

MODELLING OPTIMAL WARRANTY PRICE FOR LIFETIME POLICIES TAKING INTO ACCOUNT THE UNCERTAINTIES IN LIFE MEASURES

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Abstract: Due to assurance of longer reliable service life and greater customer peace of mind, products with lifetime warranty are becoming more and more popular. Under such policies, both the manufacturer and the buyer are exposed to uncertainties and risks of warranty pricing and product performance since products lifetime are uncertain and are not defined well in these policies. Considering the uncertainties in the measure of lifetime (useful life), this paper extends the work of Rahman and Chattopadhyay (2010) to determine the optimal warranty price. Risk preference models are developed to find the optimal warranty price through the use of the manufacturer's utility function for profit and the buyer's utility function for repair costs. Sensitivity of the risk preferences models are analysed with numerical example with respect to the factors such as the buyer's and the manufacturer/dealer's risk preferences, buyer's anticipated and manufacturer's estimated product failure intensity, the buyer's loyalty to the original manufacturer/dealer in repairing failed product and the buyer's repair costs for unwarranted products. Analysis of the developed models reveals manufacturer's decisions on warranty price are directly related to useful life, failure intensity of the product, and manufacture's risk preference. On the other hand buyer's acceptance of a lifetime warranty offer depends on expected lifetime of the product, buyer's anticipated product failure intensity, anticipated repair costs and most importantly buyer's risk preference.

Keywords: Lifetime warranties, lifetime measures, manufacturer's and customer's risk preferences

1. Introduction

Lifetime warranty is relatively new concept and is becoming more popular as this type of warranty provides assurance for a longer reliable service life, protection of customers against poor quality and the potential high cost of failure occurring during the long uncertain life of products as well as a better customer peace of mind (Rahman and Chattopadhyay, 2015). Under typical situation of a lifetime warranty transaction, a buyer of a product pays for the warranty at the time of product purchase whereas, manufacturer/dealer provides rectification service in case of product failures due to design, manufacturing and quality assurance problems during the defined lifetime of the product.

Under such situation, both the manufacturer and the buyer are exposed to uncertainties and risks of warranty pricing and product performance during the lifetime of the product (Rahman and Chattopadhyay, 2010). In a lifetime warranty transaction, a buyer of a product pays for the warranty at the time of product purchase which in some occasion is factored into the product price. The manufacturer provides rectification service in case of product failures due to design, manufacturing and quality assurance problems during the defined lifetime of the product. The manufacturer is risking in warranty pricing that whether its offer for such warranty will be accepted by the buyers. At the same time, buyers are unsure about the benefits of buying products sold with lifetime warranty policies. Anticipation of higher product failures encourages a buyer to pay for the higher warranty price which in turn encourages the manufacturer to charge a higher warranty price. In case of product covered with

warranty, the buyer returns to the original manufacturer for rectification of product failures. But when the product is not covered by warranty, the buyer may take it to other service providers for rectifications. If the original manufacturer is the only one that can repair the product because of some technological monopoly, the manufacturer could charge a higher price for services and more buyers would be interested to have warranty cover. If buyers pay higher repair price for any product failures, they might be interested to get warranty cover and even pay a higher warranty price for free repair or replacement policies [2].

In line with Chun and Tang (1995), Rahman and Chattopadhyay (2010) proposed manufacturer's and buyer's risk preference models for lifetime warranty policies where they assumed a constant lifetime of products which is not realistic as life measures for the period of coverage (lifetime) is totally uncertain and are not defined well (Chattopadhyay and Rahman, 2008).

This paper extends the work of Rahman and Chattopadhyay (2010) in developing risk preferences models by taking into consideration the uncertainties in life measures. The models are developed in finding the optimal lifetime warranty prices through the use of the manufacturer's utility function for manufacturer's profit and the buyer's utility function for repair cost. These models have been proposed with Non homogeneous Poisson's process for failure intensity function, and concave utility function. Using the exponential utility function, the decision models are developed to maximise the manufacturer/dealer's certainty profit equivalent. Finally, the sensitivity of the warranty price models are analysed using numerical examples.

