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 Removing XML Data Redundancies Using Functional and Equality-Generating Dependencies

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Abstract

We study the design issues of data-centric XML documents where (1) there are no mixed contents, i.e., each element may have some subelements and attributes, or it may have a single value in the form of a character string, but not a mixture of strings and subelements and/or attributes, (2) the ordering of subelements is of no significance. We provide a new definition of functional dependency (FD) for XML that generalizes those published previously. We also define equality-generating dependencies (EGDs) for XML, which, to our knowledge, have not been studied before. We show how to use EGDs and FDs to detect data redundancies in XML, and propose normal forms of DTDs with respect to these constraints. We show that our normal forms are necessary and sufficient to ensure all conforming XML documents have no redundancies. In passing, we define a normal form for relational databases based on EGDs in relational systems that can help remove data redundancies across multiple relations.

Keywords: XML tree, DTD, relation, functional dependency, equality-generating dependency, data redundancy, normal form, normalization.

1 Introduction

It is well known that XML documents can be regarded as a new type of database, and such data are particularly good for information exchange on the internet. The design of XML data has attracted much attention recently. As with any type of database, poorly designed documents may contain too many unnecessary redundancies and these redundancies may cause update anomalies. Data redundancies are usually due to some form of dependencies among the data, such as functional dependencies (FDs) and multi-valued dependencies in relational databases. Traditional functional dependencies are not suited for XML data because of the structural difference between the two types of database. On the other hand, dependencies naturally exist among data, no matter what format the data is in. Therefore, attempts to define data dependencies for XML and use them in the design of XML database have been made by several groups of researchers. For example, XML functional dependencies (XFDs) have been defined in (Wu et al. 2002), (Lee et al. 2002), (Arenas & Libkin 2004), and (Vincent et al. 2004) can express the constraint that the student number determines the set of addresses of the student in the document shown in Figure 1, nor can they express the constraint that the student number determines the set of addresses of the student in the entire document shown in Figure 1, because the student nodes are located in different paths (under both courses and subjects). The XFDs in (Hartmann & Link 2003), however, cannot express the constraint that the student number (sno) determines the student node in each course, that is, no two different student nodes under the same course can have the same sno. There are also natural constraints that can not be expressed by any of the previously defined XFDs. For example, in Figure 1 if we assume that a student may have several phone numbers (tel), but no phone number is shared by two or more students, then any single

Figure 1: The Course-Subject-Student example

Figure 2: The Course-Text example
phone number of the student will determine the student and hence the set of addresses of the student. This constraint cannot be expressed by any of the previously defined XFDs mentioned above. For a more detailed survey of related work, see Section 7.

In this paper, we present a new definition of functional dependencies for XML data that can express all of the constraints mentioned in the above examples. Our definition not only captures some useful constraints that can not be expressed before, but also unifies the previous definitions of XFDs under the same framework. We also define equality-generating dependencies (EGDs) for XML data, which, to the best of our knowledge, have not been studied before. We show how to use these dependencies (FDs and EGDs) to detect and remove potential data redundancies in XML documents. In passing, we propose a normal form for relational database schemas with respect to EGDs in relational systems that can help remove data redundancies across multiple relations. We believe this is important in itself because traditional normal forms (e.g, BCNF, 4NF and 5NF) only eliminate redundancies in a single relation. Our normal form is an direct extension of BCNF.

The rest of the paper is organized as follows. Section 2 provides preliminary definitions and notations. Section 3 presents our new definition of functional dependencies for XML data and shows how our FDs can, for example, express the constraints in the above-mentioned examples. Section 4 starts with a discussion of data redundancies across relations in relational databases, and defines a normal form of relational database schema with respect to EGDs that prevents such redundancies. Then it goes on to discuss similar problems in XML data and defines EGDs for XML documents. Section 5 considers FDs and EGDs in the presence of DTDs. Section 6 discusses potential data redundancies in XML documents conforming to a DTD, which are caused by a given set of FDs or EGDs, and proposes normal forms of DTDs that prevent such problems. Section 7 compares our results with related work. We conclude the paper with a discussion about unresolved issues and future work in Section 8.

2 Preliminary Definitions and Notations

2.1 XML tree

In this paper we will consider XML documents where (1) there are no mixed contents, i.e., each element may have some subelements and attributes, or it may have a single value in the form of a character string, but not a mixture of strings and subelements and/or attributes, (2) the ordering of subelements is of no significance. It should be mentioned that most data-centric XML documents (e.g., those converted from relational and OO databases) have these characteristics. We call elements that only have a single value a simple element, and elements that have subelements and/or attributes complex elements. Simple elements serve similar purposes to attributes, the only difference is that multiple simple elements with the same label may appear as children of a complex element, but each attribute of a complex element has a distinct label.

As well known, an XML document can be represented by a tree. Figures 1 and 2 are two example XML trees. In the figures, complex elements are shown as rounded rectangles, attributes are shown as filled circles, simple elements are shown as squares, and string values are quoted.

To facilitate our discussion in subsequent sections, we provide a formal definition of XML trees. Let $E_1$ and $E_2$ be disjoint sets of element names, $A$ be a set of attribute names, $E = E_1 \cup E_2$, and $A$ be disjoint element names and attribute names are called labels.

Definition 2.1 [XML tree] An XML tree is defined to be $T = (V, \text{lab}, \text{ele}, \text{att}, \text{val}, \text{root})$, where (1) $V$ is a set of nodes; (2) lab is a mapping from $V$ to $E \cup A$ which assigns a label to each node in $V$; a node $v \in V$ is called a complex element (node) if $\text{lab}(v) \in E_1$, a simple element (node) if $\text{lab}(v) \in E_2$, and an attribute (node) if $\text{lab}(v) \in A$. (3) ele and att are functions from the set of complex elements in $V$: for every $v \in V$, if $\text{lab}(v) \in E_1$ then $\text{ele}(v)$ is a set of element nodes, and $\text{att}(v)$ is a set of attribute nodes with distinct labels; (4) val is a function that assigns a value to every attribute or simple element. (5) root is the unique root node labelled with complex element name $r$. (6) If $v' \in \text{ele}(v) \cup \text{att}(v)$, then we call $v'$ a child of $v$. The parent-child relationships defined by ele and att form a tree rooted at root. □

As stated explicitly in the definition, ele and att define the child nodes of a complex element node. Child elements are also referred to as subelements, and child attribute nodes are sometimes simply referred to as attributes. The concept of ancestors and descendants are defined as usual: a node $v$ is the ancestor of another node $v'$ if $v$ is the parent of $v'$, or $v$ is the parent of an ancestor of $v'$; $v'$ is a descendant (node) of $v$ if $v$ is an ancestor of $v'$.

