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Groundwater head responses due to random stream stage fluctuations using basis splines

J. H. Knight^{1,2} and D. W. Rassam¹

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[1] Stream-aquifer interactions are becoming increasingly important processes in water resources and riparian management. The linearized Boussinesq equation describes the transient movement of a groundwater free surface in unconfined flow. Some standard solutions are those corresponding to an input, which is a delta function impulse, or to its integral, a unit step function in the time domain. For more complicated inputs, the response can be expressed as a convolution integral, which must be evaluated numerically. When the input is a time series of measured data, a spline function or piecewise polynomial can easily be fitted to the data. Any such spline function can be expressed in terms of a finite series of basis splines with numerical coefficients. The analytical groundwater response functions corresponding to these basis splines are presented, thus giving a direct and accurate way to calculate the groundwater response for a random time series input representing the stream stage. We use the technique to estimate responses due to a random stream stage time series and show that the predicted heads compare favorably to those obtained from numerical simulations using the Modular Three-Dimensional Finite-Difference Ground-Water Flow Model (MODFLOW) simulations; we then demonstrate how to calculate residence times used for estimating riparian denitrification during bank storage.

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1. Introduction

[2] During interstorm periods there is a streamward hydraulic gradient in gaining streams that maintains groundwater discharge into them. Stream water levels rise in response to runoff and, in most cases, this results in the reversing of the hydraulic gradient, which induces a net flux into the floodplain. This water is temporarily stored in the floodplain and is slowly released back to the stream when the stream water level drops and the gradient toward the stream is reestablished. Bank storage diminishes and delays flood peaks during a storm event and drainage of water into the channel sustains streamflow between storm events. Bank storage affords opportunities for transforming and immobilizing pollutants and nutrients [Peterjohn and Correll, 1984; Lamontagne *et al.*, 2005; Rassam *et al.*, 2005].

[3] The interaction between a stream and the groundwater in an adjacent unconfined aquifer has been considered by many authors, such as Glover and Balmer [1954], Venetis [1970], Hantush [2005], Rassam *et al.* [2004], and Knight *et al.* [2005]. Most studies have used the linearized Boussinesq equation to describe changes in the groundwater level. In general, the governing flow equation is either solved numerically using a program such as the Three-Dimensional

Finite-Difference Ground-Water Flow Model (MODFLOW) [McDonald and Harbaugh, 1988] or is simplified and solved analytically using a linearized form of the Boussinesq equation. Sophocleous *et al.* [1995] compared analytical results for simplified aquifer geometries with MODFLOW solutions for more realistic geometries (MODFLOW models flow in the saturated zone). We will follow these previous authors and model flow only in the saturated zone and ignore the presence of a capillary fringe. This is justified for systems with hydrogeological length and time-scales, and for floods with water level rises on a scale of meters rather than centimeters.

[4] When the input function is more complicated than a single pulse or step function of time, combinations of pulse or step functions can be used for the input. For inputs which are general functions of time the output is given as a convolution of the appropriate impulse response with the input, as used by Venetis [1970], Barlow *et al.* [2000] and others. In general the convolution integral must be evaluated numerically in the time domain, and care must be taken to avoid loss of accuracy in certain parameter ranges. If the input function is given as a set of measured values at certain values of time then some form of interpolation must be used to calculate input values at other times. Another approach is to use a combination of many finite pulse inputs, for each of which an analytical solution is known. Singh [2004] investigated the use of ramp function inputs, and found that they were superior to pulse inputs. The use of splines is relatively common in hydrology [e.g., Tsai and Yang, 2005], however, it has not been used for estimating groundwater responses.

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[5] In this paper, we propose to use cubic basis spline function inputs and show how to calculate explicitly the analytical response of groundwater levels in an unbounded aquifer to random changes in stream water level. The proposed technique overcomes the difficulties and uncertainties arising from the numerical evaluation of the convolution integrals associated with other methods. On the basis of knowledge of the groundwater response in the floodplain, we demonstrate how we can calculate residence times for estimation of riparian denitrification during bank storage.

