

# PLANNING AND TEACHING MATHEMATICS LESSONS AS A DYNAMIC, INTERACTIVE PROCESS

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*We are researching actions that teachers can take to improve mathematics learning for all students, with particular attention to specific groups of students who might experience difficulty. After identifying possible barriers to learning, we offered teachers mathematics lessons structured in a particular way. Teachers' use of the model outlined in this paper seemed productive and their resulting planning and teaching proved to be dynamic and interactive. This paper uses excerpts from a conversation between two teachers to illustrate specific aspects of the model.*

## MEASURE CUT MEASURE CUT MEASURE CUT

Many English language sayings urge the listener to plan carefully. “Measure twice, cut once”, for example, appeals to dressmakers, carpenters, and everyone else to plan ahead and prepare carefully for their tasks. Our research into inclusive mathematics lessons suggests that, despite the emphasis on linear structures for lesson planning in many pre-service teacher education courses, the process of constructing inclusive and engaging mathematics lessons needs to be more dynamic: more like “measure, cut, measure, cut, measure, cut”. The need for dynamic reaction in teaching was addressed early by Brophy (1983), in discussion of ways to overcome self-fulfilling prophecy effects. Brophy urged teachers to be reactive rather than proactive, listening to students and shaping teaching in directions suggested by their responses. In our model of inclusive mathematics teaching we use the terms dynamic interaction to describe this reactive process.

This article reports data from a discussion between two of the teachers who were part of a 3-year project investigating ways to structure lessons in order to include students who are at different stages of readiness. It illustrates ways that the teachers involved adapted our recommended model in their own lesson preparation and illustrates how their planning and teaching could be described as dynamic and interactive.

## A MODEL FOR PLANNING AND TEACHING

The framework for our model of planning and teaching inclusive mathematics lessons is based on the work of Cobb and his colleagues (e.g., Cobb & McClain, 2001), who used the terms *mathematical norms* and *socio-mathematical norms* to describe different dimensions of classroom action. We have extended the second of these and use the phrase *mathematical community norms* to encompass not only “classroom actions and interactions that are specifically mathematical” (p. 219) but also norms of practice and other factors that affect learning in mathematics

classrooms. In particular, our conceptualisation includes elements such as culture, social group, language use and comprehension, and modes of classroom organisation.

### **Some mathematical norms of the model**

While we have found that the model can be applied to most task types, we focus on open-ended mathematics tasks since they are likely to create opportunities for students' personal constructive activity. The open-endedness allows a focus on key mathematical ideas and can be used to encourage students to investigate, make decisions, generalise, seek patterns and connections, communicate, and identify alternatives (Sullivan, 1999). They also generally contribute to teachers' appreciation of students' mathematical and social learning (Stephens & Sullivan, 1997).

Our model for planning and teaching takes *mathematical norms* to be the principles, generalisations, processes, mathematical tasks and sub-tasks, and work products that form the basis of the curriculum. There are three specific aspects of our model of planning and teaching that direct teachers' attention to these mathematical dimensions: mathematical tasks and their sequencing; enabling prompts; and extending prompts (see also Sullivan, Mousley, & Zevenbergen, 2004).

*The tasks and their sequence.* In building sets of learning experiences, an important aspect of the model is the creation of a notional sequence of tasks that Simon (1995) described as a learning trajectory, made up of three components: a goal determining the desired direction of teaching and learning; the activities to be undertaken; and a hypothetical cognitive process, "a prediction of how the students' thinking and understanding will evolve in the context of the learning activities" (p. 136).

*Enabling prompts.* We argue that it is preferable to encourage students experiencing difficulty to engage in sub-tasks related to the goal task, rather than requiring them to listen to additional explanations or to pursue goals substantially different from the rest of the class. Enabling prompts temporarily divert students to lower-demand sub-tasks that allow them subsequently to re-join the class learning trajectory. We note that, even though they previously offering contrary advice, the English Department of Education and Skills (2004) now recommends task differentiation that is centred around work common to all pupils in a class, with targeted support for those who have difficulties keeping up with their peers.

