Integrating Everyday and Scientific Ways of Knowing Mathematics Through Forms of Participation in Classroom Talk

Raymond Brown
Centre For Applied Language, Literacy and Communication Studies, Griffith University
<ray.brown@griffith.edu.au>

Peter Renshaw
Centre For Applied Language, Literacy and Communication Studies, Griffith University
<p.renshaw@griffith.edu.au>

Integrating everyday and scientific discourses is regarded as essential in developing a deep understanding of specific domains of knowledge. The process of integration, however, may occur in quite different ways. In this paper we analyse two forms of the integration process – replacement and interweaving – which provide a heuristic for considering how students might develop facility in mathematical thinking. In an analysis of student talk in a Year 7 classroom we found that replacement and interweaving can facilitate deep understanding. The flexible deployment of both discourse formats is recommended.

Teaching mathematics in the contemporary classroom provides the teacher with a number of challenges. Lampert (1996, pp. 214-215) in her work on connecting student inquiry with learning mathematics provides insight into the nature of these challenges. First, it is important from a constructivist point of view that student participation in classroom mathematics propels students in "different directions through the subject matter". Diverse trajectories through mathematical content provide the potential for authentic, personal engagement in a topic of mathematics, but the teacher is responsible for student engagement in a broad range of topics. Second, students' construction of their own ways of knowing and doing mathematics can often lead to the development of "idiosyncratic systems" for structuring understanding. However, the teacher is responsible for ensuring that students are equipped with the tools necessary to communicate with a wider, often more mature, audience of mathematicians. Third, if students are to become life-long practitioners of mathematics they must learn what it means to "invent and justify assertions in the mathematical domain". However, success in school mathematics is often measured in terms of a student's capacity to reproduce others' inventions and justifications.

Each of the above challenges highlights the need for teachers to be concerned with more than just content knowledge in the domain of mathematics. They need also to be concerned with linking students' 'everyday' ways of knowing and doing with 'mathematical' ways of knowing and doing. In this paper, Vygotsky's distinction between the 'everyday' and the 'scientific' (mathematical) is examined. Vygotsky (1987) described conceptual development as an interaction between spontaneous concepts and the organised systems of concepts referred to as 'scientific' concepts. He proposed that through formal instruction, children are given access to scientific concepts that enable them to reconceptualise their everyday experiences. In this sense, scientific concepts replace children's everyday concepts and they can begin to work within the more formal and generalised conceptual frameworks associated with schooling. However, the interaction between scientific and spontaneous concepts can also be described as an interweaving process where scientific concepts grow downward through spontaneous concepts, while spontaneous concepts grow upward through scientific concepts. So we draw the distinction between 'replacement' (i.e., the substitution of an 'everyday' understanding with a more sophisticated conventionalised understanding) and 'interweaving' (i.e., the maintenance of and interaction between the everyday and the scientific concepts). We propose that 'replacement' is not necessarily inferior to 'interweaving' for enabling students to adopt more mature ways of knowing and doing mathematics, rather it is that 'replacement' and 'interweaving' are two elements of the same process of appropriation. Too greater focus on 'replacement' may mask the need for students to actively participate in socially constituted practices, such as 'conjecturing' and 'justifying'. Too greater focus on 'interweaving' may reduce student inquiry to a process that lacks mathematical substance and clarity. Both 'replacement' and 'interweaving' are necessary elements of the discourse practices of a classroom community.

Engaging students' in 'conjecturing' and 'justifying' ideas in a classroom atmosphere that promotes substantive learning are important for the enculturation of students into the 'progressive' discourse practices of a scientific community (Bereiter, 1994). Bereiter proposes four 'quasi-moral commitments' that serve to distinguish mathematical and scientific discourse from other disciplinary discourses, namely, (a) mutual understanding - a commitment to work towards a common understanding, (b) empirical testability - a commitment to frame ideas in ways that allow evidence to be brought to bear, (c) expansion - a commitment to expand the conversation, and (d) openness - a commitment to allow beliefs to be subjected to criticism. In terms of connecting student inquiry with learning mathematics, practices such as those proposed by Bereiter, emphasise the role that classroom discourse can play in supporting students' conceptual development. As such, the Initiation-Response-Evaluation (see Mehan, 1979) discourse format that Lemke (1990) used to characterise 'didactic' classroom cultures can no longer be seen as sufficient for catering for the learning needs of students in mainstream mathematics classrooms. It is the purpose of this paper to compare and contrast two discourse formats *(Replacement & Interweaving)* that may provide teachers with alternatives to the Initiation-Response-Evaluation format and assist in connecting student inquiry with learning mathematics.

