

## Two-Dimensional Digital Particle Tracking Velocimetry Algorithm Based on the Image of Particle Trace

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### ABSTRACT

Liu, C. and Tao, L., 2007. Two-dimensional digital particle tracking velocimetry algorithm based on the image of particle trace. *Journal of Coastal Research*, SI 50 (Proceedings of the 9th International Coastal Symposium), pg – pg. Gold Coast, Australia, ISBN

A digital method is developed to judge the instantaneous flow velocity by the image of the moving tracer particle formed during the exposure time. The grey-scale image of the particle trace is changed into the binary image and the pixels included in the particle trace image are marked. The length of the particle trace is obtained by determining the central principal axis of the particle trace image. The direction of the particle moving is determined by the fact that the flow velocities in a small local domain are almost the same. The instantaneous flow velocity field in the channel is measured by the method presented in this paper. The mean flow velocities along the vertical direction obtained by averaging the instantaneous velocities agree well with the logarithmic law.

**ADDITIONAL INDEX WORDS:** *Particle Image Velocimetry (PIV), Image process, Central principal axis*

### INTRODUCTION

Particle tracking velocimetry (PTV) and particle image velocimetry (PIV) are non-intrusive optical technique to measure the instantaneous velocity of the whole flow field. Due to its great advantage in measuring complex flow field, it has been widely applied to study the turbulence and multiphase flows (e.g., TANAKA et al., 2003, ANGELE and MUHAMMAD, 2006).

In the PTV system, the flow velocities are calculated by the displacement of tracer particles during the time interval. In general, there are two types of techniques to measure this displacement: (1) judging the particle images corresponding to the same particle at different exposures, (2) measuring the length of the particle trace formed at one exposure. Up to now, many digital PTV algorithms based on the first method are developed (for e.g., see COWEN, 1997, ISHIKAWA et al., 2000). However, few digital algorithms on the second method are reported. For many years, the length of the particle image and the direction of flow velocity are determined manually.

Comparing to the PTV system based on the first method, that PTV system based on the second method has considerable advantages such as low-cost, real-time, and easy to use. Some research attempts and progress have been made recently in the development of the digital PTV algorithms based on the particle image formed at one exposure. BALDASSARRE et al. (2001) measured the length of the particle trace by identifying the intersections between the grid and trace and prevented the directional ambiguity of flow velocity due to the use of two CCD cameras to acquire particle images simultaneously. The PTV system using the algorithms developed by BALDASSARRE et al. (2001) is a low-cost, real-time, relatively easy to use, safe and flexible. However, the measuring precision of Baldassarre's algorithm depends on the size of grid and the hardware system becomes a bit complex to acquire particle images simultaneously by two CCD cameras.

To overcome such a weakness inherited in the technique of BALDASSARRE et al. (2001), in this paper, we presented a new approach: the central principal axes of the particle trace image are introduced to measure the length of the particle trace. Such a approach enables us to prevent the directional ambiguity of flow velocity due to the fact that flow velocities in a small local domain are approximately the same. The instantaneous flow velocity field in a channel presented in this paper demonstrates the validity of the method. The mean flow velocity profile obtained by averaging the instantaneous velocities agrees very well with the logarithmic law.

### MEASURING METHODS

Flow tracer visualisation methods use tracer particles which are neutrally buoyant. By applying a PTV system, the velocity of each particle is assumed equal to the fluid velocity at the same location. So the flow velocities can be measured by the velocities of tracer particles. Since the size of seeding particles is always much smaller than the scale of flow structure, a suitable exposure time can be chosen to satisfy the following conditions: (1) the size of seeding particles is much smaller than the length of particle trace formed during the exposure time, (2) the particle trace is a straight line. Thus the particle trace image formed during the exposure time must be a rectangle in which the edges along one direction are much longer than that along another direction. Therefore, the vector of the particle trace can be obtained based on the particle trace image, and the velocity of the tracer particle can be calculated. Assuming that the vector of a particle trace is  $\vec{p}$  and the exposure time is  $\Delta t$ , the velocity of the particle  $\vec{u}$  can be calculated as