*Extending prompts.* Students who complete planned tasks quickly can be posed supplementary activities, to extend their thinking on those tasks. A characteristic of open-ended tasks is that they create opportunities for extension of mathematical thinking, since students can explore a range of options as well as consider forms of generalised response. The challenge for teachers is to pose prompts that extend students' thinking in ways that do not make them feel that they are getting more of the same or being punished for completing earlier work.

### **Some mathematical community norms of the model**

Our model focuses on two characteristics of *mathematical community norms*:

*Normative interactions.* These are the common practices, organisational routines, and communication modes that impact on approaches to learning, types of responses valued, views about legitimacy of knowledge produced, and responsibilities of individual learners. Through the overt and the hidden curricula of schools, students receive diverse messages (Bernstein, 1996). Sullivan, Zevenbergen, and Mousley (2002) listed a range of strategies that teachers can use to make implicit pedagogies more explicit and so address aspects of possible disadvantage of particular groups. Making these aspects explicit is a feature of our model of inclusive teaching.

*Mathematical community norms.* These conveys the idea that all students can participate and progress as part of the classroom community, including making individual decisions as well as contributing to class discussions and lesson reviews, at the same time developing a shared knowledge base for subsequent learning. We agree with Wood (2002) who proposed that all students benefit from participation in core mathematical and social experiences, and that rich social interactions with others contribute substantially to children's opportunities for learning mathematics.

## **MAXIMISING SUCCESS FOR ALL STUDENTS**

The overall project, of which the illustrative data below are a part, seeks to identify strategies for teaching mathematics to heterogeneous groups. Initially we identified and described aspects of classroom teaching that may act as barriers to mathematics learning for some students. Next we described strategies for overcoming such barriers (see Sullivan et al., 2002), including creating some scripted experiences, that were taught by participating teachers (see Sullivan et al., 2004). Analysis of these experiences allowed reconsideration of the emphasis and priority of respective teaching elements. It was found that it is possible to create sets of learning experiences that include all students in rich, challenging mathematical learning.

For the most recent stage of the research, we sought to examine whether teachers themselves could use the model to create effective sets of mathematical learning experiences, and have considered ways that specific aspects of the model contribute to the goal of inclusive experience. Our research approach was based in the action research paradigm, with its spirals of planning, acting, monitoring, and reflection (Kemmis & McTaggart, 1988) and emphasis on autonomous decision-making by participants. Essentially this stage involved 20 teachers developing, planning, and teaching sets of experiences of their own design and choice, while considering the aspects of both sets of norms outlined above.

The following illustration of these aspects uses as data a discussion between two female teachers from the same school who planned together and taught various sets of mathematical lessons as part of the project. This report refers to just one of those sets of experiences, planned formally using the model proposed. Their school serves a predominantly lower socio-economic community, and the teachers joined the project because of what they identified as a lack of engagement of their students in learning mathematics. The discussion between the two teachers was moderated by

one of the researchers, recorded, transcribed and then analysed by identifying key issues. The discussion below draws mainly on the words on one teacher, Zeta, but refers to the second teacher, Delta, where appropriate. It reports aspects of their planning, their mathematical intent, the goals and sequence of tasks, their emphasis on engagement, and their use of prompts to support students experiencing difficulty as well as those who finished quickly.

### **Knowing where you are going**

The two teachers, planning collaboratively, had chosen the topic of capacity, volume, and surface area. Zeta described the two goal tasks as follows: “Let’s get them to think about how big is 360 cm cubed and what do the dimensions of these shapes ... really look like”, and “If a shape has a surface area of 1000 square cm, what could (the dimensions) be?” Both of these tasks are open-ended and address significant mathematical content; the latter task being well above the normal expectation for a class at the level they were teaching (age 13). The teachers also considered the trajectory of experiences necessary for the students to be able to respond to the goals.

Zeta: I actually think that we knew ... that we were both headed towards what could the dimensions of an object be with the volume of whatever. ... We just had to work out what would the teaching be along the way. And so that is what this first [stage of the written plan] was about: empowering them to be able to answer that question.

(It) talks about volume, capacity, millimetres, millilitres, litres, megalitres—all that stuff—and just gets them to think ... about volume. [We said] “Let’s start with water, I just want you to start thinking about all the different sizes we can buy water in” , and ... got them to list ... you can buy it in bottles, what about dams, what about tanks? And so they started to think about water from about this big to however big ... to just get them all to the same spot where they are thinking about it.

