For many students, the experience of school mathematics is not a positive one (Clements, 1989). Processes of exclusion operate to disadvantage students along social class, race and gender lines. For students from backgrounds that are not part of the success regime, significant scaffolding by teachers is needed if they are to be successful. In this paper we discuss two key factors that shape the learning environments for learning mathematics. First, the expectations teachers (and students) have of learners of mathematics significantly shape the experiences that will be provided for learners. Through exposure to particular practices, learners come to understand themselves as learners of mathematics in predictable ways. Once they see themselves in particular ways (such as successes or failures), then they hold particular expectations of themselves and what they can expect when they enter mathematics classrooms. The other significant factor is the discourse of “ability” that permeates mathematics more than any other curriculum area. Learners are frequently described (and inscribed) as having some abilities (or not having them) that predispose them to success in mathematics. This notion is treated as unproblematic and is seen as the reason why some students are more likely to be successful (or not) in their study of mathematics. While recognising the potential of individual differences within any
group, in the second section of this paper we propose a number of features of a more inclusive pedagogy that we believe will work toward more equitable outcomes for all students.

**Teacher expectations**

Teacher expectations have been seen to be a salient and significant feature in the success of learners. In their seminal work on teacher expectations Rosenthal and Jacobsen (1969) showed how teachers’ beliefs about learners shaped the experiences that were provided to students and so produced a self-fulfilling prophecy. In the study, students were randomly assigned a number which was read as a score. At the end of the year, there was a correlation between the initial marks and where students ended up. This study was pivotal in highlighting how teachers’ expectations of students shape how they will interact with the learners, the types of activities that will be assigned to learners and how their behaviours will be interpreted and reified in assessments. Such a study would not be possible in today’s research environment but similar work has confirmed the power of teacher beliefs and expectations of learning where teachers held particular views of students from disadvantaged backgrounds (Zevenbergen, 2003). Where teachers have certain expectations of learners, such as that they will not do homework, the parents will not be involved in their children’s learning, the learner is not able to learn due to cultural or linguistic features and so on, then the stage has been set for what will be achieved.

Learners come to the school environment and have particular learning experiences. They come to see themselves within those framings provided by their teachers. For example, it is well documented that the early experiences in the home are often different from those experienced in the school setting. When students are unable to crack the code of classroom mathematics and teaching practices, they come to see themselves as “failed” learners and so develop particular dispositions towards mathematics and have minimal expectations of their achievement potential.

Raising expectations of teachers and learners is critical to reforming mathematics classrooms. Believing that students can learn mathematics enables teachers (and students) to provide rich learning experiences rather than impoverished ones and in the process provide appropriate learning environments to develop conceptual knowledge that is well connected with other areas of mathematics and knowledges beyond the discipline.

**The discourse of ability**

When thinking about why some students are successful and others not, the most frequent explanation is based on some notion of innate ability. However, such a discourse has been challenged as it fails to account for the ways in which practices in school mathematics recognise some features of culture and deny others. In their work with mothers and children, Walkerdine and Lucey (1989) showed how the interactions between working-class mothers and their children are substantially different from those of their middle-class peers. Similarly, Heath (1982) showed how the questioning practices between home and school are very different for some cultural groups. Further, the extensive work of Bernstein (1990) has shown the very different structures of working-class and middle-class families and how the pedagogical practices of schooling favour the middle-class. Drawing on the work of Bernstein, mathematics education scholars such as Cooper (Cooper & Dunne, 1999; Cooper & Harries, 2005), whose work has centred on responses to mathematical assessment items, and Dowling (1991; 1998) whose work has centred on class divisions in mathematics text book series, have shown how practices in mathematics favour middle-
class students. Through subtle processes, such as those listed above, the bias in the mathematics curriculum can be shown to permit the reification of cultural differences as if they were some innate mathematical ability. These critiques of mathematical practices call into question whether success is something innate in learners or whether there are social factors working to allow a myth of ability to be perpetuated and thus support the reproduction of social and cultural inequities.

Research conducted in classrooms where students were streamed according to perceived ability demonstrated that students were able to articulate quite strongly the expectations teachers had of them and the implications of those expectations for them as learners. In a study of secondary mathematics classrooms, this synergy of expectations and ability was evident in student comments (Zevenbergen, 2002). In terms of the students’ perceptions of their teachers’ beliefs in them as learners of mathematics, the comments are poignant. In most cases, they were of the form that the teachers had little confidence in their students’ potential to achieve, and in many cases, the students cited that they felt that their teachers held quite negative beliefs about them. This is evident in the comments below. Pseudonyms are used for the names of students and schools.

