

1 Wave interaction with a perforated circular breakwater of 2 non-uniform porosity

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9 **Abstract.** In this paper, wave interaction with a porous cylindrical breakwater is studied
10 analytically by linear potential wave theory. The breakwater is assumed to have a thin skin,
11 is bottom-mounted and surface-piercing. The porosity of the breakwater is uniform vertically
12 but varies in the circumference direction. This allows the choice of a partially impermeable
13 wall or a vertical slot in the breakwater. Three different basic configurations of the breakwater
14 are investigated, namely, (1) uniformly porous cylinder; (2) porous cylinder with partial im-
15 permeable wall; and (3) porous cylinder with an opening. The performance of these types of
16 breakwaters is studied versus wave parameters and breakwater configurations including angle
17 and position of opening or partial impermeable wall as well as porosity. Parametric studies
18 with regard to the wave amplification factor, wave forces, and elevation contours are made.
19 The results should be found useful in the design of coastal and offshore structures.

20 **Keywords:** short-crested wave, wave diffraction, circular breakwater, porous structure

21 1. Introduction

22 Porous breakwaters are often constructed to reduce the wave impact on coastal
23 and offshore structures. They can also reduce resonance more effectively than
24 an impermeable breakwater [1]. Since the early work of Jarlan [2], wave in-
25 teraction with a porous breakwater has attracted the attention of many coastal
26 and offshore researchers. In one instance among many, Dalrymple et al. [3]
27 studied the reflection and transmission of a wave train at an oblique angle of
28 incidence by an infinitely long porous breakwater. Subsequently, Huang and
29 Chao [4] reported the inertial effect of the porous breakwater based on Biot's
30 theory of poroelasticity.

31 Following the porous wavemaker theory of Chwang [5] and subsequent
32 works, investigations have primarily been concentrated on the hydrodynamic
33 effects of a porous structure on the incoming wave trains, or wave impact on
34 porous structures as a breakwater in a harbour (e.g., [1, 6, 7]). In most cases,
35 Darcy's law for a homogeneous porous medium has been applied. Yu and
36 Chwang [6] investigated the resonance in a harbour with porous breakwaters
37 subjected to an arbitrary wave angle followed by an extensive study on the

38 wave transmission characteristics past a porous structure [7]. They also in-
39 vestigated the behaviour of waves within the porous medium. It was found
40 that there is an optimum thickness for a porous structure beyond which any
41 further increase in the thickness may not lead to an appreciable improvement
42 in reducing its transmission and reflection characteristics. Wang and Ren [8]
43 studied the performance of a flexible porous breakwater, and found that hy-
44 drodynamic forces on the interior cylinder as well as wave amplitudes around
45 the windward side of the interior cylinder are reduced when compared to the
46 case of a direct wave impact on the interior cylinder. More related works can
47 be found in the review article of Chwang and Chan [1].

48 The aforementioned studies on the interaction of ocean surface waves
49 with a vertical porous breakwater have generally been two-dimensional. In
50 reality, however, the ocean waves are more complex, and better described by
51 three-dimensional (3D) short-crested waves. They also commonly arise, for
52 example, from the oblique interaction of two travelling plane waves or inter-
53 secting swell waves, or from the reflection of waves at non-normal incidence
54 off a vertical seawall, as well as from the diffraction about the surface bound-
55 aries of a structure of finite length. These multi-directional, multi-component
56 waves are of paramount importance in coastal and offshore engineering de-
57 sign. In contrast to plane waves propagating in a single direction, and the
58 standing waves fluctuating vertically in a confined region, short-crested waves
59 can be doubly periodic in two horizontal directions, one in the direction of
60 propagation and the other normal to it [9].

61 Theoretical analysis on short-crested wave interaction with a vertical cylin-
62 der can be found in [10–12]. Zhu [10] presented an analytic solution to the
63 diffraction problem for a solid circular cylinder in short-crested waves us-
64 ing linear potential wave theory and found that the pressure distribution and
65 wave run-up on the cylinder were quite different from those of plane incident
66 waves. Their patterns become very complex as ka (i.e., total incident wave
67 number k times cylinder radius a) becomes large. The hydrodynamic forces
68 on the cylinder become smaller as the short-crestedness of the incident waves
69 increases. Subsequently, Zhu and Moule [11] observed that the hydrodynamic
70 force induced by short-crested waves varies with the phase angle perpen-
71 dicular to the direction of wave propagation. Later, Zhu and Satravaha [12]
72 extended the analytical solution for the velocity potential to second-order.

73 Although efforts have been made on wave interaction with porous cylin-
74 ders and breakwaters, there is no relevant work on the wave interaction with
75 a perforated cylindrical breakwater having variable porosity and opening. In
76 this paper, analytical solutions are derived to study this problem in a quan-
77 titative manner. Detailed numerical results are presented over a broad range
78 of incident short-crested wave parameters as well as structural configurations
79 including the porosity of the breakwater and the angle and position of the
80 impermeable wall and opening. In particular, their effects on wave amplifica-

81 tion factors, wave forces, and wave elevation contours near the structure are
82 discussed.

83 2. Theoretical Consideration

84 2.1. PROBLEM DESCRIPTION

85 It is worth noting that theoretical derivation can be made on 2D plane waves,
86 and the linear solutions of 3D short-crested waves can be obtained by linear
87 superposition of two plane waves. We intend to extend this study to include
88 nonlinear effects. Therefore, in this section, the mathematical formulae are
89 derived for a general case of interaction of 3D short-crested waves with a
90 porous cylindrical breakwater of variable porosity. Note that the solutions for
91 the 2D limiting cases, i.e., a plane incident wave and a standing wave, can be
92 instantly recovered from it by letting $k_y = 0$ and $k_x = 0$ ($k_x =$ wave number
93 in x direction, $k_y =$ wave number in y direction) respectively.

94 Consider a monochromatic short-crested wave train propagating in the
95 direction of the positive x axis. A perforated cylindrical breakwater extends
96 from the sea bottom to above the free surface of the ocean along z axis. The
97 origin is placed at the centre of the cylindrical breakwater on the mean water
98 surface (see Fig. 1). A partially impermeable wall or opening is located at
99 $\theta \in (\varepsilon_1, \varepsilon_2)$ in cylindrical coordinates (r, θ, z) . The whole fluid region is
100 divided into two regions - the region inside the breakwater, Ω_1 and the region
101 outside the breakwater, Ω_2 . The following notation are used in the paper:
102 $\Phi_j =$ total velocity potential, $\Phi_j^I =$ velocity potential of incident wave,
103 $\Phi_j^S =$ velocity potential of scattered wave, $k =$ total wave number, $\omega =$
104 wave frequency, $h =$ water depth, $A =$ amplitude of incident wave, $a =$
105 radius of the cylindrical breakwater, $t =$ time, $\rho =$ mass density of water, and
106 $g =$ gravitational acceleration. The subscripts $j(j = 1, 2)$ denote the physical
107 parameters in the region $\Omega_j(j = 1, 2)$.

