Restricting Weights in Supplier Selection Decisions in the presence of Dual-Role factors

Author
Saen, Reza Farzipoor

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One of the uses of Data Envelopment Analysis (DEA) is supplier selection. Weight restrictions allow for the integration of managerial preferences in terms of relative importance levels of various inputs and outputs. As well, in some situations there is a strong argument for permitting certain factors to simultaneously play the role of both inputs and outputs. The objective of this paper is to propose a method for selecting the best suppliers in the presence of weight restrictions and dual-role factors. This paper depicts the supplier selection process through a DEA model, while allowing for the incorporation of decision maker’s preferences and considers multiple factors which simultaneously play both input and output roles. The proposed model does not demand exact weights from the decision maker. This paper presents a robust model to solve the multiple-criteria problem. A numerical example demonstrates the application of the proposed method.

Keywords: Data envelopment analysis, Supplier selection, Weight restrictions, Dual-role factors

1. Introduction

Supplier selection is the process by which suppliers are reviewed, evaluated, and chosen to become part of the company’s supply chain. Shin et al. (2000), Farzipoor Saen (2007a),
Farzipoor Saen and Zohrehbandian (2008), Farzipoor Saen (2008), and Farzipoor Saen (2009) argue that several important factors have caused the current shift to single sourcing or a reduced supplier base. First, multiple sourcing prevents suppliers from achieving the economies of scale based on order volume and learning curve effect. Second, multiple supplier system can be more expensive than a reduced supplier base. For instance, managing a large number of suppliers for a particular item directly increases costs, including the labor and order processing costs to managing multiple source inventories. Meanwhile multiple sourcing lowers overall quality level because of the increased variation in incoming quality among suppliers. Third, a reduced supplier base helps eliminate mistrust between buyers and suppliers due to lack of communication. Fourth, worldwide competition forces firms to find the best suppliers in the world.

One of the uses of Data Envelopment Analysis (DEA) is supplier selection. In original DEA formulations the assessed Decision Making Units (DMUs) can freely choose the weights or values to be assigned to each input and output in a way that maximizes its efficiency, subject to this system of weights being feasible for all other DMUs. This freedom of choice shows the DMU in the best possible light, and is equivalent to assuming that no input or output is more important than any other.

The free imputation of input-output values can be seen as an advantage, especially as far as the identification of inefficiency is concerned. If a DMU (supplier) is free to choose its own value system and some other supplier uses this same value system to show that the first supplier is not efficient, then a stronger statement is being made. The advantages of full flexibility in identifying inefficiency can be seen as disadvantages in the identification of efficiency. An efficient supplier may become so by assigning a zero weight to the inputs and/or outputs on which its performance is worst. This might not be acceptable by decision makers as well as by the analyst, who after spending time in a careful selection of inputs and outputs sees some of them being completely neglected by suppliers.
Decision makers may have in supplier selection problems value judgments that can be formalized *a priori*, and therefore should be taken into account in supplier selection. These value judgments can reflect known information about how the factors used by the suppliers behave, and/or "accepted" beliefs or preferences on the relative worth of inputs, outputs or even suppliers. For example, in supplier selection problem in general, one input (material price) usually overwhelms all other inputs, and ignoring this aspect may lead to biased efficiency results. Suppliers might also supply some outputs that require considerably more resources than others and this marginal rate of substitution between outputs should somehow be taken into account when selecting a supplier. To avoid the problem of free (and often undesirable) specialization, input and output weights should be constrained in DEA.

In some situations there is a strong argument for permitting certain factors to simultaneously play the role of both inputs and outputs. In supplier selection context, the research and development cost can be considered as both an input and an output. Remembering that the simple definition of efficiency is the ratio of output to input, an output can be defined as anything whose increase will cause an increase in efficiency. Similarly, an input can be defined as anything whose decrease will cause an increase in efficiency. If the research and development cost is considered as an output, then the increase in the research and development cost will increase the efficiency of the supplier. Likewise, if the research and development cost is considered as an input, then any decrease in the research and development cost without a proportional decrease in the outputs will increase efficiency. So, depending on how one looks at it, either increasing or decreasing the research and development cost can increase efficiency. As well, as Farzipoor Saen (in press a) discussed, the factors such as ratings for service-quality experience (EXP) and service-quality credence (CRE) were considered dual-role factors. From the perspective of decision maker who intends to select the best supplier, such measures may play the role of proxy for "high quality of services", hence can reasonably be classified as outputs. On the other hand, from the
perspective of supplier that intends to supply reverse logistics services, they can be considered as inputs that help the supplier in obtaining more customers.

Beasley (1990, 1995), in a study of the efficiency of university departments, treated research funding on both the input and output sides. However, as Cook et al. (2006) addressed, the model proposed by Beasley (1990, 1995) has two limitations. The first limitation is that in the absence of constraints (e.g., assurance region or cone ratio) on the multipliers, each DMU will be 100% efficient. The second limitation is that the dual-role factor is considered as a discretionary factor.

