disadvantage the less confident students.

The work of Scribner who explored the way people are numerate and develop strategies of operation in the work place, has a further important feature to be considered in new conceptualisations of mathematics education. Scribner's work (1975, 1985a) on classifications and taxonomic organisations, in conjunction with the way people work, showed that a desirable outcome for students is the capacity to classify on various characteristics.

Overwhelmingly, workers' dairy product associations were of a complex nature, involving several dimensions at once, such as quality and kind...

... drivers and warehouse operators also grouped by size... (with) ways of classifying linked to modality of encounter and purpose (Scribner, 1985a, pp. 311–3).

Again, current mathematics education can state that it emphasises pattern recognition, and classification of objects in various ways. However, again in assessment approaches a multiple choice item with different options available to complete a figure around a line of symmetry, becomes the teaching, learning and assessment focus (see, for example, Item 14 on the 1995 LSAY Maths/Numeracy Test, ACER). Classification tasks and completion of pattern tasks are similarly decontextualised from the situations in which it is hoped that learners will apply such information. This decontextuality of performance and directing of teaching emphasis by task practice and assessment are the reasons why such learning does not transfer beyond schooling. Students are not given the purpose for such learning, and are not driven to use such knowledge in the search for minimisation of work and efficiency, identified as needs in the workplace by Scribner. It is important for us to help all students to develop classification and pattern-recognition skills, and to compare skills across different cultural taxonomies from which they might operate. However, the context and purpose should be the instructional focus and framework for evaluation.

Equity

Schools must have a role to play in addressing equity issues related to the changing role of technology in education. Mathematics education has a specific role to play in this. As Luke (1997) has noted, new technologies tend to be used first by the affluent and in some cases can remain the domain of the affluent. While automatic teller machines are now common in our society, they are still used only by those with finances to transact. At the moment there is a contrast between rural and urban users

of technology. While the growth of use of computers and resources such as Internet in rural areas is rapid, such usage will always be dependent on simple factors such as constant electricity supply and more pervasive influences such as access to technical support, something taken for granted in urban communities. Schools will therefore be a conduit for ensuring that all students are technologically fluent, that all students understand the possible applications of software such as spreadsheets and graphical calculators, regardless of home and environmental background. An indicator of success in reading for some time has been access to books or magazines in the home. A similar indicator for future job success may depend on technological familiarity.

Cultural issues have been discussed in the section on Assessment and also in considerations of what technological society wants to be included in education for the future. It is equally important that in devising new curriculum and new numeracy education policies, that issues of cultural understanding should be addressed. In schooling, it is important to focus on ways of teaching subjects such as mathematics, in ways that consider students' literacy, language and cultural heritages. Lee and Fradd (1998) have noted that participation in the discourse of science means that students need to have a shared language and cultural understanding.

Talking science is closely related to literacy development and representational fluency, involving written, pictorial, graphic, and electronic formats (Lee & Fradd, 1998, p. 17).

Mathematics involves similar concerns. Cumming et al. (1998) identified that the use of appropriate terminology in classroom discourses appeared to assist student learning and understanding. It was easier for students to develop shared understandings about conceptual terms and relationships and to follow instruction when the discourse focused strongly on these. By contrast, the use of more general language, which can introduce confusions with the many meanings of different phrases, is more difficult to follow. Cumming et al. suggest that direct instruction in the terminology of a subject and exploration of ideas and meanings until common understandings have developed, are important for student learning. Jones (1998), providing numeracy instruction in an auto factory, similarly found that

...for this group problems are less mathematical numeracy and more numerical language caused by different ways of expressing numerical factors in different countries (Jones, 1998, p. 82).

Australia prides itself on being a multicultural country, with its own Indigenous people who have a range of language backgrounds, and people from many other backgrounds with a diversity of cultural heritages and languages. Past mathematics education has not addressed the issue of ensuring good educational outcomes for all learners. Policies and practice at all levels must consider issues and strategies to ensure that cultural bias does not develop.

Gender effects are a specific cultural issue that will also need to be counteracted. There is already a growing body of evidence that boys are making more effective use of technology than girls, in ways that will relate to vocational usefulness. While girls are outperforming boys on standard reading and writing tasks, and hence concern is being expressed about underperforming boys, concern might be more meaningfully addressed as to whether boys are attaining more technologically sophisticated and vocationally relevant skills and the girls, conforming to traditional school expectations, missing out. For example, Cumming et al. (1998) found in a survey of literacy practices with students from Years 10 to 12 that boys spent significantly more time than girls reading 'manuals and instructions', reading about 'computer sources' and 'computer games', writing email messages and playing computer games (p. 92). Girls spent more time than boys in more traditional activities such as reading books and letters, writing letters and diaries. In an Internet competition sponsored by the *Courier-Mail* newspaper for school students in 1997, the final was won by a Year 9 boy, with only one of the final sixteen contestants a girl.