The outline of this paper is as follows. Section 1 briefly introduces manufacturers' and buyers' risk preferences towards a lifetime warranty policy considering the uncertainties

of warranty duration. Section 2 provides brief overview of lifetime warranty policies. In section 3, risk preference models for both manufacturer and buyer towards a lifetime warranty policy are developed using exponential utility function. In section 4, sensitivity of the risk preferences models are analysed with numerical example with respect to the buyer's and the manufacturer/dealer's risk preferences, buyer's anticipated and manufacturer's estimated product failure intensity, the buyer's loyalty to the original manufacturer/dealer in repairing failed product and the buyer's repair costs for unwarranted products. Finally, in the concluding section contribution of this research work and some recommendation of future research scopes are discussed in Section 5.

2. Literature review

One important difference between normal warranty and lifetime warranty is the coverage period. The warranty coverage can be regarded as an important product attribute that affects both buyers' decisions and manufacturers' profit (Ye and Murthy, 2016). In the case of normal warranty, warranty coverage period is fixed whereas it is uncertain for lifetime warranty and is often difficult to tell about life measures for the period of coverage since the lifetime of the product is assumed to be uncertain and to be terminated in some finite, random time horizon. Termination of such warranty may arise from any one of the following reasons (Rahman and Chattopadhyay (2008)).

Technical life/ Physical life – the period over which the product might be expected to last physically, or when replacement or major rehabilitation is physically required.

Technological life – the period until technological obsolescence dictating replacement due to the development of a technologically superior alternative

Commercial life/ Economic life/ Functional life – the period, over which the need for the

product exists, the period until economic obsolescence dictates replacement with a lower cost alternative (economic or operational convenience).

Ownership life/ Social and legal life – the period until human desire or legal requirement dictates replacement or change of ownership occurs.

Since both manufacturer and buyer are exposed to uncertainties and risks of such warranty pricing and product performance during the lifetime of the product it is essential to develop risk preference models for both manufacture and buyer taking into account the uncertain nature of the warranty coverage period.

Limited work so far takes into account the manufacturer and buyer's risk preferences toward how much a manufacturer should charge and how much a buyer is willing to pay for such warranty service. Aggrawal et al (2014) proposed a model which integrates marketing decision of optimal pricing and warranty length in order to calculate the overall profit for the firm. Ensuring producer's profitability in the presence of warranty by optimizing warranty length and price of the good is a way to future business. Xu et al (2012) presented a review of the issues associated with a manufacturer's pricing strategies in a two echelon supply chain that comprises one manufacturer and two competing retailers, with warranty period-dependent demands. Jindal (2015) studied the relative importance of different risk preferences in explaining extended warranty purchases and the high premia paid for them. Ritchken and Tapiero (1986) proposed a framework in which warranty policies for non-repairable items can be evaluated according to risk preferences of both the manufacturer/dealer and the buyers where they emphasised the design and pricing of warranties to which the manufacturers are indifferent in an expected utility sense. Chun and Tang (1995) proposed a warranty model

for the free-replacement, fixed-period warranty policy that determines the optimal warranty price for a fixed warranty period. Zhou et al (2009) proposed dynamic pricing and warranty models for products with fixed lifetime to determine a joint dynamic pricing and warranty policy which maximises the manufacturer's expected profit. Chen and Chang (2010) proposed models for determining the optimal warranty period and expected lifetime of product based on maximizing the total expected profit. In line with Chun and Tang, Rahman and Chattopadhyay (2010) proposed risk preference models for lifetime warranty where they considered a fixed lifetime of the product which is similar to a normal warranty. They assumed an increased failure rate for the product over time, constant repair costs throughout the warranty period, and a producer's and customers' risk aversion for future repair costs. Using the exponential utility function and the gamma failure rate distribution, they derived the decision model that maximizes the producer's certainty profit equivalent. Unfortunately, they assumed a constant lifetime which is not realistic because product lifetime is totally uncertain as discussed earlier. To overcome this shortcoming, this paper attempts to capture the uncertain lifetime measures in developing the risk preference models for lifetime warranty in finding the optimal warranty price through the use of the manufacturer's utility function for manufacturer's profit and the buyer's utility function for repair cost.

3. Modelling Lifetime Warranty Price

3.1 Notations and Statistical Preliminaries

Let the number of items sold by the manufacturer, be S and p be the proportion of product sold with lifetime warranty, and $(1 - p)$ be the proportion of S products sold without

such warranty. Two conditions are considered here: products are sold with warranty or without warranty. Subscripts m and b stands for manufacturer and buyer respectively.