Cook et al. (2006) developed a new model that has not the abovementioned limitations. However, their development pertains to a single dual-role factor and does not consider multiple dual-role factors.

The objective of this paper is to propose a method for selecting the best suppliers in the presence of both weight restrictions and dual-role factors. This paper depicts the supplier selection process through a DEA model, while allowing for the incorporation of decision maker’s preferences and considers multiple factors which simultaneously play both input and output roles.

This paper proceeds as follows. In Section 2, literature review is presented. Section 3 introduces the method which selects the suppliers. Numerical example and concluding remarks are discussed in Sections 4 and 5, respectively.

2. Literature review

Some mathematical programming approaches have been used for supplier selection in the past. Table 1 categorizes the reviewed papers based on applied techniques. However, because of the complexity of the decision making process involved in supplier selection, all the aforementioned references in Table 1, rely on some form of procedures that assigns weights to various performance measures. Meanwhile, they do not consider dual-role factors.
The primary problem associated with arbitrary weights is that they are subjective, and it is often a difficult task for the decision maker to accurately assign numbers to preferences. It is a daunting task for the decision maker to assess weighting information as the number of performance criteria increased.

To this end, Weber (1996) demonstrated how DEA can be used to evaluate vendors on multiple criteria and identified benchmark values which can then be used for this purpose. Weber et al. (2000) presented an approach for evaluating the number of vendors to employ in a procurement situation using Multi-Objective Programming (MOP) and DEA. The approach advocates developing vendor-order quantity solutions (referred to as supervendors) using MOP and then evaluating the efficiency of these supervendors on multiple criteria using DEA. Braglia and Petroni (2000) described a Multi-Attribute Utility Theory (MAUT) based on the use of DEA, aimed at helping purchasing managers to formulate viable sourcing strategies in the changing market place. To evaluate the aggregate performances of suppliers, Liu et al. (2000) proposed to employ DEA. This extends Weber’s (1996) research in using DEA in supplier evaluation for an individual product. Forker and Mendez (2001) proposed an analytical method for benchmarking using DEA that can help companies identify their most efficient suppliers, the suppliers among the most efficient with the most widely applicable Total Quality Management (TQM) programs, and those suppliers who are not on the efficient frontier but who could move toward it by emulating the practices of their “best peer” supplier(s). Talluri and Sarkis (2002) focused upon the supplier performance evaluation and monitoring process, which assist in maintaining effective customer-supplier linkages. They tried to improve the discriminatory power of Banker-Charnes-Cooper (BCC) model proposed by Banker et al. (1984). To select appropriate suppliers, Talluri et al. (2006) suggested a Chance-Constrained Data Envelopment Analysis (CCDEA) approach in the presence of multiple performance measures that are uncertain. To select the best suppliers in the presence
of both cardinal and ordinal data, Farzipoor Saen (2007a) proposed an innovative method, which is based on Imprecise Data Envelopment Analysis (IDEA).

Recently, Farzipoor Saen (2008) developed a new pair of assurance region-imprecise data envelopment analysis (AR-IDEA) model for ranking the best suppliers in the presence of both weight restrictions and imprecise data. But, the proposed model is cumbersome and does not consider dual-role factors. As well, Farzipoor Saen (in press a) proposed a model for selecting third-party reverse logistics (3PL) providers in the presence of multiple dual-role factors. However, his model does not consider weight restrictions.

However, all of the abovementioned references do not consider dual-role factors and weight restrictions simultaneously. A technique that can deal with both dual-role factors and weight restrictions is needed to better model such situation.

To the best of author’s knowledge, there is not any reference that discusses supplier selection in the presence of both dual-role factors and weight restrictions. The approach presented in this paper has some distinctive features.

- The proposed model does not demand exact weights from the decision maker. Since classical techniques always require intuitive judgments that have biases, this paper helps decision makers to select the suppliers without relying on intuitive judgments.
- The proposed model considers dual-role factors for supplier selection.
- The proposed model considers both dual-role factors and weight restrictions simultaneously.
- Supplier selection is a straightforward process carried out by the proposed model.
- The increasing number of decision making criteria, complicates the supplier selection process. This paper presents a robust model to solve the multiple-criteria problem.
3. Proposed method for suppliers selection

DEA¹ proposed by Charnes et al. (1978) (Charnes-Cooper-Rhodes (CCR) model) and developed by Banker et al. (1984) (Banker-Charnes-Cooper (BCC) model) is an approach for evaluating the efficiencies of DMUs. One serious drawback of DEA applications in supplier selection has been the absence of decision maker judgment, allowing total freedom when allocating weights to input and output data of supplier under analysis. This allows suppliers to achieve artificially high efficiency scores by indulging in inappropriate input and output weights.

The most widespread method for considering judgments in DEA models is, perhaps, the weight restrictions inclusion. Weight restrictions allow for the integration of managerial preferences in terms of relative importance levels of various inputs and outputs. The idea of conditioning the DEA calculations to allow for the presence of additional information arose first in the context of bounds on factor weights in DEA’s multiplier side problem. This led to the development of the cone-ratio (Charnes et al., 1989) and assurance region models (Thompson et al., 1990). Both methods constrain the domain of feasible solutions in the space of the virtual multipliers.

