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## **Cracking and Deflection Behaviour of Partially Prestressed High Strength Concrete Beams**

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### **Abstract**

High strength concrete (HSC) has revolutionised the construction industry in the 1980's and in recent years a marked increase in the use of HSC (compressive strength,  $f'_c > 50$  MPa) has been evident in construction projects around the world.

HSC is more brittle than normal strength concrete (NSC). As such, the problem of cracking is more pronounced for HSC structures because of the higher brittleness in comparison to their NSC counterparts. Further, by using HSC, smaller cross-sections with reduced dead load and longer spans can be designed. For the smaller cross-sectional members, excessive deflections may become a problem.

In order to investigate the cracking and deflection behaviour of partially prestressed high strength concrete beams, 4 full-size beams have been tested to failure in which measurements were made of mid-span deflections, crack spacings and crack widths at different load levels. The simply-supported beams had varying span lengths, reinforcement ratios, concrete compressive strengths and degrees of prestressing. The concrete strengths for the beams varied from 99.3 to 103.1 MPa.

As expected, the beams exhibited less ductile and bordering on brittle behaviour characteristic of high strength concrete. There was less number of cracks developed but the cracks extended to much larger widths with increasing loads.

Two of the major code methods – the ACI Code and the Australian Standard - were used to predict mid-span deflections for the test beams and were compared with the experimental results. Both the code methods have been found to be inconsistent in either under- or over-predicting the deflections.

# Cracking and Deflection Behaviour of Partially Prestressed High Strength Concrete Beams

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## 1. Introduction

High strength concrete (HSC) has revolutionised the construction industry in the 1980's. In recent years a marked increase in the use of HSC (compressive strength,  $f_c' > 50$  MPa) has been evident in construction projects around the world (Mendis, 2003).

HSC is more brittle than normal strength concrete (NSC). It has been revealed by Pam *et al.* (2001) that reinforced HSC beams would fail in an extremely brittle manner if they were not designed properly. It is also reported by Li *et al.* (1991) that reinforced HSC columns with concrete cylinder strength of 100 MPa or thereabouts, are extremely brittle when they are not confined adequately and that the confinement reinforcement should be heavier than that would have been provided in NSC columns for the same level of flexural ductility. From their tests on six reinforced HSC columns designed according to BS8110 (1997), Ho & Pam (2002) found that the columns exhibited behaviour from very brittle to moderately ductile depending on the compressive axial load levels. Other researchers (Fang *et al.*, 1991; Hibi *et al.*, 1991; Marzouk & Hussein, 1991; Pendyala & Mendis, 1998) also acknowledged the brittle behaviour of HSC structures. It follows that the problem of cracking is more pronounced for HSC structures because of the higher brittleness in comparison to their NSC counterparts.

Partial prestressing, which fills the gap between fully prestressed concrete and reinforced concrete, has now been accepted and become normal practice in many regions (Au & Du, 2004). Partially prestressed concrete (PPC) permits cracking under service loads, but, to satisfy serviceability requirements, the maximum crack width should not exceed the code-recommended limits on crack width (Joint ACI-ASCE Committee 423, 2000).

Further, by using HSC, smaller cross-sections with reduced dead load and longer spans can be designed (Mendis *et al.*, 2000). For the smaller cross-sectional members, excessive deflections may become a problem. The ACI state-of-the-art report for HSC (ACI Committee 363, 1992) recommends the use of the Branson (1963)'s formula for calculating the effective moment of inertia. However, Lambotte & Taerwe (1990) suggested that the exponent of  $(M_{cr}/M_s)$  may be greater than 3 for HSC beams.

In order to investigate the cracking and deflection behaviour of partially prestressed high strength concrete beams, 4 full-size beams have been tested to failure in which measurements were made of mid-span deflections, crack spacings and crack widths at different load levels. The simply-supported beams had varying span lengths, reinforcement ratios, concrete compressive strengths and degrees of prestressing. The concrete strengths for the beams varied from 99.3 to 103.1 MPa.

