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A Theory of Forgetting in Logic Programming*

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Abstract

The study of forgetting for reasoning has attracted considerable attention in AI. However, much of the work on forgetting, and other related approaches such as independence, irrelevance and novelty, has been restricted to the classical logics. This paper describes a detailed theoretical investigation of the notion of forgetting in the context of logic programming. We first provide a semantic definition of forgetting under the answer sets for extended logic programs. We then discuss the desirable properties and some motivating examples. An important result of this study is an algorithm for computing the result of forgetting in a logic program. Furthermore, we present a modified version of the algorithm and show that the time complexity of the new algorithm is polynomial with respect to the size of the given logic program if the size of certain rules is fixed. We show how the proposed theory of forgetting can be used to characterize the logic program updates.

Introduction

The ability of discarding irrelevant information is a key feature that an intelligent agent must possess to adequately handle reasoning tasks such as query answering, planning, decision-making, reasoning about actions, knowledge update and revision. This ability is referred to as forgetting (Lin and Reiter 1994) or elimination (Brown 1990), and, often, studied under different names such as irrelevance, independence, irredundancy, novelty and separability (see Subrahmanian et al. 1997; Lang et al. 2003) for more details). For example, we have a knowledge base $K$ and a query $Q$. It may be hard to determine if $Q$ is true or false directly from $K$. However, if we discard or forget some part of $K$ that is independent of $Q$, the querying task may become much easier.

According to (Lin and Reiter 1994), if $T$ is a theory in propositional language and $p$ is a ground atom, then the result of forgetting $p$ in $T$ is denoted forget($T, p$) which can be characterized as $T_{\text{true}}(p) \lor T_{\text{false}}(p)$, i.e. the disjunction of two theories obtained from $T$ by replacing $p$ by true and false, respectively. For example, if $T = \{\text{cooking}\lor\text{cleaning}\}\land\text{singing}\}$, then forget($T, \text{singing}$) is the theory $\{\text{cooking} \lor \text{cleaning}\}$.

The notion of forgetting has found its applications in artificial intelligence. However, the existing theories of forgetting are mainly investigated in the context of classical logics. It would be interesting to establish a theory of forgetting in logic programming and nonmonotonic reasoning. This issue is first considered in (Zhang et al. 2005) and consequently two kinds of forgetting are defined (the strong and weak forgettings) by first transforming a logic program $P$ into a reduced form and then deleting some rules (and literals). While they have been used to resolve conflicts in logic programming, these approaches suffer from some shortcomings: (1) There is no semantic justification for the strong or weak forgetting. Specifically, the relationship between the semantics of a logic program and the result of the strong or weak forgetting is unclear. (2) It is not addressed in (Zhang et al. 2005) that what are the desirable properties for a reasonable notion of forgetting in logic programming. In particular, one may ask what is the difference of these notions of forgetting from traditional approaches to deletion of rules/literals in logic programming and databases. (3) More importantly, both of the strong and weak forgettings are syntax-sensitive. That is, equivalent programs may have different results of forgetting about the same literal. For example, $P = \{p \leftarrow q \leftarrow \neg p\}$ and $P' = \{p \leftarrow\}$ are equivalent programs under the answer sets. However, WForgetLP($P, p$) = $\{q \leftarrow\}$ and WForgetLP($P', p$) = $\{\}$ are not equivalent. Here WForgetLP($P, p$) denotes the result of the weak forgetting about $p$ in $P$.

Thus, a more reasonable notion of forgetting is highly desirable for nonmonotonic reasoning (and logic programming). In this paper, we choose answer set programming (ASP) (Lifschitz 2002) as the underlying nonmonotonic logic. ASP is a paradigm of logic programming under the answer sets (Gelfond and Lifschitz 1990) and it is becoming one of the major tools for knowledge representation due to its simplicity, expressive power, connection to major nonmonotonic logics and efficient implementations. First of all, we believe that a reasonable semantic notion of forgetting in ASP should satisfy the following criteria. Let $P$ be a logic program and $P'$ be the result of forgetting about a literal $l$ in $P$.

(F1) The proposed forgetting is a natural generalization of
We deal with extended logic programs (Gelfond and Lifschitz 1990) whose rules are built from some atoms where default negation not and strong negation ¬ are allowed. A literal is either an atom or its strong negation ¬a. For any atom a, we say a and ¬a are complementary literals. For any set X of literals, not X = \{not l | l ∈ X\}.

