ANALYSIS OF RAIL FAILURE DATA FOR DEVELOPING PREDICTIVE MODELS AND ESTIMATION OF MODEL PARAMETERS

Chattopadhyay G [a] and Rahman A [b],

[a,b] School of Engineering Systems, Queensland University of Technology, 2 George Street, P.O Box 2434, Brisbane, QLD 4001, AUSTRALIA

Servicing strategy of a rail network is developed by understanding reliability of rails used in the rail track system. Reliability analysis of rails can be carried out by understanding the failure mechanism of rail through modelling and analysis of failure data. These failure data are time or usage dependent for certain conditions. In a probabilistic sense, rail failure is a function of its usage in terms of Million Gross Tones (MGT) for certain conditions. This paper is to analyse real life rail industry data, deal with the limitations of available data and develop predictive models for maintenance and replacement decisions. Parameters of the model are estimated using real world data with an application of non-homogeneous Poisson process.

Key Words: Service Contract, Rail failure, Maintenance, Cost Model.

1 INTRODUCTION

Rails play a significant role in transporting goods and passengers. In Australia, railway transport industry contributes 1.6% of GDP with goods and services worth $AUD 8 billion each year which includes $ AUD 0.5 billion per year in exports [1]. Maintenance or servicing of rail track plays an important role in the reliability and safety since failure of track in operation is costly due to loss of service, properties, and loss of lives. Current trend is to contract out the maintenance and servicing actions of rail track to outside agencies rather than performing in-house maintenance [2], [3]. Servicing of such complex system which includes inspection, planned or preventive maintenance (PM) and corrective maintenance actions incurs huge cost. Therefore, before signing in contracts the infrastructure providers need to estimate costs of such contracts and analyse benefits out of it.

In a probabilistic sense, failure due to degradation Million Gross Tones (MGT) for certain conditions [4]. This paper analyses real life data for predictive models and replacement decisions. Parameters of the model are estimated. A decision model is proposed for developing service contracts useful for outsourcing.

Outline of this paper is as follows: Section 1 provides the introduction with scope and outline of this paper. Section 2 defines rail degradation process in brief. Probabilistic failure models are developed in Section 3. Overview of failure and costs models for service contract is described in Section 4. Section 5 analyses data to estimate parameters. In the final section contribution of this paper is discussed along with scope for future work.

2 DEGRADATION OR FAILURE OF RAIL TRACK

Degradation or failure of rail track is a complex process and it depends on the rail materials, traffic density, speed, curve radius, axle loads, Million Gross Tonnes (MGT), wheel rail contact, rail track geometries and importantly the servicing strategies. The rail profile and curves make large contributions to rail degradation. Rail tracks are designed to reduce the contact stresses and the twisting effect of the wheel load. Wheel loads produce bending moment and shear forces in the rail causing longitudinal compressive and tensile stresses concentrated mainly in the head and foot of the rail and shear stresses in the web.

Corrosion and surface cracks have significant influence on the rail brakes. Traffic wear, rolling contact and plastic deformation are the growing problems (For details see [5]). To predict the rail failures and decide on maintenance strategies for rail track it is necessary to model the degradation/failures and associated costs related to maintenance actions of rail.

3 FAILURE MODELLING

Ageing takes place in the line due to tonnage accumulation on track resulting from traffic movement leading to defect. It is realistic to assume that initiated defects left in the system will continue to grow with increase in cumulative MGT. Rail failures/breaks can be modelled as a point process with an intensity function $\Lambda(m)$ where m represents Millions of Gross Tonnes (MGT) and $\Lambda(m)$ is an increasing function of m indicating that the number of failures in a statistical sense increases with MGT. That means older rails with higher cumulative MGT passed through the section is expected to have more probability of initiating defects and if undetected then through further passing of traffic can lead to rail failures. As a result, the number of failures till an accumulated MGT is a function of usage MGT, m, and is a random variable and can be modelled using Weibull distribution [6]. Let cumulative MGT of rail, m, be known and $F(m)$ and $f(m)$denote the cumulative rail failure distribution and density function respectively,

$$f(m) = dF(m)/dm. \tag{1}$$

Here we have,

$$F(m) = P\{m_1 \le m\} \text{ where } m_1 \text{ is the MGT to rail failure} \tag{2}$$