As expected, the beams exhibited less ductile and bordering on brittle behaviour characteristic of high strength concrete. The cracks developed once the prestressing force in the tendons was overcome. There was less number of cracks developed but the cracks extended to much larger widths with increasing loads.

Both the ACI Code (ACI, 2005) and the Australian Standard (SAI, 2001) recommend the use of Branson (1963)'s effective moment of inertia formula to calculate deflection for simply supported beams. The code formulas were used to predict mid-span deflections for the test beams and were compared with the experimental results. Both the code methods have been found to be inconsistent in either under- or over-predicting the deflections.

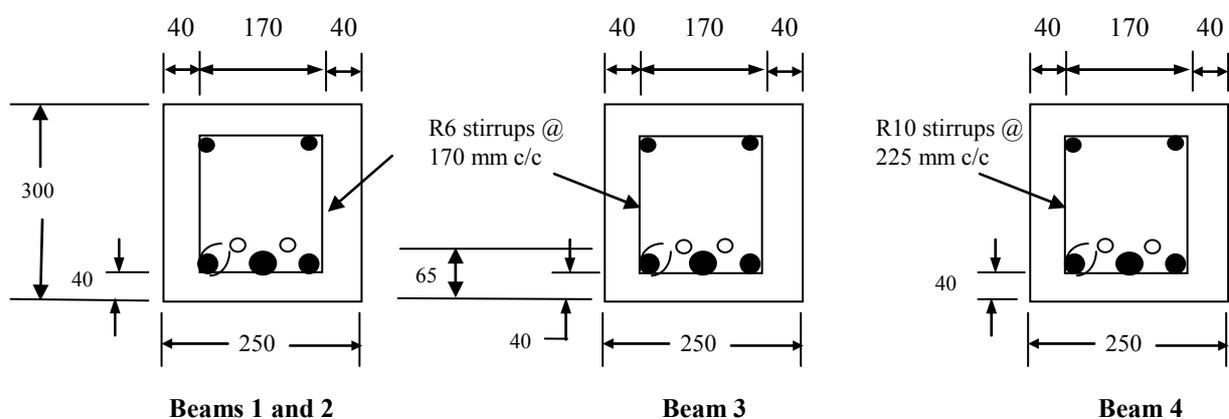
## 2. Experimental program

### 2.1 Design of test beams

Four full-scale partially prestressed concrete beams have been designed and fabricated to investigate their cracking and deflection behaviour. All the beams were 250 mm x 300 mm in cross-section and had a clear cover of 40 mm. Each beam had two 5 mm diameter prestressing tendons but had different amounts of non-prestressed longitudinal reinforcements. The prestressing tendons were pre-tensioned to the design prestress level before the pouring of concrete. The stressed tendons were released from the anchors 28 days after the pouring of concrete and thus resulting in bonded partially prestressed concrete test beams.

*Table 1 Design parameters for the test beams*

Beam number	Beam span, L in m	Non-prestressed longitudinal reinforcement			Prestressing reinforcement		
		No. of bars	Bar diameter (mm)	Steel Area, $A_s$ (mm <sup>2</sup> )	No of tendons	Tendon diameter (mm)	Steel Area $A_p$ (mm <sup>2</sup> )
1	2.50	2	12	530	2	5	39.2
		1	20				
2	2.50	2	12	530	2	5	39.2
		1	20				
3	2.00	2	12	420	2	5	39.2
		1	16				
4	3.00	2	16	710	2	5	39.2
		1	20				



Notes: All dimensions are in mm; o - prestressing steel; ● - longitudinal reinforcing bars

*Figure 1 Cross-sectional details of the test beams*

The design variables included beam spans, reinforcement ratios, concrete compressive strengths and degree of prestress. Note that Beams 1 and 2, otherwise identical, had different concrete compressive strengths. The main design parameters of the 4 test beams are summarised in Table 1, while the cross-sectional details given in Figure 1.