An extended logic program is a finite set of rules of the following form

\[ l_0 ← l_1, \ldots, l_m, \text{not } l_{m+1}, \ldots, \text{not } l_n \quad (1) \]

where \( l_0 \) is either a literal or empty, each \( l_i \) is a literal for \( i = 1, \ldots, n \), and \( 0 ≤ m ≤ n \). If \( l_0 \) is empty, then the rule is a constraint.

Without loss of generality, we consider propositional programs. For technical reasons, it should be stressed that the body of a rule is a set of literals rather than a multiset. For instance, \( a ← \text{not } b, c, c \) is not a rule in our sense while \( a ← \text{not } b, c \) is a rule. That is, we assume that any rule of a logic program has been simplified by eliminating repeated literals in its body.

If a rule of form (1) contains no default negation, it is called positive; \( P \) is a positive program if every rule of \( P \) is positive.

If a rule of form (1) contains no body atoms, it is called negative; \( P \) is a negative program if every rule of \( P \) is negative.

Given a rule \( r \) of form (1), \( \text{head}(r) = l_0 \) and \( \text{body}(r) = \text{body}^+(r) ∪ \text{not } \text{body}^-(r) \) where \( \text{body}^+(r) = \{l_1, \ldots, l_m\} \), \( \text{body}^-(r) = \{l_{m+1}, \ldots, l_n\} \). The set \( \text{head}(P) \) consists of all literals appearing in rule heads of \( P \).

In the rest of this section we assume that \( P \) is an extended logic program and \( X \) is a set of literals. If \( X \) does not contain a pair of complementary literals, we say \( X \) is an interpretation of \( P \).

A rule \( r \) in \( P \) is satisfied by an interpretation \( X \), denoted \( X \models r \), iff “if \( \text{body}^+(r) ⊆ X \) and \( \text{body}^-(r) ∩ X = ∅ \), then \( \text{head}(r) \in X \)”. An interpretation \( X \) is a model of \( P \), denoted \( X \models P \) if every rule of \( P \) is satisfied by \( X \).

The answer set semantics

The reduct of logic program \( P \) on a set \( X \) of literals, written \( P^X \), is obtained as follows:

- Delete every \( r \) from \( P \) such that there is a \( \text{not } q \in \text{body}^-(r) \) with \( q \in X \).
- Delete all negative literals from the remaining rules.

Notice that \( P^X \) is a set of rules without any negative literals. Thus \( P^X \) may have no model or have a unique minimal model, which coincides with the set of literals that can be derived by resolution.

An interpretation \( X \) is an answer set of \( P \) if \( X \) is the minimal model of \( P^X \).

A logic program may have zero, one or more answer sets. We use \( P \) to denote the collection of answer sets of \( P \). A program is consistent if it has at least one answer set. Two logic programs \( P \) and \( P' \) are equivalent, denoted \( P \equiv P' \), if they have the same answer sets.

As usual, \( B_P \) is the Herbrand base of logic program \( P \), that is, the set of all (ground) literals in \( P \).

Forgetting in Logic Programming

In this section we introduce a semantic definition of forgetting for extended logic programs. That is, we want to define what it means to forget about a literal \( l \) in a logic program \( P \). The intuition behind the forgetting theory is to obtain a logic program which is equivalent to the original logic program if we ignore the existence of the literal \( l \).

It is direct to forget a literal \( l \) in a set \( X \) of literals, that is, just remove \( l \) from \( X \) if \( l \in X \). This notion of forgetting can be easily extended to subsets. A set \( X' \) is an \( l \)-subset of \( X \),

Preliminaries

We deal with extended logic programs (Gelfond and Lifschitz 1990) whose rules are built from some atoms where default negation not and strong negation ¬ are allowed. A literal is either an atom or its strong negation ¬a. For any atom a, we say a and ¬a are complementary literals. For any set X of literals, not X = \{not l | l ∈ X\}.