Note that our definition of XML trees is different from those, for example, in (Buneman, Davidson, Fan, Hara & Tan 2001, Fan, Schwenzer & Wu 2001, Arias & Libkin 2004). We have explicitly distinguished complex and simple elements so that the special text node under a simple element is not required. We have also made the ordering of child elements insignificant by treating them as a set rather than a sequence.

2.2 Paths in XML trees

We distinguish three types of paths: downward paths, upward paths, and composite paths. These paths are subclasses of XPath.

Definition 2.2 [paths] A downward path is of the form $l_1 l_2 \cdots l_n$ where $l_i \in E \cup A \cup \{\sim\}$ $(i = 1, \ldots, n - 1)$, $l_n \in E \cup A$, and if there is a label name and a simple element name in the path, it must appear at the last position. Then, it is a downward path if the symbol represents a wildcard (which can match any label), and $\sim$ represents $\ast$, namely the Kleene closure of the wildcard. A simple path is a downward path where there is no $\sim$ or $\sim$. The number of labels in a simple path $p$ is called its length. The simple path of length 0 is called the empty path and denoted $\epsilon$.

An upward path is of the form $\uparrow \cdots \uparrow$. If there are $k > 1$ upward arrows (to distinguish from the empty path, we require $k \geq 1$), we will sometimes abbreviate the path as $\uparrow^k$.

A composite path is of the form $\xi \rho$, where $\xi$ is an upward path, and $\rho$ is a simple path. □

According to the above definition, an upward path is a special composite path, the empty path is a special simple path, which in turn is a special downward path.

A path is either a downward path or a composite path. Let us use last($p$) to denote the last symbol in path $p$.

In any XML tree $T$, only some paths are valid. Here the validity of paths is with respect to a node, as defined below.
Definition 2.3 [valid paths] Let \( T \) be an XML tree and \( v_0 \) be a node in \( T \). A path \( p \) is said to be valid wrt \( v_0 \), if one of the following is true:

- \( p \) is the empty path.
- \( p \) is the downward path \( l_1, \ldots, l_n \), and there is a sequence of nodes \( v_1, \ldots, v_n \) in \( T \) such that for \( i = 1, \ldots, n \) if \( l_i \) is in \( \mathcal{E} \cup \mathcal{A} \{ \} \), then \( v_i \) is a child of \( v_{i-1} \) and the label of \( v_i \) matches \( l_i \) (note that the label of any node matches \( \));
- \( p \) is the composite path \( \hat{\gamma}^k l_1, \ldots, l_n \) and there is a sequence of nodes \( n_1, \ldots, n_k, v_1, \ldots, v_n \) in \( T \) such that \( n_1 \) is the parent of \( v_0 \), \( n_1 \) is the parent of \( n_{i-1} \) for \( i = 2, \ldots, k \), and \( v_1 \) is a child of \( n_k \), \( v_i \) is a child of \( v_{i-1} \) for \( i = 2, \ldots, n \), and the label of \( v_i \) matches \( l_i \) for \( i = 1, \ldots, n \).

The sequence of nodes described above (for a non-empty path \( p \)) is called an instance of path \( p \) wrt \( v_0 \). Generally there may be many such instances. We will refer to the set \( \{ v_n \mid v_n \text{ is the last node of an instance of } p \text{ wrt } v_0 \} \) as the target set of \( p \) wrt \( v_0 \), denoted \( v_0[p] \). For easy presentation, we also define \( v_0[\epsilon] = \{ v_0 \} \) for any node \( v_0 \). Note that path \( p \) is not valid wrt \( v_0 \) if and only if \( v_0[p] = \emptyset \) is empty.

For example, in the XML tree shown in Figure 1, student.sno and student.address.city are simple paths that are valid wrt \( v_3 \) and \( v_4 \), but not to others; \( \hat{\gamma}^1 \hat{\gamma}^1 \) is an upward path valid wrt all nodes except \( v_1, v_2 \) and root; \( \hat{\gamma}^1 \hat{\gamma}^1 \hat{\gamma}^1 \hat{\gamma}^1 \) is a composite path which is valid path \( v_3 \); and root[\~ course.students.student = \{v_5,v_6\}, root[\~ course.students.student = \{v_5,v_6\}] \} = \{v_5,v_6,v_7\}, \{v_5,\text{address} = \{v_8,v_9\} \}, and \{v_5,\text{address} = \{v_8,v_9\} \} is the set containing the two street attribute nodes under \( v_8 \) and \( v_9 \).

2.3 Value Equality and Node Agreement

In order to compare two nodes \( n_1 \) and \( n_2 \) in an XML tree, we need to define the equality between them. Obviously, if \( n_1 \) and \( n_2 \) are the same node (denoted \( n_1 = n_2 \)), they should be considered equal, but this kind of node equality is not sufficient, because there are cases where two distinct nodes have equal values. So we need to define value equality between nodes. Since we consider the ordering of child elements insignificant, our definition of value equality is different from that in (Buneman, Davidson, Fan & Hara 2002, Buneman et al. 2001).

Definition 2.4 [value equal] Let \( n_1 \) and \( n_2 \) be two nodes in \( T \). We say \( n_1 \) and \( n_2 \) are value equal, denoted \( n_1 =_v n_2 \), if \( n_1 \) and \( n_2 \) are of the same label, and

1. \( n_1 \) and \( n_2 \) are both attribute nodes or simple element nodes, and the two nodes have the same value, or
2. \( n_1 \) and \( n_2 \) are both complex elements, and for every child node \( m_1 \) of \( n_1 \), there is a child node \( m_2 \) of \( n_2 \) such that \( m_1 =_v m_2 \), and vice versa.

For example, the two nodes \( v_8 \) and \( v_{12} \) in Figure 1 are value equal. The two nodes \( v_9 \) and \( v_{11} \) are also value equal.

Note that node equality implies value equality, but not vice versa.

We now turn to the definition of the agreement of two nodes on a path. Intuitively, given two nodes \( n_1 \) and \( n_2 \) and a path \( p \), there can be several different interpretations of agreements between \( n_1 \) and \( n_2 \) on \( p \). For example, every node in the target set \( n_1[p] \) may have a node in \( n_2[p] \) such that the two nodes are value equal and vice versa, or there may be only some nodes in the two sets that are value equal. In this paper, we are interested in the cases defined below.

Definition 2.5 [types of agreement] Let \( n_1, n_2 \) be two nodes with the same label. Let \( p \) be a simple or composite path.

- We say \( n_1 \) and \( n_2 \) node agree or N-agree on \( p \) if
  - \( p \) is a simple path, and \( n_1 \equiv n_2 \); or
  - \( p \) is an upward path, \( n_1[p] \neq \emptyset \) and \( n_1[p] = n_2[p] \); or
  - \( p \) is a composite path, and \( n_1 \) and \( n_2 \) node agree on the upward path part of \( p \).
- We say that \( n_1 \) and \( n_2 \) set agree or S-agree on \( p \) if for every node \( v_i \) in \( n_1[p] \), there is a node \( v_2 \) in \( n_2[p] \) such that \( v_1 \equiv v_2 \), and vice versa.
- We say that \( n_1 \) and \( n_2 \) intersect agree or I-agree on \( p \) if there exist nodes \( v_1 \in n_1[p] \) and \( v_2 \in n_2[p] \) such that \( v_1 \equiv v_2 \).