It is possible to see evidence here of a focus on the mathematical content goals, a sequence of experiences, and the building of community understanding. They were also drawing on student experiences, as the school is in inland Australia where available drinking water in catchments is a community issue, thus facilitating access to the content and allowing key terminology to arise naturally.

Delta: I don’t remember actually saying the word volume. I think the kids came back with the word volume.

Zeta: Well what word did you say?

Delta: I said, “In what size can water be purchased?” Exactly like that ... but they came back with the word volume and then ... I think capacity came from them too. ... and I remember writing this down: volume was the stuff inside a container or a dam or whatever, and capacity was when it was filled to the top and we left it hang like that.

Zeta: I said, “Can you think of ...” and obviously I had an answer in my head, but “Can you think of a place or a time where they talk about water capacity?” And I think kids talked about our water storage. “Does anyone know what it is at the moment?” “27%” And I said “Well what does that mean, 27% capacity?” So we talked about it in that way.

These teachers seemed to see teaching as dynamically interactive, responding to children's knowledge of the relevant mathematics.

### **Engaging with the mathematics**

To follow this introductory class discussion, the two teachers had planned a task that required their students to arrange common containers in order of their capacity. In earlier observations we had identified an additional aspect of the model that we are tentatively terming a "hook task" that is interesting to the students and which can be used to engage the students with the mathematical content. Characteristics of such a hook include that it should be within the experiences of the students, that it should be in the form of a problem, and that students should all anticipate success. These characteristics were evident in this first task for the class.

Zeta: Then we talked about the volume of a container and I pulled out a whole lot of containers. There must have been 30 or 40 containers in the room and I just chose about 10 of them and put them on the table, then asked them to rank them from smallest to largest. We actually did it as a class, because some of them were a little obvious. There were a few times where kids couldn't agree on a shape, and so I sat them on top of each other and I said, "When I get you to list them in an order, I want you to list them in your own order. You can decide then". So I got them to write them down in order and we kind of agreed on some names so that we were all talking the same language. And then I asked them to guess what the volume [of each container] was.

This is an example of actions that explicitly address some mathematical community norms: contributing to a shared knowledge base for subsequent learning. The other teacher also commented on the way that this hook engaged the students. She illustrated how the teachers were alert to a range of students' engagement in the task.

Delta: I have got all the containers up the front, and kids ... coming closer and closer and pushing up to the front of the room; and I became superfluous too, and I ended up saying to these two boys "Well, I'll step out", and stepped back, and they just took over. ... and one of them, ... can be disruptive and really aggressive at times, about work and about anything, and he was up there ... not taking over . ... There wasn't any "No, you're wrong" either, which I thought would come up.

There was also clear intention that the students experienced the mathematics that the teachers had in mind, and this was particularly evident in the way the teachers were explicit about the focus of the learning.

Zeta: I alluded to the fact that I had an ulterior motive. "So what if I told you that there is a mathematical way to calculate the volume of everything?" and "Yes, we can check using water, but I would like to teach you how to calculate it mathematically and then you can check your answer". ... So we talked about volume and then I think I picked a box that I hadn't put on the table and we looked at how we calculated the volume of that, and they all did it, and so then they were set the task of calculating the volume of one of the shapes. So in their small groups they each had to choose a shape and go off and calculate the volume of it, bring it back to the table, pick another shape. I think I wanted them all to do 3 or 4, to not calculate all of them ... some of them did 5, and others did 1.

While this task was not open-ended in that there was not a range of possible answers for any one object, it did allow variety of activity and the students were able to make individual decisions, a key aspect of the teaching model's mathematical community norms. Also important for the model is the way that the teachers respond to individual students.

Zeta: But it was interesting, one kid chose the hexagonal vase because he liked the way it looked. But then when it came down to working out the volume he had no idea of how to calculate the area of the hexagon; and I suggested he could break it up into some other shapes. "What shapes could you break it up into?" No idea. He talked about what area was and we talked about area in terms of being centimetres squared or millimetres squared, so he traced around the shapes and he decided to break it up into half centimetre squares, because centimetres squared would be too big for the shape.