An extended logic program is a finite set of rules of the following form

\[ l_0 ← l_1, \ldots, l_m, \text{not } l_{m+1}, \ldots, \text{not } l_n \quad (1) \]
denoted $X' \subseteq X$, if $X' - \{l\} \subseteq X - \{l\}$. Similarly, a set $X'$ is a true $l$-subset of $X$, denoted $X' \subset X$, if $X' - \{l\} \subset X - \{l\}$.

Two sets $X$ and $X'$ of literals are $l$-equivalent, denoted $X \sim_l X'$, if and only if ($X - X'$) $\cup$ ($X' - X$) $\subseteq \{l\}$.

Given a consistent logic program $P$ and a literal $l$, we could define a result of forgetting about $l$ in $P$ as an extended logic program $P'$ whose answer sets are exactly $\parallel P \parallel - l = \{X - \{l\} \mid X \in \parallel P \parallel\}$. However, such a notion of forgetting cannot even guarantee the existence for some simple programs. For example, consider $P = \{(a \leftarrow p, q), (q \leftarrow \neg p\}$. Then $\parallel P \parallel = \{(a, p), (a, q)\}$ and thus $\parallel P \parallel - p = \{(a, q)\}$. Since $\{a\} \subseteq \{a, q\}$ and each answer set must be minimal, $\parallel P \parallel - p$ cannot be the set of answer sets of any logic program. So we need a notion of minimality of answer sets which can naturally combine the definition of answer sets, minimality and forgetting together.

If a logic program is inconsistent, it would make no sense to talk about the result of forgetting. However, we could discuss the possibility of removing inconsistency from a logic program. For example, a logic program $P$ may have partial stable models (Saccà & Zaniolo 1991) while it is inconsistent under the answer set semantics (i.e. the stable model semantics). This topic may be interesting but not in the scope of this paper. In the rest of this paper, we assume that $P$ is a consistent program.

**Definition 1** Let $P$ be a consistent logic program, $l$ a literal in $P$ and $X$ a set of literals.

1. We say that $X$ is $l$-minimal in a collection $S$ of sets of literals if $X \in S$ and there is no $X' \in S$ such that $X' \subseteq X$ and $X' \subset X$. In particular, denote $Sp$ is the set of models of $P$, then we say $X$ is an $l$-minimal model of a logic program $P$ if $X$ is a model of $P$ and it is $l$-minimal in $Sp$.

2. An answer set $X$ of an extended logic program $P$ is $l$-minimal if it is $l$-minimal in $\parallel P \parallel$. In this case, $X$ is also called an $l$-answer set of $P$.

For $P = \{a \leftarrow q \leftarrow \neg p \leftarrow \neg q\}$, we have two answer sets $X = \{a, p\}$ and $X' = \{a, q\}$. $X$ is a $p$-answer set of $P$ but $X'$ is not. This example shows that, for a logic program and a literal $l$, an answer set may not be an $l$-answer set.

Having the notion of minimality about forgetting a literal, we are now in a position to define the result of forgetting about a literal in a logic program.

**Definition 2** Let $P$ be a consistent logic program and $l$ be a literal. A logic program $P'$ is a result of forgetting about $l$ in $P$ if the following conditions are satisfied:

1. $B_{P'} \subseteq B_P - \{l\}$.
2. For any set $X'$ of literals, $X'$ is an answer set of $P'$ if and only if there is an $l$-answer set $X$ of $P$ such that $X' \sim_l X$.

Notice that the first condition implies that $l$ does not appear in $P'$. This condition can also be replaced with $B_{P'} = B_P - \{l\}$ by adding a rule $q \leftarrow q$ to $P'$ for every $q \in B_P - B_{P'}$ with $q \neq l$.

An important difference of the notion of forgetting here from existing approaches to logic program update and merging is that only $l$ and possibly some other literals are removed. In particular, no new symbol is introduced in $P'$.

A logic program $P$ may have different logic programs as results of forgetting about the same literal $l$. However, it follows from the above definition that any two results of forgetting about the same literal in $P$ are equivalent under the answer set semantics.

**Proposition 1** Let $P$ be an extended logic program and $l$ a literal in $P$. If $P'$ and $P''$ are two results of forgetting about $l$ in $P$, then $P'$ and $P''$ are equivalent (i.e. they have the same answer sets).

We use $\text{forget}(P, l)$ to denote the result of forgetting about $l$ in $P$.