108 Assume that the fluid is inviscid and incompressible, and the flow is ir-
109 rotational. Then the fluid flow can be described by a velocity potential Φ_j ,
110 which satisfies the Laplace equation

$$\nabla^2 \Phi_j = 0 \quad \text{in } \Omega_j, \quad (1)$$

111 subject to the combined linearised free surface boundary condition

$$\Phi_{j,tt} + g\Phi_{j,z} = 0 \quad \text{at } z = 0, \quad (2)$$

112 and the bottom condition

$$\Phi_{j,z} = 0 \quad \text{at } z = -h, \quad (3)$$

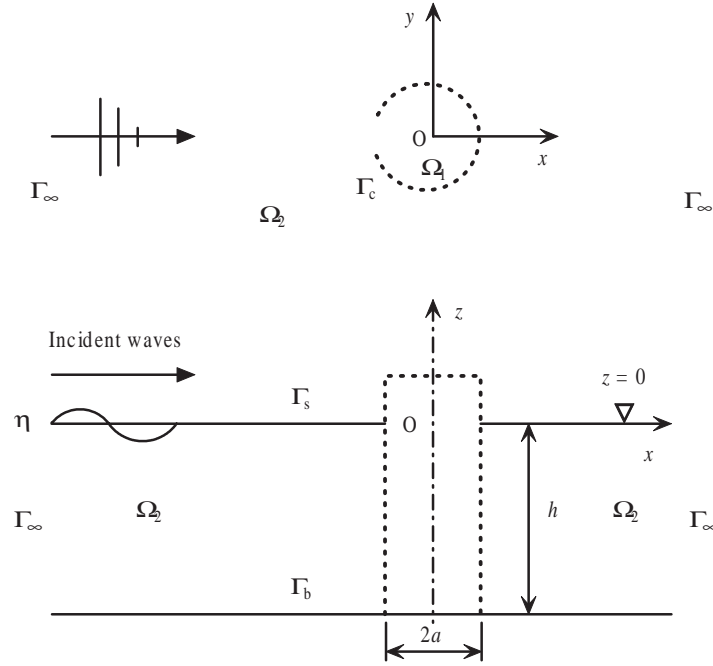


Figure 1. Definition sketch of short-crested waves on a porous cylindrical breakwater.

113 where the comma in the subscript designates partial derivative with respect
 114 to the variable following it.

115 The total velocity potential in region Ω_2 can be expressed by the summa-
 116 tion of the incident and scattered wave velocity potentials

$$\Phi_2 = \Phi_2^I + \Phi_2^S \quad \text{in } \Omega_2, \quad (4)$$

117 where Φ_2^I and Φ_2^S also satisfy (1) - (3).

118 The velocity potential of the linear short-crested incident wave [13] trav-
 119 elling principally in the positive x direction is given by the real part of

$$\Phi_2^I = -\frac{igA}{\omega} f(z, h) e^{i(k_x x - \omega t)} \cos(k_y y) \quad \text{in } \Omega_2, \quad (5)$$

120 where $k^2 = k_x^2 + k_y^2$, and

$$f(z, h) = \frac{\cosh k(z + h)}{\cosh kh}. \quad (6)$$

121 The term $f(z, h)$ leads to the sea bottom condition being automatically
 122 satisfied, while the linearised free surface boundary condition is satisfied
 123 using the following dispersion relationship

$$\omega^2 = gk \tanh kh. \quad (7)$$

124 Assuming that the fluid flow passing through the perforated breakwater
 125 as a porous boundary obeys Darcy's law [14], the boundary condition on
 126 perforated breakwater can be expressed as [5]

$$\Phi_{1,r} = \Phi_{2,r} = iG(\theta)k(\Phi_1 - \Phi_2) \quad \text{on } r = a, \quad (8)$$

$$\Phi_{2,r}^S = iG(\theta)k(\Phi_1 - \Phi_2^S - \Phi_2^I) - \Phi_{2,r}^I \quad \text{on } r = a, \quad (9)$$

127 where r is the radial axis, $i = \sqrt{-1}$, $G(\theta) = \frac{\rho\omega d(\theta)}{\mu}$ is a measure of the
 128 porosity, μ is the coefficient of dynamic viscosity, $d(\theta)$ is a material constant
 129 having the dimension of length. The porous effect parameter G is a dominant
 130 parameter in the present study. Its value depends on the geometrical parame-
 131 ters of the permeable wall and wave factors [15]. The geometrical parameters
 132 of a permeable wall consist mainly of geometrical porosity, plate thickness
 133 and porous shape. In engineering practices, the geometrical porosity is about
 134 20% and can reach as high as 60% or higher in some circumstances. Sev-
 135 eral porous shapes are common in coastal or offshore structures, including
 136 slit, screen and circular or rectangular holes. Detailed method of estimate of
 137 G could be found in [15]. In addition, the scattered potential satisfies the
 138 Sommerfeld radiation condition at infinity as follows:

$$\lim_{kr \rightarrow \infty} (kr)^{1/2} (\Phi_{2,r}^S - ik\Phi_2^S) = 0 \quad \text{in } \Omega_2. \quad (10)$$

139 Therefore, the scattered wave velocity potential Φ_2^S in Ω_2 is governed
 140 by the Laplace equation (1) with the boundary conditions (2) and (3), the
 141 boundary condition at the interface of fluid and breakwater at $r = a$ (8) and
 142 (9), and the radiation condition (10).

143 The velocity potential Φ_1 in the interior domain Ω_1 is governed by the
 144 Laplace equation (1) with the boundary conditions (2) and (3), and the bound-
 145 ary conditions at the interface of fluid and breakwater at $r = a$:

$$\Phi_{1,r} = iG(\theta)k(\Phi_1 - \Phi_2^S - \Phi_2^I) \quad \text{on } r = a. \quad (11)$$

146 These constitute two sets of the governing equation and corresponding
 147 boundary conditions for the diffraction of short-crested waves by a vertical
 148 perforated cylindrical breakwater with nonuniform porosity, corresponding
 149 to boundary-value problems in a bounded domain and an unbounded domain
 150 respectively. After obtaining Φ_2^S , Φ_2 and Φ_1 by solving the above boundary-
 151 value problems, all the physical quantities including the fluid particle velocity,
 152 free surface elevation and the dynamic pressure can be calculated respectively
 153 from

$$\mathbf{v}_j = \nabla\Phi_j, \quad (12)$$

$$\eta_j = \frac{i\omega}{g} \Phi_j|_{z=0, t=0}, \quad (13)$$

$$p_j = -\rho \Phi_{j,t}. \quad (14)$$

154 2.2. ANALYTICAL SOLUTION

155 The incident wave potential (5) can be written in the cylindrical coordinates
156 as

$$\Phi_2^I = -\frac{igA}{\omega} f(z, h) e^{-i\omega t} \left[\sum_{m=0}^{+\infty} \varepsilon_m i^m J_m(k_x r) \cos(m\theta) \right] \left[\sum_{n=0}^{+\infty} \varepsilon_n J_{2n}(k_y r) \cos(2n\theta) \right], \quad (15)$$

157 where

$$\varepsilon_m = \begin{cases} 1 & \text{for } m = 0 \\ 2 & \text{for } m \neq 0 \end{cases}, \quad (16)$$

158 and the J_m and J_{2n} are Bessel functions of m th and $2n$ th order respec-
159 tively.