2. Methods

[6] In this section, we present the governing differential equation for groundwater flow along with a relevant solution for an input power function. The final goal is to estimate the groundwater response in a floodplain due to a random stage height. The method is summarized as follows: (1) we describe this random pulse with a series of cubic basis splines; every stage height is described using three consecutive splines and some numerical coefficients; (2) the unit cubic basis spline is the unit building block, which can be described by a series of power functions; (3) because of linearity, we can find the response for the cubic basis spline from the known responses of the component power functions; (4) having used a number of basis splines to describe the random input pulse, we can use the cubic spline response and the numerical coefficients (used to fit the stage height data) to formulate an explicit analytical solution for the groundwater response due to this random input pulse (i.e., groundwater response in floodplain due to a fluctuating stage in the neighboring stream). Finally, we conduct numerical simulations to compare results of the splines technique to those obtained from MODFLOW [McDonald and Harbaugh, 1988].

2.1. Stream-Aquifer Interaction

[7] Todd [1955] and Cooper and Rorabaugh [1963] gave solutions for transient rise of the stream level in response to a pulse of floodwater in the stream. The latter authors used a smooth pulse input of finite duration in the shape of a square wave plus one period of a cosine wave. Aquifer transmissivity is considered to be uniform throughout. It is assumed that there are only small fluctuations about a mean groundwater level, and that the aquifer has a horizontal base. It is also assumed that the only bound on the aquifer is the river at $x = 0$. Another important assumption is that any changes are small relative to the overall aquifer storage. Because of this the individual changes may be summed to calculate the overall groundwater response to temporally distributed stream levels. There are no sources or sinks of water apart from the flow into or out of the river. The linearized Boussinesq equation is given by

$$\frac{\partial h}{\partial t} = \frac{Kh^*}{\phi} \frac{\partial^2 h}{\partial x^2}, \quad (1)$$

where ϕ is the specific yield, K is the hydraulic conductivity, h is the height of the water table in an unconfined aquifer, h^* is an average height of water table, x is the distance from the stream in the horizontal plane, and t is time [Bear, 1972]. Equation (1) is applicable when the fluctuations in the water table are small compared to the

water table depth. Equation (1) has the form of a diffusion equation where Kh^*/ϕ is the diffusivity D . The initial conditions assume a horizontal water level corresponding to the elevation of the stream water surface, and that the aquifer is unbounded in the positive x direction. We assume that the stream fully penetrates the aquifer and that there is no impervious layer of sediments between the stream and the aquifer. The boundary condition at $x = 0$ is the Dirichlet condition $h(0, t) = h_0(t)$, where $h_0(t)$ is the river height.

[8] Carslaw and Jaeger [1959, pp. 63–64, 77] provided solutions for the free surface height $h(x, t)$ for a power function input $h_0(t) = t^k/k!$, $t > 0$ as follows:

$$h(x, t) = (4t)^k i^{2k} \operatorname{erfc} \left[\frac{x}{2(Dt)^{1/2}} \right]. \quad (2)$$

[9] In this paper we will use third powers of time to construct spline functions, so usually $k = 3$. Knight [2006] has verified the effectiveness of using third-order basis splines to describe random flood waves. In general, smaller time intervals will provide a better response; the parameter Δt is governed by the frequency at which field data is acquired. The functions $i^n \operatorname{erfc}(z)$ are the repeated integrals of the complementary error function whose properties are given by Carslaw and Jaeger [1959, pp. 482–484].

2.2. Basis Spline Pulse

[10] We seek a form of smooth pulse input for which we can find an explicit analytical solution. The functions that we will use are the basis splines, which are a convenient way of representing the spline, or piecewise polynomial, approximations originally introduced by Schoenberg [1946]. Unser [1999] describes the use of splines in signal and image processing, and Zhou et al. [2003] use linear basis splines in electromagnetic modeling. Here, we will consider cubic basis splines only on equally spaced knots. A more detailed account of the construction and theory of basis splines on equally spaced knots, known as cardinal splines is given by Unser [1999] and de Boor [2001]. The cubic basis splines are given by

$$B_n^3(t) = \left[(t - t_{n-2})_+^3 - 4(t - t_{n-1})_+^3 + 6(t - t_n)_+^3 - 4(t - t_{n+1})_+^3 + (t - t_{n+2})_+^3 \right] / [6(\Delta t)^3], \quad (3)$$