**Examples**

1. If $P_1 = \{q \leftarrow \neg p\}$, then $\text{forget}(P_1, p) = \{q\}$ and $\text{forget}(P_1, q) = \{}$.
2. If $P_2 = \{q \leftarrow \neg p. p \leftarrow \neg q\}$, then $\text{forget}(P_2, p) = \{}$. The reason is that $P_2$ has two answer sets $\{p\}$ and $\{q\}$ but only $\{p\}$ is a $p$-answer set of $P_2$. Thus, $\text{forget}(P_2, p)$ has a unique answer set $\{\}$.
3. Consider $P_3 = \{q \leftarrow \neg p, p \leftarrow \}$, which has the unique answer $\{p\}$. Thus, $\text{forget}(P_3, p) = \{\}$ rather than $\{q\}$. This is intuitive because we are forgetting all impacts of $p$ on $P_3$. In particular, “forgetting about $p$” is different from “assuming $\neg p$”.
4. Let $P_4 = \{a \leftarrow \neg b, b \leftarrow \neg a. p \leftarrow \neg a. c \leftarrow \neg p\}$. According to (Zhang et al. 2005), the weak forgetting $\text{WForgetLP}(P_4, p) = \{a \leftarrow \neg b. b \leftarrow \neg a. c \leftarrow \}$; the strong forgetting of $\text{SForgetLP}(P_4, p) = \{a \leftarrow \neg b. b \leftarrow \neg a\}$. However, $\text{forget}(P_4, p) = \{a \leftarrow \neg b. b \leftarrow \neg a. c \leftarrow \}$. In the next section we will explain how to obtain $\text{forget}(P, l)$.

The following proposition generalizes Proposition 1 and shows that the criteria (F4) is satisfied.

**Proposition 2** Let $P$ and $P'$ be two equivalent logic programs and $l$ a literal in $P$. Then forget$(P, l)$ and forget$(P', l)$ are also equivalent under the answer sets.

Now we show that our notion of forgetting does satisfy (F3).

**Proposition 3** For any consistent program $P$ and a literal $l$ in $P$, the following two items are true:

1. An $l$-answer set $X$ of $P$ must be an answer set of $P$.
2. For any answer set $X$ of $P$, there is an $l$-answer set $X'$ of $P$ such that $X' \subseteq X$.

This result implies that, if $l$ is ignored, forget$(P, l)$ is equivalent to $P$ under both credulous and skeptical reasoning with respect to the answer set semantics.

The above definitions of forgetting about a literal $l$ can be extended to forgetting about a set $F$ of literals. Specifically, we can similarly define $X_1 \subseteq F X_2$. $X_1 = F X_2$ and $F$-answer sets of a logic program. Those properties of forgetting about a single literal can also be generalized to the
case of forgetting about a set. Moreover, the result of forgetting about a set $F$ can be obtained one by one forgetting each literal in $F$.

**Proposition 4** Let $P$ be a consistent program and $F = F' \cup \{l\}$. Then
\[
\text{forget}(P, F) \equiv \text{forget} \left( \text{forget}(P, l), P' \right).
\]

**Computation of Forgetting**

Since Definition 2 is a semantic one, it does not guarantee the result of forgetting about $l$ in $P$. So one important issue is to study the problem of computing the result of forgetting. In the following we will justify the criterion (F5) by developing algorithms for computing $\text{forget}(P, l)$ using program transformations. The basic idea is to **equivalently** transform the original program $P$ into a standard form $N$ first and then to obtain $\text{forget}(P, l)$ directly from $N$.

**Program transformations**

The program transformations in this section are introduced in (Brass et al. 2001). $P$ and $P'$ are extended logic programs.

**Elimination of Tautologies** $P'$ is obtained from $P$ by the elimination of tautologies if there is a rule $r$ in $P$ such that $\text{head}(r) \in \text{body}^+ (r)$ and $P' = P - \{r\}$.

**Elimination of Contradiction** $P'$ is obtained from $P$ by the elimination of contradiction if there is a rule $r$ in $P$ such that $\text{head}(r) \cap \text{body}^-(r) \neq \emptyset$ and $P' = P - \{r\}$.

**Positive Reduction** $P'$ is obtained from $P$ by the positive reduction if there is a rule $r$ in $P$ and $c \in \text{body}^- (r)$ such that $c \notin \text{head}(P)$ and $P'$ is obtained from $P$ by removing not $c$ from $r$.

