An Improved Evolutionary Algorithm Applied to Multi-Objective Inverse Problems

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This paper presents a methodology based on multi-objective evolutionary algorithms for the designs of electromagnetic devices. The proposed algorithm allows one to optimize multiple heterogeneous criteria in complex systems. To strike a balance between the diversity and the quality of the final solutions, the entire search process is divided into an exploration and an exploitation phases. In order to keep the diversity of the Pareto solutions in the global search phase, two approaches are proposed: a diversity-oriented mechanism and an intensification search strategy. To identify a potential point which will yield a promising solution in the local search phase, an auxiliary objective function is proposed. By examining the selected performance indicators on two different numerical examples, this improved algorithm is found to be statistically competitive with the conventional algorithms in terms of keeping the diversity of the individuals along the trade-off surface, tending to extend the Pareto front to new areas, and finding a well-approximated Pareto optimal front.

Key Words: Optimal design, Multi-objective evolutionary algorithm, Pareto optimal.

1. Introduction

In many scientific and engineering disciplines, it is not uncommon to face a design challenge when there are several, and often conflicting, criteria or design objectives to meet simultaneously. Then the problem becomes one of finding the best possible designs that satisfy the competing objectives under different tradeoff scenarios. Therefore, multi-objective optimization has become a very demanding research topic. However, there are still many open issues which are needed to answer qualitatively and quantitatively. As it is well known, the presence of multiple objectives in a problem gives rise to a set of optimal solutions (Pareto optimal solutions in terminology). This feature demands a designer to find as many Pareto solutions as possible. Consequently, the searching of the Pareto-optimal set of a multi-objective problem involves two conflicting objectives, i.e., the distance to the true Pareto front is to be minimized, while the diversity of the generated solutions is to be maximized (in sense of objective or parameter spaces). However, this ultimate goal is far from being accomplished by the traditional evolutionary algorithms and most of these methods have difficulty in dealing with the tradeoff between uniformly distributing the computational resources and finding the near-complete and near-optimal Pareto set. On the other hand, no formal assurance of an algorithm’s general effectiveness exists if insufficient knowledge of the problem characteristics is incorporated into algorithm domain. In this regard, an improved vector evolutionary algorithm, whose entire search process is divided into a local and a global search phases with the goals of finding more promising solutions in local phase and preserving diversities of the solutions in global phase, is proposed.

Without a loss of generality, the following minimization problem is considered:

\[ \text{Minimize } f_i(x) (i = 1, \ldots, N_{obj}) \]  \hspace{1cm} (1)

\[ \text{Subject to: } \begin{cases} g_j(x) & \geq 0 \ (j = 1, \ldots, M) \\ h_k(x) & \leq 0 \ (k = 1, \ldots, K) \end{cases} \]  \hspace{1cm} (2)

where, \( f_i \) is the \( i^{th} \) objective functions, \( x \) is a decision vector that represents a feasible solution, \( N_{obj} \) is the number of objectives, \( M \) and \( K \) are the numbers of equality and inequality constraints, respectively.

For a multi-objective optimization problem, any two solutions \( x^1 \) and \( x^2 \) can have one or two possibilities: one dominates the other or none dominates the other. For a minimization problem as formulated in (1) and (2), solution \( x^1 \) dominates
solution $x^2$ if the following two conditions are satisfied:

$$\forall i \in \{1, 2, ..., N_{\text{obj}}\}: f_i(x') \leq f_i(x^2)$$  \hspace{1cm} (3)

$$\exists j \in \{1, 2, ..., N_{\text{obj}}\}: f_j(x') < f_j(x^2)$$  \hspace{1cm} (4)

If any of the above conditions is violated, solution $x^1$ does not dominate solution $x^2$. If $x^1$ dominates solution $x^2$, $x^1$ is called a nondominated solution. The solutions that are nondominated within the entire search space are called Pareto-optimal and constitute the Pareto-optimal set or Pareto-optimal front.

2. An Improved Evolutionary Algorithm

To strike a best balance between the quality and the diversity of the searched Pareto solutions, some improvements on available evolutionary algorithms are proposed in this paper.