where the superscript refers to the spline order ($k = 3$), the subscript n refers to the spline number, Δt is the time interval, and the quantity $(t)_+$ is taken to be t for t positive and zero for t negative. The splines are continuous, with continuous first and second derivatives. The values of each spline at its five-knot points are 0, 1/6, 2/3, 1/6, and 0. Truncating the basis splines at the start points t_0 (see Figure 1) so that they are zero for $t < t_0$ gives

$$B_{-1}^3(t) = \left[(\Delta t)^3 (t - t_0)_+^0 - 3(\Delta t)^2 (t - t_0)_+ + 3(\Delta t) \cdot (t - t_0)_+^2 - (t - t_0)_+^3 + (t - t_1)_+^3 \right] / [6(\Delta t)^3]. \quad (4)$$

[11] The truncated splines $B_0^3(t)$ and $B_1^3(t)$ are given by Knight [2006]. We need the explicit analytical expressions of the three starting splines (and their responses) to avoid errors associated with using full splines at the beginning of the time series. Note that the truncated segment of the spline

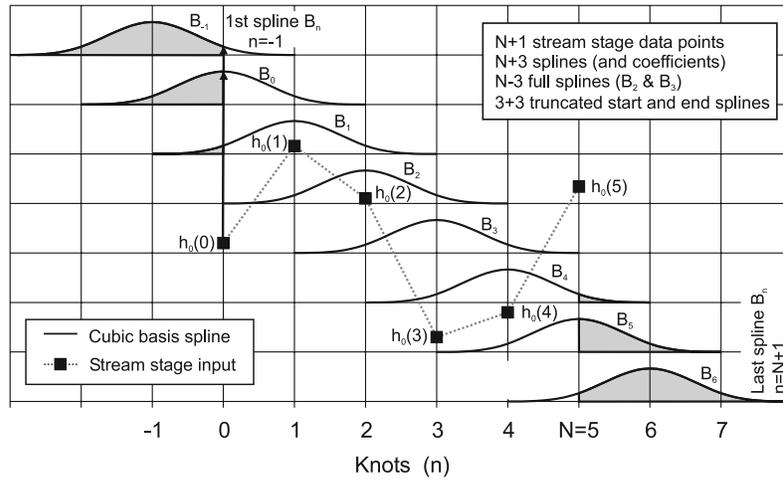


Figure 1. Stage height input and series of cubic basis splines to describe it.

(shaded areas of splines B_{-1} , B_0 , and B_1 in Figure 1) are outside the data range and hence would produce additional spurious responses when full splines are used. These three splines are required to define the first three data points ($h_0(0)$, $h_0(1)$, and $h_0(2)$) of Figure 1). Any of the stream stage data of Figure 1 can be described using three consecutive splines, a spline centered at the point itself, the preceding spline, and the following spline. For example, the first point $h_0(0)$ is equal to $a_{-1}/6 + 2a_0/3 + a_1/6$ where a_{-1} , a_0 and a_1 are numerical coefficients and $1/6$, $2/3$, and $1/6$ represent the values of different knots of the three splines (i.e., $1/6$ is the value of the fourth point of spline B_{-1} , $2/3$ is the value of the third point of spline B_0 , and $1/6$ is the value of the second point of spline B_1 ; see arrows in Figure 2).

2.3. Basis Spline Head Response

[12] Using equation (2) we can calculate the solution of equation (1) corresponding to a boundary condition $h_0(t) = B_n^3(t)$ for $1 < n < N - 1$. The response function for the full splines is (e.g., splines B_2 and B_3 in Figure 1):

$$h_n^3(x, t) = g_3(x, t - t_{n-2}) - 4g_3(x, t - t_{n-1}) + 6g_3(x, t - t_n) - 4g_3(x, t - t_{n+1}) + g_3(x, t - t_{n+2}), \quad (5)$$

where

$$g_k(x, t - t_n) \equiv \left(\frac{4(t - t_n)_+}{\Delta t} \right)^k i^{2k} \operatorname{erfc} \left[\frac{x}{2D^{1/2}(t - t_n)_+^{1/2}} \right], \quad (6)$$

with $k = 3$ in equation (6). The response functions for the truncated spline at the beginning (spline B_{-1} of Figure 1) is