For example, in Figure 1, \( v_5 \) and \( v_7 \) S-agree on address; \( v_6 \) and \( v_7 \) I-agree on address; \( v_5, v_6 \) N-agree on \( \)·.

It is straightforward to see that N-agreement implies S-agreement, which in turn implies I-agreement. Besides, using induction on the length of paths, we can easily prove the following lemmas.

Lemma 2.1 Let \( n_1 \) and \( n_2 \) be two nodes. Then \( v_1 \equiv v_2 \) if and only if \( n_1 \) and \( n_2 \) S-agree on every simple path.

Lemma 2.2 Let \( n_1 \) and \( n_2 \) be two nodes, and \( p_1 \equiv l_1, \ldots, l_m \) and \( p_2 \equiv l_1, \ldots, l_m, l_{m+1} \) be two simple or composite paths, where \( l_{m+1} \) is not \( \)·.

- We say \( n_1 \) and \( n_2 \) N-agree on \( p_1 \), implies they N-agree on \( p_1 \).
- We say \( n_1 \) and \( n_2 \) S-agree on \( p_1 \), implies they S-agree on \( p_2 \).
- We say \( n_1 \) and \( n_2 \) I-agree on \( p_1 \), implies they I-agree on \( p_2 \).

3 Functional Dependencies

Our definition of functional dependency uses the different types of agreements on paths that are defined in the previous section.

Definition 3.1 [XML functional dependency] Let \( T \) be an XML tree. A functional dependency (FD) on \( T \) is an expression of the form

\[ Q: p_1(c_1), \ldots, p_n(c_n) \rightarrow p_{n+1}(c_{n+1}) \]

where \( Q \) is a downward path, \( p_1, p_2, \ldots, p_n \) are simple or composite paths, \( p_{n+1} \) is a simple path of length 1 or 0, and \( c_i \) (for all \( i = 1, \ldots, n+1 \)) is one of \( N, S, \) and \( I \).

\( T \) is said to satisfy the above functional dependency if, for any two nodes \( n_1, n_2 \in \text{root}[Q] \), the following statement is true: if \( n_1[p_i] \) is not empty, and \( n_1, n_2, c_i \)-agree on \( p_i \) (for all \( i = 1, \ldots, n \)), then \( n_1[p_{n+1}] \) and \( n_2[p_{n+1}] \) are not empty, and \( n_1 \) and \( n_2 \) also \( c_{n+1} \)-agree on \( p_{n+1} \).
In Definition 3.1 we require that the type of agreement be specified for every path. However, some types of agreements are more common than others in practical cases. To simplify the notation, when \( c_i \) is omitted we use the following default types of agreement: for the empty path or an upward path, the default is N-agreement; for all other paths, the default is S-agreement.

Our definition of XFD generalizes those in previously published work, and it can express many different constraints, some of which cannot be expressed by any of the previous XFDs. Here we provide a few examples only.

**Example 3.1** We use the XML tree in Figure 1 in this example.

1. To say that any single telephone number of a student determines his/her set of addresses, we can use
   \[ \sim . \text{student} : \text{tel}(I) \rightarrow \text{address}(S). \]
   This constraint cannot be expressed by any of the previously defined XFDs.

2. To say that student number determines student name for all student nodes, we can use
   \[ \sim . \text{student} : \text{sno} \rightarrow \text{name}. \]
   This constraint can not be expressed using the functional dependencies defined in [Arenas & Libkin 2004] or [Vincent et al. 2004] because the student nodes are located in different paths.

3. To say that the student number determines the student’s set of addresses, we can use
   \[ \sim . \text{student} : \text{sno} \rightarrow \text{address}. \]
   Note that this is different from the multi-valued dependencies defined in [Vincent & Liu 2003], and it cannot be expressed by the XFDs in [Arenas & Libkin 2004], [Vincent et al. 2004], [Lee et al. 2002], or [Wu et al. 2002].

4. To say that the course number determines the course node, i.e., no two course nodes have the same course number, we can use
   \[ \text{course} : \text{cno} \rightarrow \epsilon. \]
   This constraint cannot be expressed by the XFDs in [Hartmann & Link 2003].

5. To say that the course code and student number determine the student grade in that course, we can use
   \[ \text{course}.\text{students}.\text{student} : \text{sno}, \text{cno} \rightarrow \text{grade}. \]

**Example 3.2** Let us now look at the XML tree about textbooks shown in Figure 2. Suppose that the title, the set of authors, and the year of publication can determine the publisher. This constraint can be expressed by the functional dependency

\[ \text{course}.\text{text}. : \text{title}, \text{author}, \text{year} \rightarrow \text{publisher}. \]

The above constraint cannot be expressed using the FDs defined in any of the previous work mentioned above except [Hartmann & Link 2003].

Note that, according to definition, if \( \text{root}(Q) = \emptyset \), that is, \( Q \) is not valid wrt root, then the FD is trivially satisfied by \( T \). Also when checking satisfaction of a FD, we only need to consider those nodes in \( \text{root}(Q) \) that have a non-empty target set for every path on the LHS. In particular, if there is a path on the LHS which is not valid to any node in \( \text{root}(Q) \), then the functional dependency is trivially satisfied by \( T \). Although it appears to be useless to consider invalid paths, this is actually necessary when we define FD assertions on DTDs later.

### 4. Equality-Generating Dependencies

We start with a discussion of EGDs in relational databases first, and define a normal form that extends BCNF to multiple relations. Then we point out similar problems in XML and define EGDs for XML data.