160 Splitting the product of the two trigonometric functions, and truncating
161 the infinite series at $m = M$ and $n = N$, (15) becomes

$$\Phi_2^I = -\frac{igA}{2\omega} f(z, h) e^{-i\omega t} \sum_{m=0}^M \sum_{n=0}^N \varepsilon_m \varepsilon_n i^m J_m(k_x r) J_{2n}(k_y r) \cdot [\cos(m+2n)\theta + \cos(m-2n)\theta]. \quad (17)$$

162 (17) can be further simplified as

$$\Phi_2^I = -\frac{igA}{\omega} f(z, h) e^{-i\omega t} \sum_{l=0}^L \psi_l(k_x r, k_y r) \cos(l\theta), \quad (18)$$

163 where $L = M + 2N$, and

$$\begin{aligned} \psi_l(k_x r, k_y r) = & \frac{1}{2} \left\{ \sum_{n=\max\{0, \lceil (l-M)/2 \rceil\}}^{\min\{N, \lfloor l/2 \rfloor\}} \varepsilon_{l-2n} \varepsilon_n i^{l-2n} J_{l-2n}(k_x r) J_{2n}(k_y r) \right. \\ & + \sum_{n=0}^{\min\{N, \lfloor (M-l)/2 \rfloor\}} \varepsilon_{l+2n} \varepsilon_n i^{l+2n} J_{l+2n}(k_x r) J_{2n}(k_y r) \\ & \left. + \sum_{n=\lceil l/2 \rceil}^{\min\{N, \lfloor (M+l)/2 \rfloor\}} \varepsilon_{2n-l} \varepsilon_n i^{2n-l} J_{2n-l}(k_x r) J_{2n}(k_y r) \right\}, \quad (19) \end{aligned}$$

164 in which $\lceil \cdot \rceil$ is a function giving the greatest integer less than or equal to
 165 its argument and $\lfloor \cdot \rfloor$ is a function returning the smallest integer greater than
 166 or equal to its argument.

167 According to [6] and [10], the evanescent waves do not exist in the ab-
 168 sence of related boundary conditions. The solution of the scattered velocity
 169 potential in region Ω_2 can be constructed by the following expression

$$\Phi_2^S = -\frac{igA}{\omega} f(z, h) e^{-i\omega t} \left\{ \sum_{l=0}^L A_l^1 \cos(l\theta) H_l(kr) + \sum_{l=1}^L A_l^2 \sin(l\theta) H_l(kr) \right\}, \quad (20)$$

170 which satisfies the Laplace equation (1), boundary conditions (2) and (3),
 171 and the Sommerfeld radiation condition (10) for all A_l^1 and A_l^2 , where H_l is
 172 the Hankel functions of the first kind, and A_l^1 and A_l^2 are unknown complex
 173 coefficients.

174 Similarly, the solution of the velocity potential in the interior region Ω_1
 175 can be constructed as

$$\Phi_1 = -\frac{igA}{\omega} f(z, h) e^{-i\omega t} \left\{ \sum_{l=0}^L B_l^1 \cos(l\theta) J_l(kr) + \sum_{l=1}^L B_l^2 \sin(l\theta) J_l(kr) \right\}, \quad (21)$$

176 where B_l^1 and B_l^2 are unknown complex coefficients.

177 Substituting (18), (20) and (21) into the body boundary conditions (8) and
 178 (11), and noting the orthogonality property of the trigonometric functions, we
 179 have

$$B_l^1 J_l'(ka) = \psi_l'(k_x a, k_y a) / k + A_l^1 H_l'(ka), \quad (22)$$

$$B_l^2 J_l'(ka) = A_l^2 H_l'(ka), \quad (23)$$

$$\begin{aligned} & \sum_{l=0}^L [B_l^1 J_l(ka) - A_l^1 H_l(ka) - \psi_l(k_x a, k_y a)] \cos(l\theta) \\ & + \sum_{l=1}^L [B_l^2 J_l(ka) - A_l^2 H_l(ka)] \sin(l\theta) \\ = & \frac{1}{iG(\theta)} \left\{ \sum_{l=0}^L B_l^1 J_l'(ka) \cos(l\theta) + \sum_{l=1}^L B_l^2 J_l'(ka) \sin(l\theta) \right\} \quad (G \neq 0), \end{aligned} \quad (24)$$

180 where the prime denotes the derivative with respect to r .

181 It should be noted that (24) is not appropriate when $G = 0$. However, if
 182 a very small value (e.g. $1e^{-12}$) is assigned to $G(\theta)$, representing the case of
 183 impermeable wall, (24) still applies and leads to highly accurate results.

184 From (22) and (23), we have

$$B_l^1 = \frac{\psi_l'(k_x a, k_y a) + k H_l'(ka) A_l^1}{kJ_l'(ka)}, \quad l = 0, 1, 2, \dots, L, \quad (25)$$

$$B_l^2 = \frac{H_l'(ka)}{J_l'(ka)} A_l^2, \quad l = 1, 2, \dots, L. \quad (26)$$

185 Multiplying both sides of (24) by $\cos(j\theta)$ ($j = 0, 1, 2, \dots, L$) and $\sin(j\theta)$
 186 ($j = 1, 2, \dots, L$), integrating with respect to θ from 0 to 2π , and further
 187 simplifying by the orthogonality property of the trigonometric functions, the
 188 following set of linear equations is obtained.

$$\mathbf{DA} + \mathbf{EB} + \mathbf{C} = 0, \quad (27)$$

189 in which

$$\mathbf{A} = [A_0^1, A_1^1, \dots, A_L^1, A_1^2, \dots, A_L^2]^T, \quad (28)$$

$$\mathbf{B} = [B_0^1, B_1^1, \dots, B_L^1, B_1^2, \dots, B_L^2]^T, \quad (29)$$

$$\mathbf{C} = -\mathbf{Q}\Psi, \quad (30)$$

$$\mathbf{D} = -\mathbf{QH}, \quad (31)$$

$$\mathbf{E} = \mathbf{QJ} + i\mathbf{SJ}', \quad (32)$$

$$\mathbf{Q} = \text{diag}[2\pi, \pi, \pi, \dots, \pi], \quad (33)$$

$$\mathbf{H} = \text{diag}[H_0(ka), H_1(ka), \dots, H_L(ka), H_1(ka), \dots, H_L(ka)], \quad (34)$$

$$\Psi = \text{diag}[\psi_0(k_x a, k_y a), \psi_1(k_x a, k_y a), \dots, \psi_L(k_x a, k_y a), 0, \dots, 0], \quad (35)$$

$$\mathbf{J} = \text{diag}[J_0(ka), J_1(ka), \dots, J_L(ka), J_1(ka), \dots, J_L(ka)], \quad (36)$$

$$\mathbf{J}' = \text{diag}[J_0'(ka), J_1'(ka), \dots, J_L'(ka), J_1'(ka), \dots, J_L'(ka)], \quad (37)$$

$$\mathbf{S}_{jl} = \begin{cases} \int_0^{2\pi} \frac{1}{G(\theta)} \cos(j\theta) \cos(l\theta) d\theta & 0 \leq j \leq L, 0 \leq l \leq L, \\ \int_0^{2\pi} \frac{1}{G(\theta)} \cos(j\theta) \sin(l-L)\theta d\theta & 0 \leq j \leq L, L+1 \leq l \leq 2L+1, \\ \int_0^{2\pi} \frac{1}{G(\theta)} \sin(j-L)\theta \cos(l\theta) d\theta & L+1 \leq j \leq 2L+1, 0 \leq l \leq L, \\ \int_0^{2\pi} \frac{1}{G(\theta)} \sin(j-L)\theta \sin(l-L)\theta d\theta & L+1 \leq j \leq 2L+1, L+1 \leq l \leq 2L+1, \end{cases} \quad (38)$$

190 where “diag” denotes a diagonal matrix with the elements in the square
191 brackets on the main diagonal.