$$h_{-1}^3(x, t) = \frac{1}{6} g_0(x, t - t_0) - \frac{1}{2} g_1(x, t - t_0) + g_2(x, t - t_0) - g_3(x, t - t_0) + g_3(x, t - t_1), \quad (7)$$

with $g_k(x, t - t_n)$ defined in equation (6). The response functions for the two other truncated splines $h_0^3(x, t)$ and $h_1^3(x, t)$ are given by Knight [2006]. The evaluation of the individual $h_n^3(x, t)$ functions can be affected by cancellation errors at large times. Suitable asymptotic large time forms of these functions valid for $t - t_n \gg \Delta t$ can be found using Laplace transforms. The approximate responses for the full and truncated splines are found in the work by Knight [2006].

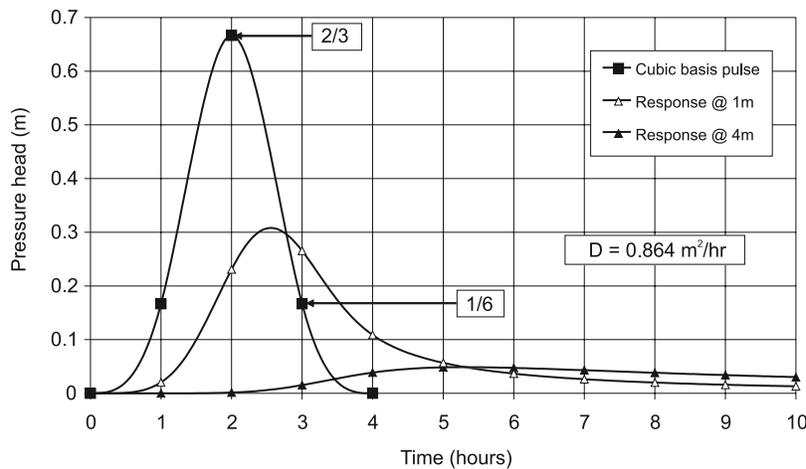


Figure 2. Head responses to a cubic basis spline input at 1 and 4 m.

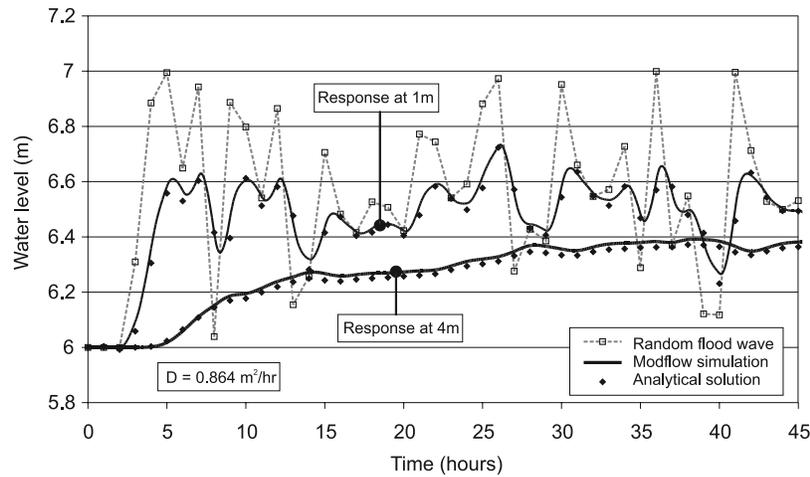


Figure 3. Random stream stage input and corresponding responses at 1 and 4 m.

2.4. Random Flood Wave and Its Response

[13] A general stream stage input function $h_0(t)$ defined on the $N + 1$ equally spaced points $\{t_n: n = 0, 1, \dots, N\}$ similar to that shown in Figure 1 (marked “Stream stage input”) can be approximated by a series of $N + 3$ basis splines $\{B_n(t): n = -1, 0, \dots, N + 1\}$ with $N + 3$ numerical coefficients $\{a_n: n = -1, 0, \dots, N + 1\}$ as

