

GA-Fuzzy Financial Model For Optimization Of A BOT Investment Decision

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Abstract

Financial modeling for investments to build/operate/transfer (BOT)-type projects is essentially intricate. The complexity stems mainly from two folds: multi-party involvement and uncertainty. Promoters need a systematic means for objective evaluation of financial performance measures in order to examine whether a certain level of profit margin and an attractive financial proposal to clients, are possible. A clear research gap is perceived in simultaneous evaluation of profitability and bid-winning potential from the promoters' perspective. By using a combination of genetic algorithms and the fuzzy set theory, an intelligent algorithm, is developed for optimization of conflicting financial interests in deriving the right mix of three key decision variables: equity ratio, concession length, and base price. Fuzzy sets are used to explicitly incorporate uncertainty in estimating economic and financial parameters due to lack of available data. Genetic algorithms is used for solving corresponding fuzzy objective function coupled with multiple constraints. A case study from prevailing literature demonstrates the excellent capability of the developed model to produce optimal financial scenario under uncertainty.

Keywords

Build/Operate/Transfer, Evolutionary Computation, Financial Management, Fuzzy Sets, Optimization.

1. Introduction

Following detailed feasibility studies, once BOT-type projects are scrutinized as possible investment opportunities, potential promoters need to submit a tender proposal along with financial offer to clients. One of the key challenges that potential promoters often face is how to design an attractive financial proposal. Although, profitability plays a central role in evaluating their financial standing, this is not the sole criterion as potential promoters also look for financial interests of government, as it should be. This is particularly very important in competitive tendering where potential promoters need to draw keen attention of clients, by offering them financial proposals as attractive as possible.

The next obvious question then arises is what would be the essential elements that promoters should emphasize from a group of available economic and financial elements, during financial modeling. It is well recognized in literature that concession length, bid mark-up and debt-to-equity ratio are three critical attributes, which possess both financial and contractual implications in evaluating long-term cash flow of BOT-type facilities. Excessive tuning of these elements in favor of promoters may even yield loss of opportunity of winning the bid. Recognizing paramount importance of previously mentioned elements in financial modeling, Lianyu and Tiong (2005) developed a minimum feasible tariff model for determining possible lowest tariff of a BOT-type water supply project. Shen and Wu (2005) and Zhang and AbouRizk (2006) developed financial models for determining a suitable concession length that satisfies financial

interests of both promoters and clients. Bakatjan et al. (2003) employed a linear programming technique to determine optimal capital structure of a BOT-type power project. Zhang (2005) also described a generic methodology for obtaining an optimal mix of debt-to-equity ratio for BOT-type projects. Unfortunately, none of these models has considered all the three attributes together as decision vectors in examining their aggregated effects to cash flow modeling, and thereafter, deriving an optimal scenario for proposing a feasible and attractive financial offer to clients. This integration is necessary to objectively identify financial situations and most importantly, to establish a baseline for further negotiations for minimizing conflicting financial interests of both promoters and clients. The issue is more challenging to prospective promoters as BOT-type projects are inherently characterized by uncertainties.

Previous studies incorporated uncertainties/risks in financial modeling by utilizing a probabilistic approach, namely Monte Carlo simulation. The primary notion is to consider decision variables as random numbers. Random numbers are used with prior assumption of adequacy in data availability. In reality, financial and economic data are often very scarce due to hidden sources, heterogeneous investment climate, and so on. Therefore, subjective judgments are often employed in defining probability distribution of model parameters that may even lead to illusion of precision of the whole process.