$$\vec{u} = \frac{\vec{p}}{\Delta t} \quad (1)$$

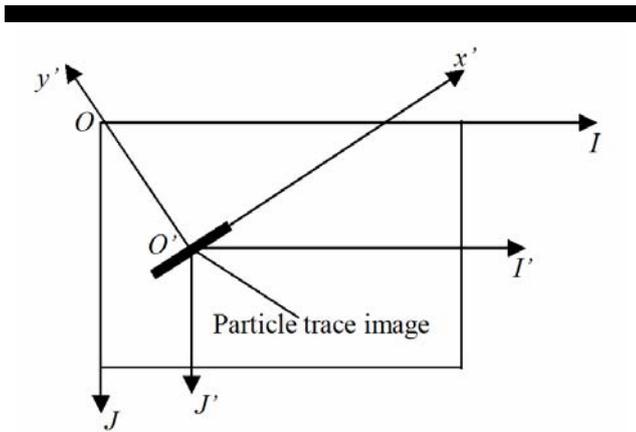


Figure 1. Scheme of the coordinate systems

In practice, the vector of the particle trace is obtained by the following three steps: (1) image acquisition and processing, (2) particle trace marking, (3) particle trace vector calculation.

### Image acquiring and processing

In order to obtain the particle trace image, a light sheet is used to illuminate the seeding particles. The particle trace images formed during the exposure time are recorded by CCD camera and transformed into digital image by a grabbing card. The ordinary grey-scale images are changed into binary images according to a certain threshold value. In order to obtain high quality binary images, the threshold value is set according to the mean grey value of the local area around the particle image trace.

### Particle trace marking

It is important to mark the pixels included in a particle trace image in order to obtain the length of it. In this paper, the pixels included in a particle trace image are marked according to the binary images. In the binary images, the grey-scale at one pixel has only two values, 0 and 1. The grey value at the pixels of the particle trace images is 1 and the other is 0. The pixels included in a particle trace image can be defined as the connecting pixels whose values are 1. For two-dimensional images, the pixels connecting with the pixel  $(i, j)$  are  $(i, j+1)$ ,  $(i, j-1)$ ,  $(i+1, j+1)$ ,  $(i+1, j)$ ,  $(i+1, j-1)$ ,  $(i-1, j+1)$ ,  $(i-1, j)$ ,  $(i-1, j-1)$ . According to the above definition, an algorithm is devised to mark the pixels included in a particle trace image. Assuming that the pixels included in a particle trace image are  $(I_1, J_1)$ ,  $(I_2, J_2) \dots (I_N, J_N)$ , the center of particle trace image  $(I_c, J_c)$  can be calculate by the following formula:

$$\begin{cases} I_c = \frac{\sum_{k=1}^N I_k}{N} \\ J_c = \frac{\sum_{k=1}^N J_k}{N} \end{cases} \quad (2)$$

### Particles trace vector calculation

In this paper, three coordinate systems is introduced to calculate the trace vector of a given particle which is labeled A in the image frame. One coordinate system is the fixed image frame coordination  $(I-O-J)$ , the others are  $(I'-O'-J')$  and  $(x'-O'-y')$

respectively. The origin of the coordinate systems  $(I'-O'-J')$  and  $(x'-O'-y')$  are the center of the particle image A, the direction of the  $I'$ -axis is the same as the  $I$ -axis and the direction of the  $x'$ -axis is the same as the trace vector of particle A. Figure 1 shows the scheme of the coordinate systems. The coordinates of a pixel in  $(I-O-J)$  can be transformed to that in  $(x'-O'-y')$  by the following formula

$$\begin{cases} x' = R[(I - I_c) \cos \alpha - (J - J_c) \sin \alpha] \\ y' = -R[(I - I_c) \cos \alpha + (J - J_c) \sin \alpha] \end{cases} \quad (3)$$

where  $\alpha$  is the angle rotating form  $I'$ -axis to  $x'$ -axis,  $R$  is the scale ratio. The particle trace vector  $\bar{p}$  can be expressed as

$$\bar{p} = (x'_{\max} - x'_{\min}) \bar{e}_{x'} \quad (4)$$

where  $x'_{\max}, x'_{\min}$  are the maximum and minimum  $x'$  of the pixels included in the particle trace image respectively,  $\bar{e}_{x'}$  is unit vector along the  $x'$ -axis. The coordinates of pixels included in a particle trace image in  $(I-O-J)$  can be obtained by the particle trace marking algorithm, and the coordinates in  $(x'-O'-y')$  can be also calculated using formula (3). Thus the value of  $x'_{\max}, x'_{\min}$  can be given. The unit vector  $\bar{e}_{x'}$  can be expressed as

$$\bar{e}_{x'} = \bar{e}_I \cos \alpha - \bar{e}_J \sin \alpha \quad (5)$$

where  $\bar{e}_I, \bar{e}_J$  are unit vectors along  $I$ -axis and  $J$ -axis respectively.