Let k be the proportion of buyers without warranty coming back to manufacturer for repairing of the faults/defects and $1-k$ portion of buyers prefer to go to third parties for repairing the failed products.

$N_b(L)$ represents the number of valid claims made by the buyer per item and $N(L)$ is the total number of possible claims for S products over the lifetime L .

$E[N_b(L)]$ is the expected number of failure per item experienced by the buyers over the lifetime.

$U_m(Y)$ and $U_b(X)$ denote the manufacturers and buyers continuous utility functions for a manufacturer's profit Y and buyer's repair cost X respectively.

U_m is an individual manufacturer's utility function

U_b is the aggregate utility function representing the entire buyer's risk preference as a whole.

$A_m(t)$ is the manufacturer's failure intensity function.

$A_b(t)$ is the per item failure intensity function for an individual buyer during the lifetime

Let r_b represents the cost of rectification (repair cost) for a buyer in each occasion of failure if the item is not warranted.

r_m represents the manufacture's per occasion cost of rectification (repair cost) which is the actual rectification cost when the item is covered under warranty.

W is the warranty price offered by the manufacturer during the time of purchase.

3.2 Assumptions

- The Item failures are statistically independent and failures in a probabilistic sense, is only a function of its age.

- Failures over the warranty period modelled at the system (or item) level.
- An item failure results in an immediate claim and all claims are valid.
- Manufacturer's cost of rectification r_m and the buyer's cost of each rectification r_b are constant over the warranty period.
- The Manufacturer's utility function $U_m(Y)$ for profit Y is concave (risk averter) and strictly increasing as most manufacturers prefer a higher profit to a lower profit. Thus, it follows from Jensen's inequality Zlobec (2004), that if Y has finite mean then $E[U_m(Y)] < U_m[E(Y)]$ and $[y_1 < y_2] \leftrightarrow [U_m(Y_1) < U_m(Y_2)]$
- The buyer's utility function $U_b(X)$ for repair cost X is concave (also risk averter) and strictly decreasing as buyer prefer a lower cost to a higher cost. Thus, it follows from Jensen's inequality that if X has finite mean then $E[U_b(X)] < U_b[E(X)]$ and $[x_1 < x_2] \leftrightarrow [U_b(X_1) > U_b(X_2)]$.

3.3 Buyer's Acceptances of Warranty

For the interest and clear understanding of new readers, in this section firstly, we will recall and briefly discuss the work and models developed by Rahman and Chattopadhyaya (2010). Then we will extend the models by capturing the uncertainties of lifetime. According to Rahman and Chattopadhyay (2010), under lifetime warranty, a buyer may first estimate the expected total repair cost $r_b N_b(L)$ of his purchased product throughout the defined lifetime and then compare it with the given warranty price W in terms of the expected utility. Since r_b is constant, buyer's total repair cost is given by $r_b N_b(L)$ which is estimated by his perceived product failure intensity $\Lambda_b(t)$. The higher the buyer's estimate of the product failure rate, the more likely he would be willing to buy the warranty Chun and Tang (1995). Let n_b^* be the number of product failure when buyers are indifferent between

the warranty price W and the total repair cost $r_b N_b(L)$ in terms of the expected utility. Certainty equivalent of similar concept can be found in Moskowitz, and Plante (1984). By definition, a certainty equivalent of W is an amount of $r_b N_b(L)$ such that the decision maker is indifferent between W and $r_b N_b(L)$. This can be expressed (Rahman and Chattopadhyay, 2010) as

$$U_b(W) = E[U_b(r_b N_b(L))]$$

Therefore,

$$U_b(W) = E[U_b(r_b N_b(L))] = \sum_{n_b=0}^{\infty} U_b(r_b n_b) P(N_b(L) = n_b^*) \quad (1)$$

When failures of product follows Non-homogeneous Poisson process (i.e. failures are time dependent) then we express the following equation

$$\Pr ob\{N_b(L) = n_b^*\} = \frac{\int_0^L \Lambda_b(t) dt \}^{n_b^*} e^{-\int_0^L \Lambda_b(t) dt}}{n_b^*!} \quad (2)$$

One form for $\Lambda_b(t)$ using Non-homogeneous Poisson Process is

$$\Lambda_b(t) = \lambda_b \beta_b (t)^{(\beta_b - 1)} \quad (3)$$

with the shape parameters $\beta_b > 1$ and inverse characteristic life $\lambda_b > 0$. This is an increasing function of time or age t . they expressed the buyers' risk aversions as

$$U_b(Y) = -e^{-cY}, \quad c > 0, \quad (4)$$

Where c is the risk parameters representing the buyers' risk preferences. A buyer's attitude toward risk can be determined by the nature of c . A buyer is more risk-averse if the risk parameter increases. In this study, the buyers' risk parameter c represents a wide spectrum of buyers' risk preferences. A risk parameter $c = 0$, indicates a risk neutral buyer.