## **2.2 Material properties**

The concrete was provided by a local commercial supplier which contained ordinary Portland cement (Type GP) and the maximum aggregate size was 10 mm. Standard (100 mm x 200 mm) concrete cylinders which were used to determine the compressive strength of concrete,  $f'_c$ , were cast at the same time as the beams and cured under the same conditions. Measured average concrete compressive strengths from cylinder testing for Beam 1 was 99.3 MPa; for Beams 2 and 3, 102.2 MPa; and for Beam 4, 103.1 MPa.

Deformed bars of (Australian) grade 500N (with a minimum yield strength,  $f_{sy}$ , of 500 MPa) were used as the non-prestressed longitudinal tensile reinforcement and 250R plain bars (with  $f_{sy} = 250$  MPa), for the stirrups. The high strength prestressing tendons were 5 mm in diameter and had yield strength of 1550 MPa.

## **2.3 Instrumentation and measurements**

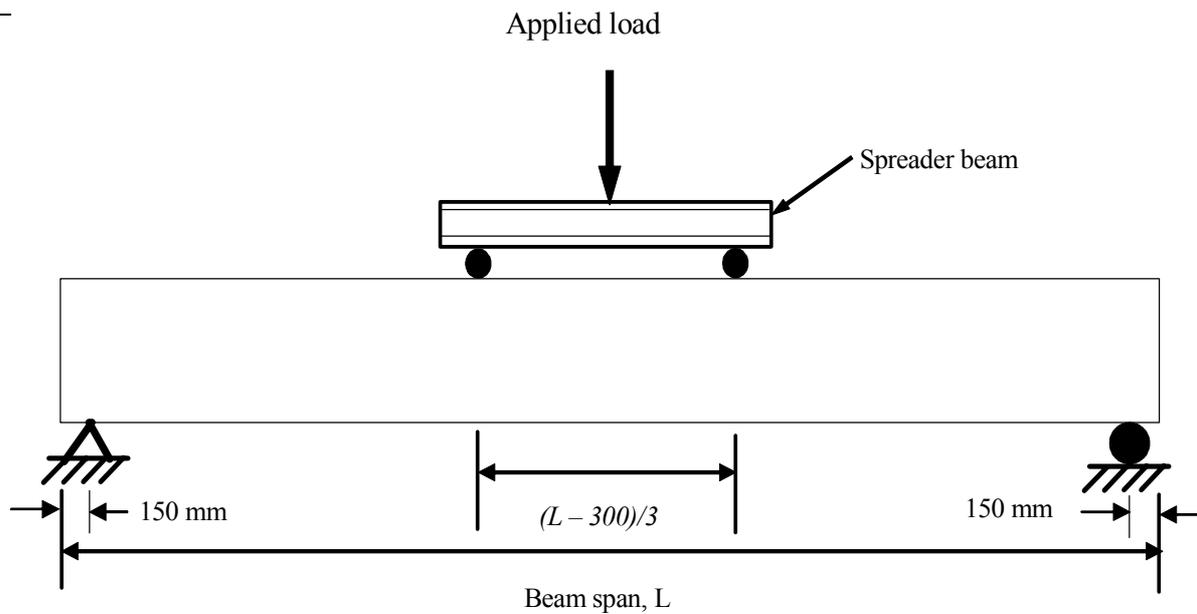
The prestressing tendons were pre-tensioned by using a hydraulic jacking system that has a capacity of 20 tonnes. The tensioning was conducted across anchor plates bolted to the strong floor at the end of beam moulds as shown in Figure 2.



*Figure 2 Applying of prestressing force using hydraulic jack in progress*

For each beam, crack widths and deflections at different static load levels were measured.