**Negative Reduction** $P'$ is obtained from $P$ by negative reduction if there are two rules $r$ and $r'$ in $P$ such that $\text{body}(r') = \emptyset$ and $\text{head}(r') \in \text{body}^-(r)$, and $P' = P - \{r\}$.

Let $r$ and $r'$ be two distinct rules in a logic program. We say $r'$ is an implication of $r$ if $\text{head}(r) = \text{head}(r')$ and $\text{body}(r) \subseteq \text{body}(r')$.

**Elimination of Implications** $P'$ is obtained from $P$ by the elimination of implications if there are two distinct rules $r$ and $r'$ of $P$ such that $r'$ is an implication of $r$ and $P' = P - \{r\}$.

For two rules $r$ and $r'$ with $\text{head}(r') \in \text{body}^+(r)$, the unfolding of $r$ with $r'$, denoted $\text{unfold}(r, r')$, is the rule $\text{head}(r) \leftarrow (\text{head}(r') - \{\text{head}(r')\}), \text{body}(r')$.

**Unfolding** $P'$ is obtained from $P$ by unfolding if there is a rule $r$ such that
\[
P' = P - \{r\} \cup \{\text{unfold}(r, r') \mid r' \in P, \text{head}(r') \in \text{body}^+(r)\}.
\]

A special case of Unfolding is that there is no rule $r'$ such that $r'$ is resolved with $r$. In this case, $P' = P - \{r\}$.

$T$ denotes the set of the program transformations introduced above.

**Lemma 1** (Brass et al. 2001) Every logic program can be transformed into a canonical form by $T$, which is a negative program.

This lemma is also true for extended logic programs.

**Algorithms for Computing** $\text{forget}(P, l)$

We are now ready to present our basic algorithm for computing a result of forgetting about a given literal in a logic program.

**Algorithm 1** (Computing a result of forgetting)

Input: logic program $P$ and a literal $l$.

Procedure:

1. Transform $P$ into its canonical form $N$.
2. Suppose that $N$ has $n$ rules having head $l$ (but body does not contain $l$ due to the Elimination of tautology) where $n \geq 0$:
\[
r_j : l \leftarrow \text{not } l_{j1}, ..., \text{not } l_{jm_j}
\]

where $j = 1, \ldots, n$ and $m_j \geq 0$ for all $j$.

   If $n = 0$, then $Q$ is the program obtained from $N$ by removing all appearances of not $l$.
   
   If $n = 1$ and $m_1 = 0$, then $l \leftarrow$ is the only rule in $N$ having head $l$. In this case $D_1$ is defined as false.

   If $n > 1$ and $m_1 > 0$, then $m_1 > 0$, for $i = 1, \ldots, s$, by the Elimination of implications. Let $D_1, \ldots, D_s$ be all possible conjunctions $(l_{1k_1}, \ldots, l_{nk_s})$ where $0 \leq k_1 \leq m_1, \ldots, 0 \leq k_n \leq m_n$.

   Replace every appearance of not $l$ in rule bodies of $N$ by all possible $D_i$ and the resulting program is denoted $Q$.

3. Step 3. Remove all rules with head $l$ from $Q$ and the resulting program is denoted $N'$.

Output $N'$.

The following result shows that this algorithm is sound wrt. the semantic forgetting.

**Theorem 1** For any consistent program $P$ and a literal $l$, let Algorithm 1 returns $N'$. If $X'$ is an answer set of $N'$, then $X'$ is also an answer set of $\text{forget}(P, l)$.

Consider two examples.

**Example 1** Let $P = \{ a \leftarrow \text{not } b. \ b \leftarrow \text{not } c. \ c \leftarrow \text{not } b. \ p \leftarrow \text{not } a, \text{not } c. \ d \leftarrow \text{not } p \}$. Since $P$ is a negative program, we can directly get $\text{forget}(P, p) = \{ a \leftarrow \text{not } b. \ b \leftarrow \text{not } a. \ c \leftarrow \text{not } b. \ d \leftarrow \text{not } a. \ d \leftarrow \text{not } c \}$.

Consider another program for which program transformations are needed.