In the exponential utility functions, the absolute risk aversion measure is constant for

all X, implying that the exponential function represents only the constant risk-averse case Chun and Tang (1995).

By combining Equations (2), (3), (4) and (5), they developed the buyer's expected optimal warranty price (details can be found in Rahman and Chattopadhyay, 2010) as

$$W = \ln \sum_{n_b^*=0}^{\infty} \exp(cr_b n_b^*) \times \frac{[(\lambda_b L)^{\beta_b}]^{n_b^*} \exp[-(\lambda_b L)^{\beta_b}]}{n_b^*} \quad (5)$$

Unfortunately, they considered lifetime duration L is constant. In real life situation which is not true as the upper limit of the warranty coverage is totally uncertain since the termination of life is uncertain and randomly variable. Product failure throughout this uncertain coverage period can exhibited through the Fig. 1.

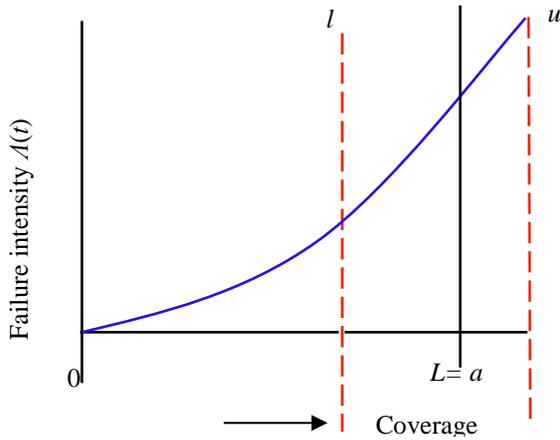


Figure.1: Failure intensity over lifetime warranty coverage period

Conditioned on the upper limit of coverage $L = a$, one can capture this uncertain coverage period by binding a with a lower limit l and upper limit u at statutory base since the termination of lifetime is assumed to be occurred in some finite, random time horizon. One can model this as a random variable with a distribution function $H(a)$ with $H(l) = P(a \leq l) = 0$ and $H(u) = P(a \leq u) = 1$

$h(a)$ is the probability density function of coverage period a associated with $H(a)$ and

$$h(a) = \frac{dH(a)}{da}$$

One form of $H(a)$, which is analytically tractable, is the following truncated exponential distribution (Chattopadhyay and Rahman, 2008)

$$H(a) \text{ is } \frac{e^{-\rho l} - e^{-\rho a}}{e^{-\rho l} - e^{-\rho u}} \text{ which gives a}$$

$$h(a) = \frac{\rho e^{-\rho a}}{e^{-\rho l} - e^{-\rho u}} \quad \text{The mean value of useful life of the sold product can be expressed by}$$

$$\mu_L = E(a) = \frac{(le^{-\rho l} - ue^{-\rho u}) + (e^{-\rho l} - e^{-\rho u})/\rho}{e^{-\rho l} - e^{-\rho u}} \quad (6)$$

ρ is Parameter for the truncated exponential distribution used in the life distribution of products. In real life distribution of lifetime coverage might not be possible to model using a particular distribution and can be modelled using a probability mass function. Therefore, the Equation 5 can now be expressed as

$$W = \ln \sum_{n_b^*=0}^{\infty} \exp(cr_b n_b^*) \times \frac{[(\lambda_b \mu_L)^{\beta_b}]^{n_b^*} \exp[-(\lambda_b \mu_L)^{\beta_b}]}{n_b^*} \quad (7)$$

From the above equation it will be worthy for the buyer to accept the warranty offer if the buyer's estimated number of failure is more than and equal to indifferent failure n_b^* . Accordingly, a buyer whose expected failure is higher than n_b^* would buy the warranty if the total estimated repair cost is higher than the warranty price W . Notice that probability to buy this warranty is determined by n_b^* , which, in turn, is determined by W . Based on the information about the buyer's willingness to pay for the warranty price W , a manufacturer may determine the warranty price such that the expected total profits during the warranty period is maximised.

