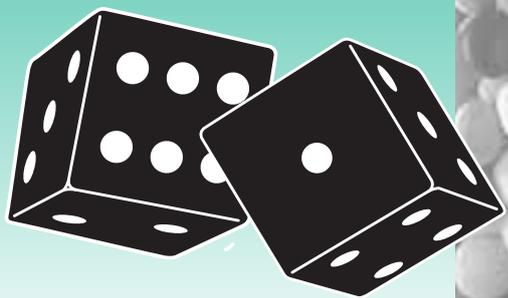# A DICEY STRATEGY
to get your
M & Ms

Over the last decade, mathematics curriculum documents have advocated the need for children to explore the notion of chance, beginning in the primary school. For example, the *National Statement on Mathematics for Australian Schools* (Australian Education Council, 1991) suggests that children in upper primary school (Years 4–7) and beyond should conduct experiments with coins, dice and spinners; record and organise the data; and compare the results with predictions (p. 170–171). Similarly, the *Principles and Standards for School Mathematics: Discussion Draft* (National Council of Teachers of Mathematics, 1998) recommends that students in Years 3–5 should begin to actively consider the likelihood of events through informal explorations of probability (p. 181). These informal explorations can occur through problem tasks, games in which strategies need to be developed, and experiments where children collect and analyse data. We have found that games which generate 'cognitive conflict' (Lakatos, 1976) are especially helpful in stimulating children's probabilistic learning.

This article describes and analyses a learning episode where two children in Year 4 interact with each other and their teacher, Miss Riley, while playing a probability game involving chocolate M & Ms. The game produces cognitive conflict for the learners and this conflict leads them to generate conjectures and engage in insightful mathematical reasoning. As a consequence of their exploration the children focus on a number of powerful probabilistic ideas.

The Probability Game

At the start of the lesson Miss Riley paired the children, passed out a gameboard for each pair (see Figure 1), and gave each child 12 M & Ms. Then she explained the game to the class.

> *This game is called* Get Your M & Ms First*. I've given each of you twelve M & Ms. You can place your M & Ms in any way that you wish in the*

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team up with

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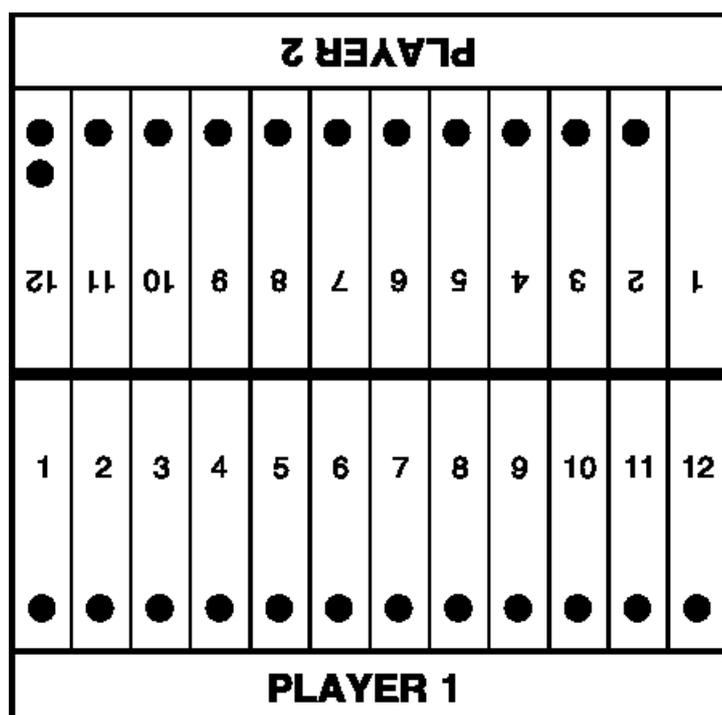
to get your teeth into.

Figure 1: The M & M Gameboard with M & Ms added

boxes on your side of the game-board. The boxes are labelled 1 to 12. You don't have to put an M & M in every box, and you can even put all your M & Ms in one box if you like.'

The object of the game is to remove all of your M & Ms from the gameboard before your partner clears all of his or hers. To get your M & Ms you take turns rolling two dice. After you have rolled the dice, you find their sum and remove one M & M if you have at least one on that sum. For example, if you roll two fours, you would remove one M & M from the 8 box if there is one there. If there is no M & M in the box, then you pass. A pass means that you can't remove an M & M and it becomes your partner's turn.'

After this explanation Miss Riley said, 'Now that you know the rules, place your M & Ms to give yourself the best chance of clearing them all off the board before your partner does. Play the game several times and see if you

can work out a "best strategy" for placing the M & Ms on the board.'

The game between Terry and Jamie

Miss Riley watched while Terry and Jamie played the game three times. In the first game, both children's strategy was to put an M & M in every box, except that Terry left out the box marked 1 and put two M & Ms in the box marked 12 — because '1 won't come up and 12 is the biggest number.'

As the first game proceeded, both children removed about 4 M & Ms without having to pass. All of these M & Ms came from the middle boxes. Then each child was surprised when there was a series of turns where both had to pass. As cognitive conflict developed, that is they became frustrated because their strategies were not working, Miss Riley said, 'What seems to be happening here?' Showing this frustration Jamie said, 'Too many 6s!'. Terry said, 'It looks like they're in the middle – from 4 to 11.' After they'd had a few more turns, Miss Riley suggested that they start a new game – and she watched carefully to see what strategy they would use in placing their M & Ms. This time the children spread them across most of the board but doubled up on 6 and one other number in the middle of the board. She noticed that the children had begun to work together on the task rather than trying to compete.