#### 4.1 EGDs in Relational Databases

In relational databases, the traditional normalization technique removes data redundancies within a single relation, but it cannot remove redundancies across relations. For example, if we have two relations

\[ \text{Graduate}(\text{sNo}, \text{stName}, \text{address}) \]

\[ \text{UMember}(\text{stNum}, \text{stName}, \text{phone}) \]

representing graduate students and student union members, where \( \text{sNo} \) and \( \text{stNum} \) both represent student number, \( \text{stName} \) and \( \text{Name} \) both represent student name, then although both relations are in BCNF, there are data redundancies across the two relations if the student information in the two relations overlap. Such redundancies can be detected using equality-generating dependencies (EGDs). An EGD is an expression of the form

\[ R_1.X_1 = R_2.X_2 \rightarrow R_1.Y_1 = R_2.Y_2 \]

where \( R_1, R_2 \) are two relation schemas, \( X_1, Y_1 \) are lists of attributes in \( R_1 \), and \( X_2, Y_2 \) are lists of attributes in \( R_2 \). The EGD specifies that, for any two tuples \( t_1 \) and \( t_2 \) in instances of \( R_1 \) and \( R_2 \) respectively, whenever \( t_1[X_1] = t_2[X_2] \), then \( t_1[Y_1] = t_2[Y_2] \). For instance, for the example above, there is an EGD

\[ \text{Graduate}.\text{sNo} = \text{UMember}.\text{stNum} \rightarrow \text{Graduate}.\text{sName} = \text{UMember}.\text{stName} \]

Due to the above EGD, if the two relations overlap, then there will be data redundancy. To remove such redundancies we can restructure the two tables as follows: if every union member appears in the graduate table, we can change the \( \text{UMember} \) table to \( \text{UMember}(\text{stNum}, \text{phone}) \) and add a foreign key \( \text{"UMember}(\text{stNum}) \) references \( \text{Graduate}(\text{sNo}) \); if only some graduates are union members, and only some union members are graduates, we can add another relation \( \text{Student}(\text{sNo}, \text{Name}) \), modify the original tables to \( \text{Graduate}(\text{sNo}, \text{address}) \) and \( \text{UMember}(\text{stNum}, \text{phone}) \), and add the foreign keys \( \text{"Graduate}(\text{sNo}) \) references \( \text{Student}(\text{sNo}) \) and \( \text{"UMember}(\text{stNo}) \) references \( \text{Student}(\text{sNo}) \).

FDs are special EGDs where \( R_1 = R_2, X_1 = X_2, \) and \( Y_1 = Y_2 \). Like FDs, an EGD can be trivial or non-trivial. An EGD is said to be trivial if it holds in every database schema. For example, every trivial FD is a trivial EGD. Also, the EGD \((*)\) will be trivial if \( Y_1 \) is a subset of \( X_1 \), and \( Y_2 \) is a corresponding subset of \( X_2 \).

Given a set \( E \) of EGDs for a database schema \( D \), we may be able to derive some other EGDs. For example, from \( R_1.x = R_2.x \rightarrow R_1.y = R_2.y \) and \( R_1.y = R_2.y \rightarrow R_1.z = R_2.z \) we can derive \( R_1.x = R_2.x \rightarrow R_1.z = R_2.z \). Let us use \( (D, E)^+ \) to denote the set of all EGDs that hold in \( D \) and that can be derived from \( E \).

We now define a new normal form of a relational database schema that directly extends BCNF.

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1. For simplicity, we consider only EGDs involving two tuples on the left-hand side, but the idea presented here readily extends to general EGDs. Also note our EGDs are not defined on a universal relation as in [Fagin & Vardi 1984].

2. We use list to stress that (1) the attributes in \( X_1 \) (and \( Y_1 \)) are ordered, and (2) an attribute in \( X_1 \) (and \( Y_1 \)) may appear more than once.
Definition 4.1 [normal form wrt EGDs in relational databases] A relational database schema $\mathcal{D}$ is said to be in normal form with respect to a given set $E$ of EGDs, if for every non-trivial EGD

$$R_1 \cdot X_1 = R_2 \cdot X_2 \rightarrow R_1 \cdot Y_1 = R_2 \cdot Y_2$$

in $(\mathcal{D}, E)^+$,

- if $R_1 = R_2$, $X_1 = X_2$, and $Y_1 = Y_2$ then $X_1$ is a superkey of $R_1$;
- Otherwise, there is a corresponding exclusion constraint $R_1 \cdot [X_1] \cap R_2 \cdot [X_2] = \emptyset$, which means that the projections $r_1[X_1]$ and $r_2[X_2]$ are disjoint, where $r_1$ and $r_2$ are instances of $R_1$ and $R_2$ respectively in every possible database instance.

We now briefly discuss about the above definition. For the EGD in the definition, if $R_1 = R_2$, $X_1 = X_2$ and $Y_1 = Y_2$, it becomes a FD $R_1 : X_1 \rightarrow Y_1$. By requiring $X_1$ to be a superkey of $R_1$ we are demanding that $R_1$ be in BCNF with respect to the FD. Therefore, the first condition in our normal form is equivalent to say that all relation schemas in $\mathcal{D}$ are in BCNF with respect to the FDs in $(\mathcal{D}, E)^+$.

Definition 4.2 [semantic value equal] Let $n_1$ and $n_2$ be two nodes in XML tree $T$. We say $n_1$ and $n_2$ are semantically value equal, denoted $n_1 =_{sv} n_2$, if $\text{lab}(n_1) = \text{lab}(n_2)$, and

1. $n_1$ and $n_2$ are both attribute nodes or simple elements, and the two nodes have the same value, or
2. $n_1$ and $n_2$ are both complex elements, and for every child node $a_1$ of $n_1$, there is a child node $a_2$ of $n_2$ such that $a_1 =_{sv} a_2$, and vice versa.

For example, in the XML tree in Figure 3, assuming sno = stno, and name = stname, then the stno attribute node under $v_5$ and the stno attribute node under $v_7$ are semantically value equal, and so are the two attributes name and stname under $v_5$ and $v_7$ respectively.

Definition 4.3 [XML equality-generating dependency] Let $T$ be an XML Tree. An equality-generating dependency (EGD) on $T$ is an expression of the form

$$Q_1 , Q_2 : 1. S_1 =_{sv} 2. S_2 \rightarrow 1. q_1 =_{sv} 2. q_2$$

where $Q_1$ and $Q_2$ are downward paths, $S_1 , S_2$ are lists of simple or composite paths, and $q_1 , q_2$ are simple paths of length 1 or 0. $T$ is said to satisfy the EGD if for every pair of nodes $n_1 \in \text{root}[Q_1]$ and $n_2 \in \text{root}[Q_2]$ the following statement is true: if $n_1[p] \neq \emptyset$ for every $p \in S_1$, and $n_1.S_1 =_{sv} n_2 \cdot S_2$, then $n_1[q_1] \neq \emptyset$, $n_2[q_2] \neq \emptyset$, and $n_1.q_1 =_{sv} n_2.q_2$. □

Example 4.1 The XML tree in Figure 3 satisfies the following EGDs:

$$\text{school}.\text{students}.\text{student}, \text{union}.\text{members}.\text{member} : 1. \text{sno} =_{sv} 2. \text{sno} \rightarrow 1. \text{name} =_{sv} 2. \text{name},$$

$$\text{school}.\text{students}.\text{student}, \text{union}.\text{members}.\text{member} : 1. \text{sno} =_{sv} 2. \text{sno} \rightarrow 1. \text{tel} =_{sv} 2. \text{tel}.$$ □

A functional dependency where the type of agreement is limited to S-agree can be regarded as a special case of EGD with $Q_1 = Q_2$, $S_1 = S_2$ and $q_1 = q_2$, assuming that for all nodes $n_1 , n_2$, $\text{lab}(n_1) = \text{lab}(n_2)$ iff $\text{lab}(n_1) = \text{lab}(n_2)$. □

Figure 3: The Uni-School-Union example
5 Dependencies over DTDs

DTDs and XML Schema documents (we call them XML scheme files) can be used to restrict the structure of XML documents. These files define the legal paths, among other things, in a conforming XML document. On top of a scheme file, we can put further restrictions on the data in conforming documents by insisting that some FDs or EGDs must hold. We call these dependencies assertions.