The discourse formats were initially identified from analyses of published research on classroom talk (see Renshaw & Brown, 2000). For this paper, we have drawn upon data from the first author's Ph.D. Thesis (see Brown, 2001). The data comprised video/audio recordings of teacher-student interactions gathered over the course of one school year as the teacher and students went about implementing an inquiry approach *(Collective Argumentation)* to the teaching and learning of mathematics within a Year 7 classroom. Drawn from the work of Miller (1987) and adapted and extended by Brown and Renshaw (1995), *Collective Argumentation* is organised around key strategies that require students to represent the task or problem alone, *compare* representations within a small group of peers, *explain* and *justify* the various representations to each other in the small group, reach *agreement* within the group, and finally *present* the group's ideas and representations to the class to test their acceptance by the wider community of peers and the teacher. It is from the 'present' phase of collective argumentation that the data presented in this paper is drawn.

Replacement

The *replacement pattern* of discourse (Renshaw & Brown, 2000) emphasises the importance of adopting precise vocabulary and acting within the ground rules of particular discourse genres. Ideally, there is attention given to the perspectives, words and values that children bring with them into the classroom talk, but these provide a temporary bridge into new forms of speaking and thinking. In the replacement format, progress in understanding is measured by the extent to which mathematical representations replace the more concrete and everyday ways of representing knowledge. The pedagogical process in the replacement format requires children to work within a system of signs and symbols with its own logic and set of meanings, rather than to oscillate between everyday (e.g., pictorial) representations and mathematical abstractions. As shown below, within this pedagogical process it is the voice of the teacher that dominates. It is the teacher who focuses on mathematical practices such as 'representing' and 'comparing' and on mathematical goals/values such as 'efficiency' and 'clarity'. We enter the script where groups of Year 7 students (Tracey's group & Robyn's group) had presented their solutions to the task presented in Figure 1.

> "The manager of a supermarket ordered 10 boxes each containing 100 cans of soup. How many cans of soup did the manager order? Use exponents to solve this problem."

Figure 1. Task represented by Tracey's and Robyn's groups.

Tracey's group had drawn pictures of 10 boxes on the black-board and written the number sentence: 100 x 10 = 1000. Robyn's group had represented the number sentence: 10^1 x 10^2 = 10^{1+2} = 1 000. We join the text where the class is comparing the group representations (See Table 1).

Table1

Class discussion of Tracey's and Robyn's representations.

Teacher	Liam? What is the difference between Tracey's group's (the picture) representation and Robyn's representation? Just note the similarities and the differences.
Liam	Robyn's has got ten to the power of one which is ten. Ten to the power of two which is one hundred. They got powers.
Teacher	So they have used powers, Robyn's group. And how has Tracey's group represented it Liam?
Liam	They just did (drew) boxes.
Teacher	They did boxes, well they represented boxes. Ten boxes with a hundred cans in each and that's what they wrote underneath in symbols (10x100 = 1 000). Which representation do you find more helpful, Allan?
Allan	Um, I find the one at the top (picture of boxes).
Teacher	Why?
Allan	Um, because it explains it a bit more clearer.
Teacher	You can see it, can you?
Allan	Yes.
Teacher	Come on class, what do you think about the two representations? Joel?
Joel	I reckon that Tracey's group's (the picture) one has a diagram so that it would be easier.
Teacher	What do you think Annie?
Annie	I reckon that Kerri's group's (the exponents) is easy, because one plus two is three, so ten to the power of three equals a thousand.

In the above extract, the key pedagogical strategy used is teacher questioning. Children are guided by leading questions *("What is the difference …"; "Which representation do you find more helpful?")* to use speech to conceptualise their representations in more abstract terms. The use of leading questions enables the children to use somewhat more abstract terms to describe their comparisons of the two representations. For example, when comparing the two representations, Allan and Joel prefer the pictorial representation because it *"explains it a bit more clearer"*; it *"would be easier"*. Other children, for example Annie, prefer the abstract 'exponent' representation because it employs a mathematical rule concerning the multiplication of exponential numbers to connect the major elements of the task *("one plus two is three")* so as to arrive at a satisfactory *("easy")* solution *("ten to the power of three equals a thousand")*. Later, the children begin to adopt the more abstract representation as promoted by the teacher (See Table 2).