This can be modelled as Weibull distribution given by:

$$F\ (m) = 1 - \exp(-(\lambda m)^{\beta}) \tag{3}$$

And from equation 1

$$f(m) = \lambda\beta(\lambda m)^{\beta-1}\exp(-(\lambda m)^{\beta}) \tag{4}$$

with the parameters β (Known as shape parameter of the distribution) > 1 and

λ (Known as inverse of characteristic function for the distribution)> 0

β greater than 1 indicates an increasing failure rate of the item under study and ageing is predominant in failure mechanism.

Then failure intensity function $\Lambda(m)$ is derived from (1) and 2 is given by

$$\Lambda(t) = \frac{f(m)}{1-F(m)} = \frac{\lambda\beta(\lambda m)^{\beta-1}\exp(-(\lambda m)^{\beta})}{1-(1-\exp(-(\lambda m)^{\beta})} = \lambda\beta(\lambda m)^{\beta-1} \tag{5}$$

Rail track is normally made operational through repair or rectification of the failed segment and no action is taken with regards to the remaining length of the rail in case of detected defects and rail breaks. Since the length of failed segment replaced at each failure is very small relative to the whole track, the rectification action having negligible impact on the failure rate of the track as a whole [7]. Based on these rail failure/break models, in the following sections we discuss the potential servicing strategies of outsourcing rail network and we also propose servicing strategies and cost models for those service contracts.

4 MODELS FOR PREDICTED FAILURES AND COST OF SERVICE CONTRACT FOR RAIL

Tables and illustrations should be arranged throughout the text and it is preferable to include them on the same page as they are first discussed. They should have a self-contained caption and be positioned in flush-left alignment with the text margin. For modelling purpose, we consider one segment in considered to be rail of 110 meter long. Here, we view planned grinding and lubrication activities as preventive maintenance for rail and repair and replacement of the cracked or broken portion/s of rail segment as minimal repair action since the repair replacement of such small part can not improve the overall reliability of the rail. The three defined policies of service contracts modelled by Rahman and Chattopadhyay [8] are used here to determine the costs of service contracts for rail. ***Policy S1***: **Short-term Service Contract:** here the expected life of rail is longer than the contract period and the contract period L is prefixed. This policy assumes that no replacement is necessary during the contract period which implies that $R > L$.

where L and R are the contract period and the first replacement (renewal) of rail due to complete failure respectively (See figure 1). Rahman and Chattopadhyay [8] proposed a cost model for policy S1 which is expressed as follows

Here, preventive maintenance actions are taken at constant interval x, which restores the reliability of the system to some extent and it is constant for each PM. In case of rail, preventive maintenance actions are mostly rail grindings and lubrications. In between two successive PM there may be a number of minimal repairs that can not improve the reliability significantly. Examples of minimal repair action are replacing a small damaged portion of rail segment or welding the cracks etc. The following notations were used in their model

N: number of times the planned grinding and lubrication are performed during the contract period.

k: number of times PM is carried up to an MGT *m*

τ age restoration after each PM. $\tau = \alpha x$, *where,* α is the quality of the maintenance, α ranges from 0 to 1.

When $\alpha = 1$ signifies– ‘as good as new” and $\alpha = 0$ is ‘as bad as old’.

C_{mr} cost for each minimal repairs

C_{pm} cost for each PM

Hazard rate $h_{pm}(m)$ can be expressed as

$$h_{pm}(m) = h(m - k\tau) \tag{6}$$

where,

$h_{pm}(m)$: hazard rate at accumulated MGT, *m*, with maintenance.