It can be seen from the above analysis that the particle trace vector can be calculated if  $\alpha$  is known. In order to solve  $\alpha$ , the concept of the central principal axis of the particle trace image is introduced in this paper. Due to the particle trace image is a rectangle in which the edges along one direction are much longer than that along another direction, it must have two central principal axes. One axis is parallel to the longer edges; the other is parallel to the shorter edges. The central principal axis parallel to the longer edges of the particle trace image coincides with the  $x'$ -axis. So  $\alpha$  must satisfy the following equation:

$$\operatorname{tg} 2\alpha = \frac{2I_{I'J'}}{I_{I'} - I_{J'}} \quad (6)$$

where  $I_{I'J'}, I_{I'}, I_{J'}$  are expressed as

$$I_{I'} = \sum_{k=1}^N (J_k - J_c)^2 \quad (7)$$

$$I_{J'} = \sum_{k=1}^N (I_k - I_c)^2 \quad (8)$$

$$I_{I'J'} = \sum_{k=1}^N (I_k - I_c)(J_k - J_c) \quad (9)$$

It can be demonstrated from equation (6) that  $\alpha$  must be one of the following values:  $\alpha_0$ ,  $\alpha_0 + \pi/2$ ,  $\alpha_0 + \pi$ ,  $\alpha_0 + 3\pi/2$ , where

$$\alpha_0 = \frac{1}{2} \operatorname{arctg} \left( \frac{2I_{I'J'}}{I_{I'} - I_{J'}} \right) \quad (10)$$

Considering that  $x'$ -axis coincides with the longer edges of the particle trace image, it can be deduced that  $\alpha$  must be one of the following values:  $\alpha'_0, \alpha'_0 + \pi$ , where

$$\alpha'_0 = \begin{cases} \alpha_0 & \text{if } I_t > I_j \\ \alpha_0 + \frac{\pi}{2} & \text{if } I_t < I_j \end{cases} \quad (11)$$

Due to  $\alpha$  appear two possible values, the direction of the particle trace vector is ambiguous. The following methods are employed to remove the directional ambiguity of particle trace vector. Generally, it is possible to find a seeding particle whose velocity direction can be determined (for example, the particle at the inlet of flow). As the direction of one particle trace vector is obtained, the direction of the particle trace vectors adjacent to it can be determined. Assuming that  $\vec{e}_k$  is the unit vector along the velocity of particle  $k$  and  $\vec{e}_{k+1}$  is the unit vector along the velocity of the particle  $k+1$  which is adjacent to particle  $k$ , it can be deduced from the fact that the flow velocities in a small local region are almost the same that  $\vec{e}_k$  and  $\vec{e}_{k+1}$  satisfy the following relationship:

$$\vec{e}_k \cdot \vec{e}_{k+1} > 0 \quad (12)$$

Considering  $\vec{e}_{k+1}$  is one of the following vectors:  $\vec{e}_I \cos \alpha'_0 - \vec{e}_J \sin \alpha'_0$ ,  $\vec{e}_I \cos(\alpha'_0 + \pi) - \vec{e}_J \sin(\alpha'_0 + \pi)$ , the expression of  $\vec{e}_{k+1}$  can be written as

$$\vec{e}_{k+1} = \begin{cases} \vec{e}_{k+1, \alpha'_0}; & \vec{e}_k \cdot \vec{e}_{k+1, \alpha'_0} > 0 \\ \vec{e}_{k+1, \alpha'_0 + \pi}; & \vec{e}_k \cdot \vec{e}_{k+1, \alpha'_0} < 0 \end{cases} \quad (13)$$

where  $\vec{e}_{k+1, \alpha'_0} = \vec{e}_I \cos \alpha'_0 - \vec{e}_J \sin \alpha'_0$ ,  $\vec{e}_{k+1, \alpha'_0 + \pi} = \vec{e}_I \cos(\alpha'_0 + \pi) - \vec{e}_J \sin(\alpha'_0 + \pi)$ . Once  $\vec{e}_{k+1}$  is known, the velocity of the particle  $k+1$  can be determined. And the velocity of another particle  $k+2$  which is adjacent to particle  $k+1$  can be obtained by the same approach. Finally, the velocity of all seeding particles in the flow field can be obtained.