Table 2

Class discussion of Tracey's and Robyn's representations, Cont'd.

Teacher	What do you think (about the two representations) Annie?
Annie	The above one (the picture) would be easier for someone who didn't have the exponents, but for someone who knew exponents the bottom one (the exponents) would be easier.
Teacher	What does the bottom (exponent representation) one help us to do?
Lisa	To understand exponents.
Teacher	Yes, it helps us work out the answer, but there's as little mental effort as possible, because we can see the relationships, can't we? Which one is more mathematical? Chris?
Chris	The bottom one (the exponents).
Teacher	Why?
Chris	Because it has ten to the power of one times ten to the power of two . . .

The teacher began with the representations that the children brought with them into the discussion, and he assisted them by leading questions *("What does the bottom [exponent representation] help us to do?")* to employ the more abstract 'exponent' representation to replace the more concrete pictorial representation of the 10 boxes each containing 100 cans of soup. In turn, the abstract 'exponent' representation was re-represented by Chris as being more mathematical because it employs the signs and symbols of mathematics to express relationships between the major elements of the task *("ten to the power of one times ten to the power of two …")*. The pedagogical process in the replacement format, therefore, requires children to work within a system of signs and symbols with its own logic and set of meanings, rather than to oscillate between everyday (pictorial) representations and mathematical abstractions. Within this pedagogical process it is the voice of the teacher that dominates. It is the teacher who focuses on the mathematical practices of 'representing' and 'comparing' *("Just note the similarities and the differences")*, and on mathematical goals/values such as 'efficiency' *("there's as little mental effort as possible")*, and 'clarity' *("we can see the relationships, can't we")*. Even the term 'mathematical' is introduced by the teacher. As such, the efficacy of the replacement format is dependant on the level of teacher expertise in mathematics and on the degree to which he/she is a representative of the mathematical community.

Interweaving

Interweaving refers to a type of classroom talk where children are enabled to populate discourse with their own purposes relating to challenge, perseverance and discovery (Renshaw & Brown, 2000). Children learn to weave together mathematical ideas into a form of discourse that reflects their specific circumstances. Interweaving can occur at a number of levels. For example, it may occur at a level where children's inventive ideas are interwoven with the conventions of mathematics through employing salient elements of a conventional approach to scientific investigation (theorising, hypothesising, testing). It may also occur at a more personal level where children's individual approaches to doing mathematics are interwoven with the more flexible representation systems employed by adult mathematicians.

The following extract provides evidence of a form of talk where children populate a discourse about the areas of circles with their own perspectives relating to challenge, perseverance and discovery. These children were able to weave together mathematical ideas into a form of discourse that reflected their specific circumstances. From the strictly mathematical perspective, the students learned a great deal about the language and practice of mathematics, including a strong desire to inquire and question rather than to seek closure. We enter the script where Cath and Tracey are presenting their ideas about how to find the area of each of three circles contained within a square (See Table 3). The radii of the circles, from smallest to largest, are 2cm, 4cm, and 6cm respectively.

In the whole class discussion that followed the group's presentation, Les (a student) wanted to know why Cath and Tracey had used this method when all they had to do was use the rule (Pi x R^2). Cath and Tracey explained that it was a challenge for them and that they had just tried to find a relationship between the circles by using trial and error (See Table: 4).

Table 3
Cath's and Tracey's presentation of the areas of three circles problem.