$h(m)$: original hazard rate at *m* when no maintenance is performed.

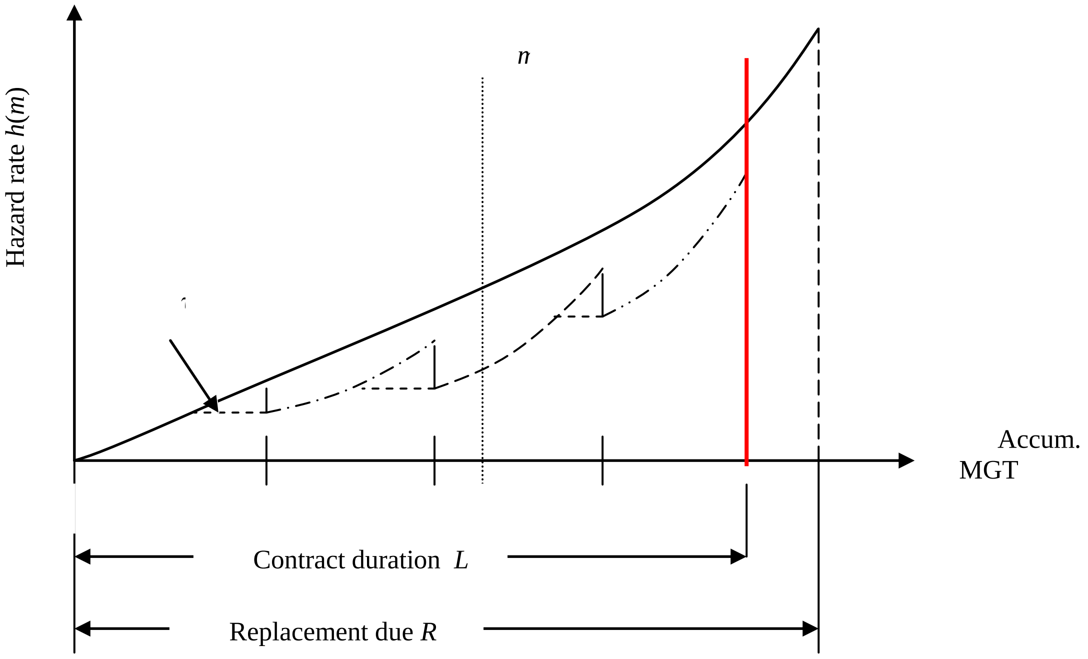

Figure 1: Graphical representation of the model S1

Legends

τ	*reliability of restoration in MGT*
$h(m)$	*failure rate distribution when there is no maintenance*
$h_1(m)$	*failure rate distribution after 1st PM*
$h_2(m)$	*failure rate distribution after 2nd PM*
$h_3(m)$	*failure rate distribution after 3rd PM*

For this type of service contract, the total expected cost per unit time $C(L,x, N_i)$ can therefore be expressed as

Total expected cost of minimal repair + total expected cost of planned PM over the contract period.

Expected total cost of all minimal repairs over the contract period can be given by

$$C_{mr}\sum_{k=0}^{N}\int_{kx}^{(k+1)x} h_{pm}(m)dm \tag{7}$$

Now substituting equation 6 in equation 2, expected total cost of minimal repair can be given by

$$C_{mr}\sum_{k=0}^{N_i}\int_{kx}^{(k+1)x} h(m-k\tau)dm \tag{8}$$

When failures are modelled as per Weibull distribution then, the failure rate is given by:

$$h(m)=\lambda^{\beta}\beta m^{\beta-1} \tag{9}$$

Therefore, from eqn.. 9 and eqn. 8,

Expected cost of minimal repair

$$C_{mr}\left\{\sum_{k=0}^{N_i}\lambda^{\beta}\beta\int_{kx}^{(k+1)x} (m-k\tau)^{\beta-1}dm\right\}$$

$$C_{mr}\left\{\sum_{k=0}^{N_i}\lambda^{\beta}x^{\beta}\left[(k-k\alpha+1)^{\beta}-(k-k\alpha)^{\beta}\right]\right\} \tag{10}$$

where, $\tau=\alpha\times x$, where, α is the quality of PM .