Non-probabilistic approach, like the fuzzy set theory may be used as an alternative method to deal with uncertainty in financial modeling. This is more reasonable to employ when uncertainty in terms of imprecision and vagueness arise due to lack of data. Kutcha (2000), Kahraman et al. (2002), to name a few, employed the fuzzy set theory for developing fuzzy equivalents of deterministic financial performance measures, such as, net present value, benefit cost ratio, and so on. Unfortunately, application of the fuzzy set theory to financial modeling of BOT-type projects is limited. Mohamed and McCowan (2001) introduced possibility measures for ranking and selection of BOT-type projects from promoters' viewpoint. Nevertheless, none of these studies pinpoints the practical need of prospective promoters for deriving a profitable yet competitive financial offer to clients within uncertain investment environment opportunity. The motivation of this paper thus arises from this clearly perceived theoretical gap. The rest of the paper is organized as follows. Section 2 provides the rationale of intended method in treating uncertainty associated with cash flow modeling parameters. Section 3 defines the reference problem. Section 4 depicts a methodology for mapping and integrating fuzzy variables into the optimization process. Section 5 succinctly describes GA as a solution tool for the optimization problem. Section 6 demonstrates applicability of the proposed model through a numerical example. Finally, section 7 concludes the paper.

2. Methods for Treatment of Uncertainty

In non-probabilistic fuzzy approach of cash flow modeling, uncertain parameters are expressed as fuzzy numbers. Consequently, fuzzy arithmetic operations are needed to evaluate the fuzzy objective function. The fuzzy extension principle, developed by Zadeh (1965) is a popular choice for executing algebraic operations on fuzzy numbers because of its simplicity in application. In the fuzzy extension principle, membership functions of output fuzzy variables are computed by using a series of discrete points on input fuzzy numbers. The vertex method is another alternative for mapping of output fuzzy variables. The vertex method is a numerical approximate method, which is based on α -cuts and standard interval analysis. Major advantages of using the vertex method may be viewed from two folds: numerical accuracy and computational burden. Regarding, quality of results, the vertex method can prevent abnormalities of output membership functions, which are often realized from using discretization technique in the fuzzy extension principle. Moreover, it can prevent widening of results due to multiple occurrences of variables in the function expression, as perceived in using conventional interval analysis method (Ross, 1995). On the other hand, the vertex method may greatly reduce computational burden of the fuzzy extension principle (Guan and Aral, 2005). Amount of saving in computational cost in employing the vertex method, is even more as the number of model parameters increases. Considering the

above, this study employs the vertex method for computing membership functions of uncertain parameters. Interested readers may refer to Dong and Shah (1987) for fundamentals of the vertex method.

3. Problem Statement

The proposed algorithm aims for deriving maximum potential of winning a fresh bid from promoters' viewpoint. A new financial performance measure; bid winning index (BWI) is developed elsewhere (Islam et al., 2006) to objectively quantify promoters' bid winning potential. BWI is defined as utility of net present value of promoters' financial gains that may be realized from unit operation period and unit base price with a certain comfort level of equity injection to stipulated initial investment outlay. From the above definition, it is clear bid winning potential depends on three key contractual-cum-financial and economic elements: concession length, base price, and equity ratio. Therefore, these three elements are considered as state variables in the optimization process. Since decision vectors comprise both integer and floating-point continuous variables, the reference optimization problem is, in fact, a continuous mixed integer optimization problem. The constrained optimization (maximization) problem can be mathematically expressed as:

$$\text{Maximize BWI : } \left\{ \text{NPV}_E \times \left(\frac{\varphi'_3}{\varphi_1 \times \varphi_2} \right) \right\} \quad 1(a)$$

where, $\varphi'_3 = f(\varphi_3)$ and *subject to the following constraints:*

$$\varphi_1 : R \geq R_E \quad 1(b)$$

$$\varphi_2 : R \leq R_G \quad 1(c)$$

$$\varphi_3 : \text{DSCR} \geq j \quad 1(d)$$

$$\varphi_4 : (\text{NPV}_E + \text{NPV}_G) \geq \text{NPV}_G \quad 1(e)$$

$$\varphi_5 : \text{NPV}_E \geq \text{NPV}_G \quad 1(f)$$

$$\varphi_6 : \varphi_1 \geq \text{LRP} \quad 1(g)$$

$$\varphi_7 : \varphi_1 \geq 0, \varphi_2 \geq 0, \varphi_3 \geq 0 \quad 1(h)$$

where φ_1 , φ_2 , and φ_3 are the three non-negative decision vectors representing concession length, base price, and equity ratio; φ'_3 is a vector of φ_3 , which represents equity comfort level φ_i s denotes i number of constraints. The objective function and constraints are defined in section 3.1 and 3.2, respectively.