Male and female enrolment patterns in mathematics subjects continue to be a concern. The *National Report on Schooling in Australia* (1998, p. 88) notes a number of gender patterns with fewer girls than boys enrolling in 'higher' mathematics. While this effect can be attributed to a number of factors such as the broad range of options available, the perceived masculine nature of mathematics, and the tendency for girls to be less risk-taking in enrolling in subjects unless they are sure that they will be successful, the figures are cause for concern. It will be important for future mathematics education to become more congruent with our technological society, and it will be important that this be done in a way that ensures equity for all groups, and not the further advancement of a particular subgroup in our culture. As Fennema and Carpenter (1998) note, reform that does not address equity, and gender equity issues specifically, is inadequate. Fennema and Carpenter report on a very interesting study, and then allow researchers from different philosophical and theoretical perspectives to comment on the study. In their study with children in Grades 1 to 3, they used mathematics teaching procedures based on current mathematical guidelines for best practice, and interviews to discuss students' learning and understanding. They found that by Grade 3, boys were superior on extension problems that had not been previously encountered, and that while girls used more concrete solution strategies like modeling and counting, the boys used more abstract solution strategies and conceptual understanding. While various interpretations of these outcomes were made, the outcome with which I am most sympathetic is one by Hyde and Jaffee who hypothesised that girls were probably following the instructional guidelines and expectations of teachers, that is exhibiting compliant behaviour, while boys were developing their own procedures regardless of teacher input. This raises a general concern that many procedures in mathematics

education and assessment in the past have rewarded compliant behaviour, while it is the student who dares or risks who may make greater learning gains.

Fennema and Carpenter's concern, however, was that they implemented a mathematics education program that had

...an underlying assumption that our program based on understanding will enable all student to learn in an equitable fashion. This assumption may not be valid (Fennema & Carpenter, 1998, p. 20).

Fennema and Carpenter's outcomes provide a warning that in the development of a new numeracy policy that will have, hopefully, the intention of being a policy based on equity principles, careful monitoring will need to be undertaken to ensure that such outcomes do occur. If the basis of the new policy is to encourage a perception of mathematics as modelling, with an element of risk taking, then it will be important to provide scenarios where all students appreciate that this is the goal. As new technologies are used, if girls and other cultural groups have had less involvement with such technologies than white Anglo-Saxon Australian boys, it will be necessary to monitor both progress and the affection of all learners for the technology.

One of the increasing concerns of national agendas for benchmarks and standards to be achieved for all students, is the implications of the agendas for groups with special learning needs. Mathematics education has never clearly identified the best ways to address the needs of these students, with conflicting emphases on conceptual or basic number training. Educational policy now allows all students to remain in schooling until at least the age of 18, regardless of the progress they make. However this policy does not help to bridge the transition of students with disabilities into the workplace, a goal which many students and their parents do hold. Phelps and Hanley-Maxwell (1997) have examined different practices for bridging this transition for youth with disabilities and found two promising practices: school supervised work experiences; and functionally oriented curricula in which occupationally specific skills, employability skills and academic skills are systematically connected for students. Both of these approaches are consistent with the proposed new directions for mathematics education, and Phelps et al. note that such programs are most likely to be beneficial for all students.

Impact of technologies on teaching and learning, virtual reality

In the past, 'high level' mathematics may have been the domain of a few because it needed considerable chunking of mathematical knowledge and memory space to manipulate this information. Technology reduces this demand. Good mathematics

education no longer needs to be a de facto test of memory. Congruence between the technologies of the world and the technologies of schooling and approaches to learning is essential for the mathematics education policy of the future.

Luke (1997) has noted the profound effect of technologies such as computers and the Internet on home entertainment and workplace practice. It is important that education keeps pace with such change. One effect has been the development of 'hypertextualities', the notions of multiple screens and formats for visualising and presenting information. She notes that the introduction of such technology, 'suggests a radically different orientation to text, information, and the organisation of time' (Luke, 1997, p. 18). However, at the same time software tools such as Internet search engines are teaching different cognitive, logic and mapping options. These logic operations are very mathematical and provide a good basis for discussion in this area. Luke's arguments emphasise the point that unless mathematics education in schools embraces technology similarly, it runs the risk of being seriously antiquated in the context of the modern world. Forthcoming generations of students will be even more difficult to persuade to spend laborious amounts of time dealing with textbook problems over and over again, when they know how to solve such situations out of school very simply and in a time effective manner. Luke provides an example of an holistic cross-curriculum activity 'Planning a hypothetical class trip to Thailand' which can be undertaken using the World Wide Web and which would form the basis for good mathematics instruction. The task would allow consideration of money (and hence numbers and decimals), percentages (discounts), timetables and time, spatial appreciation of maps and so on. This can happen now, and many students may already be exploring areas in a similar way. It can also happen now with very young children, blurring the boundaries of curriculum and grade levels. For example, the work of the Cognition and Technology Group at Vanderbilt (CTGV, 1992) which preceded the popular use of the Internet, undertakes a very similar problem-oriented approach. Such approaches using information technology can also allow interaction of student groups around the world on common problems and tasks, an activity that may be seen as enhancing both globalisation and cultural understanding.