192 (25), (26) and (27) constitute a set of linear equations for A_l^1 , A_l^2 , B_l^1 ,
193 and B_l^2 . Once the values of these coefficients are obtained, all the physical
194 quantities can be calculated accordingly.

195 2.3. PHYSICAL QUANTITIES

196 The elevations in the interior and exterior regions are

$$\eta_1 = A \left\{ \sum_{l=0}^L B_l^1 J_l(kr) \cos(l\theta) + \sum_{l=1}^L B_l^2 J_l(kr) \sin(l\theta) \right\}, \quad (39)$$

$$\eta_2 = A \left\{ \sum_{l=0}^L [\psi_l(k_x r, k_y r) + A_l^1 H_l(kr)] \cos(l\theta) + \sum_{l=1}^L A_l^2 H_l(kr) \sin(l\theta) \right\}. \quad (40)$$

197 The pressures on the boundary (interior and exterior) are

$$p_1 = \rho g A f(z, h) e^{-i\omega t} \left\{ \sum_{l=0}^L B_l^1 J_l(ka) \cos(l\theta) + \sum_{l=1}^L B_l^2 J_l(ka) \sin(l\theta) \right\}, \quad (41)$$

$$p_2 = \rho g A f(z, h) e^{-i\omega t} \left\{ \sum_{l=0}^L [\psi_l(k_x a, k_y a) + A_l^1 H_l(ka)] \cos(l\theta) + \sum_{l=1}^L A_l^2 H_l(ka) \sin(l\theta) \right\}. \quad (42)$$

198 The total force per unit length in the direction of s ($s = x, y$) is

$$\frac{dF_s}{dz} = a \left[\int_0^{2\pi} (p_1 - p_2) \cdot \varphi_s d\theta \right] = P_s(k_x, k_y, k, a) \cdot \rho g a A \cdot f(z, h) e^{-i\omega t}, \quad (43)$$

199 where the function $P_s(k_x, k_y, k, a)$ is a nondimensional parameter of $\frac{dF_s}{dz}$
200 without the constant term $\rho g a A \cdot f(z, h) e^{-i\omega t}$, and

$$\varphi_x = \cos(\theta), \quad \varphi_y = \sin(\theta). \quad (44)$$

201 By the orthogonality of the trigonometric functions, only the term $l = 1$
 202 in the series (41) and (42) remains, so that the function $P_s(k_x, k_y, k, a)$ can
 203 be expressed explicitly as

$$P_x(k_x, k_y, k, a) = \pi \cdot [B_1^1 J_1(ka) - \psi_l(k_x a, k_y a) - A_1^1 H_1(ka)], \quad (45)$$

$$P_y(k_x, k_y, k, a) = \pi \cdot [B_1^2 J_1(ka) - A_1^2 H_1(ka)]. \quad (46)$$

204 The function $P_s(k_x, k_y, k, a)$ determines the first-order total force in s
 205 ($s = x, y$) direction on the perforated cylindrical structure, F_s , which can
 206 be obtained by integrating (43) with respect to z ,

$$F_s = \int_{-h}^0 \frac{dF_s}{dz} dz = P_s(k_x, k_y, k, a) \cdot \rho g h a A e^{-i\omega t} \cdot \tanh(kh)/kh. \quad (47)$$

207 The total moments about an axis parallel to the y and x axis passing
 208 through the bottom of the cylindrical structure respectively are

$$M_y = \int_{-h}^0 (z+h) \frac{dF_x}{dz} dz = P_x(k_x, k_y, k, a) \rho g h^2 a A e^{-i\omega t} Z(kh), \quad (48)$$

$$M_x = - \int_{-h}^0 (z+h) \frac{dF_y}{dz} dz = -P_y(k_x, k_y, k, a) \rho g h^2 a A e^{-i\omega t} Z(kh), \quad (49)$$

209 where

$$Z(kh) = [kh \tanh(kh) + \operatorname{sech}(kh) - 1]/(kh)^2. \quad (50)$$

210 It is noted from (47) - (49) that only the function $P_s(k_x, k_y, k, a)$ needs to
 211 be determined in order to derive all the subsequent results.

212 2.4. LIMITING CASE

213 For uniform porous cylinder, i.e. $G(\theta) = G_0$, matrix \mathbf{S} becomes a diagonal
 214 matrix and the solution can be expressed explicitly as

$$A_l^1 = - \frac{i\pi k a G_0 [\psi_l J_l'(ka) - \varphi_l' J_l(ka)/k] + \pi a \varphi_l' J_l'(ka)}{2G_0 + \pi k a J_l'(ka) H_l'(ka)}, \quad (51)$$

$$B_l^1 = \frac{-i\pi k a G_0 [\psi_l H_l'(ka) - \varphi_l' H_l(ka)/k]}{2G_0 + \pi k a J_l'(ka) H_l'(ka)}, \quad (52)$$

$$A_l^2 = B_l^2 = 0. \quad (53)$$

215

3. Results and Discussion

216 Fig. 2 shows the variations of wave amplification factor ($|\eta|/A$) at the origin
 217 $r = 0$ (left) and nondimensional wave forces on the breakwater (right) *vs.*
 218 ka , where Figs. 2(a) and 2(d) correspond to the case of a breakwater with
 219 a uniform porosity ($G_0 = 1$), and Figs. 2(b & e) and 2(c & f) correspond
 220 to the cases of a breakwater with a partial solid wall and a partial opening
 221 respectively at $175^\circ < \theta < 185^\circ$ with the balance of the porosity remaining
 222 at $G_0 = 1$. Cases comprising of five different wave spread angles at $\beta = 0$,
 223 $\pi/8$, $\pi/4$, $3\pi/8$, and $\pi/2$ (where $\beta = \arctan(k_y/k_x)$) are calculated and
 224 the results are plotted. As can be seen in Fig. 2(a), all the curves represent-
 225 ing wave amplification factors of different wave spread angles coincide with
 226 one another. This is a clear indication that the wave amplification factor at
 227 origin is independent of the wave spread angle β for breakwaters with a
 228 uniform porosity. The wave amplification factor at origin is seen to decrease
 229 monotonically from 1 to approximately half as ka increases up to about 2.2,
 230 and then increase monotonically to about 1 before ka reaches around 3.8
 231 and afterwards fluctuate again. As shown in Fig. 2(b), the variation of wave
 232 amplification factor for a breakwater with a partial solid wall is very similar
 233 to that of a breakwater with a uniform porosity. However, waves of different
 234 β values result in slightly different amplification factors. It is seen that a
 235 standing wave ($\beta = \pi/2$) tends to result in the highest amplification factor
 236 whilst the incident short-crested wave with $k_x = k_y$ produces the lowest
 237 amplification factor for a large range of ka . As indicated in Fig. 2(c), the
 238 variation of amplification factor for the breakwater with a partial opening
 239 is similar to that for the breakwater with a uniform porosity at large ka . A
 240 distinct feature of the variation of amplification factor is that an additional
 241 peak is clearly observed for each short-crestedness at around $ka = 0.2$, and
 242 the maximum amplification factor at origin is about 1.15. Furthermore, as
 243 one would expect, a plane wave is seen to result in the highest amplification
 244 factor, while a standing wave tends to result in its lowest value for $ka \leq 3$.