3.1. Objective Function

The objective function as stated in Equation 1(a) is to maximize BWI.

3.2 Constraint Sets

3.2.1 Profitability

Equation 1(b) illustrates promoters' profit margin (defined as return on investment, R), which should at least be equal to a pre-specified limit, R_E . In contrast, equation 1(c) is constructed from government restrictions on promoters' profit margin, that is, it must be within a certain limit, R_G (Zhang and AbouRizk, 2006).

3.2.2 Debt servicing

Equation 1(d) represents debt-service-coverage ratio (DSCR) in each year of the operation period until loan maturity, and must not be less than a threshold value (Bakatjan et l., 2003, Zhang, 2005).

3.2.3 Financial return to government

Equations 1(e) and 1(f) depict financial benefits of clients (defined as net present value of government earnings, NPV_G) from running the BOT-type project after its concession period until end of economic life, which must be positive (Shen and Wu, 2005).

Equation 1(g) represents operation period available for running a project by promoters, and it cannot be less than loan maturity period, LRP. The remaining equation 1(h) imposes non-negativity condition on decision vectors. Note, detailed derivation of the financial model can be found in Islam et al. (2006).

4. Methodology

Section 4.1 presents steps of fuzzy transformation whereas section 4.2 illustrates constraint-handling mechanism used by GA.

4.1 Fuzzy transformation

Equations 1(b) through to 1(h) involve four financial performance measures, which do essentially become as fuzzy numbers when input parameters are considered as fuzzy sets. Three performance measures from promoters' perspective, namely net present value of financial gains; $NP\tilde{V}_E$; return on investment; \tilde{R}_E , debt service coverage ratio; $DSC\tilde{R}$, and one from Government perspectives $NP\tilde{V}_G$ are considered in this study. In order to evaluate uncertainties in these output fuzzy financial performance measures that propagates through fuzzy input variables, the vertex method is used.

For simplicity of demonstration, if we consider three continuous-valued fuzzy input variables: Initial Investment Outlay; \tilde{I}_C , Discount rate; \tilde{d} and demand; \tilde{Q} then by using a-cuts at the same level, we may obtain corresponding intervals of three fuzzy input variables as:

$$[I_C^{aL}, I_C^{aR}], [d^{aL}, d^{aR}], \text{ and } [Q^{aL}, Q^{aR}] \quad \forall a \in [0, 1] \quad (3)$$

where aL and aR represents left and right extreme points of an interval respectively at a specified a-cut.

Combining these extreme points of intervals will yield eight vertices (combination points) as follows:

$$\mathbf{u}_{|b=1}^8 \equiv \{ \mathbf{u}^{b=1} : (I_C^{aL}, d^{aL}, Q^{aL}), \dots, \mathbf{u}^{b=8} : (I_C^{aR}, d^{aR}, Q^{aR}) \} \quad \forall a \in [0, 1]; \quad \forall b \in [1, 8] \quad (4)$$

where β represents number of vertices, ?.

For a given set of decision vectors $O = f(?_1, ?_2, ?_3)$, using each of the vertices mentioned in equation (4), extreme points of fuzzy financial performance measures will be a function of fuzzy input parameters as:

$$\mathbf{j}_{i,a}^b = f(\mathbf{u}^b) \quad \forall a \in [0, 1]; \quad \forall b \in [1, 8] \quad (5)$$

where i is an index representing number of fuzzy outputs (financial performance measures) f.