Technology allows new ways of examining information and new ways of undertaking holistic numeracy tasks to develop problem-solving ability and domain knowledge. Technology also has a very direct impact on the way mathematics can now be taught. Use of computer technology allows a refocus of the emphasis in mathematics education, away from repetitious practice of the mundane, to application with understanding and eventually a stronger development of domain knowledge. Engebretsen (1997) notes that while the use of technology does not automatically transform teaching from the boring to the exciting, it does have the potential to do so. Calculators can now allow 'extensive symbolic manipulation

capabilities and built-in interactive geometry software', allowing students to utilise 'the power of visualization' (p. 3). Such a role is clearly congruent with a thesis that mathematics education should help students to 'envisage the world'. The use of graphical calculators, spreadsheets and other software, means that students can engage immediately with the ideas and concepts of a range of mathematical areas, can see the visual representations and manipulate these to obtain solutions, and can build immediately a bridge between the nature of the concept and its modeling, purpose and representational text. How many of the students who have laboriously studied and resolved quadratic or simultaneous equations have ever understood their modeling applications? How many students assume that data from real life situations comes in neat quadratic or linear equation forms? New computer technologies actually allow students to model real data and explore best-fitting solutions visually and symbolically. It cannot be rationally argued that repetitious solution of textbook equations will provide more meaningful mathematical development for all students.

Landman (1997) emphasised the role of modeling in mathematics education, noting that experienced mathematical modellers' expertise and skills include (p. 11):

- the art of asking probing questions to reveal and clarify the nature of the problem
- identifying analogies and patterns between well-known models and new problems
- breaking the problem up into workable pieces
- constructing the mathematical framework
- using the language and tool-box of mathematics to find methods and solutions
- interpreting and translating the results back into the industry context so that recommendations can be made and effective technology transfer carried out.

This reinforces the framework of the problem-solving and enquiry emphasis for mathematics education for the future. It clearly indicates that using the most effective tools to develop these skills is very important. And hence technologies such as spreadsheets and calculators that can circumvent laborious calculation, (which often in the past has become the end goal rather than the means to an end), will assist such a development. Geiger (1997) has noted that graphical calculators, as an example, allow teachers

...the opportunity to present concepts in multi-representational form and the chance to encourage students to make linkages between graphical, numeric and symbolic forms of mathematical ideas (which) must be embraced (Geiger, 1997, p. 142).

Cumming et al. (1998) noted the multimodal nature of school classrooms, a multimodality echoed in the workplace as workers move from one form of communication, text, visual representation to another, often with rapid interchangeability. Cumming et al. noted that the successful student has learned to cope with these. The student who cannot cope is often left behind. Teaching students in ways that help deliberately to address multimodality, and in which connections between the visual, the abstract and the discourse are considered, will assist all levels of learning and work performance.

Technology is not without problems both in terms of gains made in learning and the practicality of its use. Owston (1997, p. 29) has noted similar technological problems to Land (1997) regarding the possible failures of technology, problems encountered by most people as they first made use of the World Wide Web, which, in addition to problems with traffic and cost

...can create new kinds of barriers for students. These include computer hardware that malfunctions, difficulty in setting up software to access an educational institution or Internet service provider, and encountering constant busy signal when dialing up from home.

It has been noted previously that the use of technology in and of itself will not necessarily mean improved practice or learning outcomes. Owston (1997) cites Davies (1995) who has stated that for technology to be an effective learning tool in higher education it must address three questions:

- Does it make learning more accessible?
- Does it promote improved learning?
- Does it accomplish the above while containing, if not reducing, the per unit costs of education?

Similar questions must be addressed at all levels of schooling. Maxwell and Cumming (in preparation) discuss evidence that the use of technology to promote flexible delivery or delivery of instruction by distance, or even in anticipation of a distancing between the student and teacher and increased autonomy of the learner, can be problematic. Evidence to date suggests that students learning in these environments need more, if different in kind, support than students interacting face-to-face with the teacher. The loss of human contact and timely advice can lead to failure.

The changes in the way technology is used in working with students and enhancing student learning will have considerable impact on the learning teachers will need to undertake themselves. In many classrooms, the roles of teacher and students may well be reversed.

Finally, just as technology in schooling will change the roles and responsibilities of teachers and students, technology in the workplace has led to changing responsibilities for workers. Not only do workers need additional skills, access to technology can lead to more self-control and determination. Buckingham (1997, p. 27) noted that workers in a company with an integrated computerised management system carried and used calculators. They

...put their parts-made and time-worked into terminals as they finished each job. The set rates and rates achieved calculated by the system, could be taken off the terminals when they next logged on. Workers appeared to carry calculators and use them, in this case to check their rates, against the computer...

Where computers were not yet installed, rates were worked out by those with a supervisory role.

Cohen and Naylor (1998) have noted a similar outcome when manufacturing structures become flat and automation leads to reduction in machine operator assistants. Authoritarian supervisors are also removed and there is a decrease in the amount of control and coordination undertaken by others on behalf of workers. The remaining workers need to be much more flexible in completing the requirements of their job and do have more autonomy.