**Example 2** Let $P = \{ p \leftarrow \text{not } p_1. \ p \leftarrow \text{not } p_1. \ p \leftarrow \text{not } p_2. \ p \leftarrow \text{not } p_2 \}$. Then it can be equivalently transformed into the program $N = \{ p \leftarrow \text{not } p_1 \}$. So forget($P, p$) = { $p_1 \leftarrow$}, forget($P, q$) = { $p \leftarrow \text{not } p_1 \}$. forget($P, p_1$) = { $p \leftarrow \text{not } p_1 \}$.

For some special programs, it can be easier to compute forget($P, l$).

**Corollary 2** Let $P$ be a consistent program and $N$ be the canonical program obtained from $P$ by $T$. Then
1. If \( l \in X \) for every \( X \in \| P \| \), then forget\((P, l)\) is the program obtained from \( N \) by removing all rules containing \( l \). In this case, forget\((P, l)\) coincides with the strong forgetting defined in (Zhang et al. 2005).

2. If \( l \not\in X \) for every \( X \in \| P \| \), then forget\((P, l)\) is the program obtained from \( N \) by removing all rules with head \( l \) and all appearances of negative literal not \( l \). In this case, forget\((P, l)\) coincides with the weak forgetting defined in (Zhang et al. 2005).

While Algorithm 1 provides a canonical form for the result of forgetting, it is exponential in the worst case. This may be from two sources: the Unfolding and the construction of \( D_i \) for \( i = 1, \ldots, s \). However, we can replace the Unfolding by a restricted version, that is, only unfolding on \( l \).

**Unfolding on a given literal**: \( P' \) is obtained from \( P \) by unfolding on a literal \( l \) if there is a rule \( r \) such that

\[
P' = P - \{r\}
\]

\[\cup \{\text{unfold}(r, r') | \text{there is a rule } r' \in P \text{ such that } \text{head}(r') = l, l \in \text{body}(r)\}.
\]

Notice that we require head\((r') = l \) here. If we just want to obtain forget\((P, l)\), some program transformations are not actually needed.

**Algorithm 2** Input: logic program \( P \) and a literal \( l \).

Procedure:

- **Step 1.** Fully apply Elimination of Tautology, Elimination of Contradiction, Negative Reduction and the Unfolding on \( l \) to program \( P \) and the resulting program is denoted \( N \).

- **Step 2.** Suppose that \( N \) has \( n \) rules whose head contains \( l \) (but body does not contain \( l \)) due to the Elimination of tautology) where \( n > 0 \):

\[
r_j : l \leftarrow l_{j1}, \ldots, l_{jm_j}, \text{not } l_{j1}, \ldots, \text{not } l_{jm_j}
\]

where \( n_{j1}, n_{j} \geq 0 \) for all \( j (1 \leq j \leq n) \).

If \( n = 0 \), then \( N' \) is the program obtained from \( N \) by removing all appearances of not \( l \).

- If \( n = 1 \), \( n_1 = 0 \) and \( n_{m_1} = 0 \), then \( l \leftarrow \) is the only rule in \( N \) whose head is \( l \). In this case \( D_1 \) is false.

- If \( n \geq 1 \) and \( n_1 + n_{m_1} > 0 \), then \( n_i + n_{m_i} > 0 \) for \( i = 1, \ldots, s \), by the Elimination of implications. Let \( D_1, \ldots, D_s \) be all possible conjunctions \((L_{1k_1}, \ldots, L_{nk_n})\) where each \( L_{ik} \) is either not \( l_{ik} \) or \( l_{ik} \), for \( i = 1, \ldots, n \).

- Replace every appearance of not \( l \) in rule bodies of \( N \) by all possible \( D_i \) and the resulting program is denoted \( Q_i \).

- **Step 3.** Remove all rules with head \( l \) and the resulting program is denoted \( N' \).

- Output \( N' \) as forget\((P, l)\).

Similar to Theorem 1, we can prove that Algorithm 2 is correct but the proof is more tedious since \( N \) is not negative.

**Theorem 3** For any consistent program \( P \) and a literal \( l \), Algorithm 2 always returns a result of forgetting about \( l \) in \( P \).