For simplicity we will focus on DTDs in this paper, but the ideas presented here also apply to any scheme file including XML Schema documents. The following definition of DTDs is adapted from [Arenas & Libkin 2004].

Definition 5.1 [DTD and natural paths] A DTD is defined to be $D = (E_1, E_2, A, P, R, r)$ where $E_1 \subseteq E_2$ is a finite set of complex element names; $E_2 \subseteq E_2$ is a finite set of simple element names; $A \subseteq A$ is a finite set of attributes; $P$ is a mapping from $E_1$ to element type definitions: $\forall \tau \in E_1$, $P(\tau)$ is a regular expression

$$\alpha = \epsilon | \tau' | \alpha|\alpha | \alpha \cdot \alpha^*$$

where $\epsilon$ is the empty sequence, $\tau' \in E_1 \cup E_2$, and $\cdot = \cdot, ..$ denote union, concatenation, and the Kleene closure; $R$ is a mapping from $E_1$ to sets of attributes; $r$ is the element type of the root, which is distinct from all other symbols.

A natural path in $D$ is a string $l_1, \ldots, l_m$, where $l_i$ is in the alphabet of $P(r)$, $l_i$ is in the alphabet of $P(l_{i-1})$ for $i \in [2, m-1]$, $l_m$ is in the alphabet of $P(l_{m-1})$ or in $R(l_{m-1})$. The set of all natural paths in $D$ is denoted $paths(D)$. □

The conformance of an XML tree to a DTD is defined as follows.

Definition 5.2 An XML tree $T = (V, lab, ele, att, val, root)$ is said to conform to an XML scheme file $S = (E_1, E_2, A, P, R, r)$ if

1. lab(root) = $r$,
2. lab maps every node in $V$ to $E_1 \cup E_2 \cup A$.
3. for every complex element node $v \in V$, if ele(v) = $\{v_1, \ldots, v_n\}$, then a permutation of the sequence lab($v_1$), ..., lab($v_n$) must be in the language defined by $P(lab(v))$; if att(v) = $\{v'_1, \ldots, v'_m\}$ then lab($v'_1$), ..., lab($v'_m$) must be in the set $R(lab(v))$.

□

Clearly if XML tree $T$ conforms to DTD $D$, then every simple path of $T$, if valid wrt the root, is in $paths(D)$.

Note that in an abstract DTD of Definition 5.1, there are no constraints such as ID or IDREFS that may exist in a DTD written according to the W3C specification. Mixed contents are not allowed either. On the other hand, every abstract DTD has an equivalent W3C DTD. Since the W3C DTDs are more familiar to readers, we will use them in our examples.

Figure 4 shows an example DTD. The XML tree in Figure 1 conforms to the DTD.

As mentioned earlier, we can make assertions on a DTD $D$. Informally, an assertion is an FD or EGD defined on conforming XML trees, and it asserts that every conforming XML tree $T$ must satisfy the dependency. For an FD or EGD to qualify as an assertion, the paths it contains must be legal paths in $D$, as defined below.

\begin{verbatim}
<ELEMENT school (course|subject)* >
<ELEMENT course (students++)>
<ATTLIST course
cno CDATA #REQUIRED>
<ELEMENT subject (students++)>
<ATTLIST subject
sno CDATA #REQUIRED>
<ELEMENT students (student)>
sno CDATA #REQUIRED
<ELEMENT student
tel, address, grade?">
<ATTLIST student
name CDATA #REQUIRED>
<ELEMENT tel (#PCDATA)>
<ELEMENT address EMPTY> 
<ATTLIST address
street CDATA #REQUIRED>
city CDATA #REQUIRED>
<ELEMENT grade (#PCDATA)>
\end{verbatim}

Figure 4: The Course-Subject-Student DTD

Definition 5.3 [legal path] The legal paths in a DTD are defined as follows:

- A substring (including the empty substring) of a natural path is a legal path, called a legal simple path;
- if $p = l_1, \ldots, l_m$ is a natural path, then the string obtained by replacing some $l_i$ with $\sim$ or by replacing some substring of $p$ with $\sim$ is a legal path, called a legal downward path;
- if there is a legal simple path of length $k > 0$, then $\sim^k$ is a legal path, called a legal upward path.
- if $p$ is a natural path (which may have a length of 0 or more), and $p_{l_1}, \ldots, l_j$ and $p_{l'_1}, \ldots, l'_k$ are natural paths, then

  - $\sim^j \cdot l'_1, \ldots, l'_k$ is a legal path, called a legal composite path;
  - if $p' \cdot l_j$ is a legal downward path obtained from the natural path $p_{l_1}, \ldots, l_j$, then $p' \cdot l_j \sim^j$ and $p' \cdot l_j$. $\sim^j \cdot l'_1, \ldots, l'_k$ are legal paths.

□

Intuitively, a path is a legal path if it is valid wrt some node in at least one conforming document. For example, in the DTD of Figure 4, all of the following are legal paths. $\sim$ student, $\sim$ student.sno, $\sim$ student. $\sim^2$ cno. But name.cno is not a legal path.

We can now formally define assertions.

Definition 5.4 [Assertion] Given a DTD $D$, an assertion $A$ over $D$ is either of the following:

1. a FD assertion

   $Q : p_1(c_1), \ldots, p_n(c_n) \rightarrow p_{n+1}(c_{n+1})$

   where $Q$ is a legal downward path, $p_1, \ldots, p_n$ are legal simple, upward or composite paths, $p_{n+1}$ is a legal simple path of length 1 or 0, $Q_p$ is a legal path, and $c_i \in \{N, S, I\}$, for $i \in [1, n + 1]$.

2. an EGD assertion

   $Q_1, Q_2 : 1.S_1 =_{sv} 2.S_2 \rightarrow 1.q_1 =_{sv} 2.q_2$
where $Q_1$ and $Q_2$ are legal downward paths, $q_1, q_2$ are legal simple paths of length 0 or 1, $S_1$ and $S_2$ are lists of legal simple, upward, or composite paths, and for $i = 1, 2$ and $p_j \in S_i \cup \{q_i\}$, $Q_i \cdot p_j$ is a legal path.