3.3 Manufacturer's profit

By modifying Rahman and Chattopadhyay (2010), the manufacturer's expected total profit $E[\pi(\mu_L)]$ for warranty and rectification related services during the lifetime under such warranty policy can now be expressed as

$$E[\pi(\mu_L)] = p(SW - E[N(\mu_L)]r_m) + (1-p)k[E[N(\mu_L)]r_b - E[N(\mu_L)]r_m] = p(SW - E[N(\mu_L)]r_m) + (1-p)kE[N(\mu_L)](r_b - r_m) \quad (8)$$

For a risk-averse manufacturer with an increasing utility function $U_m(Y)$, the certainty equivalent for profit can now be modified as

$$U_m(E[\pi(\mu_L)]) = U_m\left(\frac{p(SW - E[N(\mu_L)]r_m) + (1-p)kE[N(\mu_L)](r_b - r_m)}{1}\right) \quad (9)$$

and the manufacturer estimated failure intensity can be expressed as

$$\Lambda_m(t) = \lambda_m \beta_m (t)^{\beta_m - 1} \quad (10)$$

with the shape parameters $\beta_m > 1$ and inverse characteristic life parameter $\lambda_m > 0$. This is an increasing function of t . Since the items are statistically similar then with more information on product failure manufacture can have probability of n_m failures over life L for any item as given by:

$$Pr ob\{N_m(\mu_L) = n_m\} = \frac{\int_0^{\mu_L} \Lambda_m(t) dt \}^{n_b} e^{-\int_0^{\mu_L} \Lambda_m(t) dt}}{n_m!} \quad (11)$$

Similar to the buyers' risk aversions, the manufacturer's risk aversions is assumed to be exponential utility functions, which is given by

$$U_m(Y) = e^{-aY}, \quad a > 0 \quad (12)$$

Where, a is the risk parameters representing the manufacturer's risk preferences. $a > 0$ indicates risk averse manufacturer, whereas $a < 0$ indicates risk seeker and $a = 0$ means a

risk neutral manufacturer. According to Chun and In line with Tang and Chun (1995), Rahman and Chattopadhyay (2010) developed the optimal warranty price model where they considered lifetime as constant and failed to capture the uncertainties of coverage period (for more details and their model please see Rahman and Chattopadhyay, 2010). By capturing the uncertain coverage period their warranty price model can be modified as

$$W = (\lambda_m \mu_L)^{\beta_m} r_m - \frac{1}{apS} \times \ln \sum_{n_m=0}^{\infty} e^{-adSkn_m} \cdot \frac{[(\lambda_m \mu_L)^{\beta_m}]^{n_m} e^{-(\lambda_m \mu_L)^{\beta_m}}}{n_m!} - \frac{q}{p} k (\lambda_m \mu_L)^{\beta_m} (r_b - r_m)_m \quad (13)$$

The developed Buyer's and manufacturer's risk preference model for lifetime warranty can be solved numerically by using mathematical software such as MATLAB, MAPLE etc.

4. Sensitivity Analysis of the Risk Models

In this section, we firstly present a sensitivity analysis of the buyer's intension to pay for warranty price with respect to the following factors: (1) buyer's risk preferences, (2) buyer's anticipated product failure intensity, (3) buyer's repair costs, if not warranted, and uncertainties in lifetime measures. Secondly, we present analysis of the effect of manufacturer's optimal warranty price for (1) manufacturer's risk preference, and (2) manufacturer anticipated product failure intensity. Sensitivity analysis of buyer's willingness to pay for warranty price: for all cases let us assume that 60% of the sold products are with warranty and 40% products are sold without warranty, which implies that p

= 0.6 and $q = 0.4$ for all occasions of our analysis.