Cath	We discovered that the area of the middle circle was fifty point two four square centimetres which is a multiple of the area of the smallest circle. We then looked at the smallest and largest circles to see if their was a relationship and found that the area of the largest circle, one hundred and thirteen point zero four square centimetres, is a multiple of the area of the smallest circle.
Tracey	We had to check whether our theory - that when the radius of a circle is a multiple of two, its area will be a multiple of the area of a circle with a radius of two centimetres - works. We saw that the area of the middle circle had an area four times the area of the circle with a radius of two centimetres, and that the area of the largest circle had an area nine times the area of the circle with a radius of two centimetres. So the areas of the circles were going up in line with the square numbers. The area of the circle with a radius of two centimetres was twelve point five six square centimetres. The area of the circle with a radius of four centimetres was four times that of the smallest circle and the area of the circle with a radius of six centimetres was nine times that of the smallest circle. So we had a pattern of square numbers, the areas were going up in line with being timesed by the square numbers. So to check our theory, we extended the pattern and predicted that the area of a circle with a radius of eight centimetres would be the area of a circle with a radius of two centimetres times the next square number, which is sixteen. So we predicted that the area of this circle would be two hundred point nine six square centimetres. When we checked this prediction by using the rule ($A=PixR^2$), it worked. We actually predicted the area of a circle with a radius of eight centimetres.

Table 4
Cath's and Tracey's reply to Les.

Cath	It was hard work, but then we found a relationship and it worked, the pattern worked!
Tracey	We discovered it using trial and error and when we were just about to give up, Mr Brown (the teacher) came along and helped us to keep going.

When asked by a student what the group had learnt by doing the problem this way, both Tracey and Cath replied: *"We learnt that if you understand relationships, you can use your understanding to predict answers to other problems."*

In the above text, interweaving occurs at two levels. First, it occurs at a level where children's inventive ideas (e.g., that, *"the areas were going up in line with being timesed by the square numbers"*) may be interwoven with the conventions of mathematics (Area of Circle = Pi x $Radius^2$) by employing salient elements of a conventional approach to scientific investigation (theorising, hypothesising, testing). Here, it is the voice of science that dominates as Cath and Tracey set about proving their invention *"that when the radius of a circle is a multiple of two, its area will be a multiple of the area of a circle with a radius of two centimetres"*.

Second, interweaving occurs at a more personal level where children's individual approaches to doing mathematics are interwoven with more flexible representation systems employed by adult mathematicians. In the discussion of Cath and Tracey's presentation Les refers to issues related to challenge, persistence and discovery *("Why did you use this method when all you had to do was use the rule?")*. Cath and Tracey's response refers to using challenge *("it was hard work")*, persistence *("we were just about to give up")*, and discovery *("we discovered it using trial and error")* to find patterns and relationships to understand mathematics. These utterances relate to the aesthetic elements of the investigative process and relate to the flexible representation systems that expert mathematicians (mathematics professors, graduate students, etc.) have available to them when doing mathematics (Silver & Metzger, 1989). The everyday and the scientific perspectives are both present in this aspect of the discussion. Les is upholding the everyday/pragmatic view of school mathematics that revolves around the principle that you only do what you have to. Cath and Tracey are beginning to talk within a set of scientific assumptions - they make general claims *("that when the radius of a circle is a multiple of two, its area will be a multiple of the area of a circle with a radius of two centimetres")* and appeal to experimentation *("we had to check whether our theory ... works")* and empirical evidence *("when we checked this prediction by using the rule, it worked")*. They are beginning to consider the conditions and dispositions that might enhance their growth as mathematicians – a position later supported by Cath in response to a teacher's question*: "...we may be children, but we are still great mathematicians ready to discover or invent a new rule"*.

The above text illustrates that the children in this classroom learned to populate mathematical discourse with their own voices, that is, they learned to weave together scientific and everyday notions of what it means to do mathematics into a local classroom discourse that reflected their specific circumstances. From a strictly mathematics perspective, the participants in these discussions learned a great deal about the language and practice of mathematics, including a strong desire to inquire and question rather than to seek closure. From a cultural perspective, these discussions mirror in many ways the actual practice of the mathematics community where personal values, interests, and concerns are present at various stages of the scientific work but are obscured in the final product. The interweaving of different perspectives in the classroom talk, therefore, appears productive in enabling the children to appreciate the relevance of 'mathematics' in coming to 'know' and 'do' school mathematics tasks.