### ACCURACY OF PTV METHOD

The accuracy of PTV depends on the accuracy with which exposure time and the particle displacement are measured. The exposure time is controlled by CCD camera and the measuring accuracy of it is very high. So the accuracy of PTV is mainly determined by the accuracy of the particle displacement. This is dependent on many factors such as particle size, camera resolution, exposure time and image distortion caused by CCD camera. In this paper, the effects of particle size, camera resolution and exposure time are discussed.

In the PTV algorithms proposed in this paper, the displacement of the particle is assumed to be equal to the length of the particle trace. So the error of the measuring is large if the particle size becomes large. In order to increase the measuring accuracy, very small particles are adopted.

The length of the particle trace in the image is measured by pixels. So the error of the particle trace length measurement is  $\pm 0.5$  pixels. In order to increase the measuring accuracy, it is necessary to increase the particle trace length in the image. There are three methods to increase the particle trace length in the image: (1) increasing exposure time, (2) optical magnification, (3) increasing camera resolution. The method of increasing exposure time will lead to more error vectors, which will be discussed in the following paragraph. The methods of optical magnification and increasing camera resolution are very useful to increase the measuring accuracy of PTV.

In the measuring process of PTV, the exposure time is very important. If the exposure time is not suitable, the measuring accuracy of PTV will be very low. In order to understand why error vectors occur, the motions of the seeding particles are classified into four cases: (1) the particle is moving in the light sheet during the whole exposure time, (2) the particle passes across the light sheet and is located outside at the exposure starting time, (3) the particle passes the light sheet and is located outside at the exposure ending time, (4) the particle passes the light sheet and is located outside at both the starting and ending times. It is obvious that the particle traces formed in case (1) can be used to calculate the particle velocities. The particle velocities calculated by the particle traces formed in other cases are invalid. The expectations of the number of particle traces formed in the cases (1), (2), (3), (4) can be expressed as

$$N_1 = \varepsilon S \int_{-\frac{H}{2}}^{\frac{H}{2}} dx \int_{\frac{\frac{H}{2}-x}{\Delta t}}^{\frac{\frac{H}{2}-x}{\Delta t}} f_w(w) dw \quad (14)$$

$$N_2 = \varepsilon S \int_{\frac{H}{2}}^{\frac{H}{2}} dx \int_{-\infty}^{\frac{\frac{H}{2}-x}{\Delta t}} f_w(w) dw + \varepsilon S \int_{\frac{H}{2}}^{\frac{H}{2}} dx \int_{\frac{\frac{H}{2}-x}{\Delta t}}^{\infty} f_w(w) dw \quad (15)$$

$$N_3 = \varepsilon S \int_{-\infty}^{\frac{H}{2}} dx \int_{\frac{-x-\frac{H}{2}}{\Delta t}}^{\frac{-x-\frac{H}{2}}{\Delta t}} f_w(w) dw + \varepsilon S \int_{\frac{H}{2}}^{\frac{H}{2}} dx \int_{\frac{-x-\frac{H}{2}}{\Delta t}}^{\frac{-x-\frac{H}{2}}{\Delta t}} f_w(w) dw \quad (16)$$

$$N_4 = \varepsilon S \int_{-\infty}^{\frac{H}{2}} dx \int_{\frac{-x-\frac{H}{2}}{\Delta t}}^{\frac{-x-\frac{H}{2}}{\Delta t}} f_w(w) dw + \varepsilon S \int_{\frac{H}{2}}^{\frac{H}{2}} dx \int_{\frac{-x-\frac{H}{2}}{\Delta t}}^{\frac{-x-\frac{H}{2}}{\Delta t}} f_w(w) dw \quad (17)$$