4.1 Effect of buyer's risk preferences on the warranty price

Buyer's risk parameters 'c' is varied systematically from 0 to 1.0 representing a wide spectrum of buyer's risk preferences. A buyer become more risk averse as the risk parameter 'c' increases and he/she becomes risk neutral if the parameter is zero. Let the buyer's anticipated non-homogeneous Poisson process parameters are: inverse characteristic life $\lambda_b = 0.443$ years, and shape parameter $\beta_b = 2$, buyer's repair cost of each failure $r_b = \$30$, if not warranted and the buyer's repair cost is same for all occasions whether it is repaired by the manufacturer or by an outside repairer. The expected lifetime using Equation 6 is $\mu_L = 2.17$ years (considering shape parameter $\rho = 0.4$, $l = 1.5$ years, and $u = 3$ years. A computer simulation program generates the Fig 2 for the buyer's acceptance of warranty price with variation of the buyer's risk preference parameter c over the lifetime when $n_b=2$.

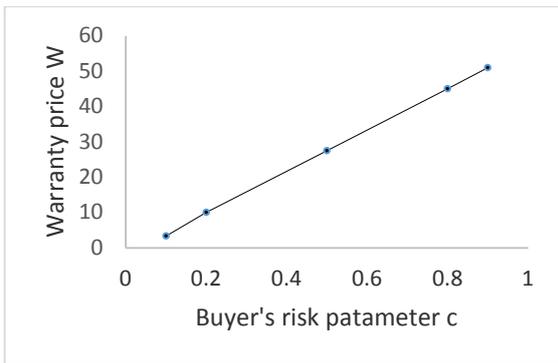


Fig 2: Effect of buyer's risk preference parameter on the warranty price

Fig 2 clearly states that the warranty price increases with the increase of the of the buyer's risk preference which implies that the buyers expected warranty price increases as the buyer becomes more risk averse. This

means that the buyer with higher risk aversness is willing to pay higher warranty price.

4.2 Effect of buyer's repair cost on the warranty price

Although the buyer's repair cost is applicable only for the buyers with non-warranted item, it has a significant effect on the warranty price. In this analysis, the buyer's rectification or repair cost for each failure is varied systematically from \$30 to \$70 representing a wide range of buyer's repair cost. It is noted that we assume this repair cost are constant for a particular product all along its lifetime. In addition to all the parameters considerations, in this case we assume a constant buyer's risk preference parameter $c = 0.5$ which is realistic because most buyers are with medium risk averse mentality in real life. The expected lifetime $\mu_L = 2.17$ years (considering shape parameter $\rho = 0.4$, $l = 1.5$ years, and $u = 3$ years. The computer program generates the Fig. 3 for the buyer's acceptance of warranty price with variation of the buyer's repair cost.

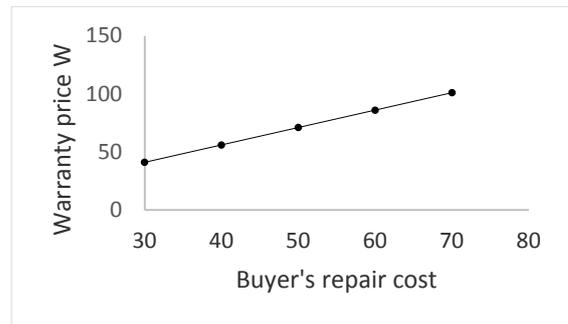


Fig 3: Effect of buyer's anticipated repair costs on the warranty price

The Fig 3 shows that the buyer's willingness for warranty price increases linearly as the buyer's repair cost increases. This means that the buyers are ready to pay higher warranty price if the repair costs are higher.

4.3 Effect of buyer's anticipated product failure intensity on the warranty price

Here, we analyse the sensitivity of buyer's anticipated product failure rate or intensity of failure on the buyer's acceptance of warranty price. To investigate the influence of product failure rate on the warranty price, we vary the λ_b from 0.125 to .525. We assume the buyer's anticipated non-homogeneous Poisson process parameters shape parameter $\beta_b = 2$, buyer's risk parameter $c = 0.5$. The expected lifetime $\mu_L = 2.17$ years (considering shape parameter $\rho = .4$, $l = 1.5$ years, and $u = 3$ years}, and average repair cost for each failure $r_b = \$30$. The computer simulation program generates the Fig 4 for the buyer's accepted warranty prices with variation of the failure intensity parameter λ_b .