2.1 Fitness Assignment

Similar to the Strength Pareto Evolutionary Algorithm (SPEA) [1], the fitness assignment in the proposed algorithm consists of two steps: (1) the individuals in the external non-dominated set ($P^E$) are ranked; (2) the individuals in the population ($P$) are evaluated. However, the strength of an external solution $\bar{x}^{(i)}$ is proportional to the number of population members $\bar{x}^{(j)} \in P$ for which $\bar{x}^{(i)}$ dominates $\bar{x}^{(j)}$, instead of the member for which $\bar{x}^{(i)}$ covers $\bar{x}^{(j)}$, lending the algorithm with the ability to find enough Pareto-optimal solutions.

2.2 Exploration Search

As the population diversity of an evolutionary algorithm is increased, the genotype of the offspring differs more from the parents. Accordingly, a highly diverse population can increase the probability of exploring more Pareto solutions and prevent a premature convergence to a part of the entire Pareto set. To guarantee the diversity of the Pareto solutions, the entire search process of the proposed algorithm is divided into an exploration (global) and an exploitation (local) phases. The goal of the exploration search is to find more Pareto solutions. For this purpose, the following two approaches are introduced.

1) A diversity-oriented mechanism

The idea behind this mechanism is described as follows:

(1) choose $m$ solution sets ($m$ is the number of objectives) in each generation according to a rule that the fitness of the $i^{th}$ set is mainly determined by the $i^{th}$ objective function;

(2) select individuals from every set of the $m$ sets as the parents to produce the offspring through crossover and mutation operations to increase the local diversity of the individuals;

(3) decide a new population from all solutions which are available in the current generation.

2) An interpolating search technique

In this method, the searched Pareto solutions are grouped into a series of clusters, and the two neighbor clusters with the maximum distance are denoted as Cluster $A$ and Cluster $B$. The proposed global search phase will then strive to search in the sub-region between Cluster $A$ and Cluster $B$ by interpolating points between point $A$ and Point $B$ to try to seek Pareto solutions in this sub-region.

2.3 Exploiting Search

In the proposed algorithm, at the end of every generation, the search will begin an exploiting search by selecting a solution from the current population. To identify a potential point which will yield a promising solution in this local search phase, an auxiliary objective function that is composed of the usual inner product and Euclidean norms of the individuals in the parameter space is proposed as

$$p_0(\bar{x}) = \frac{\nabla f_i(\bar{x}) \cdot \nabla f_i(\bar{x})}{||\nabla f_i(\bar{x})|| ||\nabla f_i(\bar{x})||} + 1$$  \hspace{1cm} (5)

Obviously, this function $p_0(\bar{x})$ reaches its minimum function value of zero at all Pareto optimal points. In other word, this function provides some metric to measure the 'closeness' of a feasible point to the nondominated solutions of the optimal problems. Therefore, if one selects the starting points using a Roulette wheel selection scheme with the probabilities which are inversely proportional to the function values as defined in (5), the proposed local search phase will intensify searches around the points with smaller function values of $p_0(\bar{x})$, equipping the algorithm with the ability to find better Pareto Solutions.

Once the starting point is selected, one then adds some small perturbations to the selected solution and evaluates their objective functions. This process will be repeated until a better or a new Pareto solution is found, or a maximum iteration number is reached.
The details of this iteration process are described as follows:

Step 1 Select, stochastically, a seed, $\bar{x}^{(i)}$, from solutions of current population according the rule as described previously.

Step 2 Generate, randomly, a set of new candidates, $\bar{y}_j^{(i)}$, according to a predefined neighbour set $H$ \{H | $h_j = h_{j,i} / c$ (j = 1, 2, ..., r}\}, and seed $\bar{x}^{(i)}$ by using

$$\bar{y}_j^{(i)} = \bar{x}^{(i)} + r_j h_j$$ (6)

Step 3 Evaluate the new candidates. If a better or a new Pareto solution is found, terminate this local search phase; Otherwise, go to next step.

Step 4 The maximum iterative number is reached? If it is true, terminate the local search phase; Otherwise, go to Step 1.