245 For a breakwater with a uniform porosity, the nondimensional wave force
 246 in the direction of wave propagation (Figs. 2(d)-2(f)), decreases as the short-
 247 crestedness increases. In fact, the wave force becomes zero when the short-
 248 crestedness arrives at its maximum value (i.e., standing waves), since the
 249 configuration is symmetric about the y -axis. Peaks and troughs occur at ap-
 250 proximately the same ka value for different short-crestedness. However, for
 251 the breakwater with a partial solid wall (Fig. 2(e)) or an opening (Fig. 2(f)),
 252 the wave forces induced by a standing wave are no longer zero, since now
 253 the configuration is nonsymmetric about the y -axis and the peaks and troughs
 254 for different short-crestedness tend to occur at slightly different values of ka .
 255 More specifically, the peaks and troughs occur at larger ka for the partial
 256 solid wall, while they occur at smaller values of ka for the opening. Due to

257 the asymmetry in the configuration, the breakwater with a partial solid wall
 258 yields the largest wave force and the one with a partial opening gives the
 259 smallest wave force, except for the case of standing incident wave.

260 Fig. 3 shows the influence of the porosity on the wave amplification factor
 261 at the origin $r = 0$ (left) and wave forces on the breakwater (right) for a break-
 262 water with uniform porosity G_0 , a partial solid wall and a partial opening at
 263 $175^\circ < \theta < 185^\circ$ with porosity of the remaining part G_0 . As can be seen from
 264 Figs. 3(a)-3(c), wave spreading angles have little effect on the amplification
 265 factor for all the breakwater configurations. The amplification factors at origin
 266 increase monotonically towards their asymptotic values. Also, Figs. 3(d)-3(f)
 267 show that a larger wave spreading angle clearly results in a smaller wave force
 268 except for the case of standing incident waves.

269 Many coastal and offshore structures are commonly designed with non-
 270 uniform porosity along the circumferential direction. Fig. 4 shows the wave
 271 amplification factor at the origin and wave forces *vs.* opening area angle for
 272 breakwaters with a partial solid wall (left) and a partial opening (right) located
 273 at $\theta = 180^\circ$, and the porosity of the remaining part at $G_0 = 1$. For the break-
 274 water with a partial solid wall (Fig. 4(a)) the amplification factors at origin
 275 generally decrease monotonically as the angle of the solid area increases with
 276 largest value for a standing wave, and reaches zero at $\theta \approx 345^\circ$. For the
 277 case of partial opening, Fig. 4(c) shows that the amplification factor at origin
 278 initially increases to a peak at the opening area reaching approximately half of
 279 the circumference then decreasing to 1 with increasing opening area angle. A
 280 plane incident wave is clearly seen to produce the largest amplification factor,
 281 while a standing wave generates the smallest. It clearly indicates that more
 282 surface disturbance occurs within the interior for the opening area angle in
 283 the range of $180^\circ \sim 360^\circ$ depending on the short-crestedness of the incident
 284 waves.

285 As clearly shown in Figs. 4(b) and 4(d), a general trend of increasing
 286 wave forces with decreasing short-crestedness is observed for breakwaters
 287 with either a partial solid wall or an opening. Though fluctuating with the
 288 solid or opening area angles, larger wave forces occur for the breakwater
 289 with a partial solid wall than that with uniform porosity without the solid
 290 part, while the breakwater with an opening tends to experience smaller wave
 291 forces. The largest wave forces occur when the solid area angle varies in the
 292 neighbourhood of 180° for a plane incident wave. As the short-crestedness
 293 increases, the wave force for the breakwater with a partially solid wall peaks
 294 for the breakwater with a larger proportion of solid wall. In contrast, for
 295 the breakwater with a partial opening, the largest wave force always occurs
 296 at zero opening area, i.e., the breakwater of uniform porosity without any
 297 opening.

298 Fig. 5 shows the variation of wave amplification factor at the origin and
 299 wave forces on the breakwater for the cases of breakwaters with a partial solid