Numeracy and employability and the world of work

Considerable research has explored the idiosyncratic ways in which workers enact numeracy within their specific contexts, and examined the relationships between these ways of working and the teaching of school mathematics. Some of the most well-known work was undertaken by Scribner who systematically explored the nature of literacy and numeracy at work, compared this with cultural and other contexts, and through emulation of approaches in semi-experimental frames, sought to theorise the nature of work cognition. Overall she emphasised a functional approach to the study of the social practices of the workplace, starting from and reconfirming her thesis that:

...people's cognitive strategies are purposive, flexible, and minimize the effort needed to accomplish a task... [People] integrated the work of the head and of the hands in the cognitive processes of purposive activities. Her work reveals a deep understanding and respect for the complex mental processes for action that workers carry out, which may or may not be facilitated by the formalisms taught in school or by those introduced by management to order the workers' process (Rogoff, Foreword, p. xv).

In her various writings Scribner has demonstrated the situatedness of learning and the extent to which it is culturally sited. In particular, her focus on minimisation of

effort is instructive in considering how to change mathematics education. In mathematics education in the past, it is difficult to identify areas where students have been encouraged to expedite their work by choosing their own strategies. This is what Scribner showed that workers do all of the time.

Much of Scribner's work in factories explored issues of cultural empowerment and effects of unionism. Buckingham (1997) examined the numeracy skills of 170 workers in light metal engineering production plants, including process workers and managers, to explore the roles and needs of workers in restructuring. One of the outcomes of Buckingham's work was to question whose numeracy needs were being met and the real role of education in workplace emancipation. She 'set out to find out about the sorts of numeracy education that were helping them to participate in decision-making in the new environment' (p. 24) and to 'learn what they felt about their scope and capacity for decision-making, with and without mathematics' (p. 24). Self-determination varied according to the context of employment, not the skills of the worker. However, the skills of the worker were found to be more varied according to the degree of individual control.

Zevenbergen (1997) explored the ways a pool designer estimated size, volume of soil, quantities of steel, and maximised strength of steel reinforcement for the concrete shell according to strict guidelines while minimising the cost and wastage. As noted earlier, it was apparent that the workers had some strong domain knowledges that were transferred into the context, such as knowledge of number and units, trigonometrical principles of angles and verticality, but also clear that the management of the information was done in site specific ways. The generic skills developed in the context of 'pool building' had to be adapted for each site. Estimation skills were necessarily strong, and even more interesting. Zevenbergen records that, 'when estimating the amount of soil to be removed from the site, the excavators could "see" the volume of dirt' (Zevenbergen, 1997, p. 90). Presumably the excavators could translate this visually into truckloads to minimise cost of transport. While this is a very specific meaning of the word 'envisaging the world', it demonstrates that as in many areas of expertise, such as with Scribner's dairy workers, a mathematical or numeracy task is transferred by experts into a spatial or visual model.

[W]hen a large array was not a solid rectangle, but had gaps, the men mentally squared off the array by visualizing phantom stacks and counting them. They then multiplied by rectangular dimensions and completed the solution by subtracting the phantoms from the product (Scribner, 1985b, p. 326).

This is the way experts 'chunk' (in the psychological sense of saving memory capacity and enabling recall and processing) such information into non-standard and often visual units. Scribner's dairy workers never used the notion of 'a case' as a unit for working out volume. Neither the pool workers nor the dairy workers used a

mathematics equation for calculating volume, recalling previous distinctions between the numeracy 'to do' and the mathematics of school. Zevenbergen's and Scribner's work raise again the issue of situatedness for learning, revisiting the question of balance between generalisation, abstraction and specificity raised by Nunes et al. (1993).

Harris (1991, p. 138) noted that:

in work... mathematical activity arises from within practical tasks, often from the spoken instruction of a supervisor and always for an obvious purpose which has nothing to do with numbers working out well.

An employee who is not numerate in the workplace is either at risk of losing employment or costing the employer or business, income. Various studies have shown the links between numeracy performance, even on standardised and constrained measures, and employability. Parsons and Bynner (1997) found, on the basis of a longitudinal National Child Development Study in the UK, that even with good literacy skills, poor numeracy skills reduced employment and training opportunities and promotion prospects. In another study tracing a group since 1958, they found (Bynner & Parsons 1997) that people without numeracy skills left school early, frequently without qualifications, and had more difficulties getting and maintaining full-time employment. Such jobs as they found were usually lower status, lower pay and with poorer prospects than for those with better skills. A particular gender effect is that women with poor numeracy skills are often excluded from clerical and sales work, a major area of occupation for women.

These overall findings of the relationship between numeracy skills as measured in these studies, educational experience and work are supported by several other studies including work by Ekinsmyth and Bynner (1994) following through a later cohort. The various national and international adult literacy surveys which document quantitative literacy, a particular aspect of numeracy related to the manipulation of numbers within text and reading of graphs and tables, similarly report strong links between performance, extent of schooling and employability. Rivera-Batiz (1992) noted that low quantitative literacy appeared to be associated with a lower probability of employment among young African Americans. Jenkins and Kirsch (1994) noted that the level of identified skill was linked to probability of employment and level of income. The ABS survey in Australia (ABS, 1997) similarly found that early school leavers were likely to have lower performance across all age groups, and difficulties in gaining or maintaining employment. This was still true for the current generation of 15 to 19 year olds, showing that there is still an important proportion of students whom schools are failing. Marks and Ainley (1997) also report correlations between numerate performance and employability over a period of time. Lamb (1997) reports for Australian youth, that fewer than 50 per cent of male students with very low numeracy test performance complete school, and fewer than

60 per cent of girls with similar performance. The relationship is clearer than for poor reading skills. Interestingly, the results show that

...weaker numeracy skills for women are a greater impediment to the chances of entering university than poor literacy skills. The opposite is true of males (Lamb, 1997, p. 11–12).