Notice that the first step in Algorithm 2 can be finished in polynomial time. In most cases, the size of rules having head \( l \) is not very large compared to the size of the whole program. So we can get a polynomial algorithm for computing forget\((P, l)\) if the size of rules having head \( l \) is bounded.

**Corollary 4** If the size of rules in \( P \) having head \( l \) is bounded, then Algorithm 2 is polynomial in the size of \( P \).

**Proof Sketch of Theorem 1**

Since \( P \) is finite, Algorithm 1 will finally terminate. We need only to show that, for any set \( X' \) of literals (with \( l \not\in X' \)), if \( X' \) is an answer set of \( N' \), then there exists an \( l \)-answer set \( X \) of \( N \) such that \( X \subseteq X' \).

\( N \) can be split into three disjoint parts: \( N_1 \cup N_2 \cup N_3 \) where \( N_1 \) consists of rules in \( N \) in which \( l \) does not appear; \( N_2 = \{ r \in N | l \not\in \text{head}(r), l \not\in \text{body}(r) \} \); \( N_3 = \{ r \in N | l \not\in \text{head}(r), l \not\in \text{body}(r) \} \).

Let \( N'_2 \) be the program obtained from \( N_2 \) by performing transformations in Step 2. Then \( N' = N_1 \cup N_3 \).

Let \( D_1, D_2, \ldots, D_s \) denote the conjunctions constructed from \( l_{ij} \) in Step 2. So each rule \( r \) in \( N \) with \( l \in \text{body}(r) \) corresponds to \( s \) rules in \( N' \): head\((r) \leftarrow D_i, (\text{body}(r) = \{\text{not } l\}) \) for \( i = 1, \ldots, s \).

If \( n = 0 \) or \( n = 1 \) and \( n_{m_1} = 0 \), the proof is direct. So we assume that \( n \geq 1 \) and \( n_{m_i} \geq 1 \) for \( i = 1, \ldots, s \). Consider two possible cases:

1. Suppose that \( X'' \) is the least model of \( N' \).

   - **Case 1.** \( X \not\models D_i \) for all \( i = 1, \ldots, s \):

     - (1.1) We first show that \( X' = X \cup \{l\} \) is a model of \( N \).

     - Since \( l \not\in X \), we have

       \( N'' = (N_1 \cup N_2 \cup N_3)' = (N_1)' \cup (N_2)' \).

     - It can be shown that \( X' \) is a model of both \( (N_1)' \) and \( (N_2)' \).

     - Thus \( X'' \) is a model of \( N' \).

   - (1.2) We then show that \( X'' = X' \cup \{l\} \) is an \( l \)-minimal model of \( N'' \).

     Suppose that \( X'' \) is the least model of \( N'' \). Notice that \( l \in X' \) because the rule \( l \rightarrow \) is in \( N' \). Thus we need only to show that \( X'' \) is the least model of \( N'' \). (1.2) We then show that \( X' = X \cup \{l\} \) is an \( l \)-minimal model of \( N'' \). By the assumption, \( (X'' - \{l\}) \not\models D_i \) for all \( i \). Thus \( (X'' - \{l\}) = (N'' - \{l\}) \), which implies that \( (X'' - \{l\}) = X'' \).

2. Suppose that \( X'' \) is the least model of \( N'' \).

   - **Case 2.** If \( X \models D_{i_0} \) for some \( i \) with \( 1 \leq i_0 \leq s \) we need only to show that \( X \) is an \( l \)-answer set of \( N \).

   - (2.1) Since \( N_X = (N_1)' \cup (N_2)' \cup (N_3)' \), we can show that \( X \) is a model of \( N_X \).

   - (2.2) If \( X \subseteq X' \) such that \( X' \models N_X \), then we can show that \( X \) is also an \( l \)-answer set of \( N \) by distinguishing two possible subcases: \( l \in X' \) and \( l \not\in X' \).

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**Logic Program Update by Forgetting**

The theory of forgetting developed in previous sections is a very general framework for update, revision, merging, inheritance hierarchy and even preference handling in logic programming. In particular, the modified PMA (Doherty et al. 1998), the update answer sets (Eiter et al. 2002), inheritance answer sets (Bucaferri et al. 1999) and dynamic answer sets (Alferes et al. 1998) can all be captured by our forgetting operator. As an example, we show in this section how to characterize the update answer sets using forgetting.