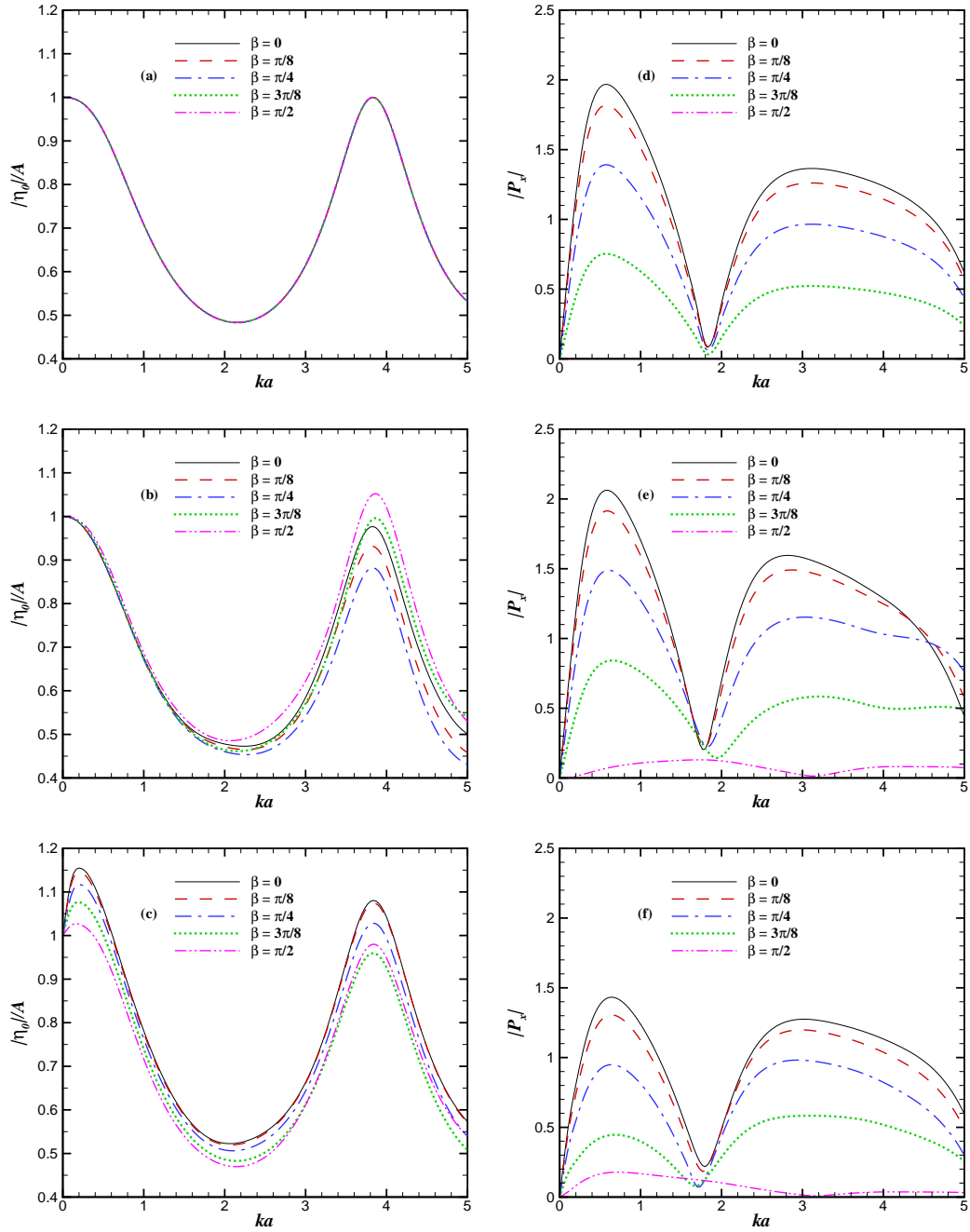


Figure 2. Variation of wave amplification factor at $r = 0$ (left) and nondimensional wave force on the breakwater (right) with porosity $G_0 = 1$ vs. ka .

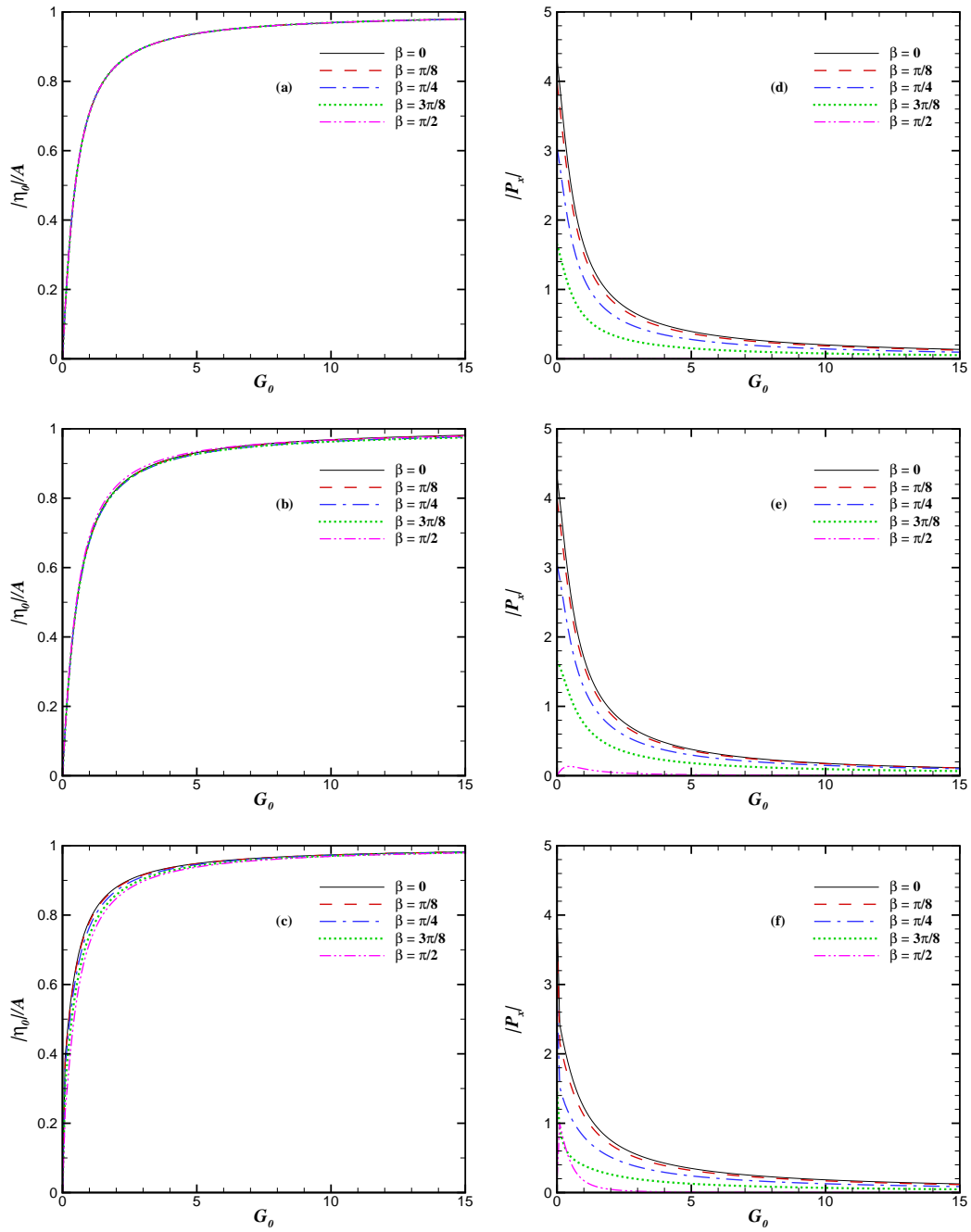


Figure 3. Variation of wave amplification factor at $r = 0$ (left) and nondimensional wave force on the breakwater (right) at $ka = 1$ vs. G_0 .