Conclusion

The 'replacement' and 'interweaving' patterns of discourse are offered as alternatives to the Initiation-Response-Evaluation format (see Mehan, 1979) that pervades classroom talk and as an initial heuristic, a device, to begin conversations with teachers on how the everyday and scientific may be linked in classroom talk to promote deep understanding. Similar to the Initiation-Response-Evaluation (I-R-E) script in that the teacher calls upon and asks students to answer questions, repeats students' responses and prods students to clarify their positions, the replacement format differs from the traditional I-R-E script in that it delays teacher evaluations of students' responses in favour of recontextualising what the students 'think' within the discourse practices of a mathematical community. The discourse format is not about transmitting mathematical knowledge to students, but about motivating students to think of themselves as capable of engaging in the co-construction and interpretation of meaning, about propelling students in different directions through the subject matter. As such, the replacement format contextualises the learning of mathematical knowledge (such as rules concerning the multiplication of exponential numbers) within a classroom discourse that foregrounds mathematical practices such as 'representing', 'comparing', and 'justifying' and evaluates student products in terms of mathematical norms (e.g., 'efficiency' & 'clarity') that relate to those practices. In this way, students are equipped with the tools necessary to communicate their ways of doing mathematics to a wider classroom audience where the concrete and experiential may be rephrased, re-represented and replaced by the more abstract and general concepts of mathematics.

The interweaving format provides a view of development as a transformative process where the everyday and scientific grow together into a hybrid form of understanding. Students using an interweaving discourse (like Cath & Tracey) adaptively deploy aspects of the signs and symbols of mathematics to initiate and extend their own approach to solving problems. In contrast to the teacher's role in the replacement format, the authority of the teacher remains 'invisible' in the interweaving format. This does not mean that the teacher has abrogated his/her authority in the learning process in favour of unguided discovery, but rather shares his/her authority with students within a classroom context that values the emergence of expertise – an emergence which the teacher has deliberately facilitated through the use of devices such as the replacement format of classroom talk.

Either format, replacement or interweaving, provides a sense that classroom talk is about assisting students to make sense of the mathematics being presented to them and about linking students' inventions to the conventions of mathematics rather than about teacher and/or textbook evaluations of student answers. The focus of the two formats is on extending student participation in mathematics beyond presenting a single, shared, homogeneous perspective to exploring how 'cultural tools' such as the exponential number system or mathematical formula may be used as thinking devices and as means to explain and to generate understanding. As in the case of Cath and Tracey, students may also enter the classroom conversation at a level that facilitates their 'imagining' of mature ways of knowing and doing mathematics.

References

Bereiter, C. (1994). Implications of postmodernism for science, or, science as progressive discourse. *Educational Psychologist, 29* (1), 3-12.

Brown, R. A .J. (2001). *A Sociocultural Study of the Emergence of a Classroom Community of Practice.* Unpublished Ph.D. Thesis University of Queensland, Brisbane.

Brown, R. A. J., & Renshaw, P. D. (1995). Developing collective mathematical thinking within the primary classroom. In B. Atweh & S. Flavel (Eds.), *Galtha* (pp. 128-134). Darwin: Mathematics Education Research Group of Australasia.

Lampert, M. (1996). Managing the tensions in connecting students' inquiry with learning mathematics in school. In D. Perkins, J. Schwartz, M. West, & M. Wiske (Eds.). *Software goes to school: Teaching for understanding with new technologies*, (pp. 213-232). NY: Oxford University Press.

Lemke, J. L. (1990). *Talking science: Language, learning, and values*, Norwood, N.J.:Ablex Publishing.

Mehan, H. (1979). *Learning lessons: Social organization in the classroom.* Cambridge, Mass.: Harvard University Press.

Miller, M. (1987). Argumentation and cognition. In M.Hickmann (Ed.), *Social and Functional Approaches to Language and Thought* (pp. 225-249). London:Academic Press.

Renshaw, P. D., & Brown, R. A. J. (2000). Four models of the process of integrating everyday and scientific discourse: Replacement, interweaving, contextual privileging, and pastiche. Paper presented as part of the Symposium (Chair, Eduardo Mortimer), *The discourse of science classrooms and popular science texts: Multiplicity in meanings, devices and rhetorical modes,* at the III Conference for Sociocultural Research, Campinas, Sao Paulo Brazil, 16th to 20th July, 2000.

Silver, E. A., & Metzger, W. (1989). In D. B. McLeod and V. M. Adams (Eds.), *Affect and Mathematical Problem Solving: A New Perspective* (pp. 59-74). New York: Springer-Verlag.

Vygotsky, L (1987) Thinking and Speech. In R.W. Rieber & A S Carton (Eds.), *The collected works of L.S. Vygotsky, Volume1: Problems of general psychology*. New York: Plenum Press.