To introduce the method for supplier selection, Table 2 lists the nomenclature used to formulate the problem under consideration.

The discussions in this paper are provided with reference to the original DEA formulation by Charnes et al. (1978) below, which assumes constant returns to scale and that all input and output levels for all DMUs are strictly positive. The CCR model measures the efficiency of DMU₀ relative to a set of peer DMUs:
\[ e_o = \max \frac{\sum_{r=1}^{s} u_r y_{r o}}{\sum_{i=1}^{m} v_i x_{i o}}, \]

subject to
\[ \frac{\sum_{r=1}^{s} u_r y_{r j}}{\sum_{i=1}^{m} v_i x_{i j}} \leq 1, \quad j = 1, \ldots, n, \quad (1) \]
\[ u_r, v_i \geq \varepsilon \quad \forall r \quad \text{and} \quad i \]

where there is a set of \( n \) peer DMUs, \( \{\text{DMU}_j : j = 1, 2, \ldots, n\} \), which produce multiple outputs \( y_{r j} (r = 1, 2, \ldots, s) \), by utilizing multiple inputs \( x_{i j} (i = 1, 2, \ldots, m) \). DMU\(_o\) is the DMU under consideration. \( u_r \) is the weight given to output \( r \) and \( v_i \) is the weight given to input \( i \). \( \varepsilon \) is a positive non-Archimedean infinitesimal. DMU\(_o\) is said to be efficient \((e_o=1)\) if no other DMU or combination of DMUs can produce more than DMU\(_o\) on at least one output without producing less in some other output or requiring more of at least one input. The linear programming equivalent of (1) is as follows:

\[ e_O = \max \sum_{r=1}^{s} y_{r o} u_r, \]

subject to
\[ \sum_{i=1}^{m} x_{i o} v_i = 1, \quad (2) \]
\[ \sum_{i=1}^{m} x_{i j} v_i - \sum_{r=1}^{s} y_{r j} u_r \geq 0, \quad \forall j, \]
\[ v_i \geq \varepsilon \quad \forall i, \]
\[ u_r \geq \varepsilon \quad \forall r. \]

In (3) the various types of weight restriction that can be applied to multiplier models are shown (Cooper et al. 2004).
Absolute weight restrictions
\[ \delta_i \leq v_i \leq \tau_i \quad (g_i) \quad \rho_r \leq u_r \leq \eta_r \quad (g_o) \]

Assurance region of type I (relative weight restrictions)
\[ \alpha_i \leq \frac{v_i}{v_i + 1} \leq \psi_i \quad (h_i) \quad \theta_r \leq \frac{u_r}{u_r + 1} \leq \zeta_r \quad (h_o) \]

Assurance regions of type II (input-output weight restrictions)
\[ \phi_i v_i \geq u_r \quad (l) \]

The Greek letters \((\delta_i, \tau_i, \rho_r, \eta_r, \alpha_i, \psi_i, \theta_r, \zeta_r, \phi_i)\) are user-specified constants to reflect value judgments the decision maker wishes to incorporate in the assessment. They may relate to the perceived importance or worth of input and output factors. The restrictions \((g_i)\) and \((h_i)\) in (3) relate on the left hand side to input weights and on the right hand side to output weights. Constraint \((l)\) links directly input and output weights. Absolute weight restrictions are the most immediate form of placing restrictions on the weights as they simply restrict them to vary within a specific range. Assurance region of type I, link either only input weights \((h_i)\) or only output weights \((h_o)\). The relationship between input and output weights are termed assurance region of type II.

Weights restrictions may be applied directly to the DEA weights or to the product of these weights with the respective input or output level, referred to as virtual input or virtual output. The virtual inputs and outputs can be seen as normalized weights reflecting the extent to which the efficiency rating of a DMU is understood by a given input or output variable. Restrictions on virtual weights were proposed first by Wong and Beasley (1990). Restrictions on virtual inputs/virtual outputs assume the form in (4), where the proportion of the total virtual output of DMU\(_j\) accounted for by output \(r\) is restricted to lie in the range \([a_r, b_r]\) and the proportion of the total virtual input of DMU\(_j\) accounted for by input \(i\) is restricted to lie in the range \([c_i, d_i]\).
\[ a_r \leq \frac{u_r y_{rj}}{\sum_{j=1}^{n} u_r y_{rj}} \leq b_r, \quad r = 1, \ldots, s \]

\[ c_i \leq \frac{v_i x_{ij}}{\sum_{j=1}^{n} v_i x_{ij}} \leq d_i, \quad i = 1, \ldots, m \]

The range is normally determined to reflect prior views on the relative “importance” of
the individual outputs and inputs. Constraints such as (5) are DMU specific meaning that the
DEA model with such constraints may become computationally expensive. Wong and
Beasley (1990) suggest some methods for implementing restrictions on virtual values:

- Method 1: Add the restrictions only in respect of DMU_0 being assessed leaving free
the relative virtual values of the comparative DMUs;

- Method 2: Add the restrictions in respect of all the DMUs being compared. This is
computationally expensive as the constraints added will be of the order of 2^n(s+m);

- Method 3: Add the restrictions (4) only in relation to the assessed DMU, and add
constraints (5) with respect to the “average” DMU, which has an average level of
the \( r \)th output equal to \( \sum_{j=1}^{n} y_{rj} / n \) and has an average level of the \( i \)th input equal to
\[ \sum_{j=1}^{n} x_{ij} / n. \]
\[ a_r \leq \frac{u_r \sum_{j=1}^{n} y_{rj} / n}{\sum_{r=1}^{s} u_r \left( \sum_{j=1}^{n} y_{rj} / n \right)} \leq b_r, \quad r = 1, \ldots, s \]

\[ c_i \leq \frac{v_i \sum_{j=1}^{n} x_{ij} / n}{\sum_{i=1}^{m} v_i \left( \sum_{j=1}^{n} x_{ij} / n \right)} \leq d_i, \quad i = 1, \ldots, m \]

Restrictions on the virtual input-output weights represent indirect absolute bounds on the DEA weights of the type shown in (g) in (3). The imposition of restrictions on virtual inputs or outputs is sensitive to the model orientation.

The multipliers formulation, with the virtual weights restrictions applying to DMU\(_o\) (method 1), is as below\(^{2,3,4}\):

\[ e_o = \max \sum_{r=1}^{s} y_{r0} u_r, \]

s.t.

\[ \sum_{i=1}^{m} x_{i0} v_i = 1, \quad (6) \]

\[ \sum_{i=1}^{m} x_{ij} v_i - \sum_{r=1}^{s} y_{rj} u_r \geq 0 \quad \forall j, \]

\[ c_i \left( \sum_{i=1}^{m} x_{i0} v_i \right) - x_{i0} v_i \leq 0, \]

\[ x_{i0} v_i - d_i \left( \sum_{i=1}^{m} x_{i0} v_i \right) \leq 0, \]

\[ v_i \geq \varepsilon \quad \forall i, \]

\[ u_r \geq \varepsilon \quad \forall r. \]

In summary, Model (6) proposes a method for selecting the best suppliers in the presence of weight restrictions.

Now, to consider dual-role factors and weight restrictions, a new model is proposed. Consider a situation where members \( j \) of a set of \( n \) DMUs are to be evaluated in terms of \( s \) outputs \( Y_j = (y_{ij})_{i=1}^{s} \) and \( m \) inputs \( X_j = (x_{ij})_{i=1}^{m} \). In addition, assume that a particular factor is
held by each DMU in the amount $w_j$, and serves as both an input and output factor. The proposed model for considering single dual-role factor is as follows (Cook et al., 2006).

$$\begin{align*}
\text{max} & \quad \frac{\sum_{r=1}^{s} u_r y_{ro} + \gamma w_o - \beta w_o}{\sum_{i=1}^{m} v_{i} x_{io}}, \\
\text{st} & \quad \sum_{r=1}^{s} u_r y_{jr} + \gamma w_j - \beta w_j - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \quad j = 1, \ldots, n \\
& \quad u_r, v_i, \gamma, \beta \geq 0.
\end{align*}$$ (7)

At this point, to demonstrate how to consider multiple dual-role factors in the model, the following new model is presented. Assume that some factors are held by each DMU in the amount $w_{jf} \ (f=1, \ldots, F)$, and serve as both an input and output factors. The proposed model for considering multiple dual-role factors is as follows:

$$\begin{align*}
\text{max} & \quad \frac{\sum_{r=1}^{s} u_r y_{ro} + \sum_{f=1}^{F} \gamma_{rf} w_{fo} - \sum_{f=1}^{F} \beta_{rf} w_{fo}}{\sum_{i=1}^{m} v_{i} x_{io}}, \\
\text{st} & \quad \sum_{r=1}^{s} u_r y_{jr} + \sum_{f=1}^{F} \gamma_{rf} w_{jf} - \sum_{f=1}^{F} \beta_{rf} w_{jf} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \quad j = 1, \ldots, n \\
& \quad u_r, v_i, \gamma_{rf}, \beta_{rf} \geq 0.
\end{align*}$$ (8)

The linear programming form of model (8) is as follows:

$$\begin{align*}
\text{max} & \quad \sum_{r=1}^{s} u_r y_{ro} + \sum_{f=1}^{F} \gamma_{rf} w_{fo} - \sum_{f=1}^{F} \beta_{rf} w_{fo}, \\
\text{st} & \quad \sum_{i=1}^{m} v_i x_{io} = 1, \\
& \quad \sum_{r=1}^{s} u_r y_{jr} + \sum_{f=1}^{F} \gamma_{rf} w_{jf} - \sum_{f=1}^{F} \beta_{rf} w_{jf} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \quad j = 1, \ldots, n \\
& \quad u_r, v_i, \gamma_{rf}, \beta_{rf} \geq 0.
\end{align*}$$ (9)
At this stage, the model that considers both dual-role factors and weight restrictions is introduced.