Expected cost of preventive maintenance during the contract

$$(N)C_{pm} \tag{11}$$

The total expected cost per unit time $C(L,x, N_i)$ can therefore be expressed as

$$C(L,x,N)=\frac{1}{L}\left[C_{mr}\left\{\sum_{k=0}^{N+1}\lambda^{\beta}x^{\beta}\left[(k-k\alpha+1)^{\beta}-(k-k\alpha)^{\beta}\right]\right\}+C_{pm}N\right] \tag{12}$$

Where, λ and β are the Weibull parameters

5 ANALYSIS OF RAIL DATA

The table 1 represents 208 rail break data in Million gross tonnes (MGT) obtained from the Sweden. The failure or breakage MGT in the analysis is generated as follows: Usage span is considered as 720 MGT. A plot of the accumulated number of rail break versus the accumulated breakage MGT is displayed in the figure 2. The plotted data indicates the usage dependent failure or breaks.

Table 1

Rail breaks in Million gross tonnes (MGT)

46	92	115	115	161	161	161	184	184	184	184	184
184	184	184	184	207	207	207	207	207	207	230	230
230	230	230	230	230	230	253	253	253	253	253	253
253	253	253	253	253	253	253	253	253	253	253	276
276	276	276	276	276	276	276	276	276	276	276	276
276	276	276	276	276	276	276	276	276	276	276	276
276	299	299	299	299	299	299	299	299	299	299	299
299	299	299	299	299	299	299	299	299	299	299	299
299	299	299	299	299	299	299	299	299	299	299	322
322	322	322	322	322	322	322	322	322	322	322	322
322	322	322	322	322	322	322	322	322	322	322	322
322	322	345	345	345	345	345	345	345	345	345	345
345	345	345	345	345	345	368	368	368	368	368	368
368	368	368	368	368	391	437	460	460	483	483	483
506	506	506	506	506	506	529	529	529	529	529	529
529	529	529	552	552	552	552	575	575	575	575	575
575	575	575	575	575	575	575	598	667	667	667	667
667	690	690	713	-	-	-	-	-	-	-	-