The membership function of output fuzzy financial performance measures then can be obtained as:

$$\mathbf{j}_{i,a} \rightarrow [\mathbf{j}_i^{aL}, \mathbf{j}_i^{aR}] = \left[\wedge_{\beta} (\mathbf{j}_{i,a}^{\beta}), \vee_{\beta} (\mathbf{j}_{i,a}^{\beta}) \right] \quad \forall a \in [0, 1]; \quad \forall b \in [1, 8] \quad (6)$$

Using algebraic rule, each of fuzzy financial performance measure can numerically be approximated as:

$$\bar{\mathbf{J}}_i = \left\{ \frac{\sum_{j=1}^N (\mathbf{a}_j \times \mathbf{j}_{i,j})}{\sum_{j=1}^N (\mathbf{a}_j)} \right\} \quad \forall a \in [0, 1]; \quad \forall b \in [1, 8] \quad (7)$$

where j represents number of a-cuts.

4.2 Constraint Handling Mechanisms

GA cannot directly handle constraints. The static penalty function approach is employed to convert constrained optimization problem into an unconstrained optimization problem by following way:

$$\text{Maximize } I = BWI - \sum_{i=1}^L \mathfrak{R} \langle \mathbf{y}_i \rangle \quad \forall i \in [1, 7] \quad (8)$$

where $\mathfrak{?}$ is the fuzzy fitness function to be optimized, \mathfrak{R} is the penalty parameter which is considered as a high positive value. $\langle \mathbf{y}_i \rangle = \mathbf{y}_i$, when \mathbf{y}_i is negative, and zero otherwise. L is total number of constraints. Note \mathbf{y}_i is defined after evaluating \mathbf{j}_i as follows:

$$\langle \mathbf{y}_i \rangle = (\mathfrak{J}_i - \mathbf{y}_i^*) \quad (9)$$

where, \mathbf{y}_i^* represents threshold values of each of the constraints

Note all constraints are normalized prior to computation of penalty term. Normalization of constraints is necessary in order to use a single penalty coefficient and most importantly, to prevent ill conditioning of optimization performance of complex systems due to dominance of large constraints.

5. Genetic Algorithms

Optimization of fuzzy fitness function as stated in equation (8) is difficult to solve analytically. Mathematically, it is a NP-hard problem. Suitable methods from the family of evolutionary computation, like genetic algorithms (GA) may facilitate the optimization process. GA is an authoritative option in producing global, optimal or near-optimal solutions. GA is a probabilistic, heuristic search technique inspired by biological evolution of nature (Goldberg, 1989). In this study, a real-coded GA is used.

The optimization process starts with generating an initial random population, and then evaluating its corresponding fuzzy fitness function. Next, GA enters into a reproduction cycle comprising selection, crossover, and mutation operation for a pre-specified number of generations. A linear-ranking selection scheme (Goldberg, 1989) is used in this study. Simulated Binary Crossover (SBX) operator, developed by (Deb and Agarwal, 1995) is applied to perform crossover operations on real-coded genes as variable-by-variable basis. In SBX operator, a distribution index, $\mathfrak{?}_c$ defines the shape and spread of polynomial probability distribution, while a probability of crossover is required for selecting a group of current chromosomes for crossover operation. After crossover, mutation operation is put into action to alter values of selected gene(s) within a specified bound. This study employs Parameter based mutation (PMB) operator (Deb, 2001). Rate of mutation is controlled by using a user-defined probability of mutation, P_m while the shape and order of perturbation is determined by a probability distribution parameters $\mathfrak{?}_m$. After mutation, elitism is applied to keep the generation monotonic. Note, details of each steps of GA reproduction cycle are intentionally skipped due to space limitation of this paper. Nevertheless, fundamentals of SBX and PMB operator can be found in Deb and Agarwal (1995) and Deb (2001), respectively. The algorithm is coded using the C++ programming language.

6. Numerical Example

The financial optimization model is tested with published data reported in Bakatjan et al. (2003). The reported case study is selected because of its comprehensiveness in data representation. The economic and financial parameters used in the optimization model are depicted in Table 1.