After they had played the second game, with much the same frustrations, Miss Riley asked whether there was a way that they could find out which sums came up more often. Terry said, 'We could make a list, but it would take a long time.'

Miss Riley urged them to go ahead, and the children listed all of the ways that they could get all the sums to 12 (See Figure 2).

1 + 1	1 + 2	1 + 3	1 + 4	1 + 5	1 + 6
2 + 1	2 + 2	2 + 3	2 + 4	2 + 5	2 + 6
3 + 1	3 + 2	3 + 3	3 + 4	3 + 5	3 + 6
4 + 1	4 + 2	4 + 3	4 + 4	4 + 5	4 + 6
5 + 1	5 + 2	5 + 3	5 + 4	5 + 5	5 + 6
6 + 1	6 + 2	6 + 3	6 + 4	6 + 5	6 + 6

Figure 2: The children's list of the ways that they could get all the sums up to

Miss Riley was pleased to note that the children included combinations and their reverses, such as 1 + 6 and 6 + 1. They had discussed reverses at some length in an earlier session on probability, so this did not produce cognitive conflict for the students. After they had finished listing, Miss Riley asked the children which sums came up the most. Jamie conjectured, '6 comes up the most. It's the middle number'. Terry counted the sums on the list and replied, '6 comes up 5 times. But that's not the most! 7 comes up 6 times. Look, I've underlined them.'

'Now that you know all this,' commented Miss Riley, 'Why don't you play the game again.' As the children placed their M & Ms on the board, she heard them say, 'Let's put most on 7, then 6 and 8 — the middle numbers come up more.' Actually they put four M & Ms on 7, two each on 6 and 8, and one each on 4, 5, 9, and 10. (See Figure 3.)

Figure 3: The children with their mats prepared for the 3rd game

Then the children played the game again. Even though they were able to complete the game, they were surprised when a 12 came up, because neither of them had put an M & M in that box. Miss Riley said, 'Why do you think that happened?'. Jamie replied, 'Because the dice go wherever they want to go'. Terry added, 'It could be 12, but there's just one chance of that — and we only had 12 M & Ms.'

Then Miss Riley said, 'Why don't you try it with 36 M & Ms and work on Jamie's gameboard'. She watched as the children carefully located the 36 M & Ms according to how many combinations they had listed for each sum. After they finished playing, the children finally got to share their M & Ms.

Classroom implications

A number of important observations for teaching and learning can be gleaned from this episode.

Powerful probability ideas were learned and reinforced

When playing this game, the children encountered cognitive conflict when more 6s and 7s than they expected began to appear. By mentally keeping track of the relative frequencies of various sums (experimental probabilities), they began to conjecture that certain sums (e.g. 'too many 6s') had greater probabilities. However, the children were not able to resolve their conflict until they began to systematically list all of the outcome pairs for the rolling of two dice. In essence, they listed the sample space for rolling two dice and, in so doing, they reinforced earlier learning that combinations like 1 + 6 and its reverse, 6 + 1, were different. Once they had systematically listed all the outcomes, they reasoned that the probabilities of the various sums were different, because they could be formed with varying numbers of combinations. Moreover, they noted that the combinations for these sums and hence their probabilities fell in a symmetrical pattern around 7 (See Figure 4).

In terms of the probability framework presented in NCTM's 1999 yearbook, *Developing Mathematical Reasoning in Years K–12* (Jones,

					6+1					
				5+1	5+2	6+2				
			4+1	4+2	4+3	5+3	6+3			
		3+1	3+2	3+3	3+4	4+4	5+4	6+4		
	2+1	2+2	2+3	2+4	2+5	3+5	4+5	5+5	6+5	
1+1	1+2	1+3	1+4	1+5	1+6	2+6	3+6	4+6	5+6	6+6
2	3	4	5	6	7	8	9	10	11	12

Figure 4: Symmetrical combinations of sums around 7

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Thornton, Langrall and Tarr), it is interesting to note that this game led the children to focus on three of the six constructs that these authors consider critical for probability learning in the primary school: experimental probability, sample space, and theoretical probability.

Children constructed probability ideas in a collaborative setting.

This episode demonstrates how children can build their own probability ideas when they are given the opportunity to work collaboratively in exploring a challenging problem. The problem was presented with materials in a game setting that had been designed to generate cognitive conflict and thereby stretch students' probabilistic thinking. Interestingly, the students initially competed with each other; however, the problematic nature of the task led them to work collaboratively to overcome the mutual frustrations they were experiencing. Note how often they worked together to solve what was seen as mutual problems arising from the game.

The teacher played a facilitating and monitoring role.

The key person in this episode was the teacher. Right from the beginning she encouraged the children to explore the situation, make conjectures, and test them. The teacher consistently used open-ended questions like, 'Why do you think that happened?' to focus the children's thinking. More importantly, she held back and listened rather than directed when they reached an impasse: 'What seems to be happening here?' In other words, the teacher adopted the role of facilitator of learning rather than transmitter of knowledge.

Concluding comments

The learning episode in this article shows children developing key ideas in probability from a game that was designed to produce cognitive conflict. In order to reduce this conflict, the children were led to systematically generate the sample space and look for structure in the pattern of probabilities. The teacher's role was pivotal in enabling the children to generate conjectures, to engage in mathematical reasoning, and to explore powerful probability ideas.

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