\[
\text{max } \sum_{r=1}^{s} u_r y_m + \sum_{f=1}^{F} \gamma_f w_{j0} - \sum_{f=1}^{F} \beta_f w_{j0},
\]

\[
st \quad \sum_{i=1}^{m} v_i x_{i0} = 1,
\]

\[
\sum_{r=1}^{s} u_r y_{rj} + \sum_{f=1}^{F} \gamma_f w_{gj} - \sum_{f=1}^{F} \beta_f w_{gj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \quad j = 1, \ldots, n
\]

\[
c_i \left( \sum_{i=1}^{m} x_{i0} v_i \right) - x_{i0} v_i \leq 0,
\]

\[
x_{i0} v_i - d_i \left( \sum_{i=1}^{m} x_{i0} v_i \right) \leq 0,
\]

\[
v_i \geq \varepsilon \quad \forall i,
\]

\[
u_r \geq \varepsilon \quad \forall r,
\]

\[
\gamma_f, \beta_f \geq 0.
\]

Therefore, one unified approach that deals with weight restrictions and dual-role factors in a direct manner have been introduced.

Now, one of three possibilities exists in regard to the sign of \( \hat{\gamma}_f - \hat{\beta}_f \), where \( \hat{\gamma}_f, \hat{\beta}_f \) are the optimal values from Model (10); \( \hat{\gamma}_f - \hat{\beta}_f > 0, = 0, \) or \( < 0 \).

**Case 1:** If \( \hat{\gamma}_f - \hat{\beta}_f < 0 \), then the dual-role factors are "behaving like inputs". Hence less of these factors are better, and would lead to an increase in efficiency.

**Case 2:** If \( \hat{\gamma}_f - \hat{\beta}_f > 0 \), then the dual-role factors are "behaving like outputs". Hence more of these factors are better, and would lead to an increase in efficiency.

**Case 3:** If \( \hat{\gamma}_f - \hat{\beta}_f = 0 \), then dual-role factors are at equilibrium or optimal level.

### 3.1 Rank-sum test

It is often necessary to test statistically the difference between two groups, i.e., in the absence of virtual weight restriction and dual-role factor, and in the presence of virtual
weight restriction and dual-role factor, in terms of efficiency. Do differences occur by chance or are they statistically significant? This subsection deals with such statistical issues. Since the theoretical distribution of the efficiency score in DEA is usually unknown, the nonparametric statistics is used. For this purpose, the rank-sum test developed by Wilcoxon-Mann-Whitney may be used to identify whether the differences between two groups are significant.

Rank-sum test is one of the nonparametric statistical tests based on the ranking of data. Given statistically independent data belonging to two groups, this test serves to test whether the hypothesis that the two groups belong to the same population or whether they differ significantly.

In the next section, a numerical example is presented.

4. Numerical example

The data set for this example is partially taken from Talluri and Baker (2002) and contains specifications on 18 suppliers. The supplier inputs considered are Total Cost of shipments (TC)$^5$, Number of Shipments per month (NS), and Research and Development cost (R&D). The outputs utilized in the study are Number of shipments to arrive On Time (NOT), Number of Bills received from the supplier without errors (NB), and R&D. R&D plays the role of both input and output.

According to the decision of decision maker, the importance of TC, as expressed by the weight $v_1$, must be as follows (method 1):

$$0.5 \leq \frac{v_1 x_{10}}{\sum_{i=1}^{m} v_i x_{i0}} \leq 3$$
Table 3 depicts the supplier's attributes and efficiency scores in the absence of virtual weight restriction and dual-role factor (applying Model (2)). Table 4 reports the results of efficiency assessments in the presence of virtual weight restriction and dual-role factor and their input/output behavior for the 18 suppliers obtained by using Model (10). Samples of Models (2) and (10) for supplier 1 have been presented in Appendix. \( \varepsilon \) has been set to be 0.0001.

Model (10) identified suppliers 3, 4, 6, 7, 17, and 18 to be efficient with a relative efficiency score of 1. The remaining 12 suppliers with relative efficiency scores of less than 1 are considered to be inefficient. Therefore, decision maker can choose one or more of these efficient suppliers.

The supplier 7 is the DMU that R&D is behaving like an input. The other suppliers are those that R&D is behaving like an output, where more of such factor would improve the efficiencies of related suppliers.