Table 1 : Economic and Financial Parameters Used in Optimization Model

Economic and financial parameters	Values	Price Variations	
		Year of Operation	Year wise Price as % of Base Price
Economic Life	20 years	Year 1	100%
Construction Period	4 years	Year 2	95%
Loan Repayment Period	10 years	Year 3	90%
Annual O & M Cost	0.60% of Initial Cost	Year 4	86%
Inflation Rate	4.1%	Year 5	81%
Loan Interest Rate	10%	Year 6	77%
Tax Rate	11%	Year 7	74%
Demand Variation	Constant	Year 8	70%
		Year 9	66%
		Year 10	63%
		Year 11-20	25%

For simplicity of demonstrating the effect of uncertainty, three input fuzzy sets are considered with triangular membership function as shown in Figures 1(a-c). For the purpose of comparison, it is assumed that most likely estimates of the input fuzzy sets: initial investment outlay, discount rate and demand are 132565 US\$, 12% and 405.8 GW.h respectively, based on their corresponding deterministic values as stated in the original paper of Bakatjan et al. (2003). As seen from Figures 1(a-c), width of the support base reflects uncertainty. Pessimistic and optimistic estimates of input fuzzy variables are set to as (-) 10% and +5 % of most likely values, respectively.

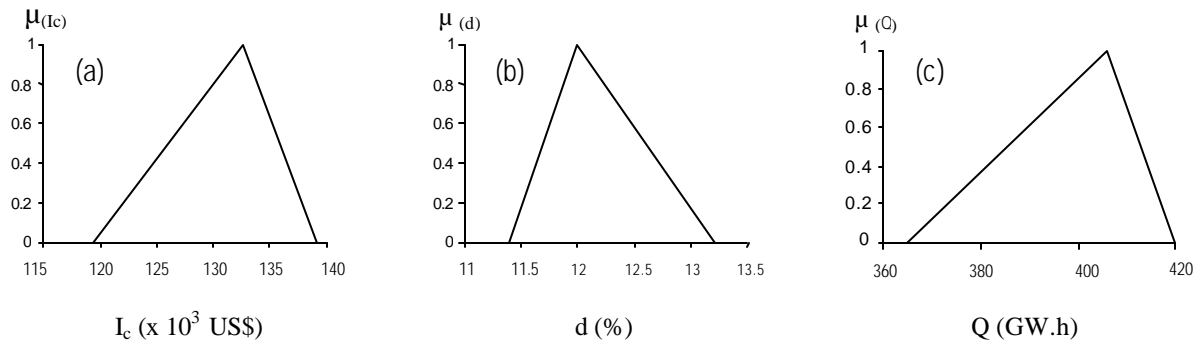


Figure 1 : Membership Functions of Initial Investment Outlays, Discount Rate, and Demand

GA is then used for finding optimal mix of decision variables from a feasible solution space that corresponds to maximum bid winning potential Table 2 depicts the values of parameters used in GA, which are selected after conducting a comprehensive sensitivity analysis.

Table 2: Parameters Used in Genetic Algorithm

Parameters	Value
Population Size	50
Maximum Number of Generations	50
Selection Pressure	1
Crossover Probability	0.8
Distribution Index for SBX operator	3.0
Mutation Probability	0.1
Distribution Index for PMB operator	200
Penalty Coefficient	10 ³

In the process of optimization, a continuous space of decision vectors for the operation period (years), base price (cents/kW.h) and equity contribution (percent) is selected as [1-20], [5-15], and [10-90], respectively. Regarding threshold values of financial constraints, a minimum value of DSCR is considered as 1.5 (Zhang, 2005). The lower and upper limits of return on investment are set to 12% and 18%, respectively. Using the values of decision vectors and thresholds of constraints, the model yields optimal solutions with corresponding financial performance measures. Due to space limitation, only the financial performance measure representing profitability is discussed further.