However, participation in TAFE was not dependent on very high literacy or numeracy achievement. Lamb notes that apprenticeships are more often undertaken by early school leavers and are likely to have higher proportions of youth with average to below average literacy and numeracy skills. Links between low numeracy skills, as measured by the test, and employment were strong.

Young people with weak literacy and numeracy skills are fundamentally disadvantaged when it comes to getting a job... (experiencing) longer spells of unemployment (Lamb, 1997, p. 19).

Poor number skills were the strongest predictor of unemployment for teenage girls, while boys with very poor number and work skills had more than twice the probability of being out of work than those with average or above skills. As in the English studies, poor skills were associated with low status and low paid jobs for those who were employed.

These findings of the relationships between poor mathematics performance, length of schooling and employability are serious. However, rather than attributing responsibility to the individual for failure, such results may be interpreted in two other ways. Firstly, the ways in which 'numeracy' is assessed in all of these studies may be questionable. In nearly all cases, the 'numeracy' assessments are based on decontextualised or school-emulating tasks and are demonstrating once again that these individuals are not successful in the context of school mathematics. The assessments do not attempt to identify the numeracy skills that these individuals have achieved. It has been noted earlier that an outcome of present mathematics education is that it selects 'out' students rather than becoming more inclusive, and many students will have been identified at an early age as failures. If this failure occurs across another area as well, such as literacy, they are very likely to be early school leavers. While it may be far-fetched, it is possible that at least previous mathematics education experience has been responsible for students' early school leaving, rather than enhancing their educational opportunities. A change in focus of the numeracy education curriculum and in pacing issues may address this problem.

Secondly, employability is dependent on how many jobs are available. It is often argued, on the basis of the correlation between poor literacy and numeracy skills and unemployment, that improving literacy and numeracy will improve unemployment. Correlation, of course, does not imply causation. Levin (1998) notes that the educational sector has used such arguments in order to drive reform, rather than

through any ‘compelling evidence on the links between specific educational standards and economic performance’ (p. 4). In fact, Levin points out that some of the evidence of the relationship between educational standards and productivity, is to the contrary,

...the enormous success of foreign manufacturers in transplanting their operations to the U.S... when such Japanese firms as Honda, Toyota, and Nissan established operations in the U.S. using local workers from areas hardly known for the quality of their education, they found that they could produce automobiles as efficiently and as high in quality as in Japan (Levin, 1998, p. 7).

Robinson (1998, p. 144) has similarly noted that

...once the vast majority of the adult population are functionally literate, which has been the case in all the advanced industrial countries for many years, any link between the attainment of literacy and numeracy and economic performance is very hard to demonstrate.

Nor does it follow that the aggregate level of employment or unemployment would be changed if the adult population overall had higher levels of literacy and numeracy.

These arguments, however, are related to economic agendas in specific areas. Arguments can also be made about the social and economic gains of enhanced general levels of literacy and numeracy for society in a number of ways — decreased health costs, decreased social welfare dependence and so on.

Statistics have also shown the rapid increase in self-employed persons in Australia. Educational institutions such as schools may have to face the reality that not every student who graduates will end up with a high status job. One of the role of schools, particularly mathematics education, can be therefore to ensure that every student who graduates, who does not have severe physical or intellectual impairments that might prevent employment, has the skills to be self-employed, self-sustaining and empowered in modern society. For those who are slightly ahead in the queue, it will be necessary to ensure that numeracy education does prepare students to be workers in the new technologies, an area where employment can be expected to grow. To achieve this will need a new approach to numeracy education, one that sets out from the start to be inclusive of all learners and their needs, even at the cost of inclusiveness of curriculum content.

The numeracy of work: new demands for flexibility, problem-solving and trainability

It was noted earlier that when adults talk about mathematics they conceptualise this in terms of what they did at school and do not relate it to their activities in everyday life and work. Research has shown that employers and employees can have similar preconceptions if asked what mathematics or numeracy employees need to undertake their jobs. Robinson (1998) has shown in a Basic Skills Survey (1994-1996) of a large number of establishments employing middle and lower level employees, employers perceived the demand for numeracy skills to be less than for literacy skills and to be low for all workers. Little information was obtained about the perceived skills needs of higher managerial or professional workers. Buckingham (1997) found similar stereotyped views of mathematics in light metal industries in Australia with workers and managers stating that 'as things stand at present there is not much need for more numeracy' (p. 26), clearly viewing mathematics as a set of basic procedural skills. Hangovers from the 'correct way' philosophy, which is difficult to expunge from community conceptions of mathematics, were shown in statements made by workers

...that they did not use a calculator because they valued 'the right way' of doing arithmetic, as in mental or written calculations. Interviewees' conceptions of the 'rightness' of a particular way of working in school mathematics, appeared to be another limitation on thinking mathematically at work (Buckingham, 1997, p. 26).