To produce cracking in the beams, static loading was applied. The beams were loaded symmetrically at two points at a distance as outlined in Figure 3. Note that all the beams were supported at a distance of 150 mm from each end. For each beam, the instantaneous crack widths at each load increment were measured along the constant moment region using an ELE crack detection microscope (model EL35-2505). The crack microscope had an accuracy of 0.02 mm. The concrete surfaces at the front of the beams were painted white to facilitate the measurement of crack



*Figure 3 Loading arrangement for the test beams*

propagation. The number of cracks and crack spacings were also measured at each loading step. Note, however, that crack spacings were stabilised at around 60 to 70% of the ultimate load. For deflection measurements, dial gauge was set up at the mid-span of the beam. Strain gauges were attached near the centre of the larger non-prestressed longitudinal bar in the middle of each beam to measure and monitor the strain values during loading.

### **3. Test results and analysis**

#### **3.1 Cracking results**

At each level of loading, for each beam measurements were made of average and maximum crack spacings, and average and maximum crack widths. The numbers of cracks were also recorded. These results are tabulated in Tables 2 to 5 for Beams 1 to 4, respectively.

An investigation of the test results reveals that the cracks were developed once the prestressing force in the tendons was overcome. This happened in the vicinity of 80 kN of applied load for Beams 1, 2 and 3. For Beam 4, it happened at a lower load because of the larger shear span resulting in the same cracking moment. Also, there was less number of cracks developed – between 8 and 12 cracks in Beams 1, 2 and 3, and 15 cracks in the larger span Beam 4. However, it can be seen from Tables 2 to 5 that the cracks extended to much larger widths with increasing loads.

As expected, crack spacings were stabilised and became constant at or around 60 to 70% of the ultimate loads for the beams.

### 3.2 Deflection results

Similar as for crack widths and spacings, at each load level, for each beam mid-span deflections were recorded from the dial gauge readings. The mid-span deflections of all 4 beams are plotted in the same graph to show their relative load-deflection behaviour as exhibited in Figure 4. Note that for Beam 3, the last deflection was measured at the load of 245 kN after which the dial gauge was taken off for the safety of the instrument.

*Table 2 Crack spacing and width data for Beam 1*

Load in kN	Number of cracks	Average crack spacing in mm	Maximum crack spacing in mm	Average crack width in mm	Maximum crack width in mm
12.75	0	---	---	---	---
16.19	0	---	---	---	---
22.56	0	---	---	---	---
29.43	0	---	---	---	---
41.69	0	---	---	---	---
51.21	0	---	---	---	---
61.80	0	---	---	---	---
72.59	0	---	---	---	---
79.50	4	196.67	290	0.120	0.16
84.50	5	147.50	262	0.128	0.20
90.00	6	143.60	262	0.097	0.12
103.00	8	124.43	192	0.123	0.18
119.50	9	142.88	272	0.136	0.20
138.50	10	127.00	192	0.164	0.28
149.50	10	127.00	192	0.234	0.38
169.50	10	127.00	192	0.274	0.52
183.50	10	127.00	192	0.352	0.80
199.00	10	127.00	192	0.282	0.44
213.50	10	127.00	192	0.418	0.80
230.50	10	127.00	192	0.585	1.20
255.00	Failure load				

*Table 3 Crack spacing and width data for Beam 2*

Load in kN	Number of cracks	Average crack spacing in mm	Maximum crack spacing in mm	Average crack width in mm	Maximum crack width in mm
31.36	0	---	---	---	---
39.20	0	---	---	---	---
61.25	0	---	---	---	---
78.40	2	307.00	307	0.050	0.06
90.74	6	190.40	307	0.100	0.14
107.91	8	135.00	232	0.118	0.18
131.45	11	124.40	172	0.147	0.24
147.15	11	124.40	172	0.222	0.32
166.77	11	124.40	172	0.269	0.40
187.37	12	113.09	172	0.287	0.44
209.93	12	113.09	172	0.332	0.54
236.42	12	113.09	172	0.582	1.30
250.16	Failure load				

An inspection of Figure 4 clearly indicates that the beams exhibited less ductile and bordering on brittle behaviour. This brittle behaviour most definitely is due to higher strengths of concrete as Beams 1 and 2 having all other parameters exactly the same had different concrete strengths and the one with higher strength i.e. Beam 2 exhibited more brittle behaviour. Note that the load of initiation of crack development and failure load for these two beams were similar too. Also, Beam 3