Considering crisp values of input parameters: initial investment outlay, discount rate and demand, the optimal solution corresponds to concession length as 21 years, base price as 9.98 cents/kW.h and equity contribution as 26% with a maximum return on investment as 18%, which is comparable to the findings of original paper of Bakatjan et al. (2003) as (24, 9.04, 31.69). However, profitability reduces to around 14%, if we consider the same situation but with the abovementioned input parameters as fuzzy sets. Resulting envelope of possibility of profitability is shown in Figure 2(a). This variation in profitability is due to possibilistic estimation of support base of input fuzzy numbers as shown in Figures 1(a-c). Profitability reduces due to a 5% increase in optimistic value and 10% decrease in pessimistic estimates of model parameters.

By using the same set of input fuzzy parameters, the obtained optimal solution (BWI) is 30.36 with the corresponding set of decision vectors as (23, 10.28, 28.57). Corresponding membership function of return on investment is shown in Figure 2(b). The decision vectors increase in order to reduce the effects of uncertainty of financial and economic parameters on return on investment. Comparing Figure 2(a) and 2(b), a 3.0 % increase of price in comparison to its counterpart crisp value is required for achieving an optimal solution. The increase in decision vectors, for example as observed in base price is due to cost of uncertainty and will be even more for a higher amount of uncertainty in parameter estimates (pessimistic).

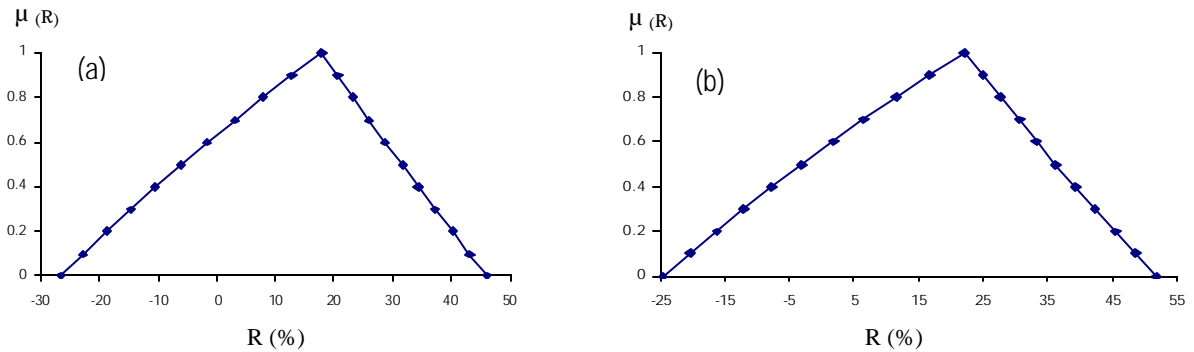


Figure 2 : Membership Functions of Return on Investment, R

Figure 3 reveals the optimization model converges to global solution. The solution improves quickly at the beginning and then gradually converges to the optimal solution.

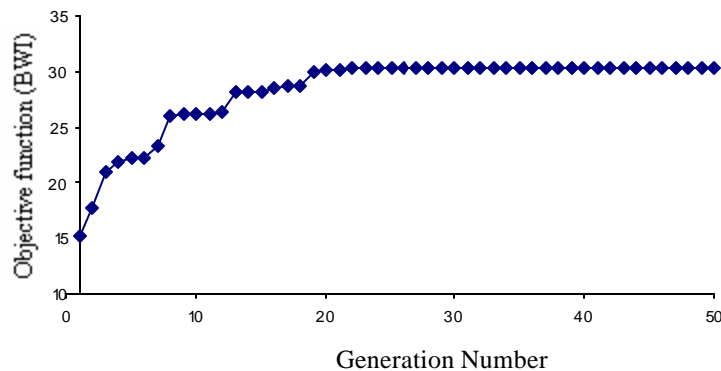


Figure 3 : Convergence Graph of GA

7. Conclusion

In this study, an intelligent optimization algorithm has been developed for financial modeling of BOT-type investment opportunities. The vertex method is embedded directly in the mixed-variable, continuous optimization problem, which is solved by using GA. The model will help meeting practical needs of prospective promoters in formulating a profitable yet competitive financial offer to clients. Propagation of uncertainty in evaluation of financial performance measures are considered by using possibility measures of continuous-valued fuzzy economic and financial parameters. Model results from the cited numerical example exhibits a clear reflection of uncertainty in the obtained set of decision vectors leading to optimal solution under fuzzy environment. Work is in progress for testing the developed model with data from other similar BOT-type investment opportunities subject to multiple uncertainties.