where  $N_1, N_2, N_3, N_4$  are the expectations of the number of particle traces formed in the cases (1), (2), (3), (4) respectively,  $\varepsilon$  is the number of particle in unit volume,  $S$  is the area of the region inquired,  $H$  is the width of the light sheet,  $w$  is the flow velocity normal to the light sheet and  $f_w(w)$  is the probability density of  $w$ . The probability that the velocity vectors obtained by PTV algorithm are correct can be expressed as  $N_1/N_t$ , where  $N_t = N_1 + N_2 + N_3 + N_4$ . Assuming that  $f_w(w)$  obeys Gaussian distribution

$$f_w(w) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{w^2}{2\sigma^2}},$$

the value of  $N_1/N_t$  can be written as

$$\frac{N_1}{N_t} = F(L_z), \quad (18)$$

where  $L_z = \sigma\Delta t/H$ . The relationship of  $N_1/N_t$  and  $L_z$  is plotted in Figure 2. It can be seen in Figure 2 that  $N_1/N_t$  decreases as  $L_z$  increases indicating that the exposure time  $\Delta t$  must be very short to ensure the measured velocity vectors are correct.

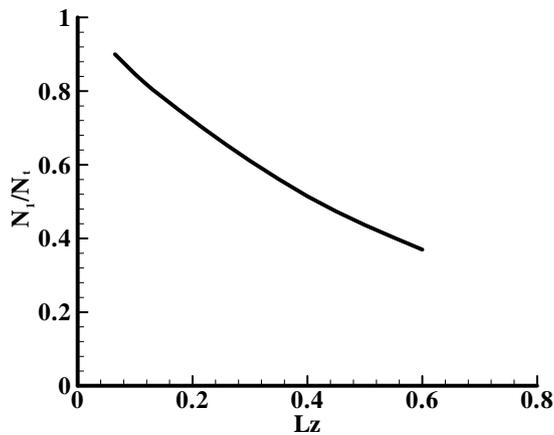


Figure 2. The relationship of  $N_1/N_t$  and  $L_z$

### RESULTS OF MEASURING

The flow fields in the vertical plane of the channel have been measured by the method presented in this paper. The experiment was carried out in the water channel of Hunan University. The experiment setup is schematized in Figure 3. The flow flux in the channel is controlled by two valves and measured by a flow meter. Pollen with a diameter about  $40\mu\text{m}$  was used as a tracer particle. The beam of light generated by a 500mw semiconducting laser was transformed to a vertical light sheet by the reflection mirror and cylindrical lens. A CCD camera was arranged at one side of the channel to record the particle trace image. The exposure time of the CCD camera was set to be 0.004s. The grey scale images, binary images of the particle trace and the instantaneous flow velocity vectors are shown in Figure 4, 5, 6 respectively. The mean velocities along the vertical line in the channel were obtained by averaging the instantaneous results and shown in Figure 7. The well-known logarithmic law in Eq. (19) is clearly observed from the mean velocity profile in such channel measured from the present experiment.

$$\frac{u}{u_*} = 2.5 \ln \frac{u_* y}{\nu} + 5.5 \quad (19)$$

where  $u_*$  is friction velocity and  $\nu$  is kinetic viscosity. According to the flow flux measured by flow meter,  $u_*$  is calculated using the following formula

$$[2.5 \ln \frac{hu_*}{\nu} + 3]u_* = \frac{Q}{Bh} \quad (20)$$

where  $Q$  is flow flux,  $B$  is the width of the channel,  $h$  is the depth of the water. The mean velocity profile obtained by logarithmic law is also plotted in Figure 7. It can be seen that the present experimental measurements agree well with that calculated by logarithmic law, a clear verification of the technique applied in the present experiment.

### CONCLUSION

We have presented a new 2D digital PTV algorithm based on the image of particle trace. Compared to the existing methods, the

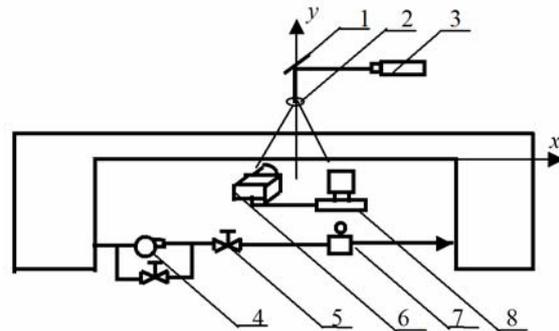


Figure 3. Schematic diagram of experiment system 1. reflection mirror 2. cylindrical lens 3. laser 4. pump 5. valve 6. CCD camera 7. flow meter 8. compute