Harris (1991), reporting on a study of the mathematics at work, is in fact critical of the methodology of the study that presented lists of mathematics activities which in themselves were strongly aligned with curriculum notions of mathematics, including statements such as 'read or write numbers', 'percentages' and so on. Mathematical focuses such as spatial relations, modelling or problem solving were not investigated. However, in follow-up analyses, she found considerable difference between reported usage and elaborations of behaviour even on such limited data.

For example, one jobholder when asked about the use of percentage calculations from the Basic Calculations questionnaire denied that she used them. When asked how she dealt with irate customers under the Communication Skills questionnaire, however, she (reported that she) calmed them down by 'knocking 15% off' (Harris, 1991, p. 136).

Harris noted that information about the use of mathematics at work, although not mentioned in the mathematics-oriented questionnaire, repeatedly arose in the communication questionnaire and 'illustrated differences between the origin, usage and techniques of mathematics at school and at work', noting that 'it was clear that much more mathematical thinking was going on than was being revealed' (p. 137).

Harris's work also shows that even nearly a decade ago, the focus in the workplace was on oral communication with reduced writing. Although a range of mathematical domain knowledge is called upon, problem-solving in specific contexts was very important. She found that fractions were called upon, which may dismay some mathematics educators, and that occupations that might be hypothesised as requiring a large and sophisticated domain knowledge, such as motor mechanics, were found to have 'very low use of arithmetic skills and a relatively high use of problem-solving skills' (p. 143).

Strasser, Barr, Evans and Wolf (1991) also explored the nature of mathematical problems that emerged as 'real life' problems from a wide range of occupations. They identified three main categories of problems (p. 162):

- budgeting in situations of uncertainty where various general decision rules must be selected and used as more or less appropriate
- allocation of time slots
- stock control and identification of trends in demand or sales from numerical data.

Jenkins and Kirsch (1994, p. 43) found similar outcomes noting that

...besides jobs requiring specific computational and measurement skills in the building trades, engineering and sales, a wide range of occupations, especially in offices, depend increasingly on the use of information technology, where some basic understanding of the logic of IT applications can increase efficiencies.

Phelps and Hanley-Maxwell (1997) undertook a comprehensive analysis of 50 entry-level occupations and found that foundational academic skills that were necessary included:

... basic computational skills, using reasoning to select an appropriate operation and applying it to practical problems... Thinking skills include problem-solving and implementing, visualizing... reasoning to find connections and relationships necessary for problem solving and continued self-learning (Phelps et al., 1997, p. 201).

Workplace competencies needed included 'identifying, organizing, planning and allocating resources (time, money, human and material/facilities)... computer technology... securing, applying, and maintaining a variety of technologies' (p. 201).

Jones (1998) examined the numeracy training needs of workers in one auto industry site, finding that workers needed

...specific skills to be integrated with all other initiatives of a Lean Manufacturing, Team Based work organisation model, where Quality of Product was identified alongside Quality of Process... (Jones, 1998, p. 75).

In the workplace training, emphasis was on use of site specific workplace materials including leave forms, job sheets, production and quality charts, and health and safety materials.

What are the implications of these findings for mathematics education that focuses on the working out by hand or calculator of simple and compound interest, following of set algorithms and procedures, completion of textbook tasks and preparation of histograms from given or collected data? Clearly an implication is that knowledge of spreadsheets and databases will facilitate most of these activities. Computer and technological tools can already address the mathematical functions of these tasks while the role of the individual is to explore the *decision parameters* and *consequences*. Envisaging the future even further, it is possible that the cards we currently use to obtain cash and credit will come with preprogrammed budget allocations, worked out in consultation with a financial adviser, in order to maintain appropriate work and lifestyle budgets. The technology of the future will reduce the calculation responsibilities of the individual even more, while enhancing the role of individual responsibility and decision making. The implication of this for future mathematics education is to assist students to develop the capacity to hold simultaneously, or in a serial manner, various pieces of information which need to be integrated and considered. Chong (1995) used a Delphi study to identify perceived numeracy demands of a technological society and found that respondents classified their requirements in three categories: social skills, blue-collar skills and white collar skills. Respondents commonly identified that workers needed the 'skills necessary for interacting with vast amounts of information available through electronic media'.

The flexibility to identify and deal with information efficiently is also important. Watson, Hall, Breen and Jeganathan (1990) found that sales, service, skilled trades and technical areas were the jobs that were expected to increase. Computational numeracy, knowledge of new technology and multiskilling were rated by employers as the most important skills needed by new employees. Similarly, Wong (1992) found that 'numeracy' was a 'non-negotiable prerequisite' along with research and problem-solving, and practical applications in living skills such as budgeting and technological usage. Both Watson et al. and Wong demonstrate that employers tend to nominate non-specific but generic skills for employees, often reminiscent of the Mayer competencies (Mayer, 1992). Other studies of the numeracy required for various areas of work, such as the study by Yasukawa (1997) of engineering numeracy, include indications that engineers need to be numerate to appreciate the world and critically numerate to appreciate society.