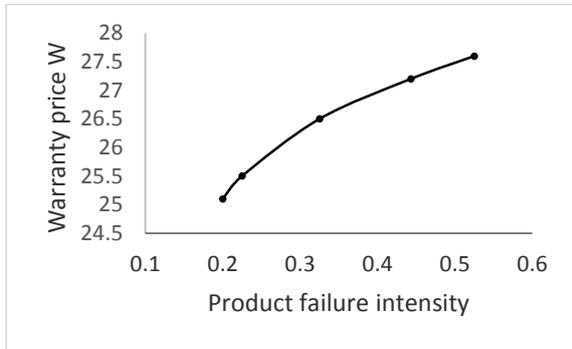


Fig 4: Effect of buyer's anticipated failure intensity (λ_b) on the warranty price

The Fig. 4 shows that the buyer's willingness for warranty price increases as the buyer's anticipated failure intensity increases. This implies that the buyers are ready to pay higher warranty price for higher failure rate. Conversely, when the buyers anticipate lower product failure intensity, they will be willing to pay less warranty price.

4.4 Effect of expected lifetime on warranty price

Now we analyse the effect of product useful life on the buyer's expected warranty price by varying the product life termination span (varying lower and upper limits of life termination). Considering all the above parameter values the Table 1 has been developed using different termination limits.

Table 1: Expected lifetime of product μ_L

		Upper limit u			
Lower limit l		1.5	2	2.5	3
1		1.25	1.45	1.65	1.87
1.5			1.74	1.97	2.17
2				2.24	2.45
2.5					2.74

Using different expected lifetime values from the Table 1 in Equation 10, we can analyse the effect of lifetime on the buyer's expected warranty price. The Fig.5 exhibits the results of the analysis

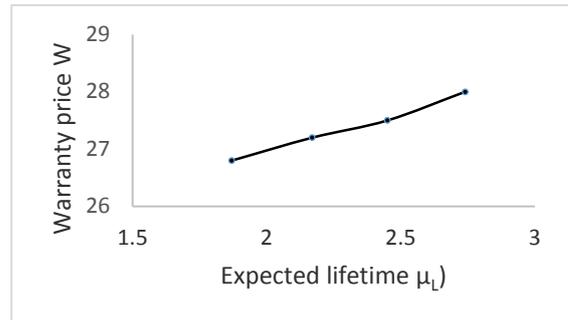


Fig 5: Effect of increase of lifetime on warranty price

According to the Fig. 5, it is clear that the buyer's willingness for warranty price increases as the expected lifetime of the product increases. This implies that buyers are more happy to pay higher warranty price when the product's expected life is higher. So the manufacturer can charge higher warranty for longer lasting products.

Sensitivity Analysis of the Manufacture's Warranty Price

In this subsection, we analyze the sensitivity of manufacturer's optimal warranty pricing with the variation of manufacturer's risk preference, manufacturer estimated product failure intensity, buyer's repair cost and buyer return rate to the manufacturer for repair of the failed product respectively.

4.5 Effect of manufacturer's risk preference on the warranty price

Similar to the sub-section 1), the manufacturer risk parameters 'a' is varied systematically from 0.05 to 1.0 representing a wide spectrum of manufacturer risk preferences. Let the manufacturer's estimated non-homogeneous Poisson process parameters are: inverse characteristic life $\lambda_m = 0.325/\text{year}$, and shape parameter $\beta_m = 2$, buyer's repair cost for each failure $r_b = \$30$ and manufacturer's actual cost of each repair $r_m = \$10$. Let the rate of returning of buyers (k) to the manufacturer for repair of failed product is 20% that is $k = 0.2$. A computer program generates the Fig. 6 for the manufacturer's optimal warranty price with variation of the manufacturer's risk parameter a.

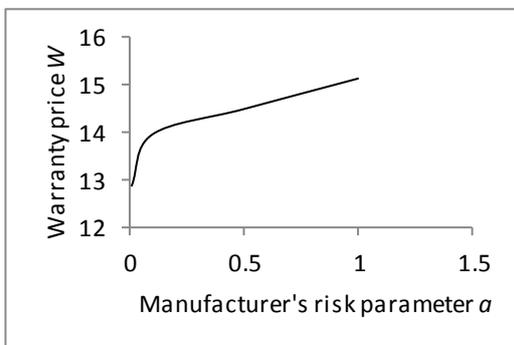


Fig. 6: Effect of manufacturer's risk parameter on the warranty price

The Fig. 6 shows that the warranty price increases with the increase of the of the manufacture risk preference parameter a. The warranty price increases as the manufacturer becomes more risk averse and the warranty price decreases as the manufacturer/dealer becomes less averse. This implies that the more a manufacturer is risk averse the more he/she will charge for the warranty price.