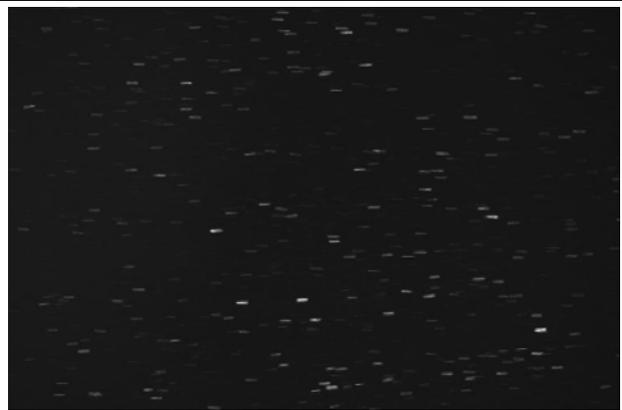


Figure 4. Grey scale images of particle trace

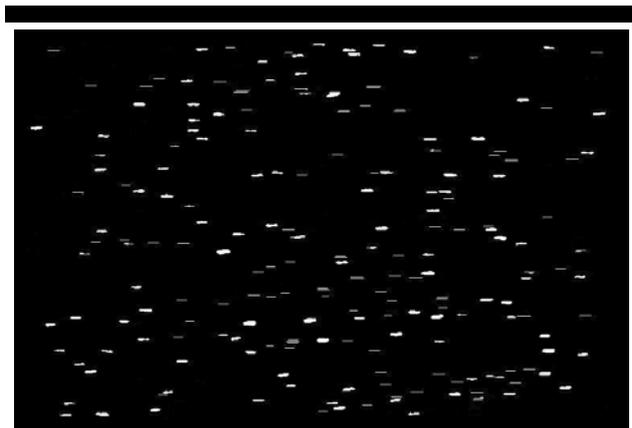


Figure 5. Binary images of particle trace

new technique is of the following advantages: (1) low cost of hardware; (b) excellent computational efficiency and accuracy; and (c) suitable to measure the flow field with high velocity gradient. Comparison of the velocity profile obtained from the present experiment and the logarithm law demonstrates the validity of the new technique.

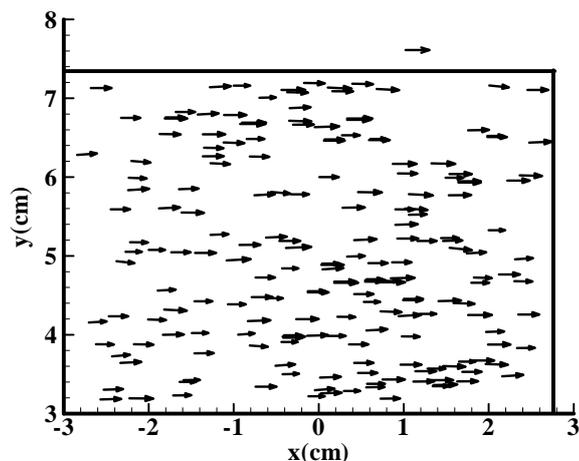


Figure 6. Vectors of instantaneous flow velocity

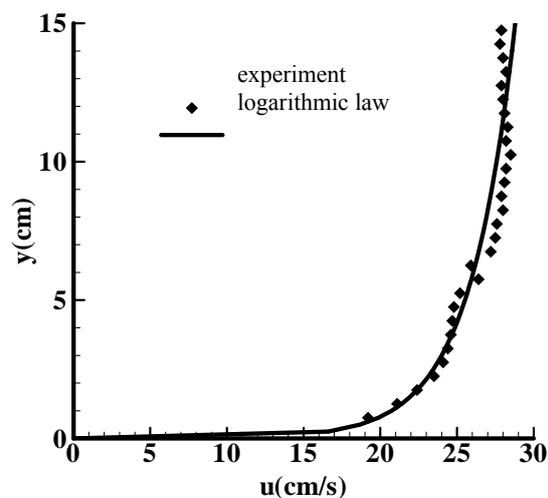


Figure 7. Mean velocity profile

### ACKNOWLEDGEMENT

This work was supported by the National Natural Science Foundation of China with Grant No.10602017.

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