The assertion states that every XML tree $T$ conforming to $\mathcal{D}$ must satisfy the corresponding dependency. □

It is easy to see the FDs in Example 3.1 are assertions over the DTD in Figure 4. Figures 5 and 6 show two more DTDs. The XML trees in Figures 2 and 3 conform to the two DTDs respectively. The FD in Example 3.2 is an assertion over the DTD in Figure 5, and the EGDs in Example 4.1 are assertions over the DTD in Figure 6.

### Definition 5.5
An assertion $\mathcal{A}$ over DTD $\mathcal{D}$ is said to be trivial if every XML tree conforming to $\mathcal{D}$ automatically satisfies $\mathcal{A}$. □

For example, the assertions mentioned above on the three DTDs are all non-trivial. On the other hand, using Lemma 2.1 and 2.2 we can show that the following FD assertions are all trivial (assuming $Q_1, \ldots, L_n \cdot L_{n+1}$ is a natural path):

- $Q : l_1, \ldots, l_n(c) \rightarrow l_1, \ldots, l_n(c)$
- $Q : l_1, \ldots, l_n(N) \rightarrow l_1, \ldots, l_n(S)$
- $Q : l_1, \ldots, l_n(S) \rightarrow l_1, \ldots, l_n(I)$
- $Q : l_1, \ldots, l_n, l_{n+1}(N) \rightarrow l_1, \ldots, l_n(N)$
- $Q : l_1, \ldots, l_n(S) \rightarrow l_1, \ldots, l_{n+1}(S)$
- $Q : l_1, \ldots, l_n(I) \rightarrow l_1, \ldots, l_{n+1}(I)$

Given a set $\mathcal{F}$ of assertions on DTD $\mathcal{D}$, we may infer other dependencies that must be satisfied by all conforming documents of $\mathcal{D}$. As usual, the set of all dependencies that can be derived from $\mathcal{D}$ and $\mathcal{F}$ is denoted $(\mathcal{D}, \mathcal{F})^+$. □

### 6 XML Normal Forms and Data Redundancy

#### 6.1 Normal Form with respect to FD Assertions

In relational databases, a table is in BCNF with respect to a set of functional dependencies if the left hand side (LHS) of every non-trivial functional dependency is a superkey. In XML documents, we can define a normal form along the same line.

We first define keys in XML. Like in relational databases, a key is a special FD.

**Definition 6.1 [key]** If there is a FD assertion $Q : p_1(c_1), \ldots, p_n(c_n) \rightarrow \epsilon$ over the DTD $\mathcal{D}$, then we call $p_1(c_1), \ldots, p_n(c_n)$ a key (of $Q$). □

Intuitively, $p_1(c_1), \ldots, p_n(c_n)$ is a key of $Q$ means that, for every XML tree $T$ conforming to $\mathcal{D}$, and for any two nodes $n_1$ and $n_2$ in $T[Q]$ (of $T$), if $n_1$ and $n_2$ agree on $p_i$ for all $i \in [1, n]$, then $n_1$ and $n_2$ must be the same node.

Our definition of keys differs from those in (Buneman et al. 2002), (Buneman et al. 2001) and (Fan et al. 2001) in two ways. First, we did not consider the ordering of subelements to be of significance, and hence the definitions of value equality are different. Second, we allow several different interpretations of agreements (as represented by the $c_i$s in the definition), while the previous definitions only consider I-agreement (i.e., every $c_i$ is $I$).

**Definition 6.2 [XML normal form wrt FDs]** A DTD $\mathcal{D}$ is said to be in normal form with respect to a set of FD assertions $\mathcal{F}$, if for every no-trivial assertion in $(\mathcal{D}, \mathcal{F})^+$, the LHS is a key. □

For example, the DTD in Figure 4 is not in normal form with respect to the FD assertion $\sim. student : sno \rightarrow name$, because the assertion is non-trivial and we cannot derive $\sim. student : sno \rightarrow \epsilon$ from the DTD and the given assertion. Similarly, the DTD in Figure 5 is not in normal form with respect to the FD assertion $\sim. course.text : title, author, year \rightarrow publisher$.

An alternative definition of normal form is as in Definition 6.3. Recall that nodes $n_1$ and $n_2$ agree on any simple path $p_{n+1}$ is defined to mean $n_1 = n_2$. Therefore Definition 6.3 is equivalent to Definition 6.2.

**Definition 6.3 [XML normal form wrt FDs]** A DTD $\mathcal{D}$ is said to be in normal form with respect to a set of FD assertions $\mathcal{F}$, if for every no-trivial assertion $Q : p_1(c_1), \ldots, p_n(c_n) \rightarrow p_{n+1}(c_{n+1})$ in $(\mathcal{D}, \mathcal{F})^+$, the functional dependency $Q : p_1(c_1), \ldots, p_n(c_n) \rightarrow p_{n+1}(N)$ is also in $(\mathcal{D}, \mathcal{F})^+$. □

---

**Figure 5: The Course-Text DTD**

```xml
courses (course+)
course (text)
course (text)
title (text)
title (text)
author (text)
year (text)
publisher (text)
```

**Figure 6: The Uni-School-Union DTD**

An assertion may or may not add any restrictions to the data in conforming XML documents. For example, the assertion $\sim. student : sno \rightarrow name$ does not add anything to the DTD in Figure 4 because every instance conforming to the DTD will automatically satisfy the assertion. We refer to those assertions that do not add restrictions on the data as trivial assertions. Formally, we have

**Definition 5.5** An assertion $\mathcal{A}$ over DTD $\mathcal{D}$ is said to be trivial if every XML tree conforming to $\mathcal{D}$ automatically satisfies $\mathcal{A}$. □
6.2 Normal Forms with Respect to EGD Assertions

Similar to the normal form with respect to EGDs in relational databases defined in Section 4.1, we can define a normal form with respect to EGDs for DTDs.

**Definition 6.4 [XML normal form wrt EGDs]**

A DTD $D$ is said to be in normal form with respect to a set $F$ of EGD assertions if for every non-trivial EGD

$$Q_1, Q_2 : 1. S_1 =_s v. 2. S_2 → 1. q_1 =_s v. 2. q_2$$

in $(D, F)^+$,

1. if $Q_1 = Q_2, S_1 = S_2$, and $q_1 = q_2$, then $S_1$ is a key of $Q_1$.
2. otherwise the following disjoint constraint holds:

$$Q_1, Q_2 : 1. S_1 =_s v. 2. S_2 → \text{False}$$

which means that in every conforming XML tree, there can not be two nodes $n_1 \in r(Q_1)$ and $n_2 \in r(Q_2)$ such that $n_1, S_1 =_s v. n_2, S_2$.

For example, the DTD in Figure 6 will be in normal form with respect to the EGD assertion

```xml
school.students.student, union.members.member : 1. sno =_s v. 2. stno =_s v. 1. name =_s v. 2. sname
```

if and only if the following constraint hold:

```xml
school.students.student, union.members.member : 1. sno =_s v. 2. stno → 1. e = 2. e
```

which means that there are no student node $s$ and member node $m$ such that $s, sno =_s v. m, stno$. In other words, school students and union members must not overlap.