4.1 Rank-sum test

This subsection deals with statistical comparison between two groups of efficiency values, i.e., in the absence of virtual weight restriction and dual-role factor, and in the presence of virtual weight restriction and dual-role factor. Using \( T \), the null hypothesis that the two groups have the same population at a level of significance \( \alpha \) can be checked. In this example, there is \( T = -1.0757 \). If \( \alpha = 0.05 \) (5\%) is chosen, then it holds that \( T_{0.025} = 1.96 \). Since \( T = -1.0757 < -1.96 = T_{0.025} \), the null hypothesis at the significance level 5\% is not rejected. Consequently, the differences among the efficiency scores obtained by Model (2) and efficiency scores obtained by Model (10) are not statistically significant\(^6\).
5. Concluding remarks

Strong competitive pressure forces many organizations to provide their products and services to customers faster, cheaper and better than the competitors. Managers have come to realize that they cannot do it alone without satisfactory suppliers. Therefore, the increasing importance of supplier selection decisions is forcing organizations to rethink their purchasing and evaluation strategies and hence the selection of suppliers has received considerable attention in the purchasing literature.

This paper has provided a model for selecting suppliers in the presence of dual-role factors and weight restriction.

Notice that, whatever we propose any possible process to improve DEA model, there always is a result that shows the best DMUs as efficient so that their efficiency scores equal to one. The reason is that DEA measures the relative efficiency of DMUs. As well, there are too many DEA models. Each DEA model has a specific assumption which should be considered beforehand. In real world, decision makers should take into consideration these assumptions. As a result, the proposed model is only a possible way to achieve better supplier selection but not sufficient. In other words, the proposed model assumes that weight restrictions and dual-role factors are present. It is obvious that if these assumptions are not applicable, the proposed model can not be used.

The problem considered in this study is at initial stage of investigation and many further researches can be done based on the results of this paper. Some of them are as follow:

- Similar research can be repeated for dealing with fuzzy data in the conditions that dual-role factors exist.

- One of the limitations of this paper is that the proposed model assumes all suppliers are completely homogeneous. As Farzipoor Saen (2007b) discussed, the assumption of classical supplier selection models is based on the principle that suppliers consume common inputs to supply common outputs. In spite of this assumption in many applications some suppliers do not comprehensively consume common inputs to comprehensively supply common outputs. In other words,
different industrial suppliers may have many differences between them. To evaluate the relative efficiency of suppliers, all the suppliers may not have identical functions. For instance, to select a supplier most of inputs and outputs (selection criteria) of suppliers are common, but there are a few input(s) and/or output(s) for some suppliers that may not be common to all. In a supplier evaluation example that buyer consumes two types of materials such as type A and type B. A supplier may not supply type B. To evaluate this supplier, considering cost as an input, cost of type B for the supplier is meaningless. It is clear that zero value allocation for this type of input, causes relative efficiency of the supplier, to increase unrealistically. In this case, it is not acceptable saying that the suppliers which do not supply material of type B, are not comparable with the suppliers which supply material of type B. Meanwhile, allocating zero value to suppliers that do not supply material of type B, is not fair. Generally, zero allocation to outputs and inputs of some suppliers, makes the efficiency evaluation unfair. That is zero allocation to output, may make a supplier inefficient, on the other hand, zero allocation to input, may make a supplier efficient, unrealistically.

Farzipoor Saen (2007b) and Farzipoor Saen (in press b) propose a model for selecting slightly non-homogeneous suppliers. However, he did not consider weight restrictions and dual-role factors. A potential extension to the methodology includes the case that some of the suppliers are slightly non-homogeneous in the presence of both weight restrictions and dual-role factors.

- This study used the proposed model for supplier selection. It seems that more fields (e.g. technology selection, personnel selection, etc) can be applied.

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Appendix

Model (2) for supplier 1:

\[
\begin{align*}
\text{max} & \quad 187u_1 + 90u_2 \\
\text{st} & \quad 253v_1 + 197v_2 = 1 \\
& \quad 187u_1 + 90u_2 - 253v_1 - 197v_2 \leq 0 \\
& \quad 194u_1 + 130u_2 - 268v_1 - 198v_2 \leq 0 \\
& \quad 220u_1 + 200u_2 - 259v_1 - 229v_2 \leq 0 \\
& \quad 160u_1 + 100u_2 - 180v_1 - 169v_2 \leq 0 \\
& \quad 204u_1 + 173u_2 - 257v_1 - 212v_2 \leq 0 \\
& \quad 192u_1 + 170u_2 - 248v_1 - 197v_2 \leq 0 \\
& \quad 194u_1 + 60u_2 - 272v_1 - 209v_2 \leq 0 \\
& \quad 195u_1 + 145u_2 - 330v_1 - 203v_2 \leq 0 \\
& \quad 200u_1 + 150u_2 - 327v_1 - 208v_2 \leq 0 \\
& \quad 171u_1 + 90u_2 - 330v_1 - 203v_2 \leq 0 \\
& \quad 174u_1 + 100u_2 - 321v_1 - 207v_2 \leq 0 \\
& \quad 209u_1 + 200u_2 - 329v_1 - 234v_2 \leq 0 \\
& \quad 165u_1 + 163u_2 - 281v_1 - 173v_2 \leq 0 \\
& \quad 199u_1 + 170u_2 - 309v_1 - 203v_2 \leq 0 \\
& \quad 188u_1 + 185u_2 - 291v_1 - 193v_2 \leq 0 \\
& \quad 168u_1 + 85u_2 - 334v_1 - 177v_2 \leq 0 \\
& \quad 177u_1 + 130u_2 - 249v_1 - 185v_2 \leq 0 \\
& \quad 167u_1 + 160u_2 - 216v_1 - 176v_2 \leq 0 \\
& \quad v_1 \geq 0.0001 \\
& \quad v_2 \geq 0.0001 \\
& \quad u_1 \geq 0.0001 \\
& \quad u_2 \geq 0.0001
\end{align*}
\]
Model (10) for supplier 1:

\[
\begin{align*}
\text{max} & \quad 187u_1 + 90u_2 + 20\gamma_1 - 20\beta_1 \\
\text{st} & \quad 253v_1 + 197v_2 = 1 \\
& \quad 187u_1 + 90u_2 + 20\gamma_1 - 20\beta_1 - 253v_1 - 197v_2 \leq 0 \\
& \quad 194u_1 + 130u_2 + 32\gamma_1 - 32\beta_1 - 268v_1 - 198v_2 \leq 0 \\
& \quad 220u_1 + 200u_2 + 15\gamma_1 - 15\beta_1 - 259v_1 - 229v_2 \leq 0 \\
& \quad 168u_1 + 100u_2 + 10\gamma_1 - 10\beta_1 - 180v_1 - 169v_2 \leq 0 \\
& \quad 204u_1 + 173u_2 + 16\gamma_1 - 16\beta_1 - 257v_1 - 212v_2 \leq 0 \\
& \quad 192u_1 + 170u_2 + 28\gamma_1 - 28\beta_1 - 248v_1 - 197v_2 \leq 0 \\
& \quad 194u_1 + 60u_2 + 12\gamma_1 - 12\beta_1 - 272v_1 - 209v_2 \leq 0 \\
& \quad 195u_1 + 145u_2 + 36\gamma_1 - 36\beta_1 - 330v_1 - 203v_2 \leq 0 \\
& \quad 200u_1 + 150u_2 + 30\gamma_1 - 30\beta_1 - 327v_1 - 208v_2 \leq 0 \\
& \quad 171u_1 + 90u_2 + 28\gamma_1 - 28\beta_1 - 330v_1 - 203v_2 \leq 0 \\
& \quad 174u_1 + 100u_2 + 19\gamma_1 - 19\beta_1 - 321v_1 - 207v_2 \leq 0 \\
& \quad 209u_1 + 200u_2 + 25\gamma_1 - 25\beta_1 - 329v_1 - 234v_2 \leq 0 \\
& \quad 165u_1 + 163u_2 + 18\gamma_1 - 18\beta_1 - 281v_1 - 173v_2 \leq 0 \\
& \quad 199u_1 + 170u_2 + 27\gamma_1 - 27\beta_1 - 309v_1 - 203v_2 \leq 0 \\
& \quad 188u_1 + 185u_2 + 22\gamma_1 - 22\beta_1 - 291v_1 - 193v_2 \leq 0 \\
& \quad 168u_1 + 85u_2 + 31\gamma_1 - 31\beta_1 - 334v_1 - 177v_2 \leq 0 \\
& \quad 177u_1 + 130u_2 + 50\gamma_1 - 50\beta_1 - 249v_1 - 185v_2 \leq 0 \\
& \quad 167u_1 + 160u_2 + 15\gamma_1 - 15\beta_1 - 216v_1 - 176v_2 \leq 0 \\
& \quad 0.5(253v_1 + 197v_2) - 253v_1 \leq 0 \\
& \quad 253v_1 - 3(253v_1 + 197v_2) \leq 0 \\
& \quad v_1 \geq 0.0001 \\
& \quad v_2 \geq 0.0001 \\
& \quad u_1 \geq 0.0001 \\
& \quad u_2 \geq 0.0001 \\
& \quad \gamma_1 \geq 0 \\
& \quad \beta_1 \geq 0
\end{align*}
\]
References


<table>
<thead>
<tr>
<th>Technique name</th>
<th>References</th>
</tr>
</thead>
</table>
Table 2. Nomenclature

**Problem parameters**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$j = 1, \ldots, n$</td>
<td>collection of suppliers (DMUs)</td>
</tr>
<tr>
<td>$r = 1, \ldots, s$</td>
<td>the set of outputs</td>
</tr>
<tr>
<td>$i = 1, \ldots, m$</td>
<td>the set of inputs</td>
</tr>
<tr>
<td>$f = 1, \ldots, F$</td>
<td>the set of dual-role factors</td>
</tr>
<tr>
<td>$y_{rj}$</td>
<td>the $r$th output of $j$th DMU</td>
</tr>
<tr>
<td>$x_{ij}$</td>
<td>the $i$th input of $j$th DMU</td>
</tr>
<tr>
<td>$y_{ro}$</td>
<td>$r$th outputs of the DMU$_o$ under investigation</td>
</tr>
<tr>
<td>$x_{io}$</td>
<td>$i$th inputs of the DMU$_o$ under investigation</td>
</tr>
<tr>
<td>$w_j$</td>
<td>the factor that plays the role of both an input and output</td>
</tr>
<tr>
<td>$w_{fj}$</td>
<td>the $f$th dual-role factor of $j$th DMU</td>
</tr>
</tbody>
</table>