$$h_0(t) \cong \sum_{n=-1}^{N+1} a_n B_n^3(t). \quad (8)$$

[14] From equation (8) the coefficients $\{a_n\}$ satisfy a tridiagonal set of $N + 1$ linear equations that can be solved using the Thomas algorithm. The first condition used to evaluate the coefficients is $a_{n-1} + 4a_n + a_{n+1} = 6h_0(t_n)$ for $0 \leq n \leq N$. Two more conditions must be imposed at the ends to determine the $N + 3$ coefficients. A commonly used assumption is the “not a knot” condition in which the third derivatives of the approximation (equation (8)) are continuous at the knot points t_1 and t_{N-1} , which is equivalent to the conditions on the coefficients $(a_{-1} - 4a_0 + 6a_1 - 4a_2 + a_3 = 0)$ and $(a_{N-3} - 4a_{N-2} + 6a_{N-1} - 4a_N + a_{N+1} = 0)$. The solution of equation (2) for input values defined by equation (8) at the boundary can be written as

$$h(x, t) = \sum_{n=-1}^{N+1} a_n h_n^3(x, t). \quad (9)$$

with each $h_n^3(x, t)$ being the cubic basis spline response corresponding to the basis spline input $B_n^3(t)$.

2.5. Numerical Simulations

[15] MODFLOW is used to simulate aquifer response to random stream stage fluctuations; it uses a finite difference approach to solve the three-dimensional equation for saturated groundwater flow. A time-varying constant head boundary condition is used to simulate the fluctuating stream stage. We assume homogenous aquifer properties as follows: saturated thickness $h^* = 6$ m, saturated hydraulic conductivity $K = 0.0288$ m/hr, and specific yield $\phi = 0.2$ thus resulting in an aquifer diffusivity $D = 0.864$ m²/hr. Cell width in the vicinity of the stream was set equal to 0.1 m,

which gradually increases to 1 m close to the opposite boundary. The flow domain, which comprises only one cell in third dimension (parallel to the stream), is hence two-dimensional. Very strict tolerances were used to exclude any numerical errors (head change criterion = 10^{-8}). A synthetic data set representing a random stream flood wave was implemented in MODFLOW. Pressure head responses in the aquifer were monitored at 1 m and 4 m from the stream. Initial and boundary conditions in the numerical simulation were identical to those implemented in the analytical solution with the exception of the boundary opposite to the stream. In numerical simulations the assumption of an infinite flow domain is invalid, that is, we need to place a bound to the domain. However, we must ensure that this artificial boundary is distant enough such that it does not interfere with groundwater responses during the simulation. A 100 m domain was deemed wide enough to exclude any effects of the inevitable no-flow boundary located at the opposite side of the constant head boundary representing the stream. Since MODFLOW is a fully distributed model that solves the groundwater flow equation using a time marching scheme, it accounts for a transient variable aquifer transmissivity resulting from water fluxes into and out of the aquifer during bank storage. However, the simplifying assumptions that underpin the analytical solution do not allow for that.

3. Results and Discussions

[16] The responses at $x = 1$ and 4 m to a basis cubic spline input are shown in Figure 2; the two responses shown are for $D = 0.864$ m²/hr with $\Delta t = 1$ and $t_0 = 0$. As mentioned earlier, equation (5) cannot be reliably used when $t - t_n \gg \Delta t$; here we used the Laplace transform long-term approximations for $t - t_n > 5\Delta t$ (equations outlined by Knight [2006]). Note that equation (5) starts to produce serious oscillations when $x^2/D > 10\Delta t$.

[17] To demonstrate the effectiveness of the proposed splines technique, we use the synthetic data set representing a random stream stage shown in Figure 3 (marked “Random flood wave”). The data is fitted with a set of splines. We use the same aquifer parameters used previously and the corresponding responses at 1 m and 4 m shown in Figure 2