4.6 Effect of manufacturer's estimated product failure intensity on the warranty price

In this analysis, we consider a range of manufacturer estimated inverse characteristic life (λ_m) of product to represent failure intensity (rate) in observing the effect on the manufacturer's optimal charge for warranty. To do so, we vary λ_m from a range of 0.125/year to 0.625. Keeping all other values as before, let the manufacturer risk parameters 'a' be 0.1. The computer program generates the Fig. 7 which exhibits the effect of failure intensity over the manufacture's charge for warranty price.

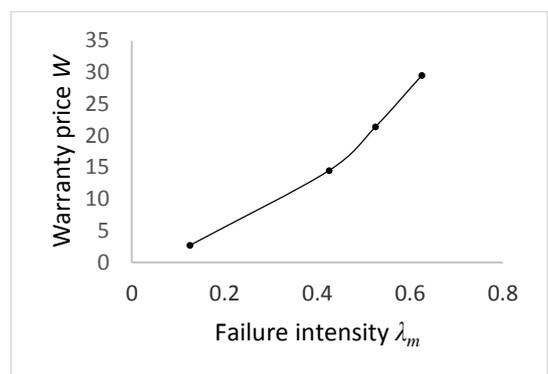


Fig. 7: Effect of manufacture's estimated failure intensity over warranty price.

Analysis of Fig. 7 shows that the manufacturer's charge for warranty price increases with the increase of the λ_m . This implies that, the higher the manufacturer estimates the failure intensity, the higher the

manufacturer charge for the warranty price to meet the higher rate of failure consequently more warranty claims as the manufacturer becomes more and more risk averse. Conversely, less failure intensity results in less warranty claims and the manufacturer is interested to offer less warranty price.

4.2.3. Effect of expected product's lifetime on the warranty price

In this time we analyse the impact of variations of expected lifetime on manufacturer's set warranty price by considering different combinations of lower and upper limits of product useful life. To do so once again we use the expected lifetime values for different combination of lower and upper limits from the Table 1. The computer program generates the Fig 8 which shows the impact of expected lifetime over the manufacture's charge for warranty price.

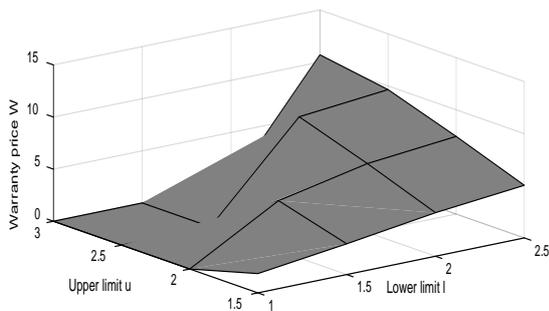


Fig. 8: Effect of product expected lifetime on warranty price

Fig.8 presents that manufacturer's warranty charge increases with the increase of product's expected lifetime. Longer useful life (lifetime) means the manufacturer/dealer has to serve more warranty claims with specific failure intensity. This implies that manufacturer/dealer has to set higher warranty price with the increase lifetime of the product as well as with the decrease of lifetime termination limits.

5. Conclusion

This paper extended the buyer's and manufacturers risk preference models developed by Chattopadhyay and Rahman [5] for product lifetime warranty by capturing uncertainty and randomness of useful life (lifetime warranty coverage). These models have been proposed with Non homogeneous Poisson's process for failure intensity function, a constant repair cost, and concave utility function. Using exponential utility function, the decision models have been developed to maximise the manufacturer/dealer's certainty profit equivalent. Risk preference models are developed to find the optimal warranty price through the use of the manufacturer's utility function for profit and the buyer's utility function for repair costs. Due to page limitations, numerical examples together with the sensitivity analysis of the models could not be presented. Numerical examples and the sensitivity of the models will be illustrated in future publication.

Longer useful life (lifetime) means the manufacturer/dealer has to serve more warranty claims with specific failure intensity. This implies that manufacturer/dealer has to set higher warranty price with the increase lifetime of the product as well as with the decrease of lifetime termination limits.

There is a scope for future research with other forms of failure distributions, impact of preventive maintenance and scope for replacements during lifetime with possible trade-ins. Another possible extension is to include discounting of the future repair costs and corrections for inflation which are important for products sold with long service life.

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