**Decision variables**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_r$</td>
<td>weight of the $r$th output</td>
</tr>
<tr>
<td>$v_i$</td>
<td>weight of the $i$th input</td>
</tr>
</tbody>
</table>
Table 3. Related attributes for 18 suppliers and efficiency scores

<table>
<thead>
<tr>
<th>Supplier No. (DMU)</th>
<th>Inputs</th>
<th>Dual-role factor</th>
<th>Outputs</th>
<th>Efficiency scores in the absence of virtual weight restriction and dual-role factor (applying Model (2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>TC (x_{i}) (1000$)</td>
<td>NS (y_{i})</td>
<td>R&amp;D (w_{i})</td>
<td>NOT (x_{j})</td>
</tr>
<tr>
<td>2</td>
<td>253</td>
<td>197</td>
<td>20</td>
<td>187</td>
</tr>
<tr>
<td>3</td>
<td>268</td>
<td>198</td>
<td>32</td>
<td>194</td>
</tr>
<tr>
<td>4</td>
<td>259</td>
<td>229</td>
<td>15</td>
<td>220</td>
</tr>
<tr>
<td>5</td>
<td>180</td>
<td>169</td>
<td>10</td>
<td>160</td>
</tr>
<tr>
<td>6</td>
<td>257</td>
<td>212</td>
<td>16</td>
<td>204</td>
</tr>
<tr>
<td>7</td>
<td>248</td>
<td>197</td>
<td>28</td>
<td>192</td>
</tr>
<tr>
<td>8</td>
<td>272</td>
<td>209</td>
<td>12</td>
<td>194</td>
</tr>
<tr>
<td>9</td>
<td>330</td>
<td>203</td>
<td>36</td>
<td>195</td>
</tr>
<tr>
<td>10</td>
<td>327</td>
<td>208</td>
<td>30</td>
<td>200</td>
</tr>
<tr>
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<td>330</td>
<td>203</td>
<td>28</td>
<td>171</td>
</tr>
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<td>12</td>
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<td>19</td>
<td>174</td>
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<td>329</td>
<td>234</td>
<td>25</td>
<td>209</td>
</tr>
<tr>
<td>14</td>
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<td>165</td>
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<td>16</td>
<td>291</td>
<td>193</td>
<td>22</td>
<td>188</td>
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<tr>
<td>17</td>
<td>334</td>
<td>177</td>
<td>31</td>
<td>168</td>
</tr>
<tr>
<td>18</td>
<td>249</td>
<td>185</td>
<td>50</td>
<td>177</td>
</tr>
</tbody>
</table>

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Table 4. Efficiency scores in the presence of virtual weight restriction and dual-role factor, and input/output behavior

<table>
<thead>
<tr>
<th>Supplier No. (DMU)</th>
<th>Efficiency score in the presence of virtual weight restriction and dual-role factor (applying Model (10))</th>
<th>$\hat{\gamma}$</th>
<th>$\hat{\beta}$</th>
<th>$\hat{\gamma} - \hat{\beta}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.934</td>
<td>0.002354147</td>
<td>0</td>
<td>0.002354147</td>
</tr>
<tr>
<td>2</td>
<td>.9695</td>
<td>0.002213107</td>
<td>0</td>
<td>0.002213107</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>0.002267565</td>
<td>0</td>
<td>0.002267565</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>0.003359652</td>
<td>0</td>
<td>0.003359652</td>
</tr>
<tr>
<td>5</td>
<td>.9705</td>
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<td>0.00228148</td>
</tr>
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<td>0</td>
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<tr>
<td>7</td>
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<td>-0.9176708</td>
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<tr>
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<td>0</td>
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<td>9</td>
<td>.8859</td>
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<td>10</td>
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<td>0.001824056</td>
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<tr>
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<td>.9053</td>
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<td>0.003981494</td>
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<tr>
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<td>0.001989893</td>
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<td>0.005784615</td>
</tr>
</tbody>
</table>

1 Since the DEA models have become common knowledge, the readers are directed to the references.

2 In this model an input orientation assumption has been considered. The use of an output-oriented model would lead to similar conclusions.

3 Note that Model (6) can control the weights of desired inputs and outputs. It is obvious that if there is not any restriction on the weight of a specific factor, there might be the weight close to zero.

4 If there are several criteria having the same weight, simply their weights are considered equal to each other (e.g. $u_1 = u_2 = v_1$, etc).

5 The inputs and outputs selected in this paper are not exhaustive by any means, but are some general measures that can be utilized to evaluate suppliers. In an actual application of this methodology, decision makers must carefully identify appropriate inputs and outputs measures to be used in the decision making process.

6 There is a rule of thumb that the number of DMUs included in a DEA model should have almost thrice the number of inputs and outputs. Therefore, because of presence of dual-role factor, the results of Model (10) have less discrimination in objective function values.