Thomas: In this class, all the dumb kids just are here to muck around. The teacher thinks we are dumb and doesn’t really care too much about what we do. (Beechwood, Year 9)

Tyler: I don’t like being in this class ’cause it is the only one I feel dumb in. I mean in English or workshop, I am doing OK, but in maths, I feel like a retard. The teacher treats us as if we know nothing. (St Michaels, Year 9)

Megan: It is like they say, “You are smart and you are dumb,” and then put us in classes where they [the teachers] make sure it happens. (Huon Pine, Year 9).

These comments encapsulate the connection between expectations and the perceptions of ability. The student comments reflect a recognition that their teachers thought they were “dumb,” “stupid,” or “idiots.” This situation has serious implications for the subsequent positioning of students. Converse comments were offered by students in the upper streams where they saw the teachers seeing them as “clever,” “smart” or “intelligent.”

**Rich mathematics or basic skills?**

With a propensity for believing that students from disadvantaged backgrounds are less likely to perform well in mathematics due to some inherently deficit feature (such as ability, work ethic or lack of parental input), there is a risk that the implemented curriculum that is made available to the students is a restricted one. By restricted, we mean that the curriculum is limited in terms of scope and pedagogy. Often the perception that there are gaps in learning means that the students are being exposed to knowledge and processes that are below what would be expected for learners of a particular year level. This is partly due to the view that learning mathematics is a linear process. However, there is now a growing body of research that shows that mathematics is more about networks than linear models. As such, there is a strong case for a “just in time” approach to curriculum rather than a “just in case” mode of learning. Providing a rich curriculum with strong connections between other areas of mathematics and beyond mathematics becomes more critical when working with students from disadvantaged backgrounds.

Past practices have been premised on models where the students are taught through a largely skill and drill approach in which basic skills were central to the curriculum. A more contemporary model, based on research, offers greater potential for deep learning of mathematics.

Drawing heavily on the work from
productive pedagogies where the intellectual quality of tasks is the focus of teaching (Hayes, Mills, Christie & Lingard, 2006), the selection or design of mathematical tasks becomes critical. Of primary importance is the richness and depth of the mathematics learning that is facilitated through the task. The task can vary in duration but it is a significant move away from the small lesson activity that dominates much contemporary practice. By creating learning opportunities that encourage depth of learning, it is recognised that learning takes time and cognitive energy. The short activities that occupy significant curriculum time in mathematics (Education Queensland, 2001), offer little opportunity for depth of learning. Further, rote and skill/drill learning is very shallow learning. Drawing on Burton’s (2004) work on research mathematicians, the task should allow for students to work mathematically. Creating opportunities for the ‘aha’ moments; for connections among mathematical ideas; to draw on early learnings (of the group) in order to build richer conceptual learning; collaborate and share knowledge; to intuit, rationalise, conjecture, hypothesise, test ideas, justify, and challenge mathematical ideas; and to represent thinking in a range of modes, are key to the selection of tasks that will enable and foster deep mathematical learning. Tasks should allow for multiple entry points and multiple pathways, and cater for the diversity in thinking and working mathematically.

On the basis of extensive work in UK and US schools, Boaler (1997; 2008) has reported that students from disadvantaged backgrounds can achieve deep conceptual understandings of mathematics when they are provided with rich mathematical experiences. Her work showed that students exposed to rich mathematics, that may be problem based, in the context of heterogeneous groupings achieve well, are motivated to learn, and can gain great affective, social and cognitive benefits. The work undertaken at ‘Railside’ in California (Boaler, 2008) showed how a school that was the poorest performing in the state was able to move to above state average in a few years when it adopted a program that encompassed these principles in the teaching of mathematics. Furthermore, there were significant social outcomes as well — the students learned how to interact positively with their peers.

Reforming mathematics classrooms for equity

Up to this point, we have provided a strong rationale for adopting an approach to mathematics teaching that is substantially different from the more traditional approaches to teaching mathematics. In this final section, we discuss some of the features that make up a more inclusive pedagogy. These are expanded elsewhere (Zevenbergen & Niesche, 2008).