*Table 4 Crack spacing and width data for Beam 3*

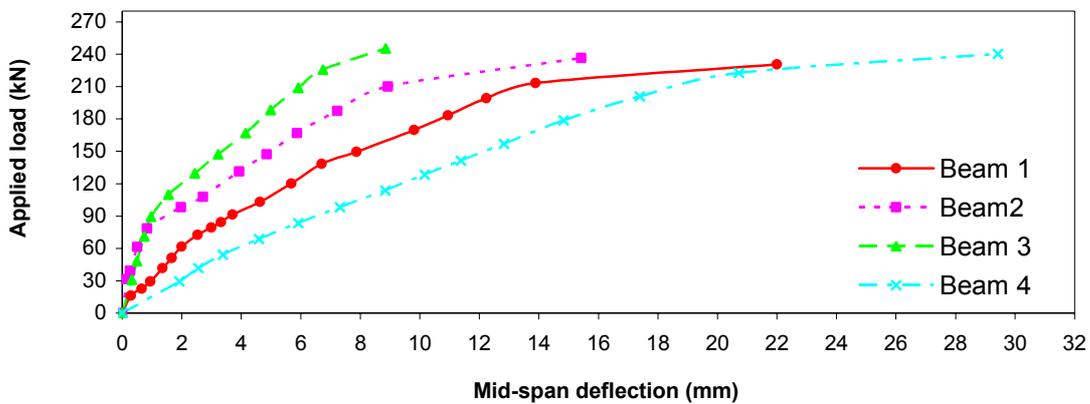
Load in kN	Number of cracks	Average crack spacing in mm	Maximum crack spacing in mm	Average crack width in mm	Maximum crack width in mm
29.43	0	---	---	---	---
48.07	0	---	---	---	---
71.12	0	---	---	---	---
89.27	2	276.00	276	0.040	0.04
109.87	4	237.00	276	0.090	0.16
129.50	5	177.75	255	0.184	0.24
147.15	7	141.83	180	0.206	0.32
167.75	7	141.83	180	0.303	0.40
188.35	7	141.83	180	0.360	0.48
208.95	7	141.83	180	0.360	0.52
225.63	8	121.57	155	0.403	0.54
245.00	8	121.57	155	0.440	0.60
274.40	8	121.57	155	2.200	3.60
282.24	Failure load				

*Table 5 Crack spacing and width data for Beam 4*

Load in kN	Number of cracks	Average crack spacing in mm	Maximum crack spacing in mm	Average crack width in mm	Maximum crack width in mm
29.40	1	---	---	0.120	0.12
41.65	2	175.00	175	0.090	0.16
53.90	4	228.33	255	0.093	0.22
68.60	6	224.20	255	0.130	0.30
83.30	8	218.57	255	0.149	0.40
98.00	9	191.25	255	0.202	0.46
113.68	11	176.20	255	0.224	0.50
128.38	12	160.18	255	0.242	0.56
141.12	13	156.33	255	0.275	0.70
156.80	13	156.33	255	0.320	0.74
178.36	15	134.00	255	0.331	0.76
200.90	15	134.00	255	0.361	0.78
222.46	15	134.00	255	0.437	0.90
240.10	15	134.00	255	0.643	3.20
250.16	Failure load				

carried the maximum load as it had the shortest shear span because of its smallest span (2.00 m) to cause the required failure moment.

The degrees of prestressing for the beams were very close to each other (0.31 for Beams 1 and 2, and 0.30 for Beam 3) except for Beam 4 for which it was 0.17. This lower degree of prestress for Beam 4 may be attributed to its relatively more ductile behaviour exhibited by its flatter and longer load-deflection curve.