In (6), $r_j$ is a random parameter out of the interval of [-1, 1].

2.4 Iterative procedures of the entire algorithm

For the convinience of the application of the proposed algorithm, its iterative procedures are described in a step by step way as follows:

Step 1 Initialization: Set algorithm parameters, initial population, and external Pareto set;
Step 2 Start the exploration search phase;
Step 3 Start the exploitation search phase;
Step 4 Terminate test. If the test is passed, stop; Otherwise, go to Step 2.

3. Numerical Examples

To test the performance of the proposed evolution algorithm for multi-objective design problems, the numerical results on two examples are reported.

3.1 A mathematical optimization problem

In order to validate the improved vector evolutionary algorithm, it is firstly used to solve a mathematical function of two variables and two objectives [2]. The problem is defined as:

$$\min \left\{ \begin{array}{l}
f_1(x_1, x_2) = \frac{1}{x_1^2 + x_2^2 + 1} \\
n_2(x_1, x_2) = x_1^2 + 3x_2^2 + 1 \\
\end{array} \right. \quad (7)$$

s.t. \begin{align*}
-3 \leq x_1 \leq 3 \\
-5 \leq x_2 \leq 5
\end{align*}

The proposed algorithm is utilized to find the Pareto solutions of this mathematical function, and the searched Pareto solutions in the objective space are given in Fig. 1. Obviously, the computed results demonstrate that the proposed method is more robust in view of uniformly sampling the entire Pareto-optimal surface.

![Fig. 1. The searched Pareto solutions by using the proposed algorithm for the mathematical function.](image)

3.2 Case study—an inverse problem

To further certify the performance of the proposed multi-objective evolutionary algorithm, it is then used to solve a case study—an inverse problem in the optimal design of a rotating generator with no load [3]. This vector design problem is specifically described as: with optimizing the geometrical design of the winding, it is able to maximize the magnetic field of $y$ direction in the air gap ($B_y$), and to minimize the magnetic field of $x$ direction in the interface of winding and air ($B_x$).

The geometrical model of the generator is shown in Fig. 2. For this case study, the widths of the winding, $(X_1, X_2)$, are considered as the decision variables. The domain of the decision variables are set as: $0.01 < x_1 < 0.034$, $x_1 + 0.006 < x_2 < x_1 + 0.01768$. The current density in the field windings is set to a constant value of $J = 0.1A/m^2$. In the numerical implementation, $B_y$ and $B_x$ are directly computed from the finite element solution of the generator. In the optimization process, a fixed mesh of 1404 triangular elements and 2917 nodes is used.

The searched Pareto solutions by using the proposed algorithm in both the parameter space and the objective space are given, respectively, in Fig. 3 and Fig. 4. The mesh and the equipotential lines of a Pareto solution searched by the proposed algorithm are shown, respectively, in Fig. 5 (a) and Fig. 5 (b). Again, it is evident that the proposed method is more
effective in uniformly sampling the entire Pareto-optimal surface and preventing a premature convergence of the algorithm. Moreover, to compare the performance of the proposed algorithm with other well developed vector optimizers, this problem is also solved by using a simulated annealing based one [4]. The numerical results shown that the Pareto solutions searched by the two algorithms are virtually the same, but the iterative number used by the proposed one is only about 75% of that used by the simulated annealing based algorithm.

Fig. 2. The schematic diagram of the geometrical model of the generator

Fig. 3. The searched Pareto solutions by using the proposed algorithm in the parameter space.

Fig. 4. The searched Pareto solutions by using the proposed algorithm in the objective space.

Fig. 5. (a) The mesh of a specific Pareto solution, (b) the equipotential lines of a specific Pareto solution.

4. Conclusions

Some new ideas to improve the solution quality while preserving the diversity of evolutionary algorithms are proposed in this paper. The two numerical examples as reported in this paper elucidate that the proposed improved evolutionary algorithm can not only efficiently find the Pareto solutions, but also distribute them uniformly in the Pareto surface, of a multiobjective optimal problem. Consequently, this paper provides a feasible vector optimizer for studying engineering multi-objective design problems.

References


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