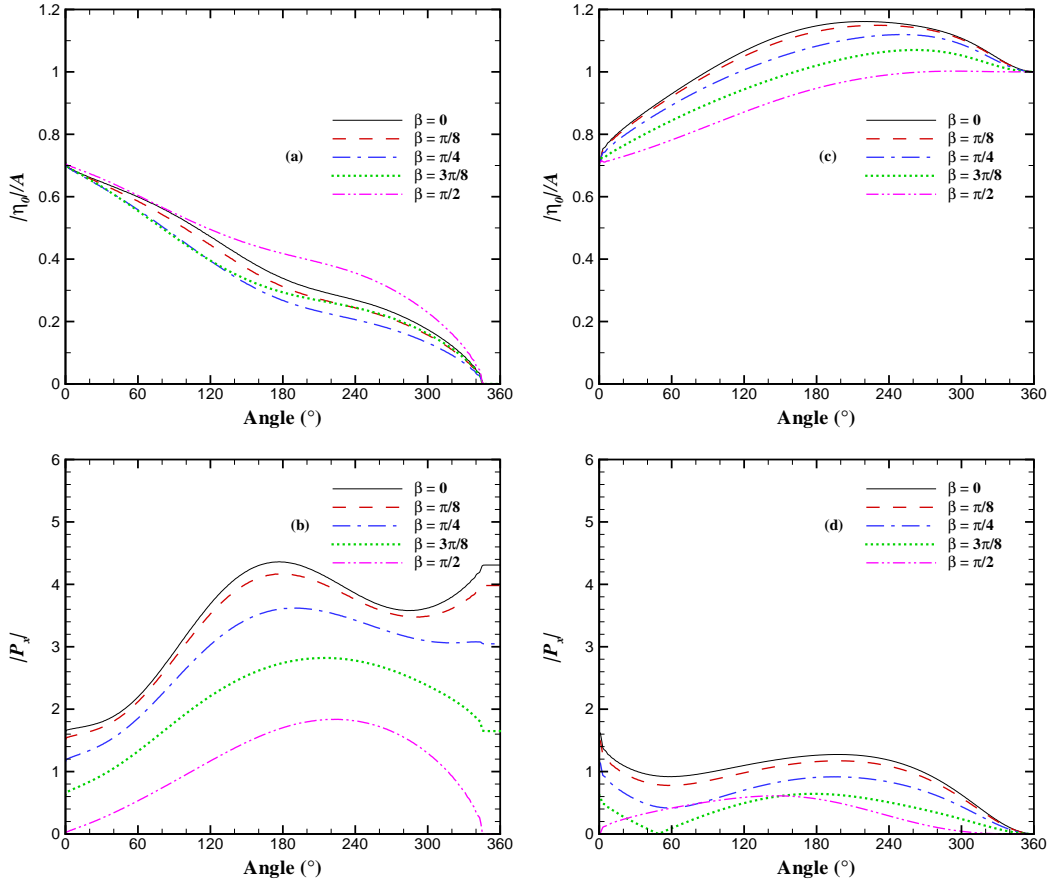


Figure 4. Variation of wave amplification factor at $r = 0$ and nondimensional wave force on the breakwater with solid (left) or opening (right) centre at $\theta = 180^\circ$, $ka = 1$ and $G_0 = 1$ vs. solid or opening area angle.

300 wall (left) and a partial opening (right) vs. their centre location with a solid
 301 or an opening area angle of 10° with the remaining part at $G_0 = 1$. As the
 302 location of the solid or opening centre varies, the amplification factor fluctu-
 303 ates whilst the largest amplification factor is often induced by either plane
 304 or standing incident waves. At some positions, different short-crestedness
 305 results in almost the same amplification factor (e.g., 70° for breakwater with
 306 a solid wall, and 40° and 120° for breakwater with an opening). As for the
 307 wave forces in the x direction, the earlier observations about smaller short-
 308 crestedness and solid wall inducing larger wave forces still hold. However,
 309 the variation of the location of the solid or opening centre does not affect the
 310 magnitude of the inline force much. When the solid or opening centre are at
 311 $\theta = 90^\circ$, the breakwater becomes symmetric along the y axis leading to zero
 312 wave force in the x direction due to standing waves. The wave force in the

313 y direction is rather small compared to its counterpart in the x direction, and
 314 the largest wave force often occurs in either plane or standing incident waves.
 315 Fig. 6 shows equi-amplitude (left) and equi-phase (right) contours for the
 316 interior region of the breakwater generated by incident plane, short-crested,
 317 and standing waves corresponding to the wave spreading angles $\beta = 0, \pi/4$
 318 and $\pi/2$ respectively. The breakwater has a partial opening at $175^\circ < \theta <$
 319 185° , and porosity of the remaining part is at $G_0 = 1$. Also wave number
 320 $k = 1 \text{ m}^{-1}$ and $a = 5 \text{ m}$. It can be seen that the wave patterns for short-
 321 crested and standing incident waves are much more complex than the one for
 322 plane incident waves. The surface elevation within the breakwater is seen to
 323 decrease as β increases. In addition to symmetry to the x -axis, wave elevation
 324 pattern due to a standing wave is seen almost symmetric to the y -axis as
 325 well. In this case, the slightly asymmetry to the y -axis is introduced by the
 326 small opening. The thick lines in phase contours represent changes from π
 327 to $-\pi$. The amphidromic points, where equi-phase lines converge and the
 328 wave amplitude vanishes, clearly form for short-crested and standing inci-
 329 dent waves. The phases near two adjacent amphidromic points rotate from
 330 $-\pi$ to $+\pi$ clockwise and counter-clockwise around the points respectively.
 331 For the standing incident wave component, the amplitudes in the transverse
 332 directions are small compared to their inline values, with a faster variation in
 333 the corresponding phase contours.

334 4. Conclusions

335 A general 3D short-crested wave interaction with a porous cylindrical break-
 336 water is studied analytically by linear potential wave theory. Three basic con-
 337 figurations of the breakwater are investigated. The performance of the break-
 338 water is examined by the effects of short-crested wave parameters, structural
 339 porosity, and the angle and position of the partial impermeable wall and open-
 340 ing on wave amplification factor, wave forces, and wave elevation contours.
 341 It is found that by making the porosity nonuniform, the amplification factor,
 342 wave forces, and elevation contours become more complex than its counter-
 343 part of uniform porosity. Incident waves with smaller short-crestedness along
 344 with solid walls generally result in larger wave forces, whilst an opening
 345 on the breakwater and limiting incident waves, i.e. plane or standing waves
 346 clearly lead to larger amplification factors within the breakwater. The effect
 347 of the location of the solid or opening centre appears to be insignificant on
 348 the inline wave force (P_x), but rather significant on the transverse wave force
 349 (P_y). However, since P_y is one order smaller than P_x , we can conclude that
 350 the wave force is insensitive to the location of the solid or opening centre. Due
 351 to asymmetrical geometry, wave forces induced by standing incident waves