6.3 Data Redundancies

Normal forms are aimed to reduce data redundancies caused by the assertions. In this section we provide a result on the relationship between our DTD normal forms and XML data redundancies.

We need to formally define data redundancy in XML trees first.

**Definition 6.5 [Data Redundancy in XML]**

Let $D$ be a DTD, $F$ be a set of assertions over $D$, and $T$ be an XML tree conforming to $(D, F)$ (i.e., $T$ conforms to $D$ and satisfies $F$). We say that $T$ has data redundancies with respect to $F$ if there is a node $n$ of $T$ such that the subtree rooted at $n$, if removed from $T$, can be fully recovered using other parts of $T$, $D$, and the assertions in $F$. That is, we can construct a tree $T'$ (to be rooted at the position of $n$) such that $n$ and the root of $T'$ are value equal.

For example, the XML tree in Figure 2, which conforms to the DTD in Figure 5 and satisfies the assertion in Example 3.2, has data redundancy. This is because we can restore the publisher node under the rightmost `text` node if it is removed, by using the DTD and the assertion as well as the publisher node under the leftmost `text` node. Similarly, the XML tree in Figure 1 which conforms to the DTD in Figure 4 and the FD assertion $\sim student \rightarrow sno$, has data redundancies. The XML tree in Figure 3 which conforms to the DTD in Figure 6 and the EGD assertion in Example 4.1 has data redundancies, because if we remove the `sname` attribute under node $v_7$, we can restore it from the corresponding attribute under $v_9$.

The next theorem explains why normal-form DTDs are preferred.

**Theorem 6.1**

Let $D$ be a DTD, $F$ be a set of FD assertions over $D$, and $E$ be a set of EGD assertions over $D$. Then

- There exists an XML tree conforming to $(D, F)$ which has data redundancies iff $D$ is not in normal form with respect to $F$.
- There exists an XML tree conforming to $(D, E)$ which has data redundancies iff $D$ is not in normal form with respect to $E$.

It is important to note that we cannot claim that every XML tree conforming to $(D, F)^+$ will have data redundancy if $D$ is not in normal form. This is particularly true if some FDs use I-agreement on the RHS.

7 Comparison with Related Work

Apparently the earliest work on using XML functional dependencies in the normalization of XML documents appeared in (Wu et al. 2002). The paper defines an XML normal form based on partial and transitive dependencies that try to resemble those in the relational system. The authors made the assumption that the scheme tree of the XML document has the property that there is a unique path from the root to every label. Unfortunately this assumption is not very realistic because, for example, both a subject and a student may have an attribute “sno” as in Figure 4. Clearly the two attributes have different paths from the root. Furthermore, the FDs are limited in expressive power because of the limitation the authors put on the types of agreements of two nodes. In effect, only I-agreement is allowed. More recently, (Lee et al. 2002), (Hartmann & Link 2003), (Arenas & Libkin 2004), and (Vincent et al. 2004), and (the later two both have earlier conference versions) all provide definitions of XML functional dependencies, and (Arenas & Libkin 2004) and (Vincent et al. 2004) went further to define XML normal forms based on their FDs.

(Lee et al. 2002) uses XPaths to define functional dependencies among a set of XML subtrees. An XFD is defined as an expression $(Q, e_1, \cdots, e_n)$ where $Q$ is an XPath, and $e_i$, for $i \in [1, n+1]$, is either an element or an element followed by dot and a set of key attributes of the element. An XML tree is said to satisfy the XFD if for any two subtrees rooted at a node in $root(Q)$, if they agree on the value of $e_1, \cdots, e_n$, then they also agree on the value of $e_{n+1}$, provided these values exist. Unfortunately, it seems that the authors have implicitly assumed that each element name in the XFD corresponds to a single node in the subtree (which is not always the case), because there is no exact definition of the “value” of an element is provided. There is also a strong structural restriction on the path and elements in the FD: if an element’s ancestor appears in the XFD, then so must its parent. It is not hard to see that the expression power suffered from this restriction.

Both (Arenas & Libkin 2004) and (Vincent et al. 2004) regard a simple element as having a child node labelled $S$, under which is attached the value of the element. The definition of XML FDs in (Arenas & Libkin 2004) is based on the so-called tree-tuples. Essentially, the authors treat a DTD as a single relation schema, a distinct path (the path in both (Arenas & Libkin 2004) and (Vincent et al. 2004) are of the form $r.p$ where $p$ is a natural path) in a DTD as an attribute, and a tree-tuple a tuple in that relation. In a tree tuple, each path which ends with an element name is mapped to a distinct node or the null value (⊥), and every other path (ending with an attribute
name or S) is mapped to either a string (PCDATA) or \(\perp\). An XML data tree \(T\) conforming to the DTD can then be regarded as consisting of a set of maximal tree tuples, here maximal means, roughly, that the tree tuple can not be extended by replacing null with non-null values while still being a subtree of \(T\). For example, the XML tree shown in Figure 7 can be regarded as an instance of the relation schema with attributes

\[
\begin{align*}
\text{school}, & \\
\text{school.course}, & \\
\text{school.course} \circ \text{cno}, & \\
\text{school.course.student}, & \\
\text{school.course.student.student}, & \\
\text{school.course.student.student.name}, & \\
\text{school.course.student.student.name.S}, & \\
\text{and the instance consists of the (maximal tree) tuples} (\text{root}, v_1, 01, v_3, v_5, 001, v_8, \text{Mary}), (\text{root}, v_1, 01, v_3, v_5, 002, v_9, \text{Ken}), \text{and} (\text{root}, v_1, 02, v_4, v_7, 001, v_{10}, \text{Mary}).
\end{align*}
\]

An XFD is defined as \(S_1 \rightarrow S_2\), where \(S_1\) and \(S_2\) are sets of paths. An XML document conforming to the DTD is said to satisfy the functional dependency if for every two maximal tree-tuples \(t_1\) and \(t_2\), and every path \(p_2 \in S_2\), whenever \(t_1.S_1 = t_2.S_1\), then \(t_1.p_2 = t_2.p_2\). For example, the XML tree in Figure 7 satisfies the functional dependency

\[
\text{school.course.student.student.name} \rightarrow \text{school.course.student.student.name.s}
\]

but not

\[
\text{school.course.student.student.name.sno} \rightarrow \text{school.course.student.student.name}
\]