An update sequence is an ordered sequence \( \mathcal{P} = [P_1, P_2, \ldots, P_t] \) where each \( P_i \) is an extended logic program for \( i = 1, \ldots, t \) and \( t \geq 1 \).
Informally, \( P_{t+1} \) is assumed to update the information represented by \([P_1, \ldots, P_t]\). So \( P_{t+1} \) represents more recent information than \( P_t \) and thus the rules in \( P_{t+1} \) are assigned higher priority in case of a conflict.

In Eiter et al.’s approach, an update sequence \( P \) is first translated into a single logic program \( P' \) and the answer set semantics of \( P' \) is defined as the set of answer sets of \( P' \). So this is basically a syntactic approach.

To define the translated program \( P' \), for each literal \( l_i \), we introduce new literals \( l_i \) and \( l_i^r \) where \( i = 1, \ldots, t \). For each rule \( r \), a new literal \( rej(r) \) is also introduced. The extended Herbrand base is denoted \( B' \).

**Definition 3** The update program \( P_{t+1} = P_1 \sqcup \cdots \sqcup P_t \) over \( B' \) consists of the following rules:

1. All constraints in \( P_i \) (1 \( \leq i \leq t \)).
2. For each \( r \in P_i \) (1 \( \leq i \leq t \)):
   \[ l_i \leftarrow \text{body}(r), \text{not rej}(r) \quad \text{if head}(r) = l \]
   \[ l_i^r \leftarrow \text{body}(r), \text{not rej}(r) \quad \text{if head}(r) = \text{not } l \]
3. For each \( r \in P_i \) (1 \( \leq i \leq t \)):
   \[ rej(r) \leftarrow \text{body}(r), l_{i+1} \quad \text{if head}(r) = l \]
   \[ l_i^r \leftarrow \text{body}(r), l_{i+1} \quad \text{if head}(r) = \text{not } l \]
4. For each literal \( l \) in \( P \),
   \[ l_i \leftarrow l_{i+1}; \quad l_i \leftarrow l_i \quad \text{if } l_i \in l_i \]

A set \( X \) of literals is an answer set of the update sequence \( P \) if \( X = X' \cap B' \) for some answer set \( X' \) of \( P_{t+1} \).

To characterize the update semantics, we define
\[
F(X) = \bigcup_{1 \leq i \leq t} F_i(X)
\]
where the sequence \( F_1(X), \ldots, F_t(X) \) for \( P \) are recursively defined as \( F_i(X) = \emptyset \), and for \( i < t \),
\[
F_i(X) = \{ \text{head}(r) \mid \text{there exist } r \in P_i \text{ and } r' \in P_j \text{ s.t. } \text{head}(r) \text{ and } \text{head}(r') \text{ are complementary, } X \models (\text{body}(r) \cap \text{body}(r')), \text{head}(r') \notin F_{i+1}(X) \cup \ldots \cup F_t(X) \}.
\]

The intuition behind \( F(X) \) is that a literal will be forgotten provided that its validity causes conflict with more recent information.

**Theorem 5** Let \( P = [P_1, \ldots, P_t] \) be an update sequence and \( X \) a set of literals. Denote \( \hat{P} = P_1 \cup \cdots \cup P_t \). Then \( X \) is an update answer set of \( P \) iff \( X \) is an answer set of \( \text{forget}(P, F(X)) \).

**Conclusion**

We have proposed a novel semantic approach to forgetting for reasoning with logic programs. The suitability of this notion of forgetting is justified against four criteria as well as illustrating examples. An important result is the algorithm for computing the result of forgetting in a logic program. Furthermore, we show that a variant of the algorithm is polynomial in the size of the given logic program if the size of certain rules is fixed. The proposed theory of forgetting is a very general framework for a variety of AI tasks including merging, update and revision of logic programs. In particular, we show that the update answer sets can be intuitively captured in our framework. Issues for future research include (1) The relation of our approach to relevance, independence and novelty. Note that the notion of relevance for reasoning can be naturally defined once the forgetting is defined. (2) Applications of our theory of forgetting in characterizing some other approaches to conflict resolving, for example, extended abduction introduced in (Sakama and Inoue 2003). (3) Determining classes of logic programs such that the computing of forgetting is computationally tractable.

**References**