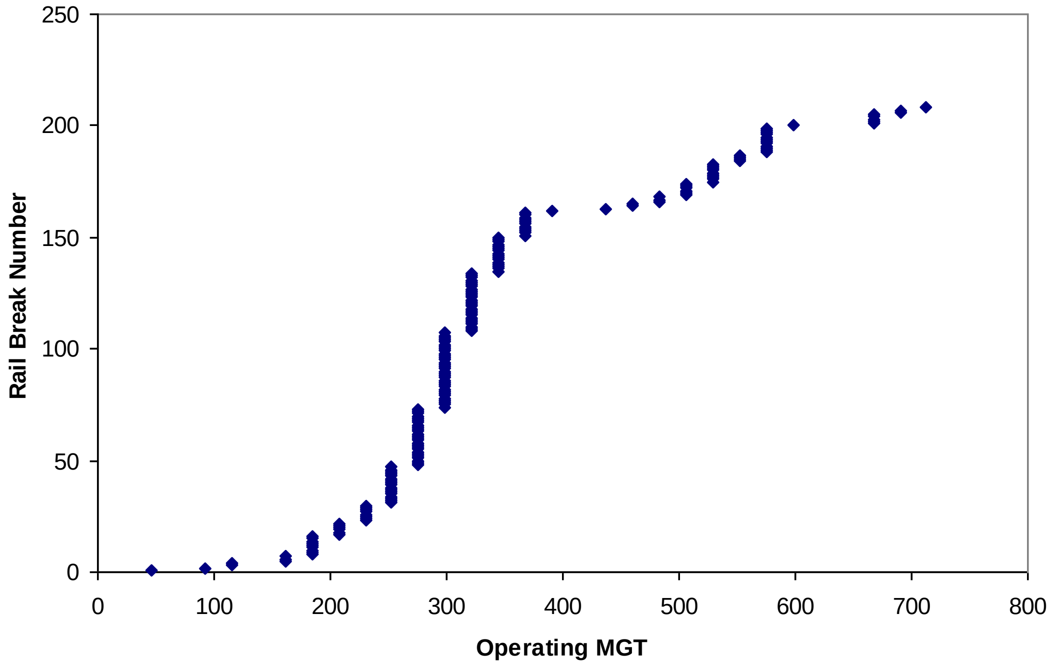

Figure 2: Cumulated Rail break vs accumulated MGT.

5.1 Parameter estimation

Some of the common methods used in estimating parameters are: method of Least square, method of Moments, and method of Maximum likelihood. However, non-parametric analysis is applied if the data requires. Suzuki [9] proposed parametric and non parametric methods of estimating lifetime distribution from field failure data with supplementary information about censoring times.

We apply the method of Maximum Likelihood (MLE) here to estimate the parameters λ and β.

Let

m MGT of rail at its *ith* failure

r number of failures over contract period

S observation period in terms of usage (MGT).

and

$0 < m_1 < m_2 \ldots\ldots < S$

This implies that there is no failure in $(0, m_1]$, one failure in $(m_1, m_1 + \delta m_1]$, one failure in $(m_2, m_2 + \delta m_2]$ for small time intervals of δm_1, δm_2. Suppose failures occur independently and according to a non-homogeneous Poisson process with intensity function Λ(m).

$$\text{Prob.}\{\text{no failures in } (0, m_1]\} = \exp\left\{-\int_0^{m_1} \Lambda(m)\,dm\right\}$$

$$\text{Prob.}\{\text{one of failures in } (m_1, m_1 + \delta m_1]\} = \Lambda(m_1)\delta m$$

Prob.{no failures in $(m_1 + \delta m_{1,} m_{2,}]$ } = $\exp\left\{-\int_{m_1+\delta m_1}^{m_2} \Lambda(m)dm\right\}$

Prob.{one of failures in $(m_{2,}\ m_2 + \delta m_2]$} = $\Lambda(m_2)\delta\ m$

The probabilities for other failures can be derived similarly. As a result, the likelihood function for failures the product to occur is (Crowder et al 1991)

$$L(\theta) = \prod_{i=1}^{r}\left[\Lambda(m_i)\exp\left\{-\int_{m_{i-1}}^{m_i}\Lambda(m)dm\right\}\right]$$

$$= \left\{\prod_{i=1}^{r}\Lambda(m_i)\right\}\exp\left\{-\int_0^S \Lambda(m)dm\right\}$$

$$= \left\{\prod_{i=1}^{r}\Lambda(m_i)\right\}\exp\left\{-\int_0^S \Lambda(m)dm\right\} \quad (13)$$

Let $l(\theta)$ be the log transformation of the likelihood function

$$l(\theta) = \log\{L(\theta)\} = \sum_{i=1}^{r}\log\Lambda(m_i) - \int_0^S \Lambda(m)dm \quad (14)$$

Where θ is the model parameter for set (λ, β).

For Non-Homogeneous Poisson Process (Power Law Process) $\Lambda(m)$ is given by:

$\lambda\beta(\lambda m)^{\beta-1}$ Then,

$$l(\theta) = \sum_{i=1}^{r}\log\left\{\lambda^{\beta}\beta\left((m_i)^{\beta-1}\right)\right\} - \lambda^{\beta}(S\ )^{\beta} \quad (15)$$

The maximum likelihood estimates are the values of the parameters that maximise the log likelihood function. These are given by the solution to the following equations (obtained from the first order necessary condition for maximisation):

$$\frac{\delta l(\theta)}{\delta\lambda} = 0 \quad \text{and} \quad \frac{\delta l(\theta)}{\delta\beta} = 0$$