*Figure 4 Mid-span deflections for the test beams*

#### 4. Code formulas for deflection prediction

Mid-span deflection of a simply-supported beam due to its uniformly distributed self-weight,  $w$ , can be obtained from:

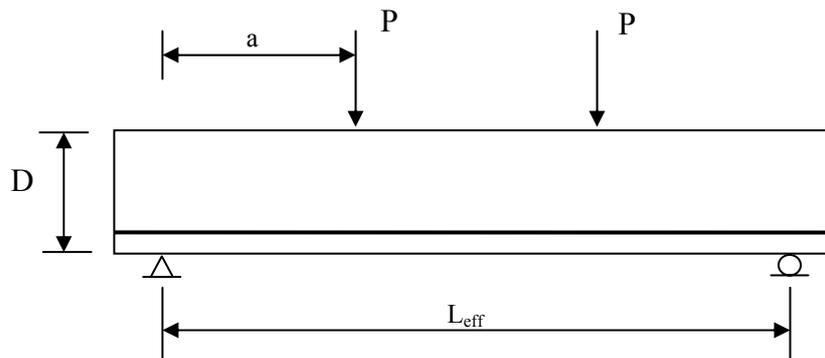
$$\Delta = \alpha_w w L_{eff}^4 / E_c I_{eff} \quad (1)$$

where  $\alpha_w = 5/384$ ,  $L_{eff}$  = the effective beam span as defined in Figure 5,  $E_c$  = modulus of elasticity for concrete and  $I_{eff}$  = effective moment of inertia for the beam cross-section.

Similarly, deflection due to two-point (middle-third) loading,  $P$  as defined in Figure 5, can be calculated as:

$$\Delta = \alpha_p P L_{eff}^3 / E_c I_{eff} \quad (2)$$

where  $\alpha_p = 1/24 (3a - 4a^3)$ , in which the term “ $a$ ” is as defined in Figure 5. Note that for the current test beams,  $a = 1/3 L_{eff}$ .



*Figure 5 Two-point loading with definition of  $L_{eff}$  and  $a$*

The effective moment of inertia,  $I_{eff}$ , as recommended by both the ACI Code (ACI, 2005) and Australian Standard (SAI, 2001), is calculated using Branson (1963)’s formula:

$$I_{eff} = I_{cr} + (M_{cr}/M_s)^3 (I_g - I_{cr}) \leq I_g \quad (3)$$

where  $I_{eff}$  is defined by the following limits:

$$I_{cr} \leq I_{eff} \leq I_g \quad (4)$$

and  $I_g$  is the moment of inertia of the gross concrete cross section about the centroidal axis,  $I_{cr}$  is the moment of inertia of a cracked section with the reinforcement transformed to an equivalent area of concrete,  $M_{cr}$  is the cracking moment at the section and  $M_s$  is the applied bending moment at the section for the loading increment being considered.

While  $I_{eff}$  was calculated the same way for both the methods,  $E_c$  values were computed using the following formula for the Australian Standard:

$$E_c = 0.043 \rho_c^{1.5} \sqrt{f'_c} \quad (5)$$

and using Equation (6) for the ACI method:

$$E_c = [3.32\sqrt{f'_c} + 6895] (\rho_c/2320)^{1.5} \quad (6)$$

In Equations (5) and (6),  $\rho_c$  is the density of concrete. Note that for the concrete used in the test beams, the measured average  $\rho_c$  was 2356 kg/m<sup>3</sup> and this value was used in  $E_c$  calculation using above formulas.

## **5. Comparison with test data**

For each beam, the theoretical values of the mid-span deflections at each load level were determined using both the ACI Code and Australian Standard methods. These theoretical values are compared with the measured values corresponding to various load levels for all the four test beams. The measured versus calculated deflection values are presented graphically in Figures 6 and 7, respectively for Australian Standard and ACI Code methods. The data points representing the four different beams are shown separately in each of these diagrams. Also shown in each figure, is an envelope of  $\pm 20\%$  variation lines.