Landman (1997) has noted the importance of integration of mathematics knowledge within other contexts of knowledge and application.

Mathematics is almost never practised in industry as an independent discipline; it is often an important supporting discipline, the value of which is determined

by its power to solve problems in improving the primary activities of the company... Problem formulation, time and cost estimates, project planning and reporting all play a role, as does the formulation of mathematical models, mathematical solution techniques, and the interpretation of the mathematical results (Landman, 1997, p. 17).

The need for worker flexibility is raised again by Landman and also by Scribner (1985b) who noted that, 'skilled practical thinking is marked by flexibility' and 'only novices use[d] algorithmic procedures to solve problems' (Scribner, 1985b, p. 328). The generic nature of numeracy work requirements supports research which has shown the degree to which performance, as discussed previously, becomes highly site and context specific, but contrasts with current training agendas in Australia and worldwide, whereby certification and training are taking place within very specific work contexts. Training packages and qualifications frameworks are being developed for different industry groups, such as forestry and light manufacturing, with the expectation that workers trained in one industry site will obtain certification which has transportability across sites. However, it is already being acknowledged in the initial implementation of these packages, that site specificity, whereby the range of materials with which the employee might interact and the range of responsibilities which they are expected to undertake, impact upon the skills employees are developing. Overall, the consensus is that workers need a range of skills and attributes to enter the workforce and the flexibility to learn job and site specific tasks quickly. This enforces notions of a change in mathematics education from the specific content knowledge to an emphasis on problem-solving and information integration. As Nash (1992) found in a survey of employers' perceptions:

...the main implication of the survey results for education policy and teaching methods is that courses need to increase the amount of practical, project type content, including work experience in industry including basic problem-solving techniques.

As Scribner (1985b, p. 329) noted, what is needed is

...not so much a matter of becoming efficient in running off all-purpose algorithms as it is in building up a repertoire of solution modes fitted to properties of specific problems and particular circumstances.

Levin (1998) had noted that shifting of factories from highly educated areas to less educated areas had not decreased productivity. He concluded that this was because of

...how these firms are organized and managed... Organization and management place special emphasis on incentives for working productively in teams and for rewarding quality in production. Training is intensive and continuous, constantly updating the skills of workers, not only increasing the value of the worker to the firm, but increasing the attachment of the worker through job-

specific skills that may not be directly transferable to other firms (Levin, 1998, p. 8).

What is perhaps most interesting in the convergence of these findings regarding the skills that employees demonstrate when in jobs, and the types of skills employers state as desirable, is that narrow job training in terms of perceived job skills, and the mathematics education that might be developed to meet these, do not appear desirable goals. This may have implications for current vocational education and competency-based education and should be considered carefully in the formation of a new numeracy policy and education strategy. The educational rationale for schooling to increase standards to improve the economy lies not in supplying skilled workers, but in providing people with sufficient skills and adaptability who can be trained to learn what is specific to a site, quickly on the job. Companies will achieve economies if they do not need to undertake remedial work but if they can immediately update skills and make them site specific. Companies will not want to redress basic computational skills and procedures, graph reading ability, ability to access and use information from a range of sources, or, possibly, using spreadsheets and graph production.

Overall improvement to the economy through improved standards for all school leavers could ensue through the empowerment of the life choices of the individual, as noted previously — the ability to identify and undertake self-employment, for example in the growing service industries, to maintain undemanding, healthy lifestyles, and to participate in lifelong learning and education. It is interesting to reflect that in the middle of such a concerted call for flexibility, much traditional schooling, particularly mathematics education, seems to actively discourage diversity and flexibility. With our Platonic view of formal knowledge and systems of assessment that divorce action from meaning, the school sector suffers from 'funnel' vision, taking students who demonstrate diversity of experience and knowledge, making them pass through a narrowly conceived curriculum and assessment regime, but then expecting students to be able to diversify their skills and apply school knowledge widely once again when they are through.

Implications for teacher education and inservice of teachers, implementing change

The previous discussion has highlighted the nature of our technological society, the world of work with its visual and oral framework, and the ways in which numeracy education needs not only to prepare for such a society, but to also be inclusive of it. Teachers are integral to the process of preparing students to be active participants in all aspects of the technological society. In order to do this, teachers must themselves

embrace this society. Clearly, some of the transitions that this paper has indicated are necessary for the future, demonstrate that change, in some cases dramatic change, will be required for numeracy education for the future. Such change is, of course, dependent on the responsiveness of teacher education institutions to prepare graduates for the future, as well as on the responsiveness of current teachers to change. As a national report on issues in science and technology education states (Brennan (Chair), 1993), a major need may be to enhance 'the experiential base of teachers so that they are able to provide students with a greater appreciation of the application of the material being taught' (Brennan, 1993, p. 6).