This teacher had provided support for an individual student, based on her specific needs. This differential support for students working on common tasks is a vital component of our model. It forms the basis of whole-class shared experience and hence the building of a mathematical community, and also enables learning from whole-class discussion about experience.

Zeta: Then I had to have a discussion. "Well this is our ranking. How are we going to check them?" Some said "Maybe if you looked at the height, and kind of the width and the length of the shapes, maybe you could work it out". "Say it again: length, by width, by height". And I mean it was just absolutely classic, ... and then we went on to a discussion about regular prisms.

Again the focus on the mathematics and dynamic interactivity are evident, although the teacher's agenda is also apparent. These preliminary tasks were leading toward the first of the open-ended goal tasks that allowed the students to calculate volumes, and particularly to recognise that there can be many shapes with the same volume.

Zeta: I posed this question of the volume being 360 cm cubed, and probably half the class just went for it and just knew what to do, and used the question I had up on the board and just changed the numbers, and so on.

Of course, not all students could readily engage in the task, and the teacher prompted them by posing variations to the initial task with a reduced cognitive demand.

Zeta: I said, "Well, we know the volume has something to do with the area, it has to do with the area at the base. Where is the base on this picture?" [They were looking at a rectangular prism.] They pointed to it, and I said, "Well, if the volume of the whole thing has to be 360, what could the area of the base be?" And they chose a number. And I said "Well okay, if the area of that base is whatever they chose, ... what would the height of that shape be if the base is this and the body is that?"

There were also students who completed the basic task readily. We have noted in many lessons that an important challenge is to engage such students, but we have found consistently that this is much easier said than done.

Zeta: And there were obviously kids I suggested doing the triangular base, or a cylinder. The cylinder was interesting ... The only problem with it probably was that there were some kids that did a couple and then thought “Well I can do any shape now and I don’t need to do this any more.”

The teachers then followed a similar sequence of interactions and tasks for the second of the goal tasks, that of describing a box with a surface area of 1000 cm<sup>2</sup>. There was an initial exploration with class discussion, some calculations using actual objects, and the open-ended task with provision of both enabling and extending prompts. Again, the emphasis on engagement, the focus on the mathematics, the interactive pedagogy, the dynamic nature of the teachers’ planning, and the differentiation of the task were all evident.

## SUMMARY

Basically our model for planning and teaching mathematics includes (a) mathematical norms, including tasks facilitating student engagement in meaningful mathematics, the sequence of the tasks, enabling prompts, and extending prompts; and (b) mathematical community norms, including making normative interactions more explicit and focusing on the development of a mathematical learning community.

We have worked with a number of teachers on a variety of sets of learning experiences, with the above being an example of the type of planning and teaching that have resulted. The above data, along with the other experiences observed, suggest that teachers are able to use the model. Interviews with all of the teachers involved indicate that the model does allow them to focus on the challenge of engaging all students in productive mathematical explorations and provides key principles and strategies for doing this.

In terms of the *mathematical norms*, the teachers were able to create open-ended tasks that addressed important aspects of mathematics, they considered the trajectory of tasks for the class, and they offered suitable variations of the tasks for those experiencing difficulty and those who completed the tasks quickly. (The latter proved the more difficult so the next phase of the research will address this issue.) The *mathematical community norms* were also explicitly addressed by the teachers, not only in the instructions about ways of approaching the tasks and formulating responses, but also in the building of community, by working on common tasks and through the interactive responses and discussions. These particular teachers focused on the engagement of the students in their learning, considering this in their planning and celebrating it in reflection on the experience, just as other participating teachers did.

Two aspects emerged from these data, and indeed observations of other project teachers. The first was the interactivity of the teachers with each other in their planning, and with the students during their teaching. They were clearly willing to

observe and listen to the students and to respond accordingly. The second was the dynamic nature of this interaction. Rather than feeling constrained by any preparation or the hypothetical learning trajectory, they were willing to adjust the tasks, the emphases, the timing, and the supports offered. The metaphor is measure, cut, see if it fits, measure, cut, see if it fits, ... . We suspect that the model gives teachers a structure that allows them freedom for a dynamic, interactive approach to teaching.

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