From equation (11) and equation (12) we obtain the estimators $\hat{\lambda}$ and $\hat{\beta}$ as follows

$$\hat{\lambda} = \left[\frac{r}{S^{\beta}}\right]^{\frac{1}{\beta}} \quad (16)$$

and,

$$\hat{\beta} = \frac{r}{r\ln S - \sum_{i=1}^{r}\ln(m_i)} \quad (17)$$

Now the parameters λ and β of the given data can be estimated by using the expressions in MATLAB 7.0. From figure 3, we get $\lambda = 0.00259$ per MGT and $\beta = 2.789$ at 95% confidence interval.

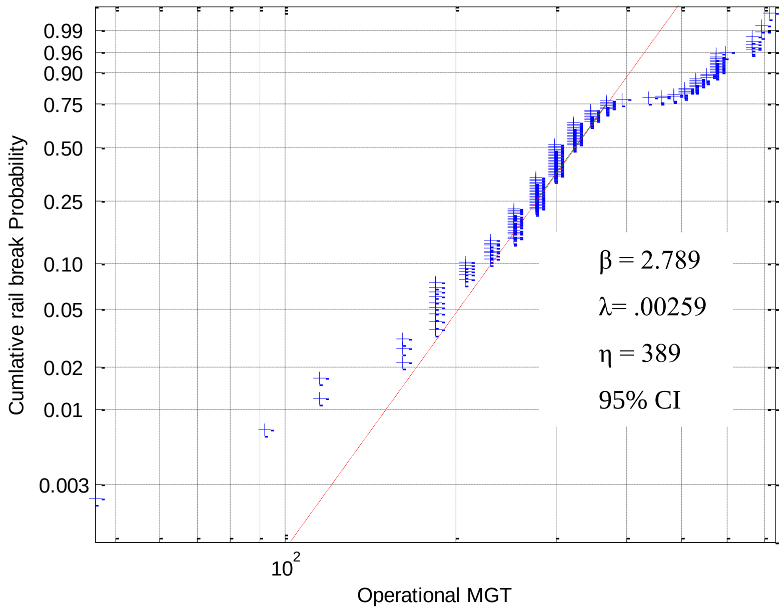

Figure 3: Analysis rail failure data

5.2 Application of collected data in determining the service contract

In this section estimated parameters are used in determining the cost of service contract for three different policies.

Here $\beta = 2.789$ and $\lambda = .00259$ per MGT.
Let,
Cost of minimal repair, C_{mr} = \$1500
Cost of each preventive maintenance (rail grinding and Lubrication), C_{pm} = \$2000
Cost of replacement, C_{re} = \$10000
Quality of each PM, $\alpha = .9$
Let Contracted usage in MGT , L = 600 MGT.
This gives us the following results
Optimal interval between preventive maintenance x* = 300 MGT
Optimal number of PMs $N^* = 1$
Expected total cost of Service Contract $C^*(L,x,N)$= \$3706 per segment over the contract period.

This implies that for short term contract with the presented rail failure distribution, each segment needs at least one preventive grinding and lubrication at maximum interval of 300 MGT but for long-term contract there may be a number of preventive maintenance (rail grinding and lubrication to get a reliable long service. However the actual total costs can be varied based on the grinding process and lubricants to be used during the contract period.

6 CONCLUSIONS

In this paper, cost model is developed for long-term service contracts. Here, real life failure data is collected and analysed. Parameters were estimated and applied to analyse cost for developing the model. Total costs of servicing strategies and cost per unit of service provided can be considered for managerial decisions. This model is applicable to outsourcing rail maintenance. There is huge scope for

I) Developing integrated decision model for Longterm service contracts applicable to rail and other asset intensive industries.
II) Development of penalty rates for train operators and infrastructure providers not complying with maintenance standards.

Authors are currently working in those areas and results will be published in the future

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