In considering the future educational needs of youth with disabilities Phelps, Allen and Hanley-Maxwell (1997) also noted the impact of changing agenda for teacher education and professional development:

...all educators... working in secondary schools must develop a critical understanding of the new workplace skills required of students... Both future and practicing teachers need to be educated about continuing assessment of employment requirements (Phelps & Hanley-Maxwell, 1997, p. 221).

Cumming et al. (1998) realised that their suggestions for changed focus of curriculum and closer integration of in-school and out-of-school activities, would also place demands on teacher education and professional development. One specific recommendation, as in the previous work, was to suggest that all teachers should be able to participate in non-school work environments in order to have more effective knowledge of the demands, in this case numeracy demands, of such environments and better ways of linking with school learning.

It will be extremely important for the future of numeracy education to have state government and sectoral initiatives to provide inservice professional development for teachers. This development needs to address all of the aspects of the discussion so far. Equity and assessment issues are equally important as familiarity with technology. Teacher education programs need to address the training of mathematics teachers at primary and secondary level, and the training of all teachers to have responsibility for numeracy. Recommendations regarding the teaching of literacy, which may be paralleled in numeracy, have been made in Cumming et al. (1998). Changing either teachers' beliefs and practices or teacher education programs in such fundamental areas is not easy. For example, the Christie Report (1991) recommended needed changes to literacy education in pre-service teacher education that may still not be being met by most institutions.

The difficulty of implementing change is recognised by the focus of much research on how to implement change (see, for example, Fullan, 1991, 1993). Changed agendas and policy for numeracy or mathematics education will not automatically lead to

successful change in teaching policy. Knapp (1997) discussed issues regarding systemic reforms and the mathematics and science classroom, raising core concerns.

The content of (the teachers') professional development opportunities focused on mathematics for all students but failed to delve deeply into relevant issues of culture, language, and ethnicity (Knapp, 1997, p. 243).

Knapp asks the basic question:

What do policies and policymakers do to instruct actors throughout the system regarding the meaning of the policy and how to put it into practice? (Knapp, 1997, p. 253).

Knapp (1997) raises the concept of social capital as it encapsulates notions of 'social relations and structures that seem to affect the capabilities for individual and collective action' (p. 254), in order

...to consider how societal relations and structures might determine, for example, what parents and others know of mathematics... and how this social capital might influence the prospects for reforms within classrooms (Knapp, 1997, p. 254).

Hence Knapp improves on the notion of inclusivity of society to not only bring the outside in and encompass cultural diversity, but also to use that society in itself as a process for change.

A further area where change issues will need to be addressed is the relationship between school mathematics and higher education studies. In recent times, this relationship has appeared to suffer as universities are unclear about their changing expectations from school graduates to meet the needs of newly-structured courses of study. They still tend to reify traditional mathematical content knowledge as the essential learnings. This debate or discussion will need to be much more substantial from both sides.

Such issues and questions will have to be addressed very seriously in the quest for a new direction in mathematics education.

Conclusion

In the *Abstract* it is noted that the focus of this paper was provided through the title *Our changing technological society: Demands and links between numeracy performance and life outcomes for employment, education and training*, with life outcomes for personal contexts also considered. The readings that inform the paper have encompassed several dimensions. Firstly, the emphasis on technological society drew attention to readings which highlighted the nature of our society but also the need for these technologies to be an integral part of education. One author (Landman, 1997) in fact

refers to the mathematical sciences as a part of the new technologies, showing that the linkages between mathematics education and society should be very strong.

The term 'demands' of our technological society also led to two major conclusions. Firstly, the nature of the demands of working numerately and operating effectively in personal life showed once again that flexibility, working with technology such as visual display screens, memorisation and classificatory skills, and problem-solving strategies, were key outcomes of education. However, the readings also showed that society was demanding to be included as a key component of education, not marginalised and distanced from the role and functions of education. Education for a technological society is education that is inclusive of this society, a society that expects to be involved, a society that is prepared to state publicly and clearly that current curriculum

... is essentially irrelevant — too formalised, too sterilised, too disconnected from the lives people live (Brennan 1993, p. 29).

Secondly, examination of links with work and personal life has shown that mathematics education to date has tended to create gaps between the mathematics learned at school and the numeracy and mathematical demands of the workplace and personal contexts of being. These gaps have served to alienate many learners from mathematics, from seeing the utility of mathematics in various contexts, and in many cases from recognising that what they do is in fact mathematical. The assessment approaches still practised in much mathematics education exacerbate this alienation and decontextualisation. The new approaches to assessment (Gipps, 1994) must be more rigorously pursued. In addition, much of the separation of mathematics from work and life contexts has served to perpetuate cultural biases and to define intelligent performance in ways that are peculiar to education. Just as Scribner (1975, p. 81) notes on the cultural comparative research of Jensen, 'that the evidence offered by Jensen from this set of experiments to establish the inherent stupidity of millions of children must be considered totally inadequate by any scientific standards', so mathematics educators must question to what degree we have allowed children and others to demonstrate the mathematics they possess and can use, rather than to categorise students as poor mathematicians and failures in education and our Platonic society.

If we are willing to accept all of these challenges, then we are ready to envisage the future.

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