8. References

- Bakatjan, S., Arikan, M., and Tiong, R. L. K. (2003). "Optimal Capital Structure Model for BOT Power Projects in Turkey". *Journal of Construction Engineering and Management*, Vol. 129, No. 1, pp 89-97.
- Deb, K. (2001). *Multi-Objective Optimization Using Evolutionary Algorithms*, 2nd edition, Wiley, New York, USA.
- Deb, K., and Agarwal, (1995). "Simulated binary crossover for continuous search space." *Complex Systems*, Vol. 9, No. 1, pp 115–148.
- Dong, W., and Shah, H. (1987). "Vertex method for computing functions of fuzzy variables". *Fuzzy Sets and Systems*, Vol. 24, No. 1, pp 65–78.
- Goldberg, D. E. (1989). *Genetic algorithms in search, optimization, and machine learning*, 1st edition, Addison-Wesley Pub. Co., Reading, Mass, USA.
- Guan, J., and Aral, M. M. (2005). "Remediation system design with multiple uncertain parameters using fuzzy sets and genetic algorithm." *Journal of Hydrologic Engineering*, Vol. 10, No. 5, pp 386–394.
- Kahraman C., Ruan D., and Tolga E. (2002). "Capital budgeting techniques using discounted fuzzy versus probabilistic cash flows". *Information Sciences*, Vol. 142, No. 1, pp 57-76.
- Kuchta, D. (2000). "Fuzzy capital budgeting". *Fuzzy Sets and Systems*, Vol. 111, No. 3, pp 367-385.
- Islam, M. M., Mohamed, S., and Blumenstein, M. (2006). "Towards an Optimal Financial Investment Decision in Build-Operate-Transfer Projects Using Genetic Algorithms". *Proceedings of Eleventh Joint International Conference on Computing and Decision Making in Civil and Building Engineering*, Montreal, Canada, June 14-16, pp. 1554–1563.
- Lianyu, C., and Tiong, R. L. K. (2005). "Minimum feasible tariff model for BOT water supply projects in Malaysia". *Construction Management and Economics*, Vol. 23, No. 3, pp 255 - 263.
- Mohamed, S., and McCowan, A. K. (2001). "Modeling Project Investment Decisions under Uncertainty Using Possibility Theory." *International Journal of Project Management*, Vol. 19, No. 4, pp 231-241.
- Ngee, L., Tiong, R. L. K., and Alum, J. (1997). "Automated Approach to Negotiation of BOT Contracts". *Journal of Computing and Civil Engineering*, Vol. 11, No. 2, pp 121-128.
- Ross, T. J. (1995). *Fuzzy logic with engineering applications*, 2nd edition, McGraw-Hill, New York, USA.
- Shen, L. Y., and Wu, Y. Z. (2005). "Risk Concession Model for Build/Operate/Transfer Contract Projects". *Journal of Construction Engineering and Management*, Vol. 131, No. 2, pp 211-220.
- Tiong, R. L. K. (1995). "Competitive Advantage of Equity in BOT Tender". *Journal of Construction Engineering and Management*, Vol. 121, No. 3, pp 282-289.
- Zadeh, L. A. (1965). "Fuzzy sets." *Inf. Control*, Vol. 8, pp 338–353.
- Zhang, X. (2005). "Financial Viability Analysis and Capital Structure Optimization in Privatized Public Infrastructure Projects". *Journal of Construction Engineering and Management*, Vol. 131, No. 6, pp 656-668.
- Zhang, X., and AbouRizk, S. M. (2006). "Determining a reasonable concession period for private sector provision of public works and services". *Canadian Journal of Civil Engineering*, Vol. 33, No. 5, pp 622 - 631.