(Vincent et al. 2004) defines an XFD that is close to that in (Arenas & Libkin 2004), with a few significant differences nevertheless. Rather than using DTDs, the authors consider a scheme file as a set of closed paths; rather than using a tree tuple to link the values in both sides of the FD, they use the concept of “closest node” to link the values. More specifically, an XFD is of the form

\[
p_1, p_2, \ldots, p_n \rightarrow q
\]

where \(p_i\) and \(q\) are paths. The satisfaction of the above XFD by an XML Tree \(T\) can be checked as follows: [Step 1] For \(i \in [1, n]\), (1) find the common prefix of \(p_i\) and \(q\), denoted \(pre(p_i, q)\); (2) find the target set \(root[pre(p_i, q)]\); (3) for each distinct path instance \(I\) of \(q\), find the unique node \(x_{I,q}\) which is common to \(I\) and \(root[pre(p_i, q)]\); (4) if \(last(p_i)\) is not an element, then find the set \(N_{I,q}\) of nodes which are descendants of \(x_{I,q}\) and are in the target set \(root[p_i]\). Also find the set \(val(N_{I,q})\) of values of the nodes in \(N_{I,q}\). [Step 2] If we cannot find two distinct path instances \(I\) and \(J\), such that for all \(i \in [1, n]\), \(x_{I,i} = x_{J,i}\) (when \(last(p_i)\) is an element), or \(val(N_{I,i}) \cup val(N_{J,i}) \neq \emptyset\) (when \(last(p_i)\) is an attribute or \(S\)), but the value of last node in \(I\) and \(J\) are not equal, then the XNF is satisfied by \(T\). For example, in the XML tree of Figure 7, the XFD (***) can be shown to hold as follows: [Step 1] (1) The common prefix of the two paths in (***) is \(pre = \text{school.course.student.students.student}\), and (2) the target set \(root[pre] = \{v_3, v_6, v_7\}\); (3) for the path instances of the RHS \(I = \text{root}.v_1.v_3.v_5.v_6.v_8.S\), \(J = \text{root}.v_1.v_3.v_5.v_6.v_8.S\), and \(K = \text{root}.v_2.v_4.v_7.v_9.v_{10}.S\), find the nodes \(X_I = v_3, X_J = v_6\) and \(X_K = v_7\); (4) find \(N_I, N_J\) and \(N_K\), which are the singleton sets containing the \(sno\) node under \(v_5, v_6\) and \(v_7\) respectively; [Step 2] Only the values of the nodes in \(N_I, N_J\) and \(N_K\) are equal, but so are the values of last node in the corresponding path instances \(I\) and \(K\). A major difference between (Vincent et al. 2004) and (Arenas & Libkin 2004) lies in the treatment of null values (missing nodes). In the XML tree of Figure 7, if we remove both \(name\) values under \(v_5\) and \(v_7\) (or alternatively, if we remove the \(sno\) value under \(v_5\)), then the XFD (***) is still considered satisfied by (Arenas & Libkin 2004), but not by (Vincent et al. 2004). Another important difference between (Vincent et al. 2004) and (Arenas & Libkin 2004) lies in the set of trivial XFDs. Since the XFDs in (Arenas & Libkin 2004) are defined for a DTD, while the XFDs in (Vincent et al. 2004) are defined for a set of closed paths, some trivial XFDs in (Arenas & Libkin 2004) may turn out to be non-trivial in (Vincent et al. 2004). This is because a DTD puts more restrictions on conforming XML trees than a set of closed paths.

The main problem with the definitions of XFDs in (Arenas & Libkin 2004) and (Vincent et al. 2004) is that some natural constraints can not be expressed, as already seen in Section 3. In addition, since no value equality between two nodes is defined, it is sometimes cumbersome to express some functional dependencies. For instance, in the DTD below (PCDATA elements are omitted),

\[
\begin{align*}
\text{school}\ast & \rightarrow \\
\text{course} & \rightarrow \\
\text{students} & \rightarrow \\
\text{name} & \rightarrow \\
\text{tel} & \rightarrow \\
\text{name} & \rightarrow \\
\text{address} & \rightarrow
\end{align*}
\]

\[
\begin{align*}
\text{name} & \rightarrow \\
\text{tel} & \rightarrow
\end{align*}
\]

\[
\begin{align*}
\text{name} & \rightarrow \text{tel}
\end{align*}
\]
if we want to say student number determines student details, rather than using a simple expression 

\[\text{student number} \rightarrow \text{details}\]

we have to use either a long XFD expression that has 9 paths on the RHS or 9 separate XFDs.

The XFD of (Vincent et al. 2004) is equivalent to a special case of our XML FD except for the treatment of null values for non-element paths on the LHS. We are yet to find a meaningful example where a constraint can be expressed by the FDs in (Arenas & Libkin 2004) but not ours.

Unlike other previous work that use paths to define XFDs, (Hartmann & Link 2003) defines two types of XFDs using *homomorphism*, *v*-subtrees and *isomorphism* of XML trees. Homomorphism between two trees is a mapping from the nodes of one tree to another that preserves the root, label, and kind of nodes, and it is used to define conformity of an XML tree to an XML schema file (represented by a schema tree). A *v*-subtree is a subtree which roots at node *v* and it is determined by the paths from *v* to a subset of leaves of the original tree. The isomorphism of two subtrees is a 1-1 mapping between the two sets of nodes which is homomorphic in both directions. Isomorphism is used to define equivalence between two XML trees. The equivalence of two subtrees is similar to the value equality of their roots. An XFD is defined to be of the form \(v \rightarrow X \rightarrow Y\), where *v* is a node in the schema tree, and *X* and *Y* are *v*-subgraphs in the schema tree. Two types of satisfaction by a constraint involving set equality (like our XFDs) as well as some constraints similar to those in (Arenas & Libkin 2004) and (Lee et al. 2002).

None of the XFDs in the other previous works takes set equality into consideration. We believe that set-equality is natural and common in real applications and should be included in defining data dependencies. Notably both (Roth, Korth & Silberschatz 1988) and (Hara & Davidson n.d.) have considered set equality in their definitions of functional dependencies in nested relations.

We are not aware of any formal definitions of EGDs for XML data, although (Lee & Wu 2000) describes a totally different type of constraints in XML, which they call equality-generating dependencies: if an element *v* can have at most one subelement, then when \(v_1\) and \(v_2\) are known to be subelements of *v*, they must be the same element.

### 8 Conclusion and Future Work

We have studied a new type of FDs as well as EGs for data-centric XML documents, and proposed normal forms of DTDs that prevent data redundancies with respect to these dependencies. However, many issues remain to be resolved, and we plan to investigate these issues in our future work.

As an immediate task, we would like to find efficient algorithms for the implication problem and computation of the closure of our data dependencies. This must be done before we can efficiently check a DTD is in normal form. The normalization process has to be carefully designed too.

We would also like to extend our work to the design of XML documents in which the ordering of subelements is important or where there are mixed contents, because such XML documents are ubiquitous.

### References


Wang, J. (2004), Database design using equality generating dependencies, Technical report, School of Information Technology, Griffith University, Gold Coast, Australia.