Working as a mathematician

We draw heavily on Burton’s work (Burton, 2001; 2004) with research mathematicians and how they go about their work. Her work posed serious challenges to the pedagogies found in so many classrooms. By showing how mathematicians work, Burton proposed that the pedagogical practices of contemporary classrooms needed to be changed. Her work showed that mathematicians value highly collaborative work; that mathematicians have emotional, aesthetic and personal responses to mathematics; that intuition and ‘aha’ moments are common; and that mathematicians desire to seek and see rich connections between the various branches of mathematics and between mathematics and other disciplines.

Group work

The value of collaboration in learning is widely recognised and yet in many mathematics classrooms learning mathematics is an individual pursuit. By enabling students to work in groups where each member is able to bring their own particular strengths and
knowledges to the situation, there is greater opportunity for students to build on each others’ thinking and so come to a richer understanding than would be possible if working alone. However, the group work must be well structured so that it is not the case of students sitting in a group working individually. The tasks must be carefully chosen so that a variety of skills are needed for the resolution of the task. In this context, collective strengths enable the group to complete a task that would be more difficult (if not impossible) by working alone. The group assumes responsibility for the learning of all members in the group so that if one student does not appear to understand the concepts that are the focus of the lesson or activity, then they need to be supported by their peers in order to understand the work.

Roles defined
The introduction of group work entails considerable background work to be undertaken so that students are able to make the most of collaborative learning. In part this is due to the widely held view that mathematics is an individual pursuit. From Cohen and Lotan’s (1997) work, roles within the group are defined. They argue that one of the key roles is that of group leader who assumes responsibility for identifying when all members of the group seem to have developed the appropriate understandings that will be robust enough for teacher scrutiny. The group leader will make a decision as to when to call the teacher to the group.

Teacher as facilitator
The teacher’s locus of control is substantially different in this approach. Rather than directing the lesson, the teacher must select or design activities that will enable students to work independently of the teacher. Appropriate scaffolds need to be developed in advance so that students are able to take control of their own learning.

Questioning
The role of questioning is a key aspect of this approach. The questions posed in mathematics classrooms are often low order, recall type questions that result in low levels of intellectual quality. To shift to a higher level of thinking, questions that foster deeper knowledge and access deeper understandings are required. A simple taxonomy of questions (Biggs & Collis, 1982) can be used to develop questions to stimulate rich conversations — either in the group work or at other phases of a lesson; questions that, for example, ask students to justify, clarify, or extend their thinking strongly align with the ways of working as a mathematician.

Rich mathematical tasks
As discussed earlier, the selection or design of mathematical tasks is critical. Of primary importance is the richness and depth of the mathematics learning that is facilitated through the task. Tasks can vary in duration but, in general, a move away from the small lesson activity that dominates much contemporary practice is recommended.

Multiple representations
Creating opportunities and scope for students to express their mathematical thinking and reasoning in ways that suit the individual, the context, and/or the task allows for greater inclusion in activities. Learners may have preferences in terms of how they think through problems. For example, there may be preferences for using logic and reasoning, drawing pictures, using mathematical notations and so on. Creating space in the curriculum to cater for these different ways of thinking, learning, and representing opens up learning opportunities for students. This is particularly the case when students are encouraged to share their ways of thinking with their peers. Such communications can happen within the small group work or at the end of the lesson. By sharing their representations with peers students can access other ways of thinking and representing.
mathematics thereby extending their current modes of operating.

**Reporting back**
Rather than using the final session of a lesson for the students to ‘show and tell’ this session can be an important opportunity for learning. Not only should students report back on their group work, but interactions between groups should also be a central aspect of the dialogue. Students may need to be scaffolded in learning how to pose questions that will support their peers in articulating their thinking and working as mathematicians. With appropriate guidance, students can be expected to pose questions to the reporting group that will seek to clarify the processes they used as they came to their understandings. Their peers can: prompt them to justify the processes used and/or knowledge created; seek clarification about aspects that are unclear; provide scaffolding when it appears that there has been an error or misunderstanding; and support their peers to move towards deeper understandings about the work that has been undertaken. Both the reporting group and the other members of the class are assisted to develop richer understandings and connections.

**Conclusion**
The approach we have suggested in this paper can produce enhanced learning outcomes for students from disadvantaged backgrounds and for students generally. We also acknowledge that leadership is critical to the implementation of any reform and, as we have shown elsewhere (Zevenbergen, Walsh & Niesche, 2009 in press), it is important that teachers become involved in the implementation process and develop strong support mechanisms to ensure that reforms are sustained. In making changes that enhance equity, teachers can be assured that all students will benefit.

**References**
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