As can be seen from Figures 6 and 7, except for a few points for Beam 1, majority of the data points are outside the  $\pm 20\%$  lines for both the prediction methods. Both methods generally under-predicted deflections for Beams 1 and 3 while mostly over-predicting for the other two beams. The code formulas are at best inconsistent in deflection predictions. May be, Lambotte & Taerwe (1990)'s suggestion of using a value greater than 3 for the exponent of  $(M_{cr}/M_s)$  in the  $I_{eff}$  determination formula needs further investigation especially for partially prestressed high strength concrete beams. This is even more justified for the fact that both formulas performed the best for Beam 1 which had the lowest concrete strength thus the closest to normal strength among the four tested beams.

Also, both methods performed poorly especially at the earlier stages of loading for each beam. This maybe attributed to the prestressing effect of delaying the crack development in partially prestressed beams.

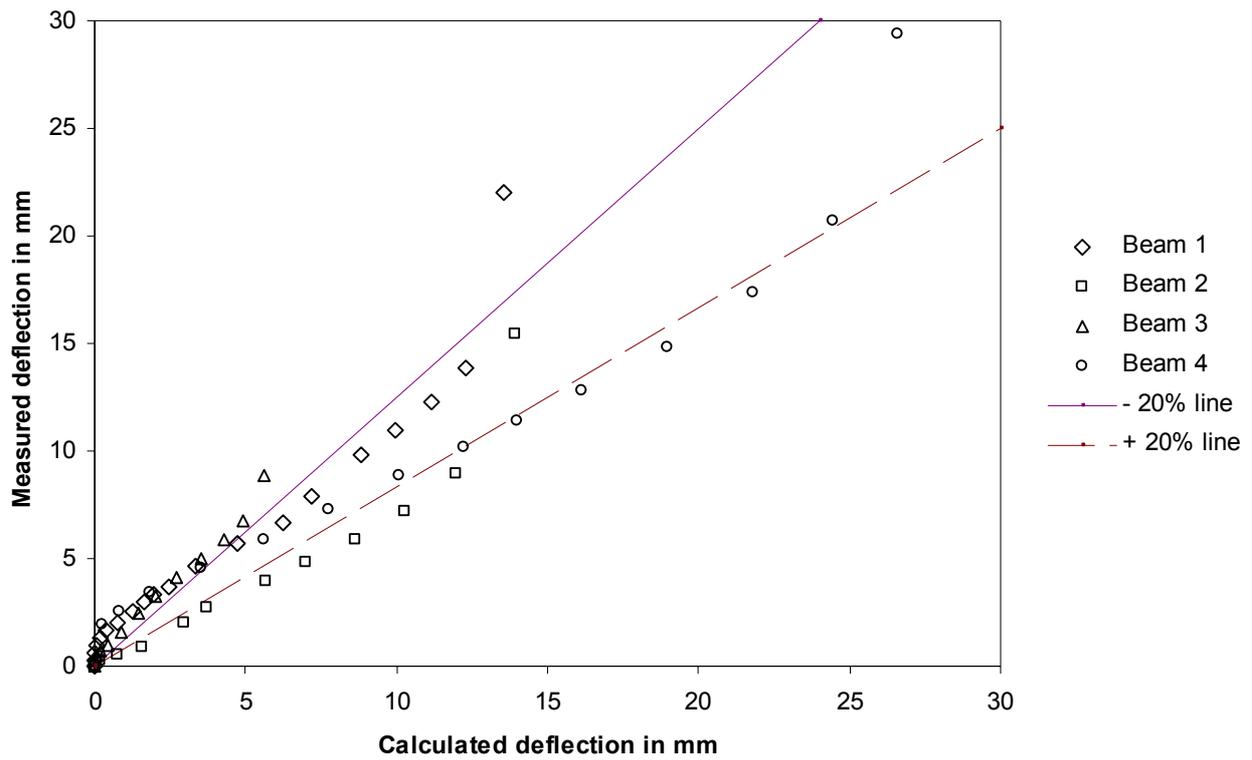


Figure 6 Measured versus calculated deflections – Australian Standard (SAI, 2001) method