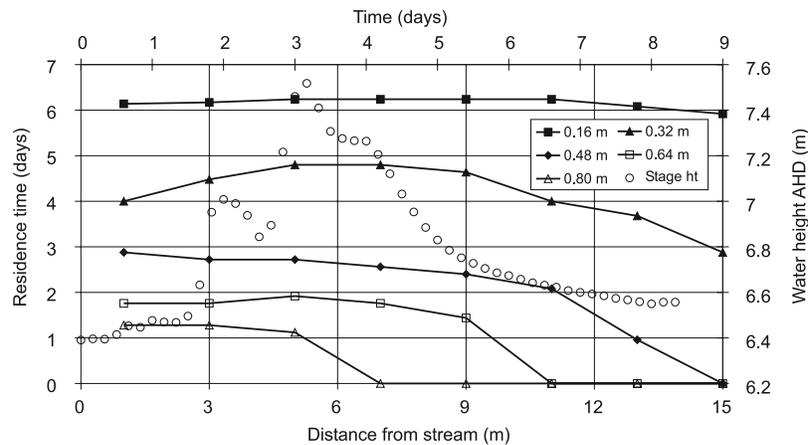


Figure 4. Estimated residence times (left-hand y axis and bottom x axis) and stage height (right-hand y axis and top x axis).

to predict the groundwater response to the random input pulse shown in Figure 3. The predicted groundwater responses at 1m and 4m shown in Figure 3 compare favorably to numerical simulations obtained from MODFLOW [McDonald and Harbaugh, 1988].

[18] As the analytical solution neglects the effect of a variable aquifer transmissivity during bank storage, we expect some error to arise. The accuracy of the linearized approach depends on the extent to which field conditions violate the underpinning assumptions of linearity. That is, if the head changes are small relative to aquifer thickness, the resulting errors will be minor. The assumption of a straight boundary representing the stream means that the approach cannot be used for meandering rivers. The current solution does not allow for a low-conductivity layer representing the resistance to flow along the stream bank.

[19] Here, we use the responses obtained from the splines method to estimate residence times that are required for estimating riparian denitrification as stream water is temporarily stored within a floodplain. Rassam *et al.* [2005] presented a model for riparian denitrification during bank storage, which simplistically assume that stream water level is mirrored in the riparian zone. On the basis of field observations, Rassam *et al.* [2006a] then proposed using an exponential decay function to model the drainage of water during a flood event. Using the proposed spline method is superior to the previous two approaches as it dynamically models the head responses and correctly accounts for the lag times for inflow and outflow. Rassam *et al.* [2005] modeled denitrification using first-order kinetics to model a diminishing denitrification rate with depth (corresponding to lower organic carbon levels) as well as modeling nitrate removal as a function of time. As heads vary across the floodplain during stream stage fluctuations and denitrification rate varies with depth, this means that we need to discretize the floodplain in both the horizontal and vertical directions. We use 1 m wide by 0.15-m-high compartments and consider a 15-m-wide riparian zone; preevent water level is assumed to be a zero datum level (i.e., the first lowermost compartment is 0–0.15 m). As a case study, we use the stream water levels reported by Rassam *et al.* [2006a]; time series starting 24 January 2005 shown in Figure 4. We use the proposed splines method to

fit the input stream pulse then predict groundwater responses at various distances from the stream. The head response at each location are then used to estimate the residence time for each vertical compartment. The results shown in Figure 4 can then be used to calculate denitrification (e.g., using first-order kinetics); note that the high-activity areas close to the soil surface (e.g., compartment 0.6–0.75 m) have much shorter residence times and drop down to zero ≥ 7 m from the stream.

[20] Such explicit analytical solutions may easily be incorporated into GIS-based models that estimate riparian denitrification such as the Riparian Nitrogen Model of Rassam *et al.* [2005]. Rassam *et al.* [2006b] also used this technique to evaluate potential denitrification and the associated ^{15}N – ^{18}O isotope fractionation during bank storage.

4. Conclusions

[21] Studying stream-aquifer interactions is relevant to processes in water resources and riparian management. In this paper, we presented a novel technique to estimate groundwater response to a random flood wave in a stream. The symmetric pulse solution of Cooper and Rorabaugh [1963] is replaced by a cubic basis spline input pulse solution, which is much easier to calculate. Spline approximations of various orders can be fitted to numerical data for river heights on equally spaced intervals, with numerical coefficients easily calculated. An analytical response function for the groundwater height corresponding to each basis spline input has been found, and so the groundwater head response for a given input can be found as a finite series with known numerical coefficients. Testing the proposed method using a random flood wave has shown that it is capable of producing reliable groundwater responses, which compared favorably to numerical simulation results. In an example application, we demonstrated how to calculate residence time for bank-stored water, which is relevant to assessing riparian denitrification.

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