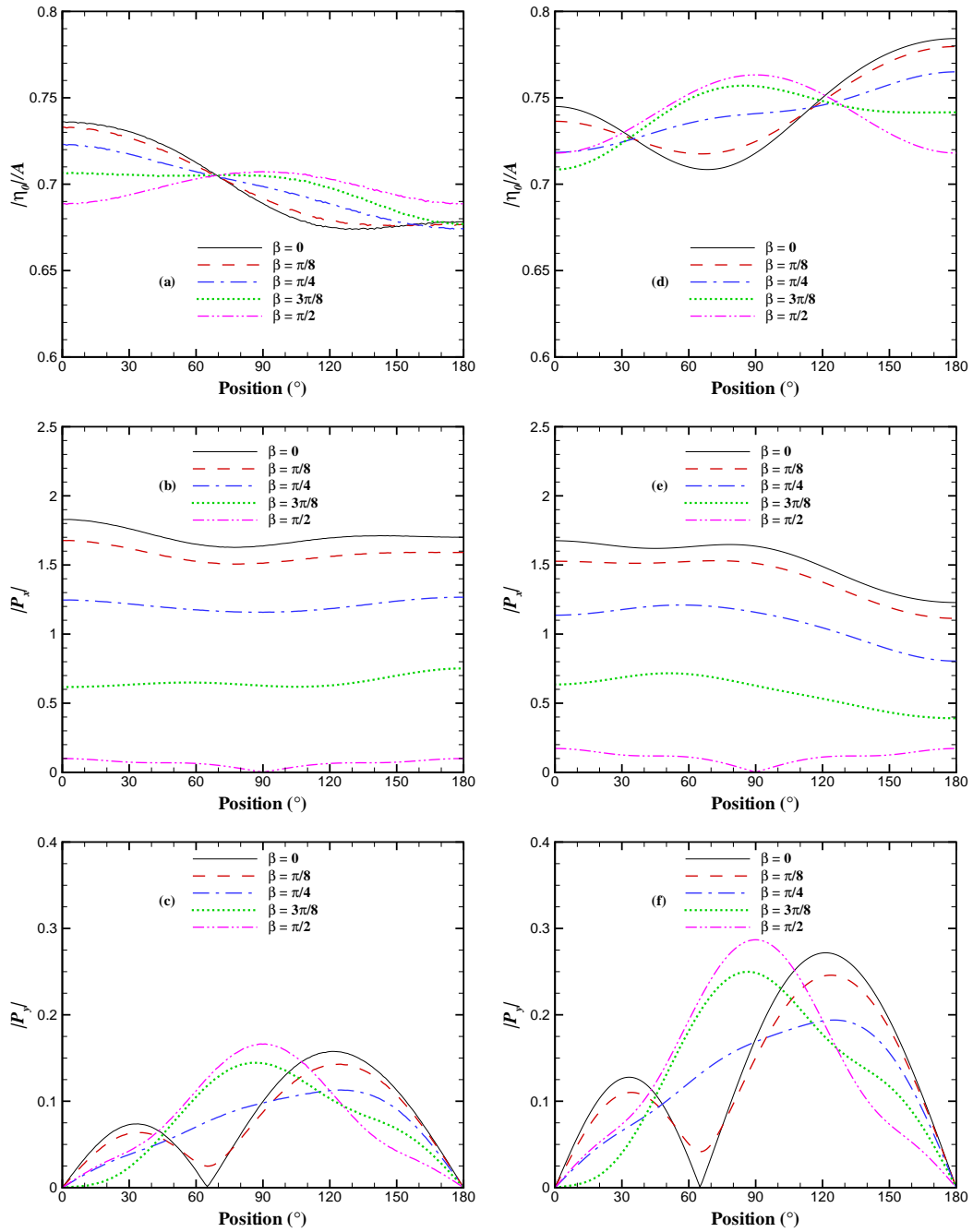


Figure 5. Variation of wave amplification factor at $r = 0$ and nondimensional wave force on the breakwater with solid (left) or opening (right) area angle 10° at $ka = 1$ and $G_0 = 1$ vs. the location of the solid or opening centre.

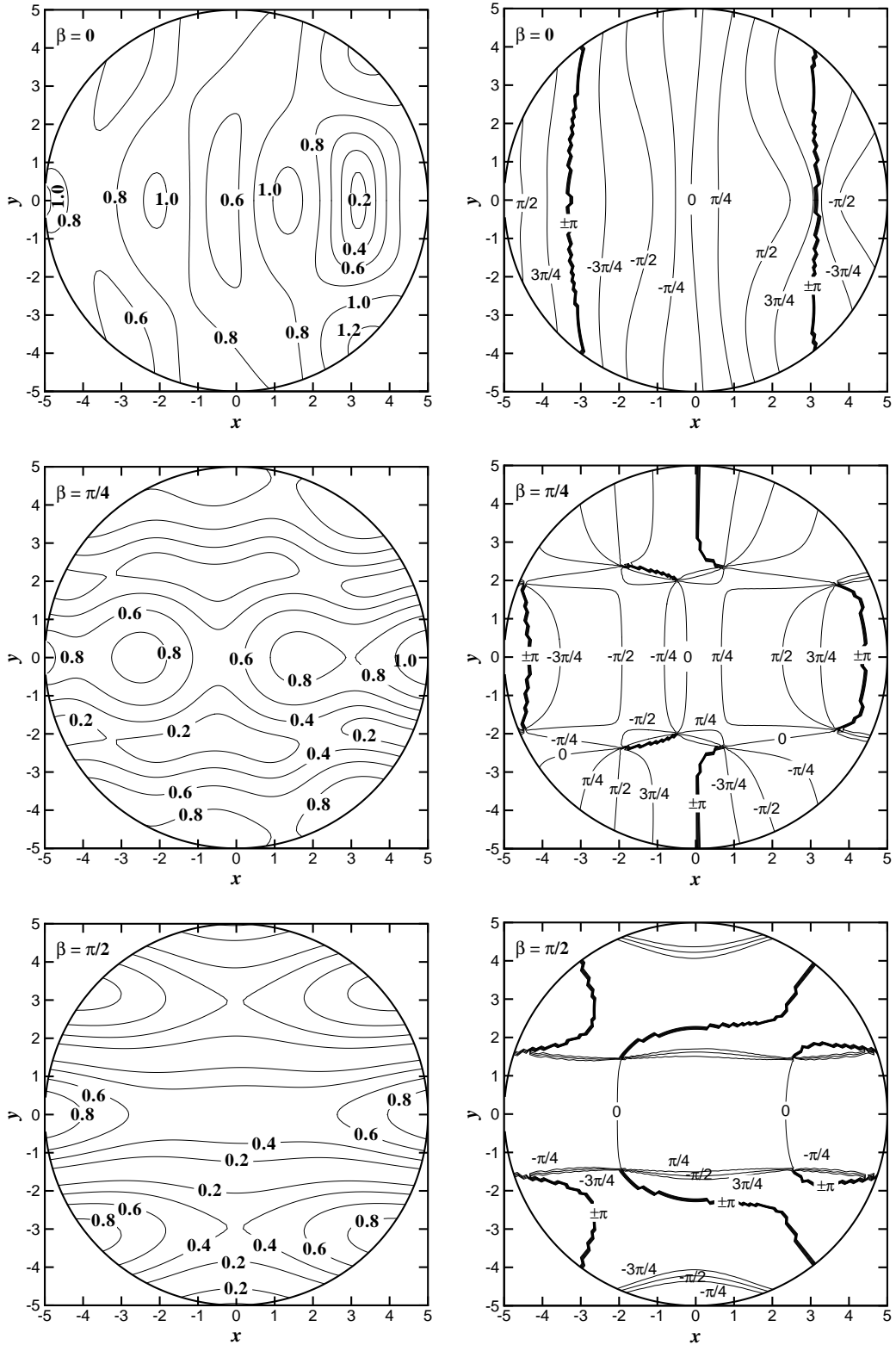


Figure 6. Equi-amplitude (left) and Equi-phase (right) contours for incident short-crested wave with short-crestedness angles $\beta = 0, \pi/4, \pi/2$ and $k = 1 \text{ m}^{-1}$, and breakwater with partial opening at $175^\circ < \theta < 185^\circ$, $a = 5 \text{ m}$, and porosity $G_0 = 1$.

352 are no longer zero. Here the component in the direction perpendicular to the
353 incident wave may come forth, though the magnitude is normally small.

354 It is hoped that the analysis presented and the results of the parametric
355 study in the paper will be found useful in the design of coastal and offshore
356 structures. They should be useful in selecting a suitable circular breakwater
357 for a particular application.

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