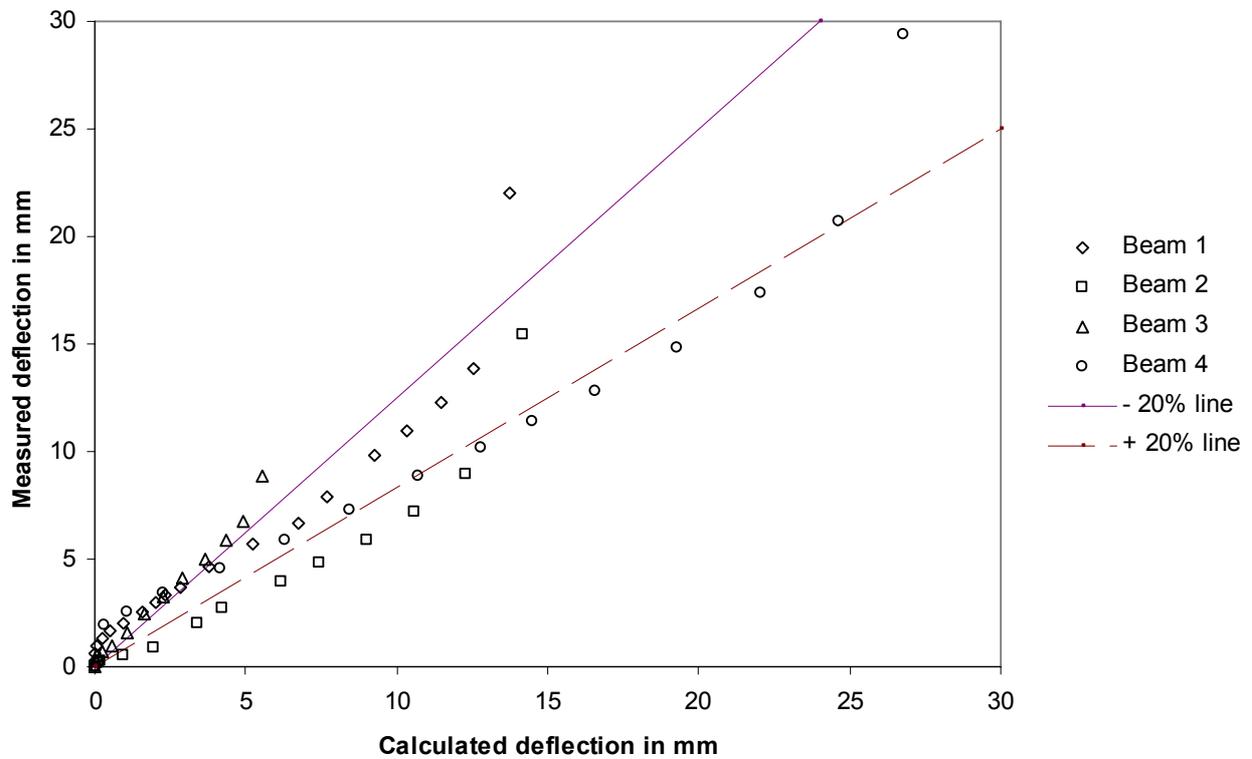


Figure 7 Measured versus calculated deflections – ACI Code (ACI, 2005) method

## **6. Conclusion**

Four full-size partially prestressed high strength concrete beams were tested to failure to investigate their cracking and deflection behaviour. For each beam, measurements were made of mid-span deflections, crack spacings and widths, and number of cracks at different load levels. The simply-supported beams had varying span lengths, reinforcement ratios, concrete compressive strengths and degrees of prestressing. The concrete strengths for the beams varied from 99.3 to 103.1 MPa and, degree of prestressing from 0.17 to 0.31.

As expected, the beams exhibited less ductile and bordering on brittle behaviour characteristic of high strength concrete. There was less number of cracks developed but the cracks extended to much larger widths with increasing loads.

Two of the major code methods - the ACI Code and the Australian Standard – were used to calculate mid-span deflections for the test beams. The theoretical values were compared with the experimental results. Both the code methods have been found to be inconsistent in either under- or over-predicting the deflections.

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